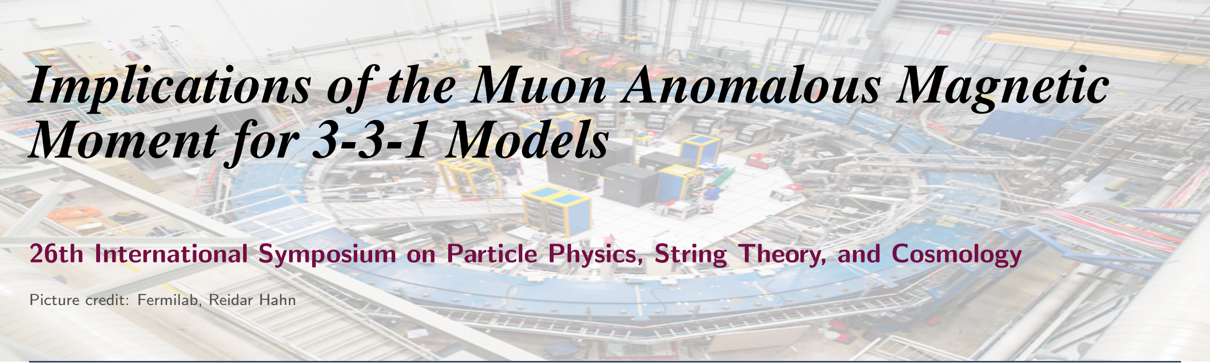

Implications of the Muon Anomalous Magnetic Moment for 3-3-1 Models

26th International Symposium on Particle Physics, String Theory, and Cosmology

Picture credit: Fermilab, Reidar Hahn

Yoxara S. Villamizar

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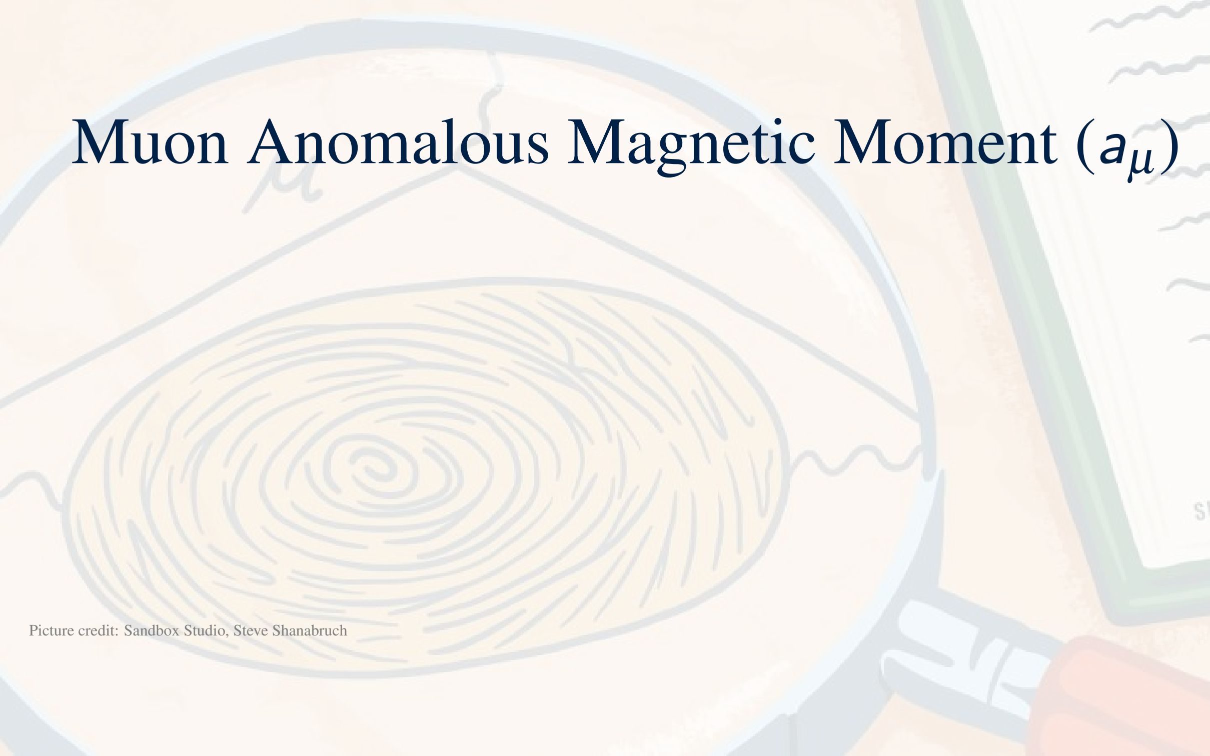
In collaboration with:

Álvaro S. de Jesus, Sergey Kovalenko,
C. A. de S. Pires and Farinaldo S. Queiroz.

Overview

Muon Anomalous Magnetic Moment (a_μ)



Picture credit: Sandbox Studio, Steve Shanabruch

According to Quantum Electrodynamics, the Dirac equation predicts at tree level the muon magnetic moment of any charged fermion as follows,

$$\vec{\mu_\mu} = g_\mu \frac{q}{2m_\mu} \vec{S}.$$

Where $g_\mu = 2$ is the gyromagnetic ratio, m_μ, q and S are the muon mass, the electric charge and the spin respectively. However, through quantum corrections at the loop $g_\mu \neq 2$, letting us define the Muon Anomalous Magnetic Moment as

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 116591802(2)(42)(26) \times 10^{-11}.$$

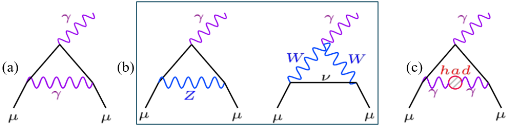


Figure 1: Feynman diagram of the corrections to a_μ on SM interactions: (a) first order QED, (b) lowest-order weak, and (c) lowest-order hadronic effects. $a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{QCD}}$

Comparing the SM prediction with the measurements from Brookhaven National Lab,we get Δa_μ [1]:

$$\Delta a_\mu = (261 \pm 78) \times 10^{-11}\ (3.3\sigma) - (2009)^a$$
$$\Delta a_\mu = (325 \pm 80) \times 10^{-11}\ (4.05\sigma) - (2012)^b$$
$$\Delta a_\mu = (287 \pm 80) \times 10^{-11}\ (3.6\sigma) - (2013)^c$$
$$\Delta a_\mu = (377 \pm 75) \times 10^{-11}\ (5.02\sigma) - (2015)^d$$
$$\Delta a_\mu = (313 \pm 77) \times 10^{-11}\ (4.1\sigma) - (2017)^e$$
$$\Delta a_\mu = (270 \pm 36) \times 10^{-11}\ (3.7\sigma) - (2018)^f$$
$$\Delta a_\mu = (251 \pm 59) \times 10^{-11}\ (4.2\sigma) - (2021)^g$$

We will explore new physics contributions to a_μ on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry and will use the following a_μ discrepancies,

$$\Delta a_{\mu\text{Current}} = (261 \pm 78) \times 10^{-11}\ (3.3\sigma)$$
$$\Delta a_{\mu\text{Projected}} = (261 \pm 34) \times 10^{-11}\ (5\sigma)$$

[1] Refs: [a]Prades, Joaquim, Eduardo De Rafael, and Arkady Vainshtein., Tanabashi, Masaharu, et al.; [b]Benayoun, M., et al.; [c]Blum, Thomas, et al. ; [d] Benayoun, M., et al.; [e]Jegerlehner, Fred.; [f]Keshavarzi, Alexander, Daisuke Nomura, and Thomas Teubner.; [7]B. Abi, et al. (Muon g-2 Collaboration)

$SU(3)_C \times SU(3)_L \times U(1)_X$

(3-3-1) Models

Models based on 3-3-1 gauge symmetry[2]:

1. Minimal 3-3-1 Model[a]
2. 3-3-1 with right-handed neutrinos, (r.h.n)[b]
3. 3-3-1 with neutral lepton (3-3-1 LHN)[c],
4. Economical 3-3-1[d]
5. 3-3-1 with exotic leptons[e],

The electric charge operator for 3-3-1 Models is,

$$\frac{Q}{e} = \frac{1}{2}(\lambda_3 + \alpha\lambda_8) + \mathrm{X}, \qquad \alpha = -\sqrt{3}, \pm\frac{1}{\sqrt{3}}$$

These models are quite popular because they can explain:

- neutrino masses,
- dark matter,
- flavor violation,
- collider physics,
- among others.

where $\lambda_{3,8}$ and I are the generators of $SU(3)_C$ and $U(1)_X$, respectively.

[2] Refs: Pisano, F., and Vicente Pleitez.[a]; Hoang Ngoc Long[b]; Martinez, R., and F. Ochoa., Mizukoshi, J. K., et al. [c]; Model, Dong, P. V., et al. , R. Martínez and F. Ochoa, Dong, P. V., and H. N. Long.[d]; Ponce, William A., Juan B. Florez, and Luis A. Sanchez., Anderson, David L., and Marc Sher., Cabarcas, J. M., J. Duarte, and J-Alexis Rodriguez.[e].

The scalar sector contains between 2 or 3 scalar triplets (χ, η, ρ) to give the masses of the fermions. The 3-3-1 gauge symmetry experiences the following spontaneous symmetry breaking: $\mathrm{SU(3)_L} \times \mathrm{U(1)_X} \xrightarrow{\langle\chi\rangle} \mathrm{SU(2)_L} \times \mathrm{U(1)_Y} \xrightarrow{\langle\eta\rangle,\ \langle\rho\rangle} \mathrm{U(1)_Q}$, with VEV different scales: $v_\chi \gg v_\eta, v_\rho$.

The fermionic sector of each 3-3-1 model contains leptonic triplets,

1. Minimal 3-3-1 Model
2. 3-3-1 with right-handed neutrinos, (r.h.n)
3. 3-3-1 with neutral lepton (3-3-1 LHN),
4. Economical 3-3-1 Model,
5. 3-3-1 with exotic leptons,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ (\ell^c)^a \end{pmatrix};$$

where $a = 1, 2, 3$ is the generation index and ν and ℓ are the SM particles.

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3. 3-3-1 with neutral lepton (3-3-1 LHN),
4. Economical 3-3-1 Model,
5. 3-3-1 with exotic leptons,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ (\nu^c)^a \end{pmatrix} ; \ell_R^a$$

where $a = 1, 2, 3$ is the generation index and ν^c is the r.h.n.

The fermionic sector of each 3-3-1 model contains leptonic triplets,

1. Minimal 3-3-1 Model
2. 3-3-1 with right-handed neutrinos, (r.h.n)
3. 3-3-1 with neutral lepton (3-3-1 LHN),
4. Economical 3-3-1 Model,
5. 3-3-1 with exotic leptons,

$$f_L^a = \begin{pmatrix} \nu^a \\ \ell^a \\ N^a \end{pmatrix} ; N_R^a \ \ell_R^a$$

where $a = 1, 2, 3$ is the generation index and N is the heavy neutral lepton.

The fermionic sector of each 3-3-1 model contains leptonic triplets,

1. Minimal 3-3-1 Model
2. 3-3-1 with right-handed neutrinos, (r.h.n)
3. 3-3-1 with neutral lepton (3-3-1 LHN),
4. Economical 3-3-1 Model,
5. 3-3-1 with exotic leptons,

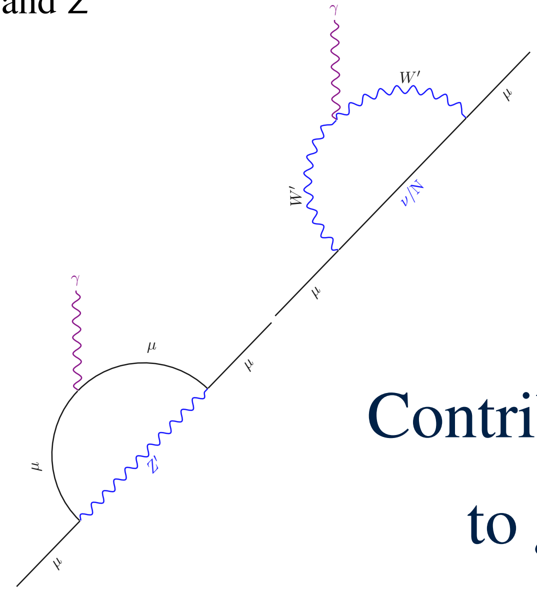
$$f_{1L} = \begin{pmatrix} \nu_1 \\ \ell_1 \\ E_1^- \end{pmatrix}; \quad f_{1,2L} = \begin{pmatrix} \nu_{1,2} \\ \ell_{1,2} \\ N_{1,2} \end{pmatrix};$$

$$f_{4L} = \begin{pmatrix} E_2^- \\ N_3 \\ N_4 \end{pmatrix}; \quad f_{5L} = \begin{pmatrix} N_5 \\ E_3^+ \\ \ell_3^+ \end{pmatrix};$$

$$\ell_1^c; \ \ell_{1,2}^c; \ E_2^c; \ E_3^c$$

where N and E are the exotic neutral and charged leptons, respectively.

- Besides, new known gauge bosons appear, such as $U^{\pm\pm}$, W'^{-}, and Z'



Contributions to $g_\mu - 2$

We make our Mathematica numerical codes of the analytical expressions to Muon Anomalous Magnetic Moment(Δa_μ) corresponding to the 3-3-1 Models available at https://bit.ly/2vFZLnG

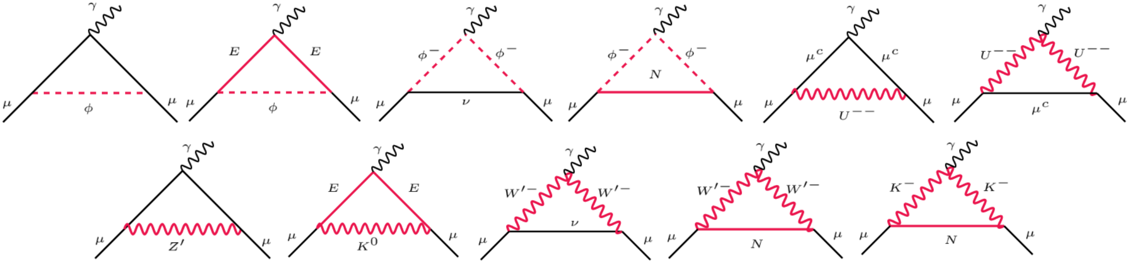


Figure 2: Feynman diagrams that contribute to the $g_\mu - 2$ in the 3-3-1 models investigated in this work. Where $U^{\pm\pm}$, W'^{-}, K^{-}, K^{0} and Z' are new gauge bosons. With ϕ and ϕ^{-} are the neutral and singlet charged scalars fields, and correspond to the scalars χ^{0}, S_2, η_1^{+}, h_1^{+}, h_2^{+}, and χ^{+}

New Physics Contributions to the Muon Anomalous Magnetic Moment: A Numerical Code (arXiv:1403.2309)

Neutral Gauge Boson Mediator:

$$\Delta a_\mu\left(f,Z'\right)=\frac{1}{8\pi^2}\frac{m_\mu^2}{M_{Z'}^2}\int_0^1 \mathrm{d}x\sum_f\left[\frac{\left|g_{v1}^{f\mu}\right|^2 P_1^+(x)+\left|g_{a1}^{f\mu}\right|^2 P_1^-(x)}{(1-x)\left(1-\lambda^2 x\right)+\epsilon_f^2\lambda^2 x}\right],$$

$P_1^{\pm}=2x(1-x)(x-2\pm 2\epsilon_f)+\lambda^2x^2(1\mp\epsilon_f)^2(1-x\pm\epsilon_f)$, $\epsilon_f\equiv\frac{m_f}{m_\mu}$, $\lambda\equiv\frac{m_\mu}{M_{Z'}}$. $g_{v1}^{f\mu}$ and $g_{a1}^{f\mu}$ are the vector and vector-axial coupling constants.

Charged Gauge Boson Mediator:

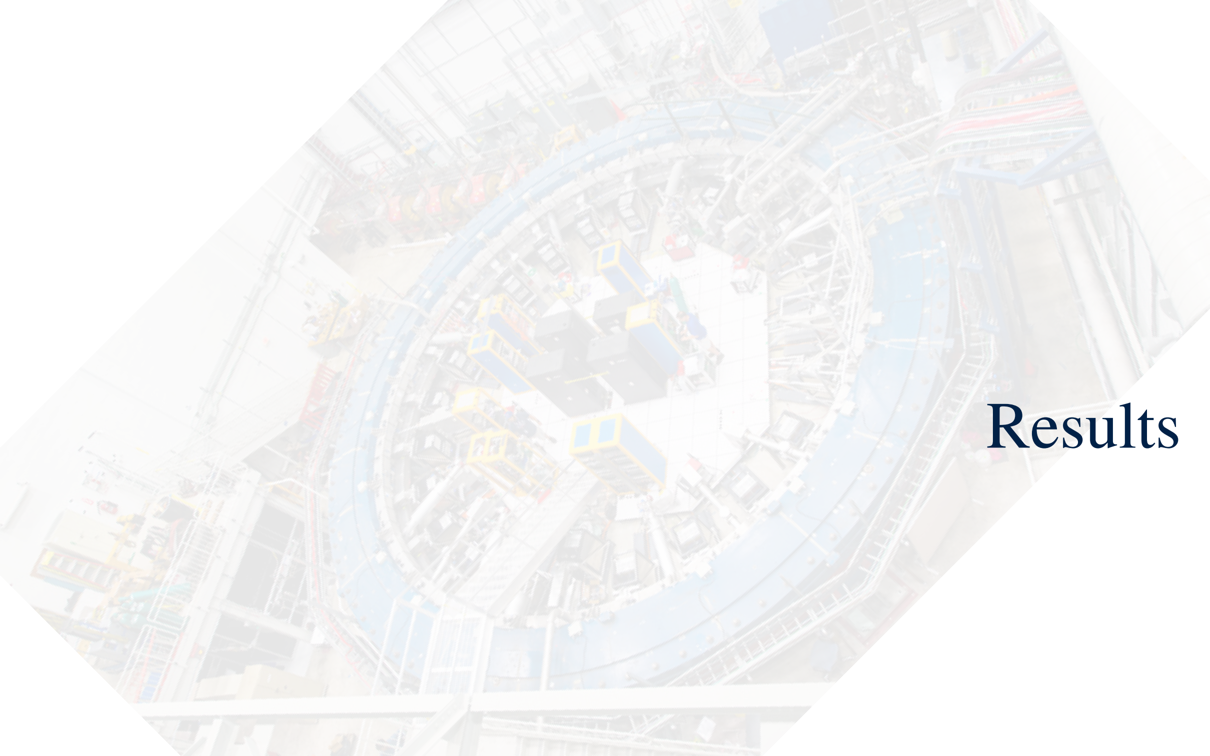
$$\Delta a_\mu\left(N,W'\right)=\frac{-1}{8\pi^2}\frac{m_\mu^2}{M_{W'}^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{v2}^{f\mu}\right|^2 P_2^+(x)+\left|g_{a2}^{f\mu}\right|^2 P_2^-(x)}{\epsilon_f^2\lambda^2(1-x)\left(1-\epsilon_f^{-2}x\right)+x},$$

with $P_2^{\pm}=-2x^2(1+x\mp 2\epsilon_f)+\lambda^2x(1-x)(1\mp\epsilon_f)^2(x\pm\epsilon_f)$, where and $\epsilon_f\equiv\frac{m_{N_f}}{m_\mu}$, $\lambda\equiv\frac{m_\mu}{M_{W'}}$. g_{v2} and g_{a2} are again the vector and vector-axial couplings. In 3-3-1 models, we have either $m_{N_f}=m_\nu$ or $m_{N_f}=m_N$.

Doubly Charged Vector Boson Mediator:

$$\Delta a_\mu\left(U^{++}\right)=\frac{8}{8\pi^2}\frac{m_\mu^2}{M_U^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{v3}^{f\mu}\right|^2 P_2^+(x)+\left|g_{a3}^{f\mu}\right|^2 P_2^-(x)}{\epsilon_f^2\lambda^2(1-x)\left(1-\epsilon_f^{-2}x\right)+x}-\frac{4}{8\pi^2}\frac{m_\mu^2}{M_U^2}\int_0^1 \mathrm{d}x\sum_f\frac{\left|g_{v3}^{f\mu}\right|^2 P_1^+(x)+\left|g_{a3}^{f\mu}\right|^2 P_1^-(x)}{(1-x)\left(1-\lambda^2x\right)+\epsilon_f^2\lambda^2x},$$

where $\epsilon_f\equiv\frac{m_f}{m_\mu}$, $\lambda\equiv\frac{m_\mu}{M_U}$, and $g_{a3}^{f\mu}$ ($g_{v3}^{f\mu}$) are symmetric and anti-symmetric couplings in flavor space.



Results

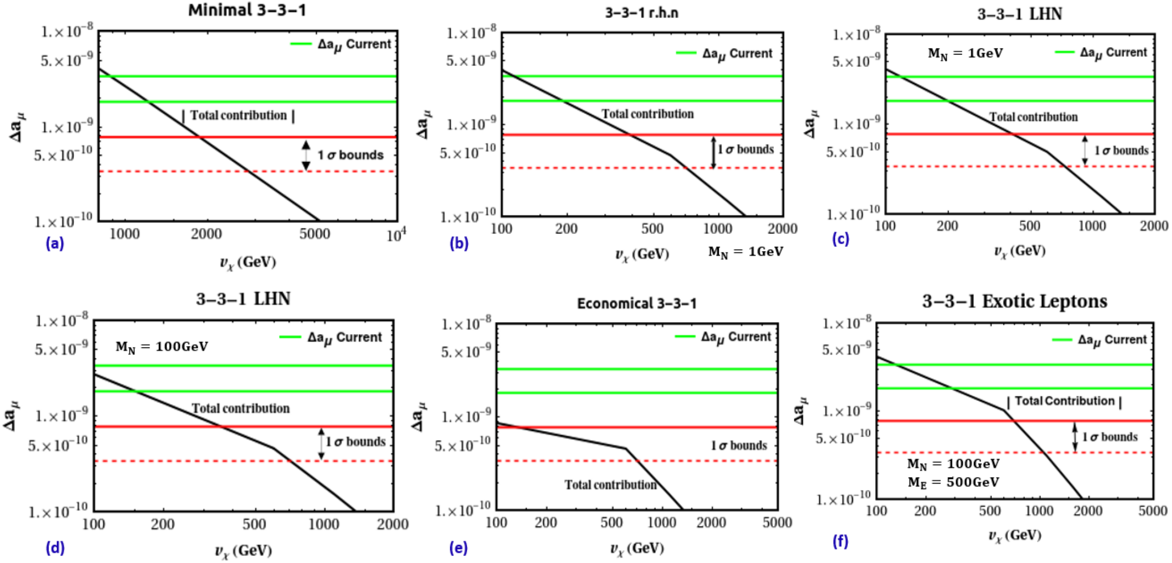


Figure 3: Overall contribution to Δa_μ from the 3-3-1 models. The green bands are delimited by $\Delta a_\mu = (261 \pm 78) \times 10^{-11}$ (3.3σ). The projected 1σ bound is found by requiring $\Delta a_\mu < 78 \times 10^{-11}$ while the bound is obtained for $\Delta a_\mu < 34 \times 10^{-11}$.

Model	LHC-13TeV	g-2 current	g-2 projected
Minimal 3-3-1	$M_{Z'} > 3.7$ TeV [1] $M_{W'} > 3.2$ TeV [1]	$M_{Z'} > 434.5$ GeV $M_{W'} > 646$ GeV	$M_{Z'} > 632$ GeV $M_{W'} > 996.1$ GeV
3-3-1 r.h.n	$^{*}M_{Z'} > 2.64$ TeV [2] ——-	$M_{Z'} > 158$ GeV $M_{W'} > 133$ GeV	$M_{Z'} > 276.5$ GeV $M_{W'} > 239$ GeV
3-3-1 LHN for $M_N = 1$ GeV	$^{*}M_{Z'} > 2$ TeV [2] —-	$M_{Z'} > 160$ GeV $M_{W'} > 134.3$ GeV	$M_{Z'} > 285$ GeV $M_{W'} > 238.3$ GeV
3-3-1 LHN for $M_N = 100$ GeV	$^{*}M_{Z'} > 2$ TeV [2] ——	$M_{Z'} > 136.7$ GeV $M_{W'} > 114.2$ GeV	$M_{Z'} > 276.5$ GeV $M_{W'} > 231$ GeV
Economical 3-3-1	$^{*}M_{Z'} > 2.64$ TeV [2] ——	$M_{Z'} > 59.3$ GeV $M_{W'} > 49.5$ GeV	$M_{Z'} > 271.4$ GeV $M_{W'} > 226.7$ GeV
3-3-1 exotic leptons for $M_N(M_E) = 10(150)$ GeV	$^{*}M_{Z'} > 2.91$ TeV [3] ——	$M_{Z'} > 429$ GeV $M_{W'} > 359$ GeV	$M_{Z'} > 693$ GeV $M_{W'} > 579.6$ GeV
3-3-1 exotic leptons for $M_N(M_E) = 100(150)$ GeV	$^{*}M_{Z'} > 2.91$ TeV [3] ——	$M_{Z'} > 369$ GeV $M_{W'} > 309.1$ GeV	$M_{Z'} > 600$ GeV $M_{W'} > 501.4$ GeV

Table 1: Summary of the lower bounds based on our calculations. For comparison we include the LHC bounds at 13 TeV center-of-mass energy.

[1] Nepomuceno, A. A., and Bernhard Meirose, [2] Lindner, Manfred, Moritz Platscher, and Farinaldo S. Queiroz., [3] Salazar, Camilo, et al.

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Table 1: Summary of the lower bounds based on our calculations. For comparison we include the LHC bounds at 13 TeV center-of-mass energy.

None of the five models investigated here can accommodate the anomaly in agreement with existing bounds.

The 3-3-1 LHN model augmented by an inert scalar triplet

The inert scalar triplet allows us to include $\mathscr{L} \supset y_{ab}\overline{f_a}\phi e_{bR}$, taking $y_{22} = 1$. Such scalar triplet gets a mass from the quartic coupling in the scalar potential $(\lambda\phi^\dagger\phi\chi^\dagger\chi)$, after the scalar triplet χ acquires a *vev*.

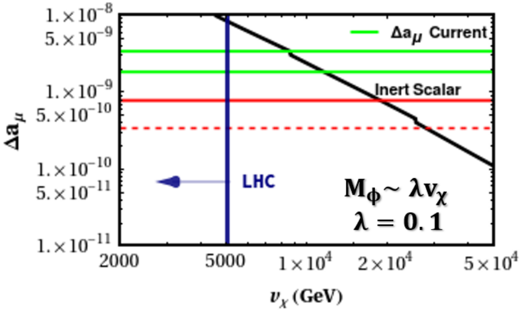

Figure 4: Overall contribution of the 3-3-1 LHN Model augmented by an inert scalar triplet ϕ.

The extended version of the 3-3-1 LHN Model successfully accommodates the a_μ anomaly for $v_\chi \sim 10$ TeV, while being consistent with LHC constraint.

Conclusions

1. We concluded that none of the five models investigated here can accommodate the anomaly.
2. We derived robust and complementary 1σ lower mass bounds on the masses of the new gauge bosons, namely the Z' and W' bosons, that contribute to muon anomalous magnetic moment assuming the anomaly is otherwise resolved.
3. The 3-3-1 models must be extended to explain the anomaly observed in the muon anomalous magnetic moment.
4. We presented a plausible extension to the 3-3-1 LHN model, which features an inert scalar triplet.

Yoxara S. Villamizar
IIP & Physics Department -UFRN
Particle and Astroparticle Group
yoxara@ufrn.edu.br

Thank you!

Questions?