

# Axion Quality from Superconformal Dynamics

**Yuichiro Nakai**

**T. D. Lee Institute & Shanghai Jiao Tong U.**

**Based on YN and M. Suzuki (TDLI), PLB 2021.**

# Strong CP Problem

QCD Lagrangian for strong interactions allows

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

explicitly violating **CP** symmetry.

The physical strong CP phase :  $\bar{\theta} \equiv \theta - \arg \det (M_u M_d)$

The current upper bound on the neutron electric dipole moment

$$\Rightarrow |\bar{\theta}| < 10^{-11}$$

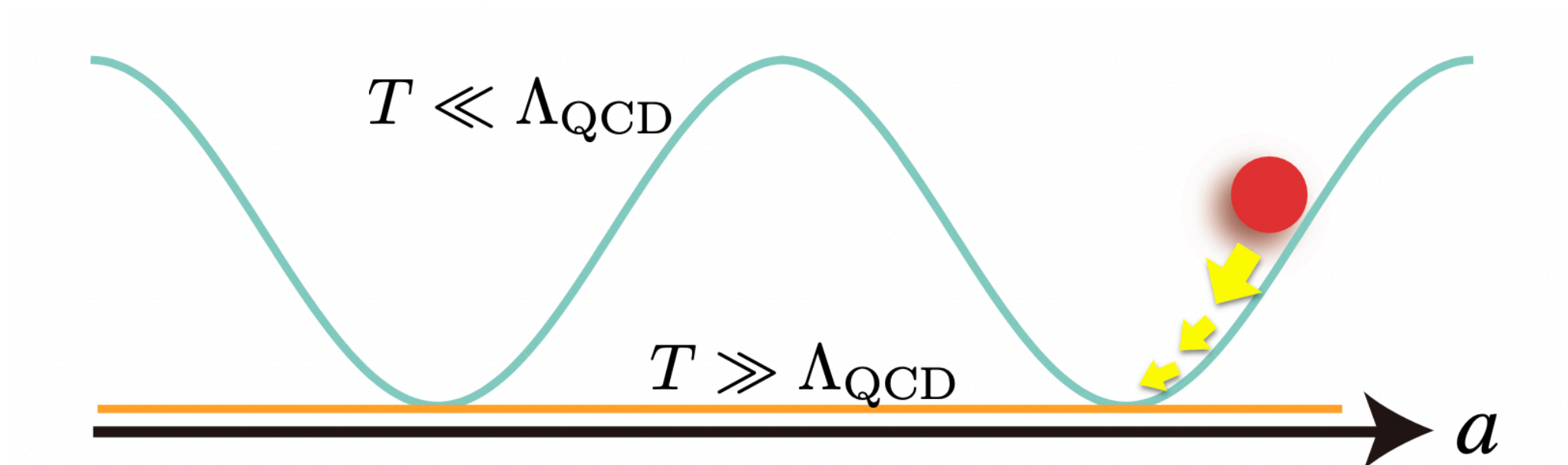
**Why is  $\bar{\theta}$  so small ??**

Some shifts of  $\bar{\theta}$  would not provide a visible change in our world.

# Axion Solution

The most common explanation is **the Peccei-Quinn mechanism** that the strong CP phase is promoted to a dynamical variable.

$$\mathcal{L}_\theta = \left( \theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$



Fuminobu Takahashi slide

The **axion  $a$**  dynamically cancels the strong CP phase !

# Axion Solution

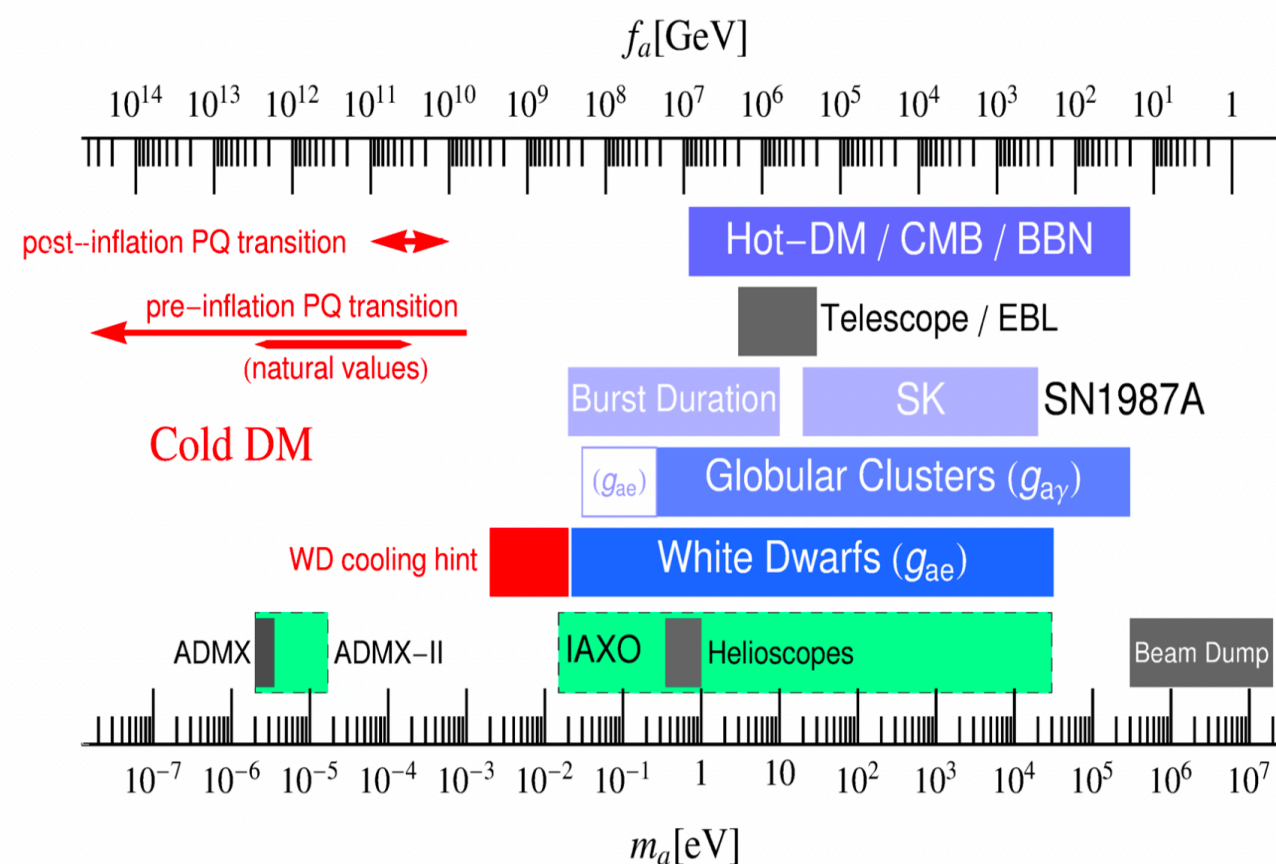
Axion is a pseudo-Nambu-Goldstone boson associated with spontaneous breaking of a **global  $U(1)_{PQ}$  symmetry**.

Non-perturbative QCD effects break the  $U(1)_{PQ}$  explicitly and generate the axion potential :

$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

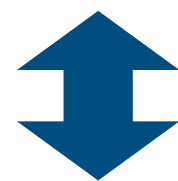
Astrophysical observations put a lower limit :

$$f_a \gtrsim 10^8 \text{ GeV}$$



# Axion Quality Problem

The small strong CP phase requires the  $U(1)_{PQ}$  to be realized to an extraordinary high degree.



**Quantum gravity effects** do not respect such a global symmetry.

Planck suppressed  $U(1)_{PQ}$ -violating operators are expected.

$$\Delta V(\phi) \sim \frac{|\phi|^{k+3}}{M_{Pl}^k} \phi + \text{h.c.}$$

➡ Destroy the Peccei-Quinn mechanism.

$$\Delta V(a) \sim f_a^4 \left( \frac{f_a}{M_{Pl}} \right)^k \cos \left( \frac{a}{f_a} - \varphi \right) \quad \langle \phi \rangle \equiv \frac{f_a}{\sqrt{2}}$$

# Axion Quality Problem

The small strong CP phase requires the  $U(1)_{PQ}$  to be realized to an extraordinary high degree.

Quantum gravity eff.  $\leftrightarrow$  Symmetry.

Planck

**Superconformal dynamics !**

$M_{Pl}^k \phi + \text{h.c.}$

➔ Destroy the Peccei-Quinn mechanism.

$$\Delta V(a) \sim f_a^4 \left( \frac{f_a}{M_{Pl}} \right)^k \cos \left( \frac{a}{f_a} - \varphi \right) \quad \langle \phi \rangle \equiv \frac{f_a}{\sqrt{2}}$$

# Conformal Dynamics

The PQ breaking field marginally couples to **CFT sector fields**.

$$W_{\text{int}} = \lambda \phi \mathcal{O}_{\text{CFT}}$$

The PQ breaking field holds a large anomalous dimension.

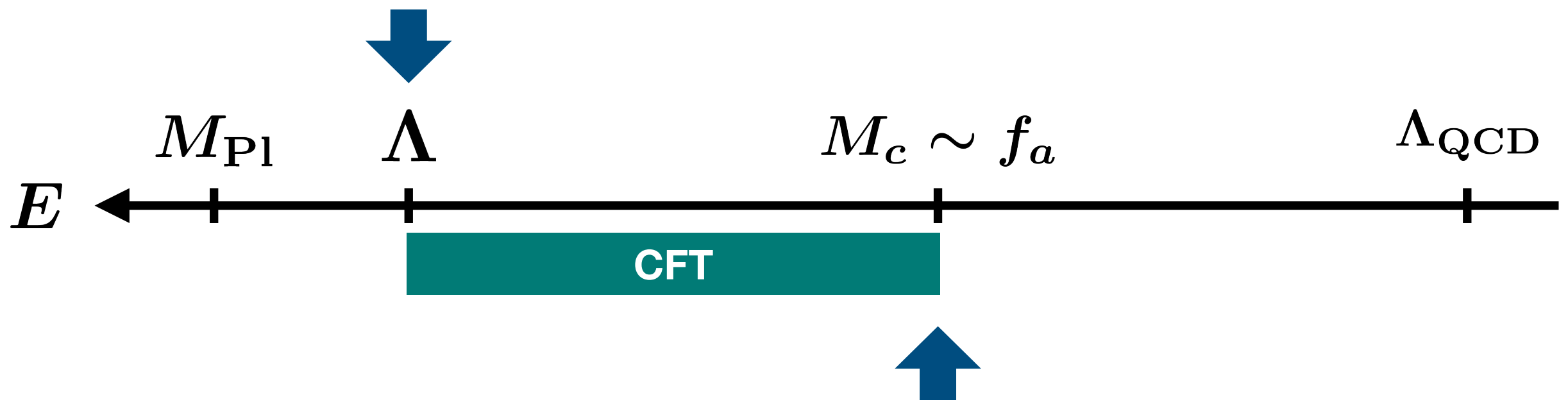
$$\Rightarrow \epsilon_{\phi} \equiv Z_{\phi}^{-1/2}(\mu) = \left( \frac{\mu}{\Lambda} \right)^{\frac{\gamma_{\phi}}{2}} \ll 1$$

The  $U(1)_{\text{PQ}}$ -violating operators are significantly suppressed at low-energies :

$$\Delta W \sim \frac{\phi^k}{M_{\text{Pl}}^{k-3}} \rightarrow \epsilon_{\phi}^k \frac{\phi^k}{M_{\text{Pl}}^{k-3}}$$

# Conformal Dynamics

The theory flows into a conformal fixed point.



PQ breaking drives conformal breaking.

All the CFT sector fields become massive.

Integrating out the CFT sector fields generates

$$\mathcal{L}_\theta = \left( \theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a$$

*cf.* The KSVZ axion model



# The Model

A SUSY SU(N) gauge theory with  $N_f$  vector-like quarks :

$$Q_I, \bar{Q}_I \quad (I = 1, \dots, N_f) \quad N_f : \text{even}$$

The theory is in **conformal window** :  $\frac{3}{2}N < N_f < 3N$

PQ singlet chiral superfields :  $\Phi, \bar{\Phi}$

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$
$$m = 1, \dots, N_f/2 \quad k = N_f/2 + 1, \dots, N_f$$

**The ordinary color** is embedded in flavor symmetries :

$$\underline{SU(N_f/2)_1} \times SU(N_f/2)_2$$
$$\supset SU(3)_C$$

# The Model

	$Q_m$	$\bar{Q}_m$	$Q_k$	$\bar{Q}_k$	$\Phi$	$\bar{\Phi}$
$SU(N)$	$N$	$\bar{N}$	$N$	$\bar{N}$	$1$	$1$
$U(1)_{PQ} (Z_N)$	$+1$	$0$	$-1$	$0$	$-1$	$+1$
$U(1)_R$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{N_f - N}{N_f}$	$\frac{2N}{N_f}$	$\frac{2N}{N_f}$

- The  $U(1)_{PQ}$  symmetry is not anomalous under the  $SU(N)$ .

➡ Axion does not couple to the  $SU(N)$  gauge field so that no new axion potential is generated.

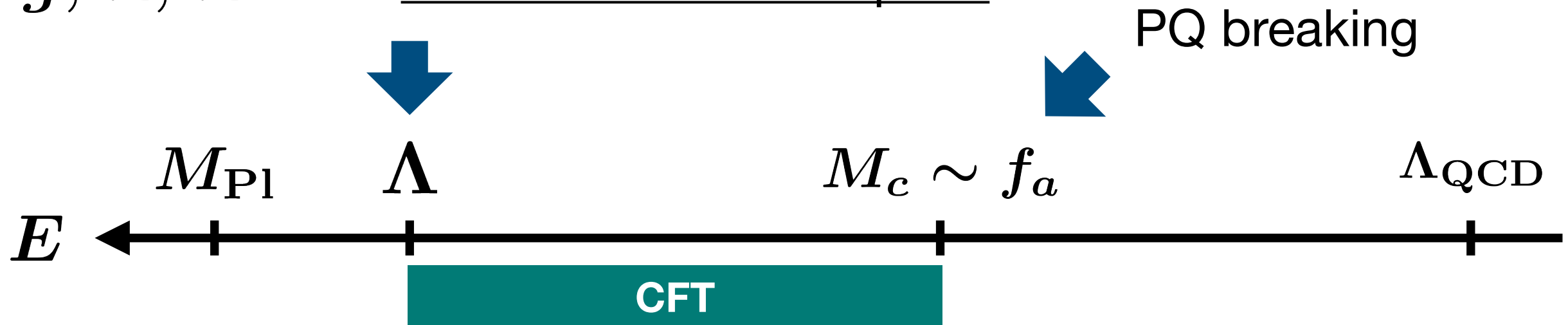
- Anomaly coefficient :  $A_{U(1)_{PQ}-SU(3)_C-SU(3)_C} = N$

➡  $Z_N \subset U(1)_{PQ}$  is **an anomaly-free discrete symmetry**.

It ensures the  $U(1)_{PQ}$  at the renormalizable level.

# Anomalous Dimension

$g, \lambda, \bar{\lambda}$  enter a non-trivial IR fixed point.

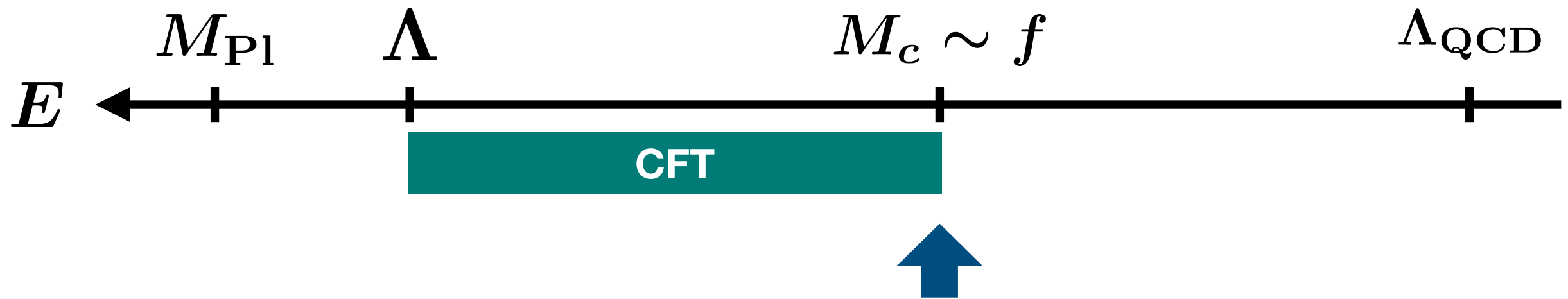


Anomalous dimension is determined by **the  $\text{U}(1)_R$  charge**.

$$Z_\Phi = \left( \frac{M_c}{\Lambda} \right)^{-\gamma_\Phi} \quad \gamma_\Phi = 6 \frac{N}{N_f} - 2$$

Canonical normalization :  $\Phi = \left( \frac{M_c}{\Lambda} \right)^{\gamma_\Phi/2} \hat{\Phi}$

# Axion Potential



$\Phi, \bar{\Phi}$  obtain nonzero VEVs.

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$

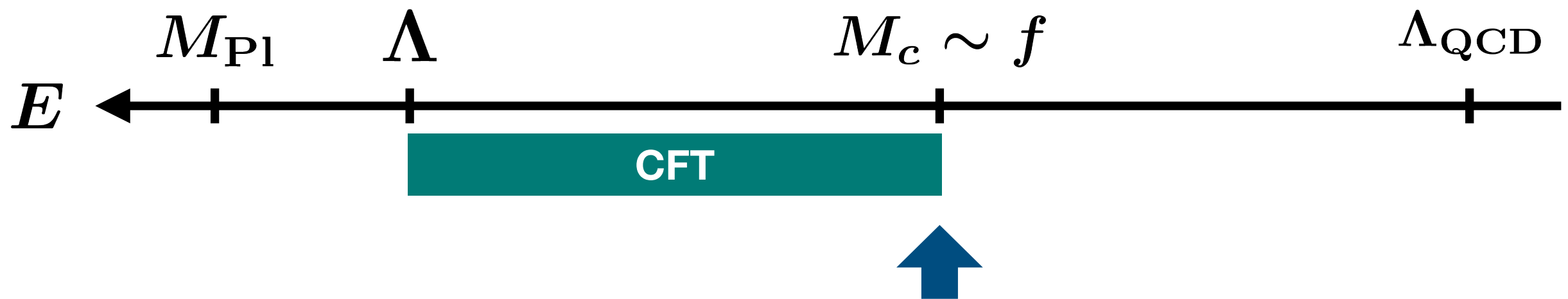
➡ All the new quarks become massive.

Integrating out the quarks ➡  $\mathcal{L}_{\text{eff}} \supset N \frac{a}{F_a} \frac{g_c^2}{32\pi^2} G \tilde{G}$

$$F_a/N = \sqrt{2} f/N$$

Axion potential :  $V \sim m_\pi^2 f_\pi^2 \cos \left( N \frac{a}{F_a} \right)$   $m_\pi^2 f_\pi^2 = (0.1 \text{ GeV})^4$

# Hidden Glueballs



$\Phi, \bar{\Phi}$  obtain nonzero VEVs.

$$W_Q = \lambda \Phi Q_m \bar{Q}_m + \bar{\lambda} \bar{\Phi} Q_k \bar{Q}_k$$

➡ All the new quarks become massive.

The model becomes a **SU(N) pure Yang-Mills theory**.

It confines just below the conformal breaking scale.

➡ Heavy **SU(N) glueballs** and their superpartners.

# Emergent PQ

The most dangerous operator respecting the  $\mathbf{Z}_N$  symmetry :

$$W_{\cancel{\text{PQ}}} \sim \frac{\Phi^N}{M_{\text{Pl}}^{N-3}} \sim \left( \frac{M_c}{\Lambda} \right)^{\frac{N\gamma_\Phi}{2}} \frac{\hat{\Phi}^N}{M_{\text{Pl}}^{N-3}}$$

The scalar potential in supergravity  $V \supset -3WW^*/M_{\text{Pl}}^2$

$$W = m_{3/2} M_{\text{Pl}}^2$$

➡  $V_{\cancel{\text{PQ}}} = \left( \frac{M_c}{\Lambda} \right)^{\frac{N\gamma_\Phi}{2}} \frac{\kappa_{\cancel{\text{PQ}}} m_{3/2} \hat{\Phi}^N}{M_{\text{Pl}}^{N-3}}$   $\kappa_{\cancel{\text{PQ}}}$  : model dependent coefficient

The  $U(1)_{\text{PQ}}$ -violating axion potential :

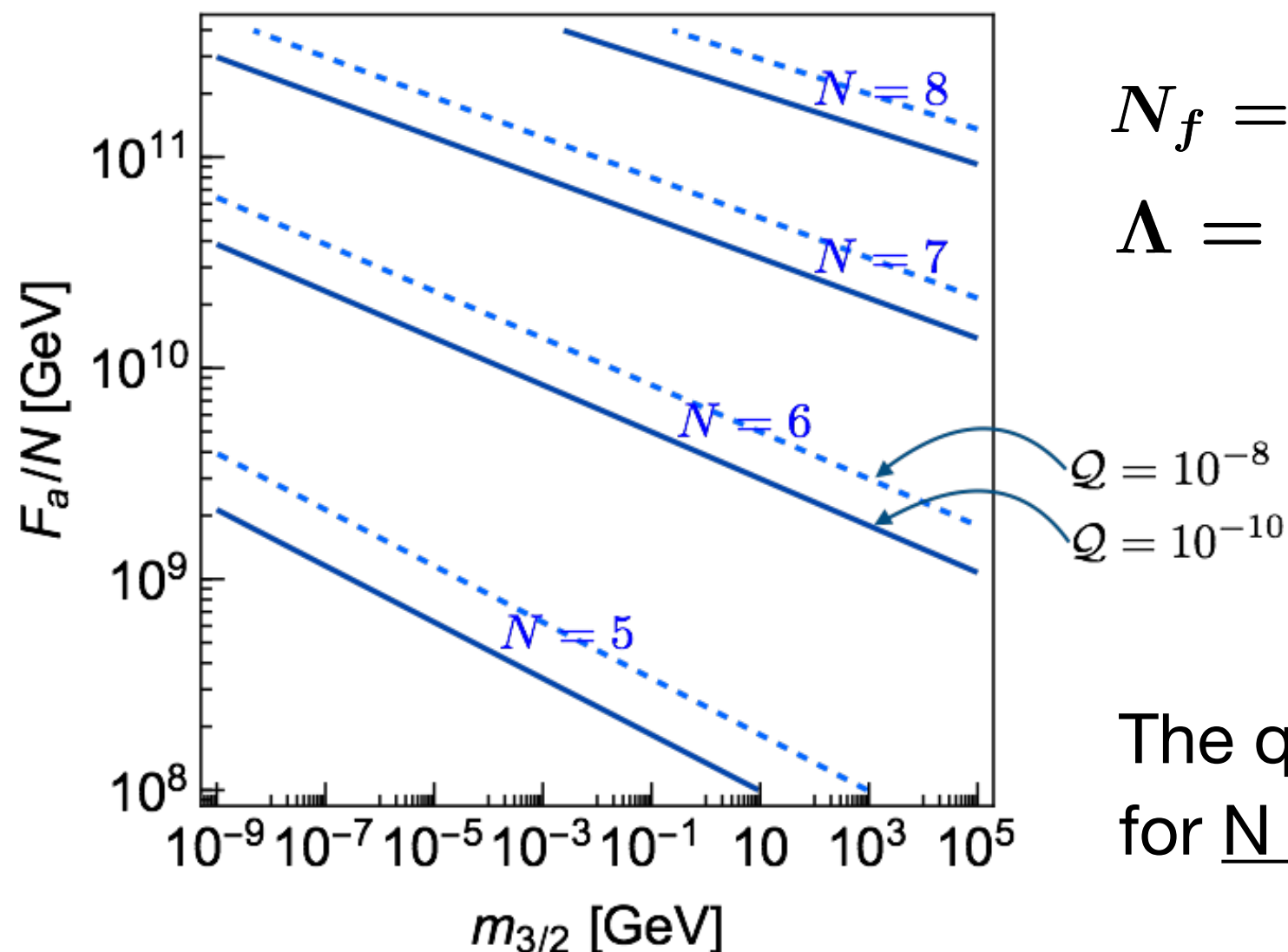
Arbitrary !

$$V_{\cancel{\text{PQ}}} \supset \left( \frac{M_c}{\Lambda} \right)^{N(3N/N_f-1)} \frac{\kappa_{\cancel{\text{PQ}}} m_{3/2} F_a^N}{M_{\text{Pl}}^{N-3}} \cos \left( N \frac{a}{F_a} + \varphi \right)$$

# Emergent PQ

Axion quality factor :  $V_{PQ} \equiv \mathcal{Q} m_\pi^2 f_\pi^2 \cos \left( N \frac{a}{F_a} + \varphi \right)$

Experimental upper bound requires  $\mathcal{Q} \lesssim 10^{-10}$



$$N_f = 2N, M_c = F_a, \kappa_{PQ} = 1$$

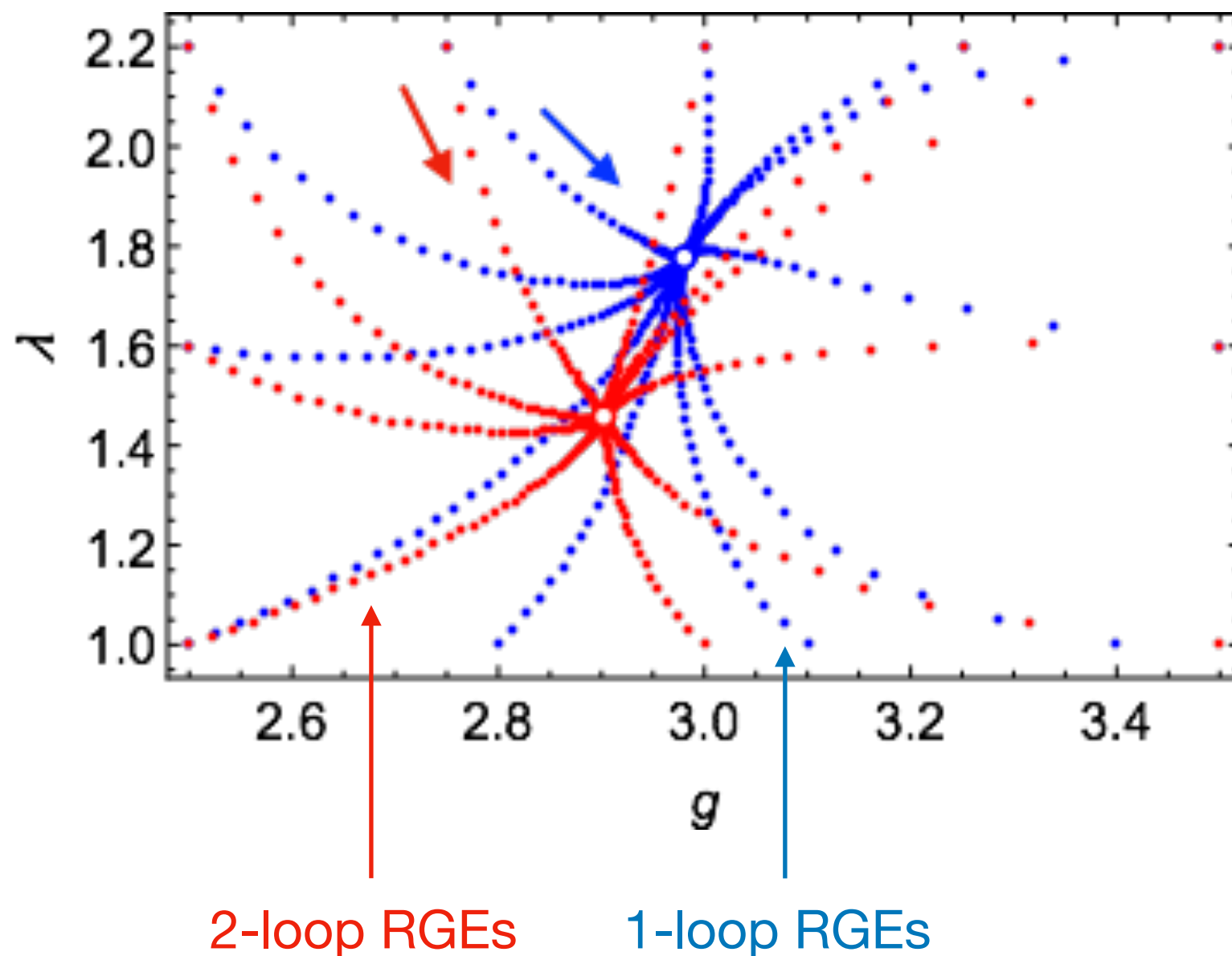
$$\Lambda = 0.1 M_{Pl}$$

The quality problem is solved  
for  $N = 5$  or larger !

# The IR fixed Point

Check the existence of the IR fixed point for  $g$ ,  $\lambda$ ,  $\bar{\lambda}$

RG flows from  $\Lambda_0$  to  $\mu = 10^{-9} \Lambda_0$



$$N = 5 \quad N_f = 10$$
$$\bar{\lambda} = 2 \text{ at } \Lambda_0$$

The effect of SU(3)<sub>c</sub> gauge coupling is ignored.

Both couplings flow into a non-trivial IR fixed point.

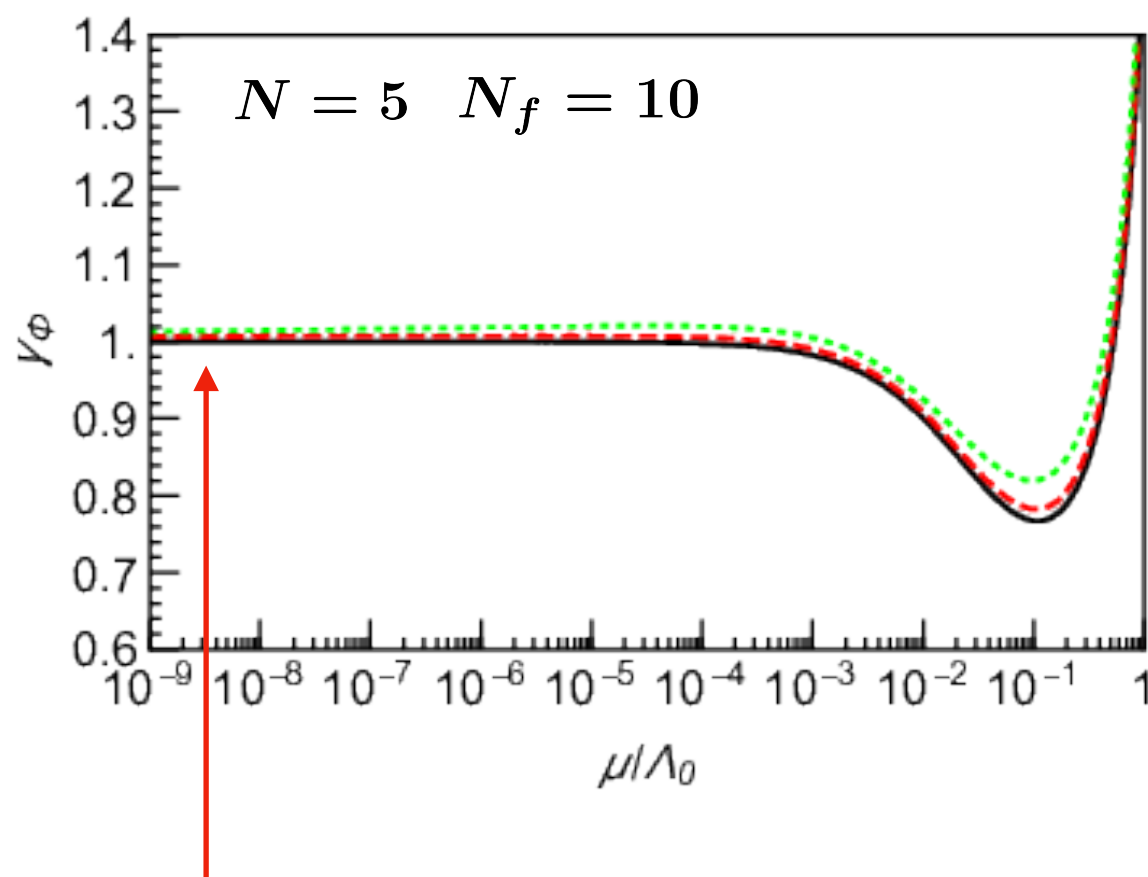


# The IR fixed Point

Include the effect of the SU(3)<sub>c</sub> gauge coupling.

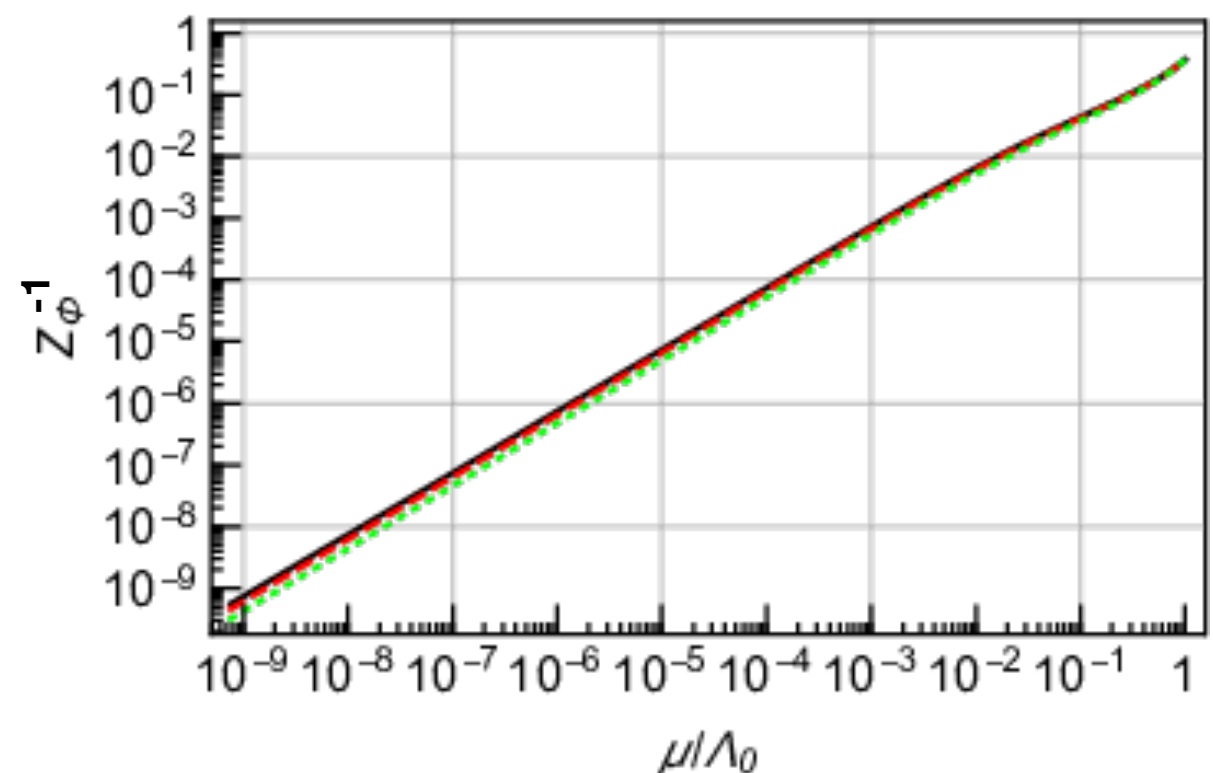
$$g = \lambda = \bar{\lambda} = 2 \quad g_c = 0, 1, 2 \quad \text{at } \Lambda_0$$

Anomalous dimension at 2-loop



**The value without the QCD effect**

Wave function renormalization factor



The smallness enables to solve the axion quality problem.

# Summary



- **Superconformal dynamics** can address the axion quality problem.
- PQ breaking fields marginally couple to new quarks charged under the  $SU(3)_c$  and a new  $SU(N)$ .
- A large anomalous dimension of PQ breaking fields leads to a strong suppression of explicit  $U(1)_{\text{PQ}}$ -violating operators.
- PQ breaking drives conformal breaking and integrating out the new heavy quarks generates the desired axion coupling to gluons.

*Thank you.*

# **Backup Material**

# PQ Breaking

PQ breaking :  $W'_X = \kappa' X (2\Phi\bar{\Phi} - f'^2)$

Canonical normalization :  $\Phi = \left(\frac{M_c}{\Lambda}\right)^{\gamma_\Phi/2} \hat{\Phi}$

➔  $W_X = \kappa \left(\frac{M_c}{\Lambda}\right)^{\gamma_\Phi} X (2\hat{\Phi}\hat{\bar{\Phi}} - f^2)$

$$\kappa \sim \kappa' \quad \boxed{f \sim \left(\frac{M_c}{\Lambda}\right)^{-\gamma_\Phi/2} f'}$$

**PQ (and conformal) breaking scale  $M_c \sim f$**