Cosmological implications of EW vacuum instability: constraints on the Higgs-curvature coupling from inflation

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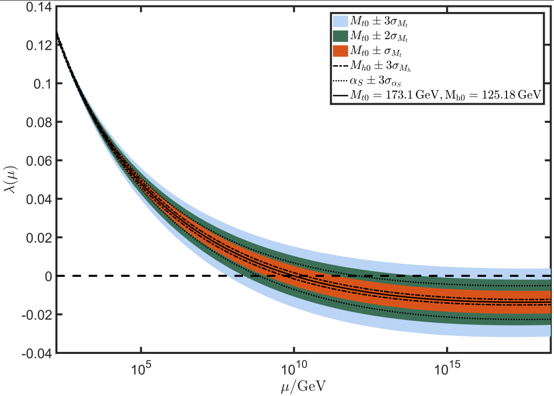
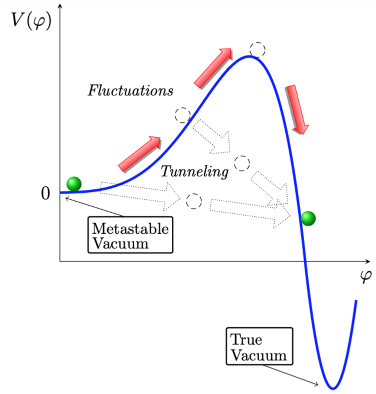
Introduction

Experimental values of SM particle masses m_h, m_t indicate that:

- SM may be valid up to $\mu_{\rm QG}$; early Universe consistent minimal model.
- currently in metastable EW vacuum $\rightarrow$ constrain fundamental physics.





Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Introduction

- Decay expands at c with singularity within $\rightarrow$ true vacuum bubbles:

$$d\langle\mathcal{N}\rangle = \Gamma d\mathcal{V} \;\Rightarrow\; \langle\mathcal{N}\rangle = \int_{\text{past}} d^4x\sqrt{-g}\,\Gamma(x)$$

- Universe still in metastable vacuum $\rightarrow$ no bubbles in past light-cone:

$$P(\mathcal{N}=0) \propto e^{-\langle\mathcal{N}\rangle} \sim \mathcal{O}(1) \;\Rightarrow\; \langle\mathcal{N}\rangle \lesssim 1$$

- Low decay rate Γ today, but higher rates in the early Universe.

Vacuum bubbles expectation value (during inflation)

$$\langle\mathcal{N}\rangle = \frac{4\pi}{3}\int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}\left(\eta_0 - \eta\left(N\right)\right)}{e^N}\right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

Tree-level curvature corrections

- Classical solutions to the tunneling process from false to true vacuum.

- High H's during inflation, CdL→HM instanton with action difference

$$B_{\rm HM}(R) \approx \frac{384\pi^2 \Delta V_{\rm H}}{R^2}$$

where $\Delta V_{\rm H} = V_{\rm H}(h_{\rm bar}) - V_{\rm H}(h_{\rm fv})$: barrier height → decay rate

$$\Gamma_{\rm HM}(R) \approx \left(\frac{R}{12}\right)^2 e^{-B_{\rm HM}(R)}$$

- Curvature effects enter at tree level via non-minimal coupling ξ:

$$V_{\rm H}(h, \mu, R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4$$

Coleman (1977), Coleman and De Lucia (1980), Hawking and Moss (1987).

One-loop curvature corrections

- Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_{\rm H}(h,\mu,R) = \frac{\xi(\mu)}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4 + \frac{\alpha(\mu)}{144} R^2 + \Delta V_{\rm loops}(h,\mu,R)\,,$$

where the loop contribution can be parametrized as

$$\Delta V_{\rm loops} = \frac{1}{64\pi^2} \sum_{i=1}^{31} \left\{ n_i \mathcal{M}_i^4 \left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2} \right) - d_i \right] + \frac{n_i' R^2}{144} \log\left(\frac{|\mathcal{M}_i^2|}{\mu^2} \right) \right\}$$

- RGI: choose $\mu = \mu_*(h,R)$ such that $\Delta V_{\rm loops}(h,\mu_*,R) = 0 \rightarrow$

RGI effective Higgs potential

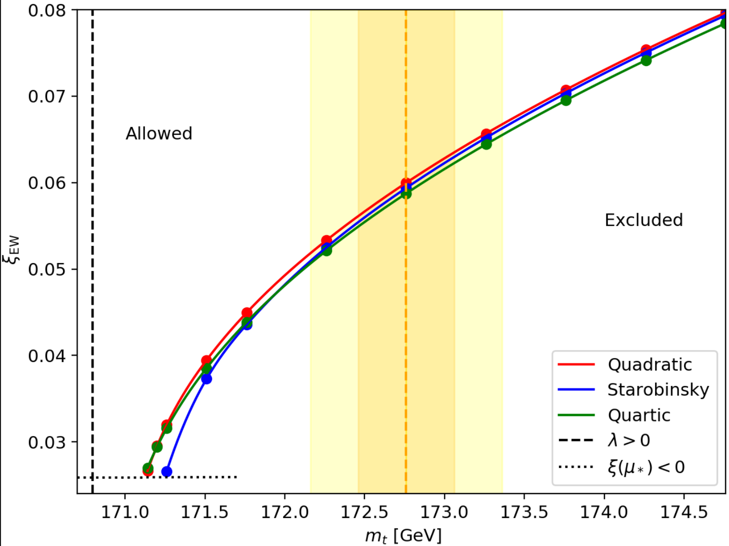
$$V_{\rm H}^{\rm RGI}(h,R) = \frac{\xi(\mu_*(h,R))}{2} R h^2 + \frac{\lambda(\mu_*(h,R))}{4} h^4 + \frac{\alpha(\mu_*(h,R))}{144} R^2$$

Markkanen *et al*, "The 1-loop effective potential for the Standard Model in curved spacetime", 2018.

Overview of computation

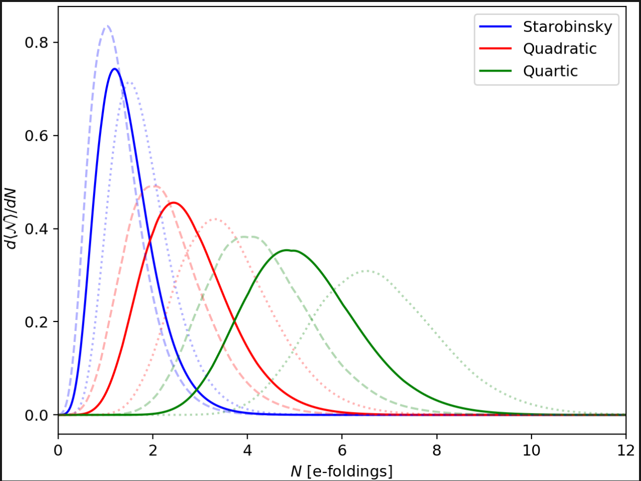
1. Calculate $\Delta V_{\rm H}$ and plug it in Γ.
2. Choose inflationary model by specifying $V(\phi)$ for the inflaton.
3. Complete calculation of $\langle\mathcal{N}\rangle$ imposing the condition $\langle\mathcal{N}\rangle \leq 1$.
4. Result: constraint on $\xi \geq \xi_{\langle\mathcal{N}\rangle=1}$.

Results: Bounds on ξ



Results: Bubble nucleation time

- If bubbles form at $N < 1$ $\rightarrow$ bounds maybe unreliable due to $B_{\mathrm{HM}}^{\mathrm{dS}}$.
- If bubbles form at $N \gg 60$ $\rightarrow$ bounds would depend on early times.



Results: Significance of the total duration of inflation

- Inflation can last for many orders of magnitude longer than 60 e-folds.

- We study early time behavior by splitting the $\langle \mathcal{N} \rangle$-integral

$$\langle \mathcal{N} \rangle (N_{\text{start}}) = \langle \mathcal{N} \rangle (60) + \int_{60}^{N_{\text{start}}} \frac{d\mathcal{V}}{dN} \Gamma(N)\, dN \ ,$$

  where we set $\langle \mathcal{N} \rangle (60) = 1$ and slow roll applies to the 2nd term.

- $B_{\text{HM}} \approx$ constant at early times, so that

$$\langle \mathcal{N} \rangle (N_{\text{start}}) \approx 1 + \frac{4\pi e^{-B_{\text{HM}}}}{3} N_{\text{start}} \ .$$

- Contributing if $N_{\text{start}} \gtrsim e^{B_{\text{HM}}} \sim 10^{60} \gg 60\, e$-folds but not infinite.

Conclusions

Included 1-loop curv. corrections beyond dS → most accurate constraints:

ξ-bounds for $m_t \pm 2\sigma$ in each model (numerical errors$< 1\%$)

$$\begin{aligned} \text{Quadratic}: \ & \xi_{\text{EW}} \geq 0.060^{+0.007}_{-0.008}, \\ \text{Quartic}: \ & \xi_{\text{EW}} \geq 0.059^{+0.007}_{-0.008}, \\ \text{Starobinsky}: \ & \xi_{\text{EW}} \geq 0.059^{+0.007}_{-0.009}, \end{aligned}$$

with the minimal assumption that inflation lasts $N = 60$ e-foldings.

that are $V(\phi)$-independent, N_{start}-independent and m_t-dependent.

Next step: consider Starobinsky Inflation (R^2-model):

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(1 - \frac{\xi h^2}{M_P^2} \right) R_J + \frac{1}{12\alpha^2} R_J^2 + \frac{1}{2} g_J^{\mu\nu} \partial_\mu h \partial_\nu h - \frac{\lambda}{4} h^4 \right]$$

Starobinsky/R^2 inflation

Due to the conformal transformation $m_i^2 \to m_i^2 e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}$, meaning that the RG scale μ_* will be different because ΔV_{loops} will change.

$$\mathcal{L} \approx \frac{M_P^2}{2} R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\rho\partial^\mu\rho - V^{\text{total}}(\rho,\phi,R)$$

with $V^{\text{total}} = V_{\text{Star}} + m_{\text{eff}}\frac{\rho^2}{2} + \lambda_{\text{eff}}\frac{\rho^4}{4}$, where $\Xi = \xi - \frac{1}{6}$ and

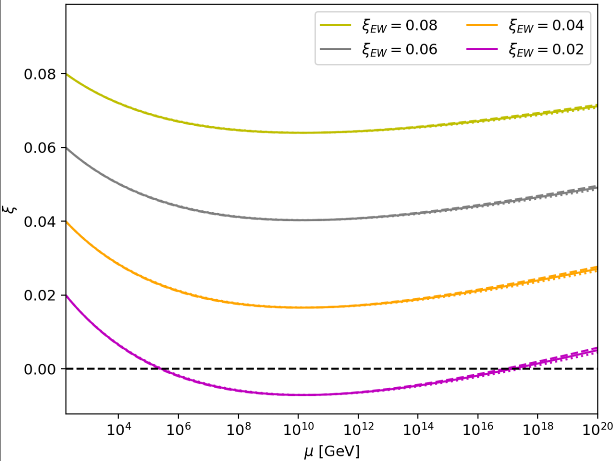
$$V_{\text{Star}} = \frac{3\alpha^2 M_P^4}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2,$$

$$m_{\text{eff}} = \xi R + 3\alpha^2 M_P^2 \Xi \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right) e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} + \frac{\Xi}{M_P^2}\partial_\mu\phi\partial^\mu\phi\,,$$

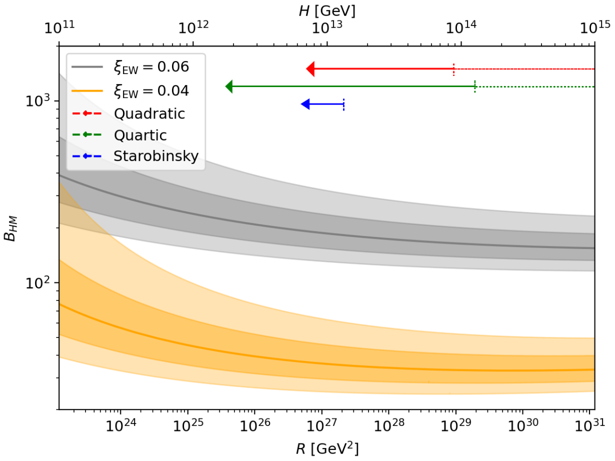
$$\lambda_{\text{eff}} = \lambda + 3\alpha^2\Xi^2 e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} + \frac{4\,(\xi R)_{\text{eff}}}{\Xi^{-2}M_P^2} + \frac{4\Xi^3}{M_P^4}\partial_\mu\phi\partial^\mu\phi\,.$$

Additional slides - Running of non-minimal coupling ξ

$$16\pi^2\beta_\xi = 16\pi^2\frac{d\xi}{d\ln\mu} = \left(\xi - \frac{1}{6}\right)\left(12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2\right)$$



Additional slides - Curvature effects on the bounce action



Shaded areas: 1σ, 2σ deviation from the central m_t; a heavier top quark decreases the value of $B_{\rm HM}$ and vice versa.

Solid red, blue and green arrows: last 60 e-foldings in quadratic, Starobinsky and quartic inflation.

Additional slides - Numerical solution

Solve the system of coupled differential equations

$$\frac{d^2\phi}{dN^2} = \frac{V(\phi)^2}{M_P^2 H^2}\left(\frac{d\phi}{dN} - M_P^2 \frac{V'(\phi)}{V(\phi)}\right)$$

$$\frac{d\tilde{\eta}}{dN} = -\tilde{\eta}(N) - \frac{1}{a_{\rm inf} H(N)}$$

$$\frac{d\langle\mathcal{N}\rangle}{dN} = \gamma(N) = \frac{4\pi}{3}\left[a_{\rm inf}\left(\frac{3.21e^{-N}}{a_0 H_0} - \tilde{\eta}(N)\right)\right]^3 \frac{\Gamma(N)}{H(N)}$$

where $\tilde{\eta} = e^{-N}\eta$ and η: conformal time and we set the end of inflation at

$$\left.\frac{\ddot{a}}{a}\right|_{\phi=\phi_{\rm inf}} = H^2\left[1 - \frac{1}{2M_P^2}\left(\frac{d\phi}{dN}\right)^2\right]\Bigg|_{\phi=\phi_{\rm inf}} = 0$$

Additional Slides - Inflationary Models

- *Quadratic inflation*, where $m = 1.4 \times 10^{13}$ GeV, with

$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

- *Quartic inflation*, where $\lambda = 1.4 \times 10^{-13}$, with

$$V(\phi) = \frac{1}{4} \lambda \phi^4$$

- *Starobinsky inflation* (Starobinsky-like power-law model), where $\alpha = 1.1 \times 10^{-5}$, with

$$V(\phi) = \frac{3}{4} \alpha^2 M_P^4 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_P}}\right)^2$$

Quadratic and quartic models are simple but not realistic; Starobinsky inflation complies with data and can link different inflationary models.