The Casimir Effect in the Presence of Infrared Transparency

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Motivation

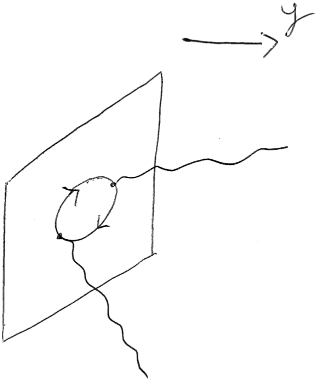
- Quantum field in vacuum with 2 parallel plates (Dirichlet boundary conditions)
  $\longrightarrow$ Casimir force $\propto -\frac{1}{R^D}$ (D: spacetime dimension)
- Replace fixed boundary conditions with localized kinetic terms
  $\longrightarrow$ infrared transparency (first introduced in DGP model to modify large distance gravity)
- Plates (or branes) act as "effective" boundary conditions
  $\longrightarrow$ modified Casimir force $\propto -\frac{1}{R^D} + \frac{1}{R^D\sqrt{Rr_c}}$
- Why interesting? Chance for gravitational Casimir effect!
  $\longrightarrow$ detection of gravitons?

Why study the DGP model?

- Why is gravity so weak? (hierarchy problem)
  $\longrightarrow$ possible solution: lower the (fundamental) Planck scale (Dvali-Gabadadze-Porrati, 2000)
- DGP modifies gravity at large distances
  $\longrightarrow$ cosmic acceleration?
- Non-perturbative model for (quasi-)massive gravity

The DGP model

$$S = M_*^3 \int \mathrm{d}^4x\mathrm{d}y \sqrt{|G|}\mathcal{R}_5 + M_\mathrm{P}^2 \int \mathrm{d}^4x \sqrt{|g|}\mathcal{R}$$
$$+ \text{(localized SM fields on ''our'' 3-brane)}$$



- Induced term due to localized matter fields running in the loop

Infrared transparency

DGP toy model

$$S = \int \mathrm{d}^4x\mathrm{d}y\,\frac{1}{2}\left[(\partial_A\Phi)^2 + r_c\,\delta(y)\,(\partial_\mu\Phi)^2\right], \qquad r_c = \frac{M_\mathrm{P}^2}{M_*^3}$$

- Kaluza-Klein decomposition:

$$\Phi(x^\mu, y) = \sum_m \psi_m(y)\phi_m(x^\mu), \qquad \phi_m\text{: Klein-Gordon field}$$

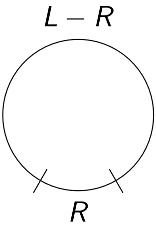
$$|\psi_m(y=0)| \propto \frac{1}{\sqrt{4 + m^2 r_c^2}}$$

- Potential between 2 static point sources at distance r

$$V(r, y=0) \propto -\frac{1}{M_*^3}\int_0^\infty \mathrm{d}m\,|\psi(y=0)|^2\frac{\mathrm{e}^{-mr}}{r} \sim \begin{cases} -\dfrac{1}{M_\mathrm{P}^2}\dfrac{1}{r}, & r \ll r_c \\ -\dfrac{1}{M_\mathrm{P}^2}\dfrac{1}{r}\dfrac{r_c}{r}, & r \gg r_c \end{cases}$$

The DGP model with two parallel branes

$$S = \int \mathrm{d}^4x \, \mathrm{d}y \, \frac{1}{2} \left\{ (\partial_A \Phi)^2 + r_c \left[\delta\left(y + R/2\right) + \delta\left(y - R/2\right) \right] (\partial_\mu \Phi)^2 \right\}$$



$$L > r_c \gg R$$

$$\Phi(x^\mu, y) = \sum_{\alpha=1}^{2} \sum_m \psi_{m,\alpha}(y) \phi_{m,\alpha}(x^\mu)$$

Mass quantization

(Schroedinger type) equation for $\psi_{m,\alpha}(y)$

$$\left\{\partial_y^2 + m^2\left[1 + r_c\delta\left(y + R/2\right) + r_c\delta\left(y - R/2\right)\right]\right\}\psi_{m,\alpha}(y) = 0$$

- Boundary conditions: continuity at the branes, discontinuity of the derivatives at the branes, periodicity
- Periodicity (in the presence of branes) $\longrightarrow$ mass quantization

$$\tan\frac{ml}{2} = -\frac{mr_c}{2}\frac{1 \pm \cos mR}{1 \pm \frac{mr_c}{2}\sin mR} \qquad \begin{array}{lll} + : & \text{even} & (\alpha = 1) \\ - : & \text{odd} & (\alpha = 2) \end{array}$$

Two extreme regimes

"Infrared transparency" regime ($m \ll 1/r_c$)

$$m = \frac{2\pi n}{L}, \quad n = 1, 2, \ldots$$

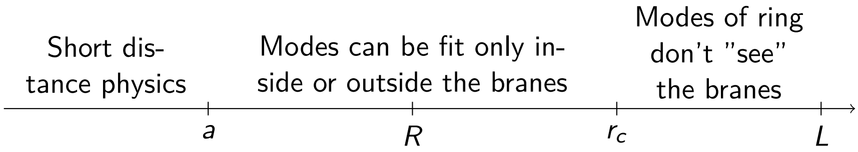
- Ordinary masses on ring of size L
- Branes are transparent.

"Opaque" regime ($m \gg 1/r_c$)

$$m = \begin{cases} \dfrac{\pi n}{L - R}, & n \gg \dfrac{L-R}{\pi r_c} \\ \dfrac{\pi n}{R}, & n = 1, 2, \ldots \end{cases}$$

- Branes act as Dirichlet boundary conditions.

- UV regulator: frequency cutoff at $\frac{1}{a}$ (like "Plasma frequency")



Casimir energy

- Regularized vacuum energy (per unit 3-volume on the brane)

$$E^{\mathrm{reg}} = \frac{1}{4\pi^2} \sum_{\alpha=1}^{2} \sum_{m} \int_0^\infty \mathrm{d}k\, k^2 \omega(m_\alpha, k) \mathrm{e}^{-\omega(m_\alpha, k)\frac{a}{\pi}}$$

$$\omega(m_\alpha, k) = \sqrt{m_\alpha^2 + k^2}$$

- Subtract divergent part (vacuum energy without branes)

$$E_0^{\mathrm{reg}} = E^{\mathrm{reg}}\big|_{m \to m_0}$$

$$m_0 = \frac{2\pi n}{L}, \qquad n = 1, 2, \ldots$$

- (Finite) Casimir energy

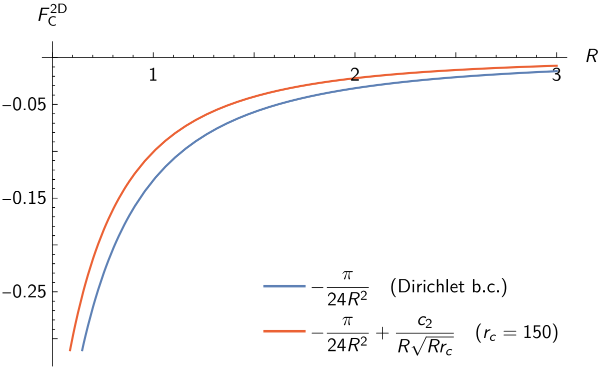
$$E_C = \lim_{\substack{a \to 0 \\ L \to \infty}} \left(E^{\mathrm{reg}} - E_0^{\mathrm{reg}} \right)$$

1+1-dimensional Casimir force

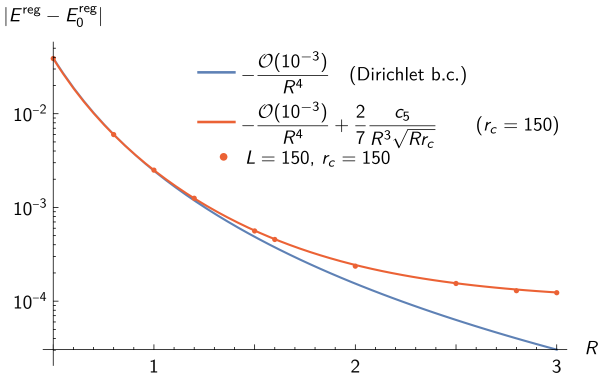
$$F_C^{2D} = -\frac{\partial E_C^{2D}}{\partial R} = -\frac{\pi}{24R^2} + \frac{c_2}{R\sqrt{Rr_c}} + \mathcal{O}\left(\frac{1}{Rr_c}\right)$$

- $c_2 \simeq 0.37$ is positive.
- Branes act (in leading order) as effective (Dirichlet) boundary conditions.
  $\longrightarrow$ Casimir force
- (In subleading order) branes "leak" some of the modes.
  $\longrightarrow$ Casimir force is weakened

Plotting the 1+1-dimensional Casimir force



4+1-dimensional Casimir force



$$F_C = -\frac{3\zeta(5)}{32\pi^2}\frac{1}{R^5} + \frac{c_5}{R^4\sqrt{Rr_c}} + \mathcal{O}\left(\frac{1}{R^4 r_c}\right)$$

- $c_5 \simeq 2.6 \cdot 10^{-3}$ is positive.

Summary

- Generalization to spin-2 theory straightforward (2 polarization states instead of 1)
- DGP branes can lead to gravitational Casimir effect (due to effective Dirichlet boundary condition)
- Infrared transparency phenomenon gives a weakening correction to the Casimir force ("leakage" of soft modes).
- Result only depends on localized kinetic terms on codimension-one branes.
  $\longrightarrow$ No dependence on number of dimensions.

Thank You

Questions?