Running Hubble Tension

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Outline

- A review of FLRW cosmology
- Standard Model of Cosmology
- Hubble tension today
- **Running** H_0
- Summary and Outlook

The FLRW cosmology

- The ubiquitous assumption is that universe is to a "very good extent" homogeneous and isotropic at cosmological distances.

- That is distances larger than sizes of galaxy clusters, superclusters, voids, bigger than few 100 MPc, associated with redshifts $z \gtrsim 0.1$.

- It is hence governed by FLRW background cosmology:

$$ds^2 = -dt^2 + a^2(t)(\frac{dr^2}{1+kr^2} + r^2 d\Omega^2)$$

with some fluctuations on this background.

- $k = -1, 0, +1$ denotes the curvature of constant time slices, respectively corresponding to open, flat and closed cosmologies.

- $a(t)$ is the scale factor and r measures the comoving distances.

- Hubble expansion rate is then

$$H(t) := \frac{da/dt}{a}, \qquad a(t) = \frac{1}{z+1}$$

- One may view $H = H(z)$ and noting,

$$ds^2 = \frac{1}{(1+z)^2}\left[-\frac{dz^2}{H(z)^2} + (\frac{dr^2}{1+kr^2} + r^2 d\Omega^2)\right],$$

one can in principle measure $H(z)$ using cosmological data.

Background cosmology models

- We usually assume specific functional forms for $H(z)$ and specify the coefficients by fitting this form into the data.

- This specific functional forms, through Friedmann equations, are related to the matter content of the cosmic fluid:

$$H^2 + k(1+z)^2 = \frac{8\pi G}{3} \sum_i \rho_i, \qquad \frac{d\rho_i}{dz} - \frac{3}{(1+z)}(\rho_i + P_i) = 0,$$

  where H, ρ_i, P_i are functions of z.

- The above can be solved once we specify the matter content by giving the Equation of State (EoS) ω_i:

$$P_i = \omega_i \ \rho_i$$

- Some typical EoS:
  - usual (baryonic) matter and usual dark matter $\omega = 0$,
  - radiation (gas of photons or relativistic particles) $\omega = 1/3$,
  - cosmological constant $\omega = -1$,
  - curvature k is an "effective matter" with $\omega = -1/3$.

- For simple cases where w is a constant,

$$\rho(z) = \rho_0 (1 + z)^{3(1+\omega)}.$$

- ω_i can in principle be a function of z, e.g. as in dynamical dark energy [*cf.* previous talk by Lu Yin].

Therefore,

$$H^2(z) = H_0^2\ E(z)^2, \qquad E(z)^2 = \sum_i \Omega_i (1+z)^{3(1+\omega_i)}$$

where

$$H_0^2 = \frac{8\pi G}{3} \sum_i \rho_{i0}, \qquad \Omega_i := \frac{\rho_{i0}}{\sum_i \rho_{i0}}, \qquad \sum_i \Omega_i = 1.$$

- So, for an N component cosmic fluid model, $H(z)$ involves
  - $N+1$ model parameters k, ω_i, and
  - $N+1$ parameters H_0, Ω_i are integration constants of the Friedmann plus continuity equations which are subject to the sum-rule $\sum_i \Omega_i = 1$.
  - H_0 dimensionful whereas Ω_i are dimensionless.
  - These N parameters are determined through fitting into the data.

Flat ΛCDM Cosmology

- Flat ΛCDM with $k = 0$, baryonic or dark matter with $\omega = 0$ and dark energy with $\omega = -1$ is a simple model for late time ($z < 1100$) cosmological background:

$$H^2(z) = H_0^2\left[\Omega_{m0}(1+z)^3 + \Omega_\Lambda\right], \qquad \Omega_{m0} + \Omega_\Lambda = 1$$

- It has only two independent parameters to be specified by observations.

- At late times $H(z)$ is dominated by dark energy and in early times by (dark) matter.

- At the equality time $z_{\rm eq}$, DE and DM parts contribute equally to $H(z)$:

$$(1 + z_{\rm eq})^3 = \frac{\Omega_\Lambda}{\Omega_{m0}} = \frac{1}{\Omega_{m0}} - 1.$$

■ Standard Model of Cosmology (SMC), the background

- Various sets of cosmological data, notably Planck CMB data, seem to be very well described by the Standard Model of Cosmology (SMC).

- The background of SMC is flat ΛCDM with

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc}, \qquad \Omega_{m0} = 0.315 \pm 0.007$$

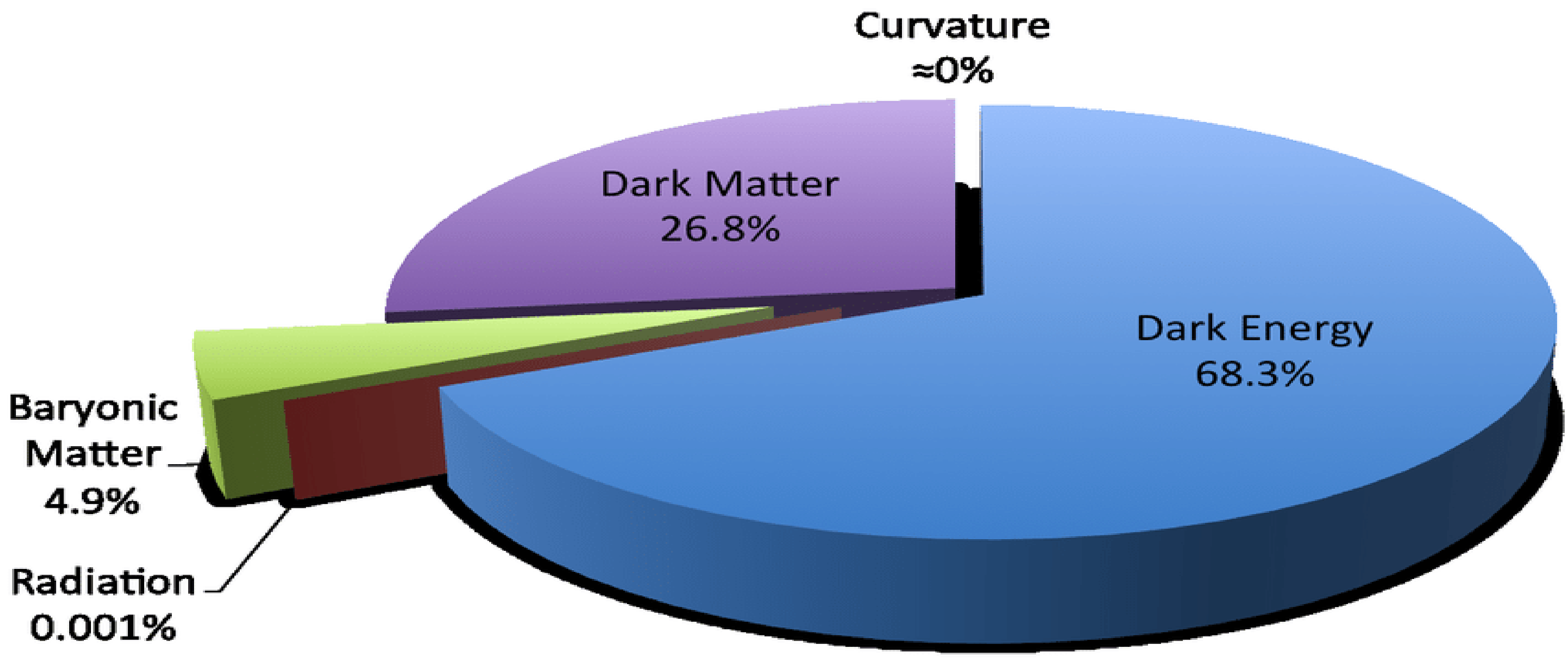
  These parameters are determined with better than 2% precision.

- Therefore, $z_{\rm eq} \sim 0.3$ and for $z \gtrsim 0.3$ Universe is matter dominated.

- To describe higher redshifts before CMB time ($z \gtrsim few \times 1000$), a radiation component, a $(1+z)^4$ term should be added in the $H^2(z)$. Planck collaboration gives $\Omega_r \sim 10^{-4} - 10^{-5}$.

- The Planck results gives $\Omega_k = 0.001 \pm 0.002$.

Cosmic Pie, energy density ratios today within concordance cosmology model



Hubble Tension

- We can reconstruct $H(z)$ in the ranges we have data and the precision of this reconstructed H depends on the precision of the available data.

- Cosmologically reconstructed $H(z)$ may then be extrapolated to $z = 0$ to read $H_0 = H(z = 0)$.

- Within FLRW framework H_0 can also in principle be directly measured by local observations, without invoking any cosmological model

$$H(z) = H_0 + H_0(1 - q)z + \mathcal{O}(z^2),$$

where q is the acceleration parameter [Riess et al (2019)]

$H_0 = 74.03 \pm 1.42$ km/s/Mpc, $\quad q \sim 0.55$ (with about 15-20% error)

- Within flat ΛCDM ,

$$H(z) = H_0 \left[1 + (\frac{3}{2}\Omega_{m0} - 1)z \right] + \mathcal{O}(z^2) = H_0 \left[1 + (1 - q)z \right] + \mathcal{O}(z^2)$$

- Note that H_0, Ω_{m0}, q are defined within FLRW framework.

- Locally measured q has large error which matches the cosmologically inferred value from Ω_{m0}.

- The situation is very much different for H_0, both local and cosmological determinations have competitive % level precision.

- Note again that

any measurement/inference of parameters are made within certain assumptions and framework. Cosmology progresses in a cycle of assumptions and the effort to contradict these assumptions...

■ H_0 Tension, as usually stated

- Riess et al (2019) directly measured

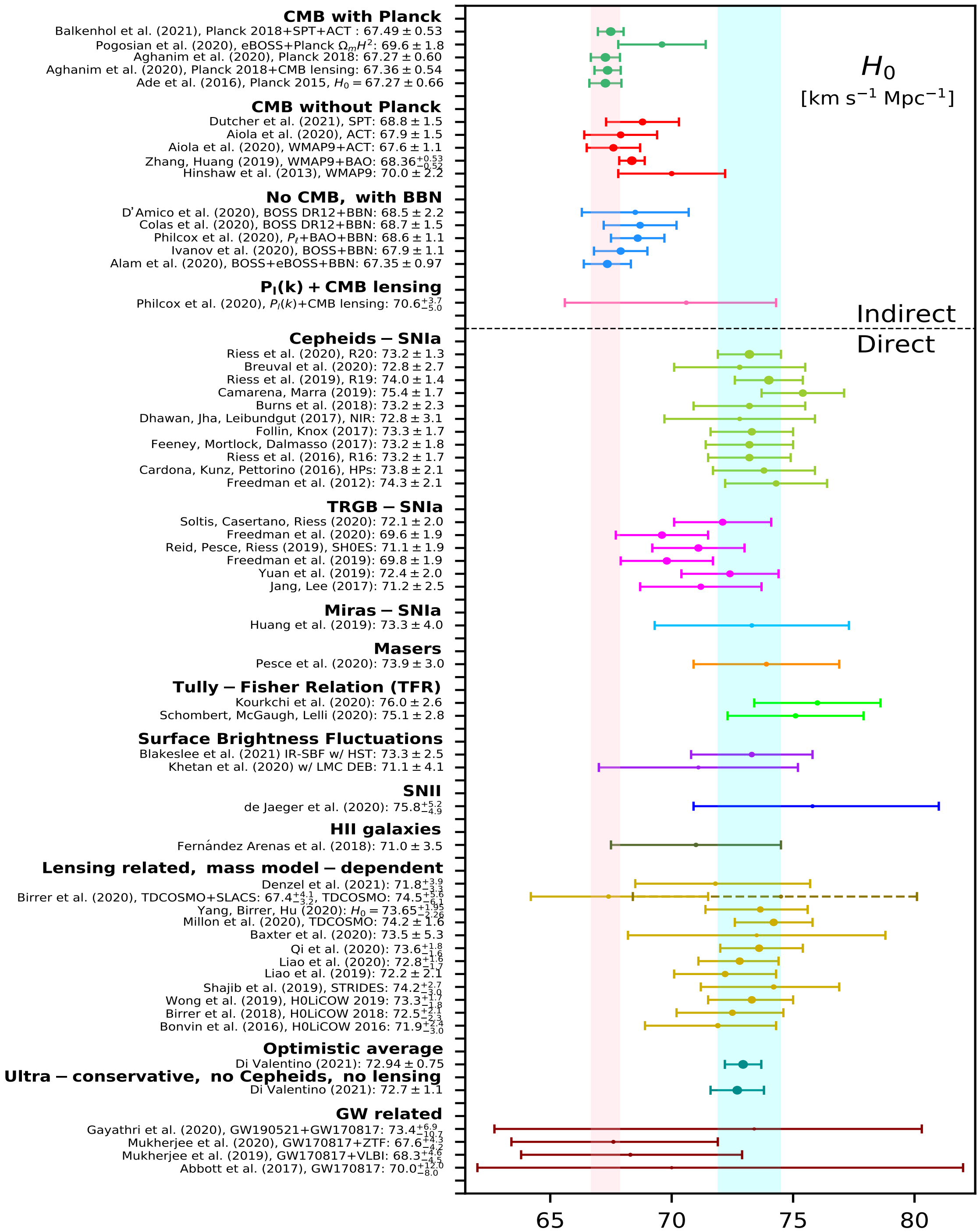
$$H_0 = 74.03 \pm 1.42 \text{ km/s/ Mpc.}$$

- Extractions from Planck collaboration (2018) within flat ΛCDM yield

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc,}$$

- H_0 tension as usually stated is the mismatch of direct local measurements and extrapolation of $H(z)$ read from cosmological data to $z = 0$ *within a cosmological model*.

- Tension between the two has reached over 10% ($4 - 5\sigma$) while each of these is determined by 1% level error. It needs to be tackled.



Courtesy of arXiv:2103.01183[astro-ph]

- The tension has been scrutinized in many different ways, checking for systematics, various calibrations and possibly unaccounted physics....

- Within the given framework, the tension seems to be real and presumably of cosmological origin.

- Many many different cosmological models, mainly playing with the dark energy sector, early or late DE models, have been proposed.

- Cosmologically, BAO+SNe, act as guardrails on Planck ΛCDM, leaving little room for late time DE models to increase H_0.

- Early Dark Energy while reducing the tension to 2σ level, worsens other cosmological tensions....

■ A fresh view on Hubble tension:

Running H_0 tension & H_0 diagnostic [arXiv:2011.02858]

- Considering age of universe constraints, none of the existing proposals quite work. C. Krishnan, R. Mohayaee, E.O. Colgain, MM.Sh-J & Lu Yin 2105.09790 [astro-ph.CO].

- Motivated by this, one may ask if at all a resolution to cosmological tensions can be found within the FLRW framework.

- To this end one should study implications of the FLRW framework, regardless of the details of cosmological model invoked.

  See the next talk by Eoin O Colgain for more details.....

- Consider FLRW cosmology and Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho, \qquad (1+z)\frac{H'}{H} = \frac{3(\rho+p)}{2\rho} := \frac{3}{2}(1+w_{\text{eff}}),$$

$w_{\text{eff}} = w_{\text{eff}}(z)$ is the effective equation of state (EoS) of the universe.

- These can be integrated to get

$$H_0 = \frac{H(z)}{\exp\left(\frac{3}{2}\int_0^z \frac{1+w_{\text{eff}}(z')}{1+z'}\mathrm{d}z'\right)} := \frac{H(z)}{E(z)}.$$

- H_0 does not appear in Friedmann equations and ought to be a constant.

- In practice, this equation may be viewed as definition of H_0, where
  - $H(z)$ is read from the OHD data and,
  - the integral in the denominator can be computed once we specify a model by giving $w_{\text{eff}}(z)$. For example, for flat ΛCDM

$$\exp\left(\frac{3}{2}\int_0^z \frac{1+w_{\text{eff}}(z')}{1+z'}\mathrm{d}z'\right) = \Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}$$

- So, H_0 is the ratio of two functions of z, one entirely read from the data and other fixed by the data for a specific model.

- That H_0 remains z-independent is non-trivially tied with the "correctness" of the model.

- Alternatively, consider two models A, B, since both should produce the same $H(z)$, then

$$\frac{{H_0}^{(B)}}{{H_0}^{(A)}} = \frac{E^{(A)}(z)}{E^{(B)}(z)} = \exp\left(\frac{3}{2}\int_0^z \frac{\Delta w_{\text{eff}}(z')}{1+z'}\mathrm{d}z'\right).$$

where $\Delta w_{\text{eff}} = {w_{\text{eff}}}^{(A)}(z) - {w_{\text{eff}}}^{(B)}(z)$.

- Suppose model B is the "correct model" which reproduces $H(z)$ read from the data and addresses the tension.

- One can in particular take model A to be flat ΛCDM .

- Within FLRW cosmology, LHS should be z-independent while RHS must have a z-dependence, unless $A \equiv B$.

- Not seeing H_0 running, can only be attributed to breakdown of FLRW framework.

- So, within FLRW it is inevitable that H_0 should run with z.

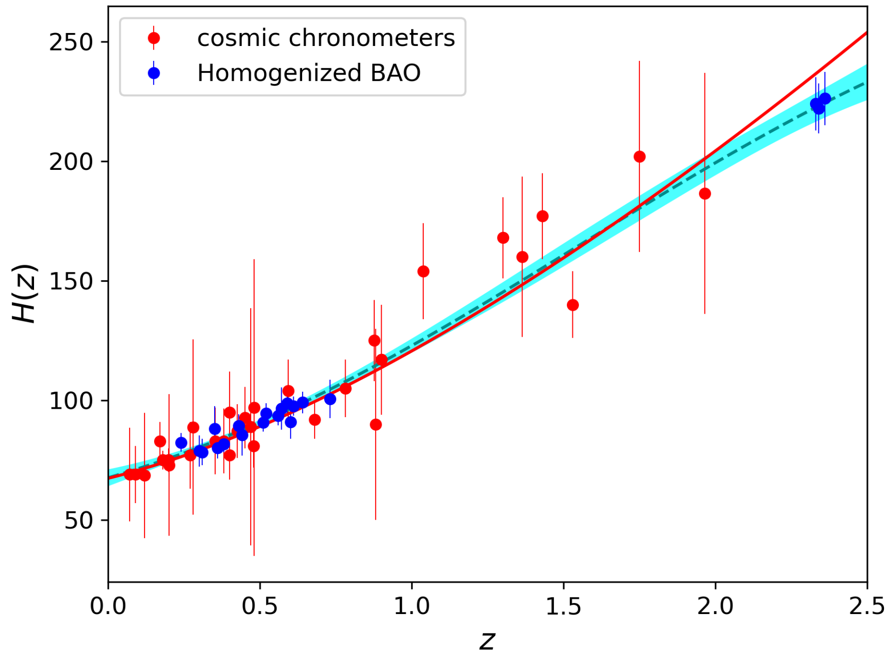
- This observation can be presented in a way closer to be put to practice by introducing a H_0 diagnostic.

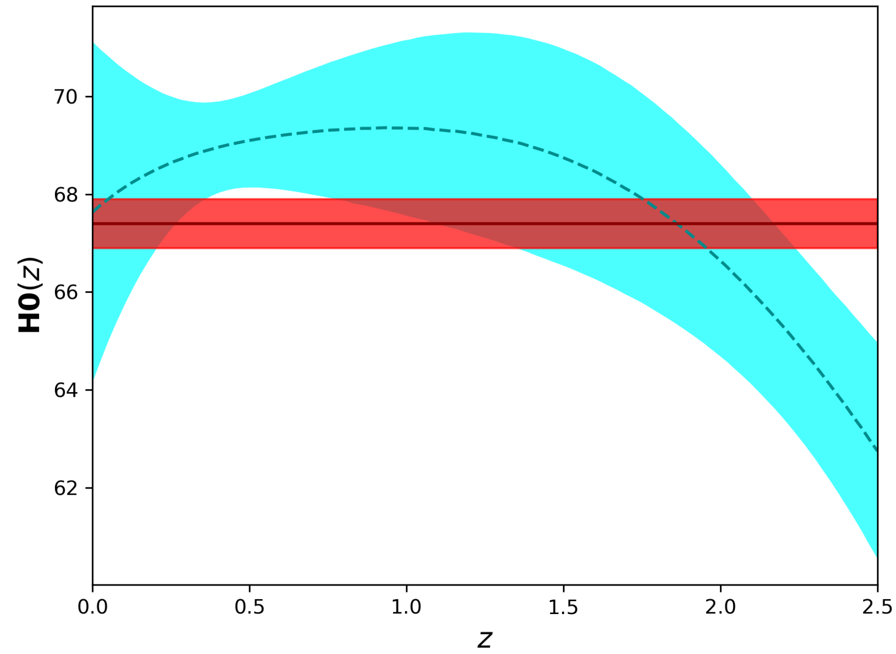
- Consider the "reference model" to be ΛCDM, then

$$\mathbb{H0}(z) := \frac{H(z)}{\sqrt{1 - \Omega_{m0} + \Omega_{m0}(1+z)^3}}.$$

- In the above $H(z)$ is inferred from OHD data and in the denominator the Planck MCMC results is used to read Ω_{m0}.

- $\mathbb{H0}(z)$ is our H_0 diagnostic. If it is a constant, flat ΛCDM is a good fit and if not, variations of $\mathbb{H0}$ in z give deviations from the SMC.





Left plot: reconstructed Hubble parameter from the CC and BAO data. The Planck-ΛCDM cosmology (red line) is given for comparison.

Right plot: The inferred value of $\mathbb{H}0$ diagnostic in units of km/s/Mpc from the current CC and homogenised BAO data compared against the Planck-ΛCDM value in red. The maximum deviation occurs at $z \sim 2.5$ with the statistical significance $\sim 2\sigma$.

Concluding Remarks and Outlook

⊛ The standard model of cosmology, the concordance model, is very remarkable as it describes the observed cosmos with only six parameters.

⊛ Two of these six parameters, H_0, Ω_{m0} describe the background cosmology which is taken to be flat ΛCDM model.

⊛ The six parameters of SMC are now determined with % level precision and they are improving as we gather further and better quality data.

⊛ There are various different cosmological datasets which may be used to determine the SMC parameters.

⊛ There could hence be some different values of these model parameters. If these different values differ significantly (compared to the error bars) our model is under tension.

⊛ Hubble expansion rate today H_0 is quite special: it can be measured using local (astronomical) data, or be determined through various cosmological datasets within SMC or a general background FLRW cosmology.

⊛ Local measurements, depending on the ladder distance calibration method used, yield different values for H_0. In particular, Riess et al (2019) report,

$$H_0 = 74.03 \pm 1.42 \text{ km/s/ Mpc.}$$

⊛ H_0 can be determined from fitting cosmological data into cosmological models. In particular, Planck collaboration (2018) using CMB(+BAO) data report,

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc,}$$

⊛ Difference between the two values is over 10% while the precision of each one is better than 1%

⊛ This Hubble tension has been explored from various angles with many different viewpoints.

⊛ Some people are scrutinizing measurements/determinations for hidden (e.g. systematic) errors.

⊛ Hubble tension is not just difference between SH0ES & Planck, but more generally local vs. cosmological H_0 determinations.

⊛ There is a growing consensus that Hubble tension may be pointing to something deeper than a problem in ΛCDM, a breakdown in FLRW framework.

Running H_0 proposal is a good way to quantify a departure from FLRW.

What if the background cosmology is not homogeneous and isotropic?!

There are already notable results pointing to this direction.
See the next talk by Eoin O Colgain.

That who is on the right track is still open to debate and will hopefully be resolved as we gather more precise data over the whole sky.

Thank You For Your Attention