16/06/2021

An effective 2-body approach to the hierarchical 3-body problem in GR

Francesco Serra

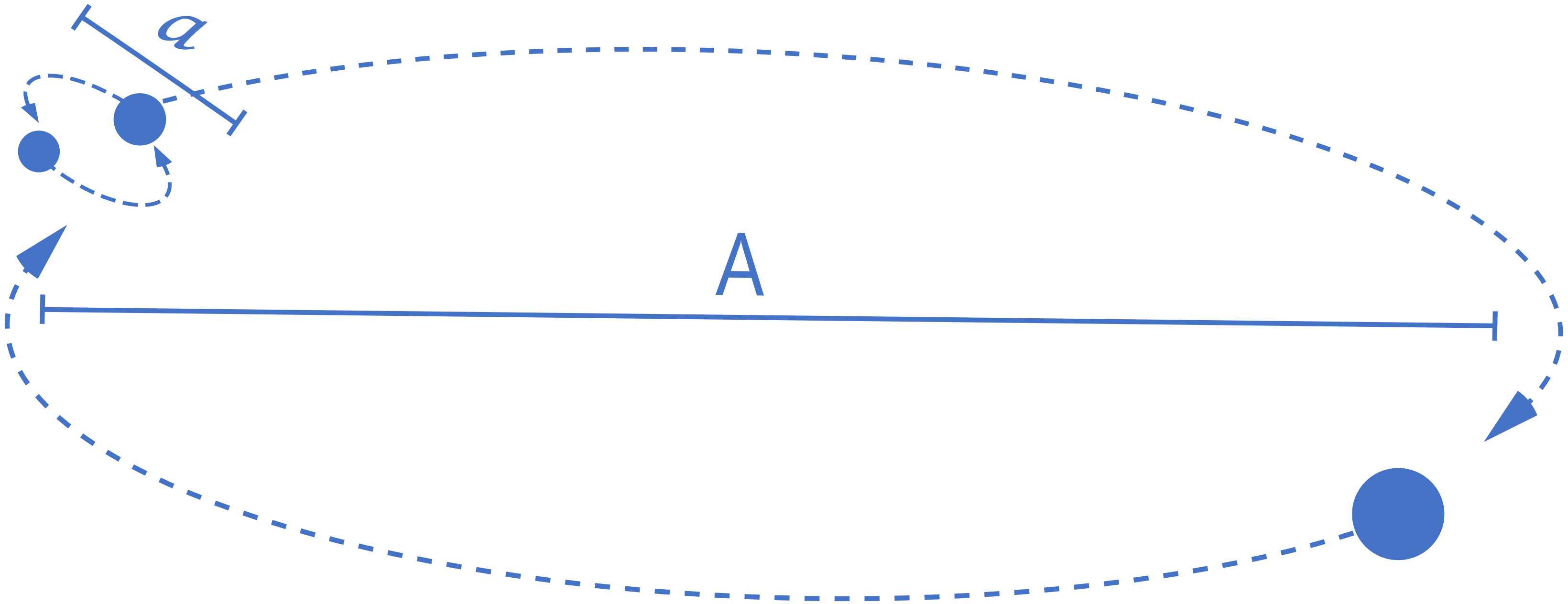
in collaboration with Adrien Kuntz and Enrico Trincherini

Scuola Normale Superiore, INFN Pisa

SCUOLA NORMALE SUPERIORE

INFN Istituto Nazionale di Fisica Nucleare

The hierarchical three body problem

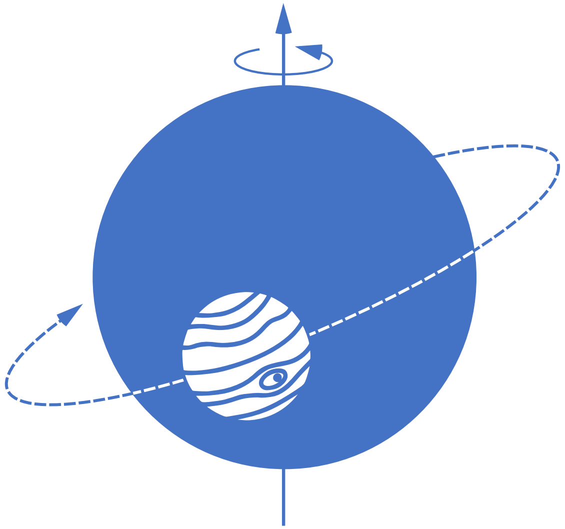


Perturbative setting

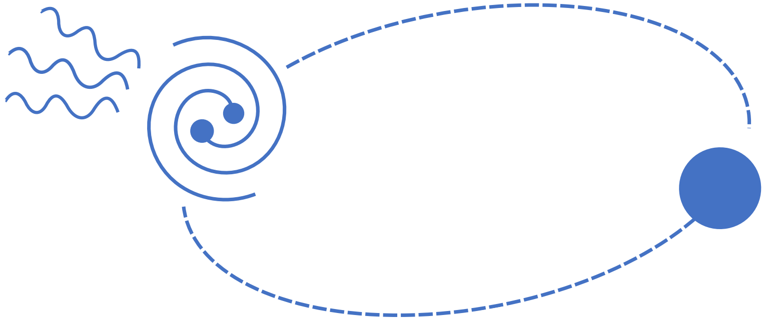
$$\epsilon = \frac{a}{A} \ll 1 \qquad \mathcal{L} = \mathcal{L}_{binary} + \mathcal{R} \qquad \mathcal{R} \sim \mathcal{O}(\epsilon)$$

Hierarchical triples in real life

Planetary satellites



Exoplanets



Compact binaries

Secular dynamics

Old and new surprises:

Eccentricity/inclination oscillations

Lidov 1961 Kozai 1962;

Orbital flips

Naoz et al 2013;

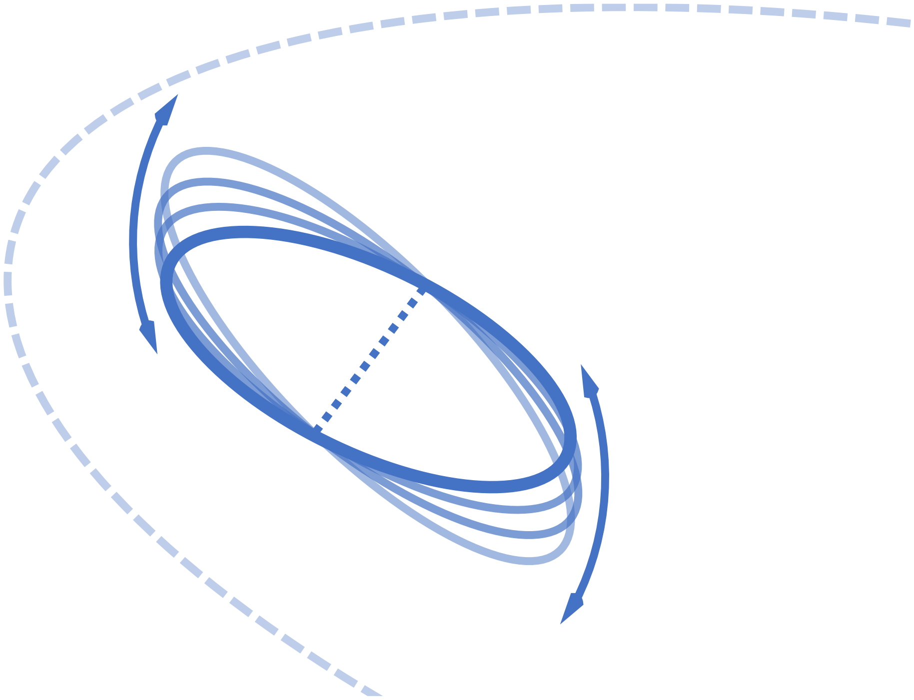
Nonlinearities

Lim et al 2016; Will 2020;

Relativistic corrections

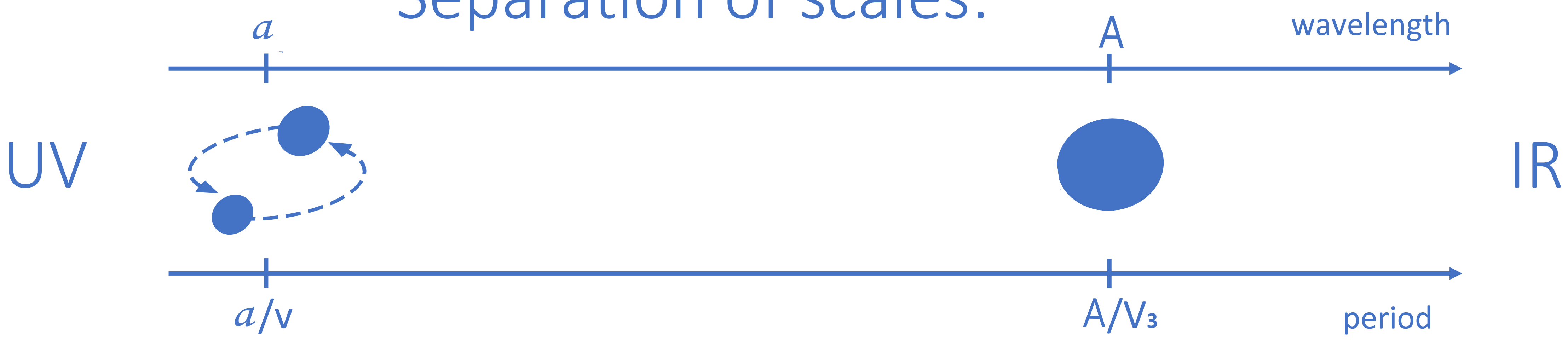
Naoz et al 2012; Will 2014;

Can we get to a more systematic understanding?



Perturbations in Effective Field Theory

Separation of scales:



$$e^{iS_{eff}[\phi_{IR}]} = \int \mathcal{D}[\phi_{UV}] e^{iS[\phi_{IR}, \phi_{UV}]}$$

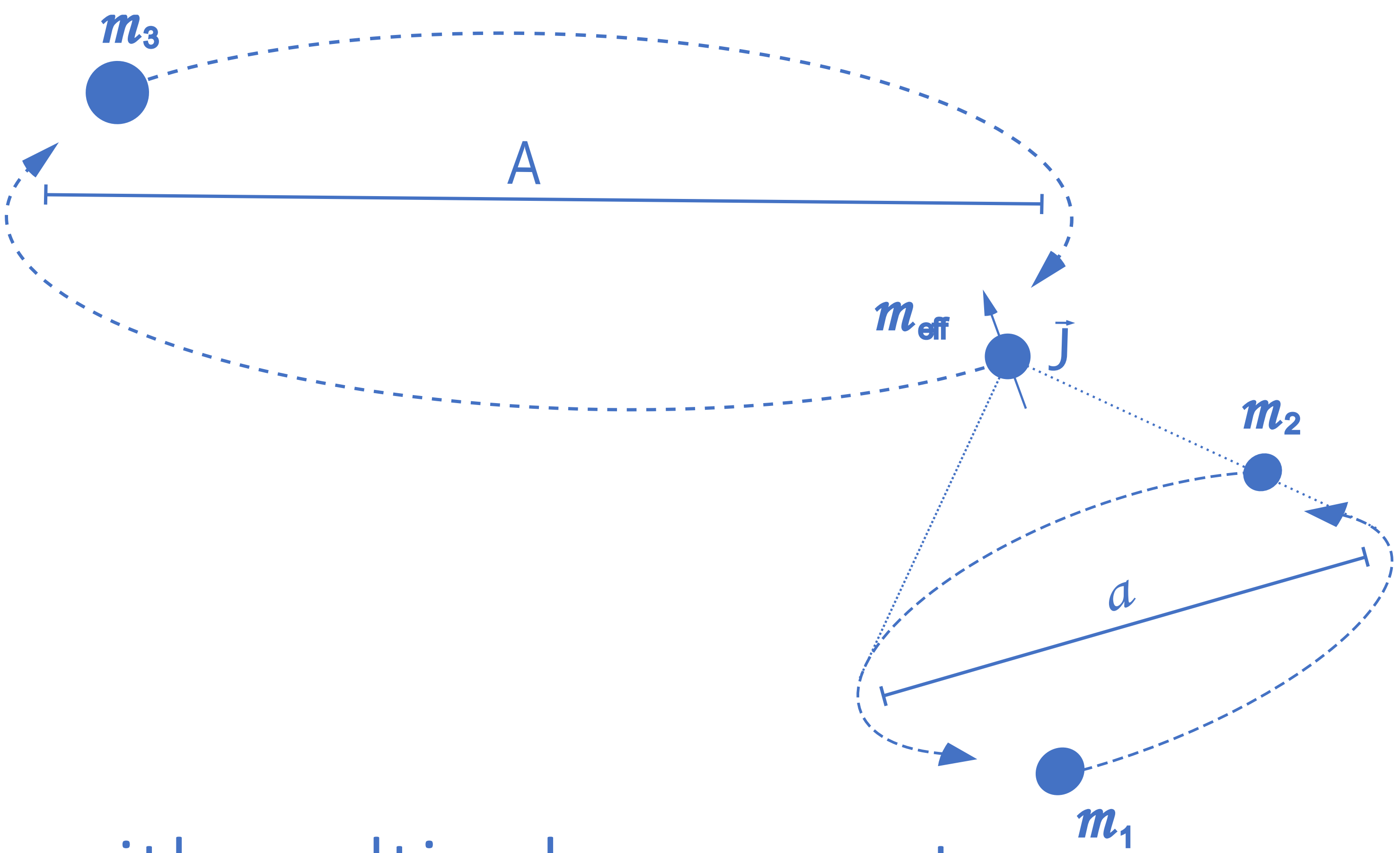
$$\phi(t) = \sum_{\omega<\Lambda} \phi_\omega e^{i\omega t} + \sum_{\omega>\Lambda} \phi_\omega e^{i\omega t} = \phi_{IR}(t) + \phi_{UV}(t)$$

EFT as a tool in General Relativity: Goldberger, Rothstein 2005

From three bodies to two bodies

Integrating out short scale modes of inner binary:

- Graviton modes
- Orbital modes



Result:
composite particle with multipole moments

Non-relativistic GR

Goldberger, Rothstein 2005

Worldlines minimally coupled to gravity:

$$S = \frac{M_{pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,R - \sum_{A=1}^{3} m_A \int \mathrm{d}t\sqrt{-g_{\mu\nu}v_A^\mu v_A^\nu} + S_{GF}$$

1PN approximation: $\frac{G_N m}{a} \sim v^2 \ll 1$

Integrate out graviton potential modes of the inner orbit:

$$g_{\mu\nu} = \tilde{g}_{\mu\nu} + H_{\mu\nu}$$

$$\mathcal{L}_{1PN}^{interaction} = -m_1\tilde{\phi}(\mathbf{x}_1)\left(1+\frac{3}{2}v_1^2\right) - \frac{m_1}{2}\tilde{\phi}(\mathbf{x}_1)^2 + m_1\tilde{\mathbf{A}}(\mathbf{x}_1)\cdot\mathbf{v}_1 + \frac{G_N m_1 m_2}{r}\tilde{\phi}(\mathbf{x}_1) + (1\leftrightarrow 2)$$

The inner binary as a composite particle

Multipole expansion: $\tilde{\phi}(\mathbf{x}_1) = \tilde{\phi} + (x_1 - X_{\rm CM})^i \partial_i \tilde{\phi} + \frac{1}{2}(x_1 - X_{\rm CM})^i (x_1 - X_{\rm CM})^j \partial_i \partial_j \tilde{\phi} + \dots$

Integrate out orbital modes beyond $\omega \sim v/a$ ⟷ Average

Monopole: $\langle \mathcal{L}_{\rm monopole} \rangle = \mathcal{L}_{\rm precession} - m_{eff} \sqrt{-\tilde{g}_{\mu\nu} V^{\mu}_{\rm CM} V^{\nu}_{\rm CM}}$ $\quad m_{eff} = m - \frac{G_N m \mu}{2a}$

The inner binary as a composite particle

Multipole expansion: $\tilde{\phi}(\mathbf{x}_1) = \tilde{\phi} + (x_1 - X_{\rm CM})^i \partial_i \tilde{\phi} + \frac{1}{2}(x_1 - X_{\rm CM})^i (x_1 - X_{\rm CM})^j \partial_i \partial_j \tilde{\phi} + \dots$

Integrate out orbital modes beyond $\omega \sim v/a$ ⟷ Average

Monopole: $\langle \mathcal{L}_{\rm monopole} \rangle = \mathcal{L}_{\rm precession} - m_{eff} \sqrt{-\tilde{g}_{\mu\nu} V^\mu_{\rm CM} V^\nu_{\rm CM}}$ $\quad m_{eff} = m - \frac{G_N m \mu}{2a}$

Dipole: $\langle \mathcal{L}_{\rm dipole} \rangle + \dots = \frac{1}{2} J_{\mu\nu} \Omega^{\mu\nu}$ $= \boldsymbol{J} \cdot \boldsymbol{\Omega} + \frac{1}{2} J_{ij} A^i_{CM} V^j_{CM} + \frac{1}{2} J_{ij} (2 V^i_{CM} \partial^j \tilde{\phi} + \partial^i \tilde{A}^j)$ $\quad J = \mu \sqrt{G_N m a (1 - e^2)}$

The inner binary as a composite particle

Multipole expansion: $\tilde{\phi}(\mathbf{x}_1) = \tilde{\phi} + (x_1 - X_{\rm CM})^i \partial_i \tilde{\phi} + \frac{1}{2}(x_1 - X_{\rm CM})^i (x_1 - X_{\rm CM})^j \partial_i \partial_j \tilde{\phi} + \dots$

Integrate out orbital modes beyond $\omega \sim v/a$ ⟷ Average

Monopole: $\langle \mathcal{L}_{\rm monopole} \rangle = \mathcal{L}_{\rm precession} - m_{eff} \sqrt{-\tilde{g}_{\mu\nu} V^{\mu}_{\rm CM} V^{\nu}_{\rm CM}}$ $\quad m_{eff} = m - \frac{G_N m \mu}{2a}$

Dipole: $\langle \mathcal{L}_{\rm dipole} \rangle + \dots = \frac{1}{2} J_{\mu\nu} \Omega^{\mu\nu}$ $= \boldsymbol{J} \cdot \boldsymbol{\Omega} + \frac{1}{2} J_{ij} A^i_{CM} V^j_{CM} + \frac{1}{2} J_{ij} (2 V^i_{CM} \partial^j \tilde{\phi} + \partial^i \tilde{A}^j)$ $\quad J = \mu \sqrt{G_N m a (1 - e^2)}$

Quadrupole: $S_{quadrupole} = \int \mathrm{d}\tau Q^{ij}_E E_{ij} - \int \mathrm{d}\tau Q^{ij}_B B_{ij}$ $\quad Q^{ij}_{E/B} = \ ?$

Secular dynamics at 1PN

Integrate out graviton potential modes of the outer orbit;

Average orbital modes $\omega \sim V_3/A$

1PN spin-orbit coupling:

$$\langle \mathcal{L}_{v^2\varepsilon^{3/2}} \rangle = -\frac{4m+3m_3}{2m} \frac{G_N}{A^3(1-E^2)^{3/2}} \mathbf{J}\cdot\mathbf{J}_3$$

$$\frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t} = \Omega_{\mathrm{prec}} \boldsymbol{J}_3 \times \boldsymbol{J}$$

Conclusions

- EFT can provide a systematic description of hierarchical triples
- Next: quadrupole and more on the composite point-particle

Conclusions

- EFT can provide a systematic description of hierarchical triples
- Next: quadrupole and more on the composite point-particle