Global symmetry and the black hole information problem

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June 14, 2021

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- Requiring $\Lambda < 0$ is clearly too restrictive an assumption for an idea that we hope applies to our universe, but on the other hand we will soon see that *some* kind of nontrivial assumption is necessary: at least in lower dimensions there do exist theories of quantum gravity with global symmetries!

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- We will show that a version of the argument of Harlow/Ooguri can be applied to exclude global symmetries in any theory of quantum gravity where the Page curve calculations of Penington, Almheiri/Engelhardt/Marolf/Maxfield 2019 are valid.

Thus our proposal is reduced to the assumption that this is the only way for black hole evaporation to be unitary and compatible with the Bekenstein-Hawking formula.

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- **String worldsheet:**

$$S = -\frac{1}{4\pi\alpha'} \int d^2x \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X_\mu,$$

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Neither has black holes, so they are "allowed" to have global symmetry.

A more interesting example is the AdS version of Jackiw-Teitelboim gravity coupled to conformal matter:

$$S = \int_M d^2x \sqrt{-g}\,(\Phi_0 R + \Phi(R+2)) + 2\int_{\partial M} dt \sqrt{-\gamma}\,(\Phi_0 K + \Phi(K-1)) + S_{CFT}(\psi_i, g).$$

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This is a renormalizable theory of quantum gravity, and after canonical quantization it can be exactly solved via a Weyl transformation to flat space. In particular any global symmetry of the matter CFT which does not have a mixed anomaly with gravity, e.g. the $U(N)$ flavor symmetry of N Dirac fermions, survives canonical quantization and becomes an exact global symmetry of the quantum theory.

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Unlike the previous two examples this theory does have black hole solutions, but we will soon see that in the quantum theory their entropy is infinite, and (like in the CGHS/RST models) their evaporation leads to remnants.

Pure 2+1 gravity

Our last example is the oriented version of pure gravity in $2+1$ dimensions, which we'll take to have negative cosmological constant:

$$S = \frac{1}{16\pi G}\int_M d^3x\sqrt{-g}\,(R+2) + \frac{1}{8\pi G}\int_{\partial M}\int d^2x\sqrt{-\gamma}(K-1).$$

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This theory has black hole solutions, but they again have infinite entropy Maloney, Kim/Porrati so our proposal allows for global symmetry.

Holography

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- Instead all UV-complete theories of quantum gravity in $3+1$ dimensions or higher come from string theory, and those which we understand non-perturbatively are all *holographic*: their fundamental description lives in a lower number of spacetime dimensions at some asymptotic boundary. 't Hooft 1993, Susskind 1994

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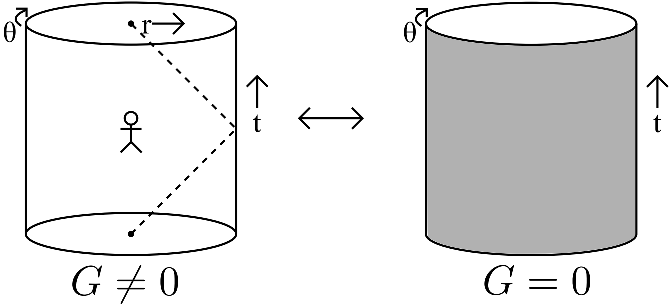

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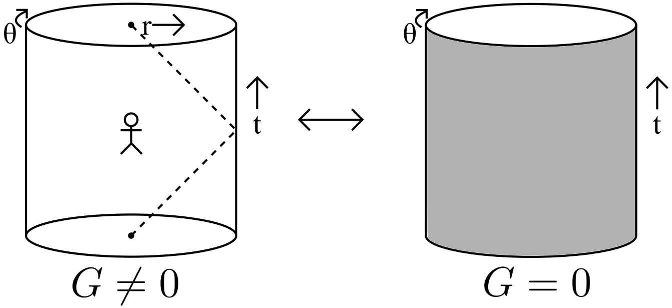
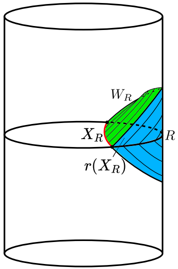


The number of boundary dimensions is one less, as expected from the holographic principle.

One of the most important features of this correspondence is the *quantum extremal surface formula*, which gives a bulk formula for boundary Von Neumann entropy: Ryu/Takayanagi 2006, Hubeny/Rangamani/Takayanagi 2007, Faulkner/Lewkowycz/Maldacena 2013, Engelhardt/Wall 2014



$$S(\rho_R) = \min_{X_R}\left[\operatorname*{ext}_{X_R}\left(\frac{\mathrm{Area}(X_R)+\dots}{4G} + S_{bulk}(r(X_R))\right)\right].$$

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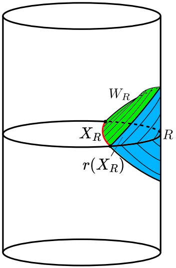

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This implies *entanglement wedge reconstruction*, which says that any bulk operator in the wedge can be represented on R alone in the CFT.

Dong/Harlow/Wall 2016

Over the last decade the QES formula has been understood from many points of view:

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Inspired by the generality of these arguments, in 2019 the formula was applied to a reservoir system coupled to a holographic CFT, giving a remarkable derivation of the "Page curve" for certain evaporating black holes. Penington, Almheiri/Engelhardt/Marolf/Maxfield 2019

The basic idea of the black hole information problem is that Hawking's calculation of the rate of particle production by black holes also tells us the quantum state of those particles: they are (basically) in a mixed thermal state. As time proceeds there are more and more of them, so the von Neumann entropy of the radiation cloud grows with time. Eventually the radiation is all that is left, so a pure state has evolved to a mixed state and quantum mechanics has been violated.

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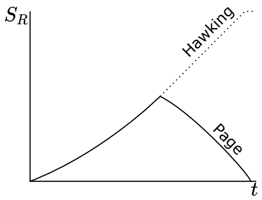

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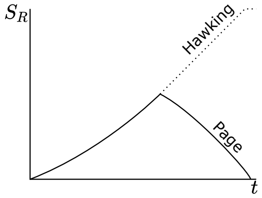
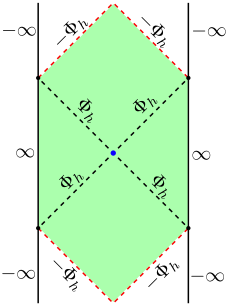


This can be represented graphically using the "Page curve", which shows the von Neumann entropy of the radiation as a function of time. Reliably showing that the Page curve bends downwards has been one of the key requirements for solving the information problem, so seeing this come from the QES formula is major progress!

JT black holes

To prepare for our discussion of global symmetry it is useful to first review how to compute a Page curve for JT gravity coupled to conformal matter:

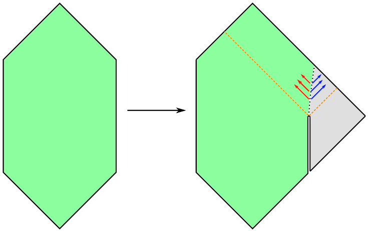


$$\Phi = \phi_b r$$

$$ds^2 = -\left(r^2 - 4\pi^2 T^2\right) dt^2 + \frac{dr^2}{r^2 - 4\pi^2 T^2},$$

$$S = 4\pi \left(\Phi_0 + \Phi(r_s)\right) = 4\pi \left(\Phi_0 + 2\pi\phi_b T\right).$$

Almheiri/Engelhardt/Marolf/Maxfield realized an information problem for this black hole by coupling to a reservoir on one side and allowing the conformal matter to leak out:



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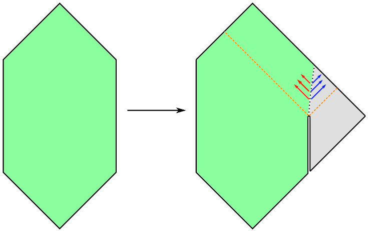

The entropy of the reservoir system in canonical quantization can be computed,

$$S_{res}(t) = 16\pi^2 \phi_b T_1^2 \left(1 - e^{-\frac{ct}{96\pi\phi_b}}\right),$$

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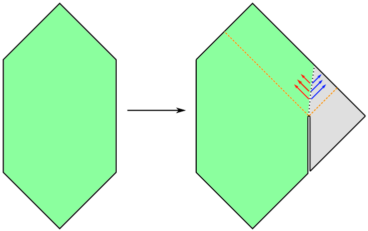

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but for $\phi_b T_0 \gg \Phi_0$ this eventually exceeds the coarse-grained entropy of the remaining black holes! Thus in this theory the entropy formula is wrong (and therefore global symmetry is allowed).

The key point however is that if we only view this theory as a low-energy effective field theory, to be UV completed into some fundamental holographic description, then we should instead use the QES formula!

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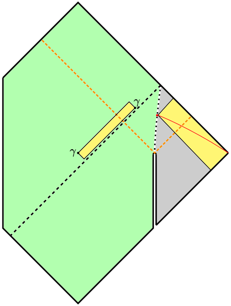

This leads to

$$S_{res} = \min\left[16\pi^2\phi_b T_1^2\left(1 - e^{-\frac{ct}{96\pi\phi_b}}\right), 8\pi\Phi_0 + 8\pi^2\phi_b T_0\left(1 + \frac{T_1}{T_0}e^{-\frac{ct}{96\pi\phi_b}}\right)\right],$$

which never contradicts the entropy formula.

No global symmetries

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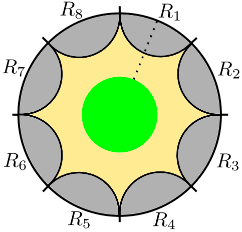

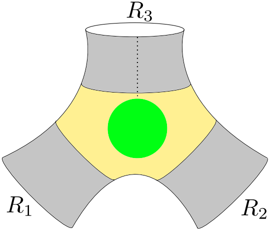

By boundary locality we have

$$U(g) = U(g, R_1)U(g, R_2)\dots U_{edge},$$

but the green charged operator isn't in the entanglement wedge of any of the R_i so it can't actually be charged.

To extend this beyond AdS/CFT, we need a few assumptions. Let S be a quantum gravity system which has black hole solutions whose semiclassical description lives in d dimensions, and let R be a "reservoir" system consisting of weakly interacting quantum fields on $\mathbb{R}^d$ (possibly including linearized gravitons). We then assume the following:

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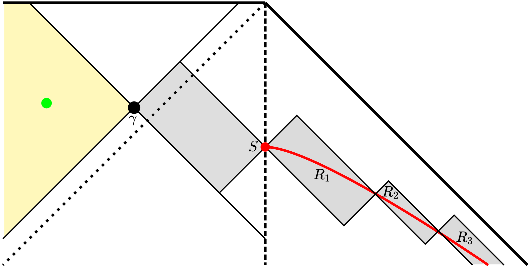
Roughly speaking, S is a black hole and R is its Hawking radiation.

We then have the same contradiction:



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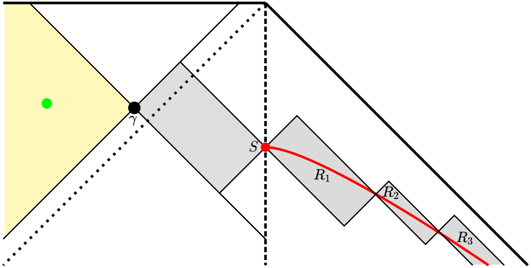

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This contradiction can be quantified by studying the relative entropy $S(\rho_R|U(g,R)^\dagger \rho_R U(g,R))$, which should vanish if there is a (unbroken) global symmetry but can't since $U(g,R)$ can't implement the symmetry on the island (also follows from wormholes). Chen/Lin 2020 (see also Hsin/Ilisiu/Yang 2020).

Some comments on Euclidean gravity

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We conjecture that this is because we are only allowed to Euclidean gravity in holographic theories:

- The Euclidean path integral in a gravitational effective field theory with a quantum-mechanical UV completion correctly computes von Neumann entropies if and only if that UV completion is holographic, in which case the entropies are those of the holographic theory.

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In other words Euclidean gravity and holography are in some sense equivalent.

To motivate this, we can first recall that in quantum field theory the path integral representation of a thermal trace

$$Z = \mathrm{Tr} e^{-\beta H}$$

always lives on a Euclidean spacetime of topology $\mathbb{S}^1 \times \Sigma$.

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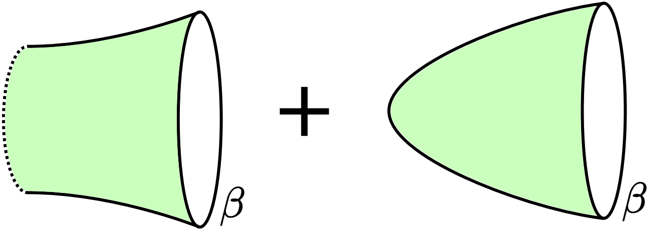
There is a simple argument due to Hawking that no such geometry can ever produce an entropy which is of order $1/G$: as long as time-translation around the thermal circle is unbroken, the Euclidean action must obey $I \propto \beta$, and thus at this order

$$S = (1 - \beta\partial_\beta) \log Z \approx -(1 - \beta\partial_\beta) I = 0.$$

The standard fix for this is to also include Euclidean geometries which are not of the form $\mathbb{S}^1 \times \Sigma$, instead requiring only that the *boundary* topology is $\mathbb{S}^1 \times \partial\Sigma$:



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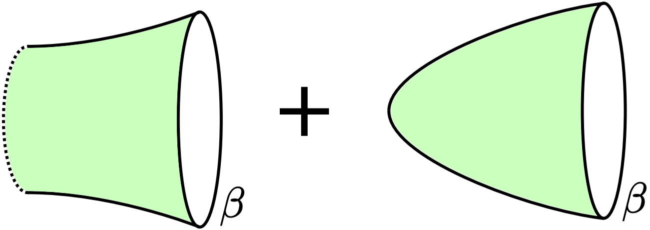

The geometry on the right would not be included by canonical quantization, but if we include it nonetheless then we get $S = A/4G$!

Gibbons/Hawking 1977

But why are we allowed to include the geometry on the right?

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- In a holographic theory the answer seems clear: since the microscopic description lives at the asymptotic boundary, so does the true thermal circle! Away from the boundary there is no particular reason to prevent the $\mathbb{S}^1$ from contracting, and indeed in AdS/CFT if we don't allow this we are unable to reproduce the high-temperature density of states of the dual CFT. Witten 1998

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- This is still somewhat mysterious however: how can mere low-energy effective field theory have access to such deep non-perturbative information about quantum gravity?

- We don't have a complete answer to this question, but in AdS/CFT we can say a bit more. Indeed from the boundary point of view high-temperature/low-temperature duality relates geometries where the thermal circle contracts to geometries where it doesn't. Strominger 1998, Shaghoulian 2015

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Thanks for listening!