Factorization of the N-jettiness in electroweak processes

Taewook Ha
Korea University

with J. Chay and T. Kwon

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Introduction

- Strongly interacting particles are observed as hadrons, that are color singlets.
- What happens if free quarks or color non-singlet particles are observed?
- In the same reasoning, what if we turn off the strong interaction? See what happens with the weak interaction.
- As a simple example, we will consider $e^+e^- \to \mu^+\mu^-$ using the effective field theory in SU(2).

Introduction

- At first, we have to consider the factorization of the process.

- Factorization of the weak processes consists of not only weak singlets but also weak non-singlets, which make factorization more difficult.

- Except for the difficulty of the factorization, some issues are encountered in the presence of the non-singlets.

  - non-cancellation of the large logs, additional rapidity evolution, ⋯

  → See next talk

Effective Field Theory (EFT)

- Concentrate on the parts of the physics that we hope to understand.

- Relevant degrees of freedom.

$$l_e = \begin{pmatrix} e \\ \nu_e \end{pmatrix}, l_\mu = \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, W_{1,2,3} \text{ in } \mathrm{SU}(2)$$

- Transparent method to derive factorization theorem.

- Simplify the problem containing scale hierarchies.

Soft-Collinear Effective Theory (SCET)

- Expanding 4-momentum.

$$p^\mu = (n\cdot p)\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\frac{n^\mu}{2} + p_\perp^\mu = p^+\frac{\bar{n}^\mu}{2} + p^-\frac{n^\mu}{2} + p_\perp^\mu.$$

- Light-cone coordinate.

$$n^2 = \bar{n}^2 = 0, \qquad n\cdot\bar{n} = 2, \qquad n^\mu = (1,0,0,1), \qquad \bar{n}^\mu = (1,0,0,-1).$$

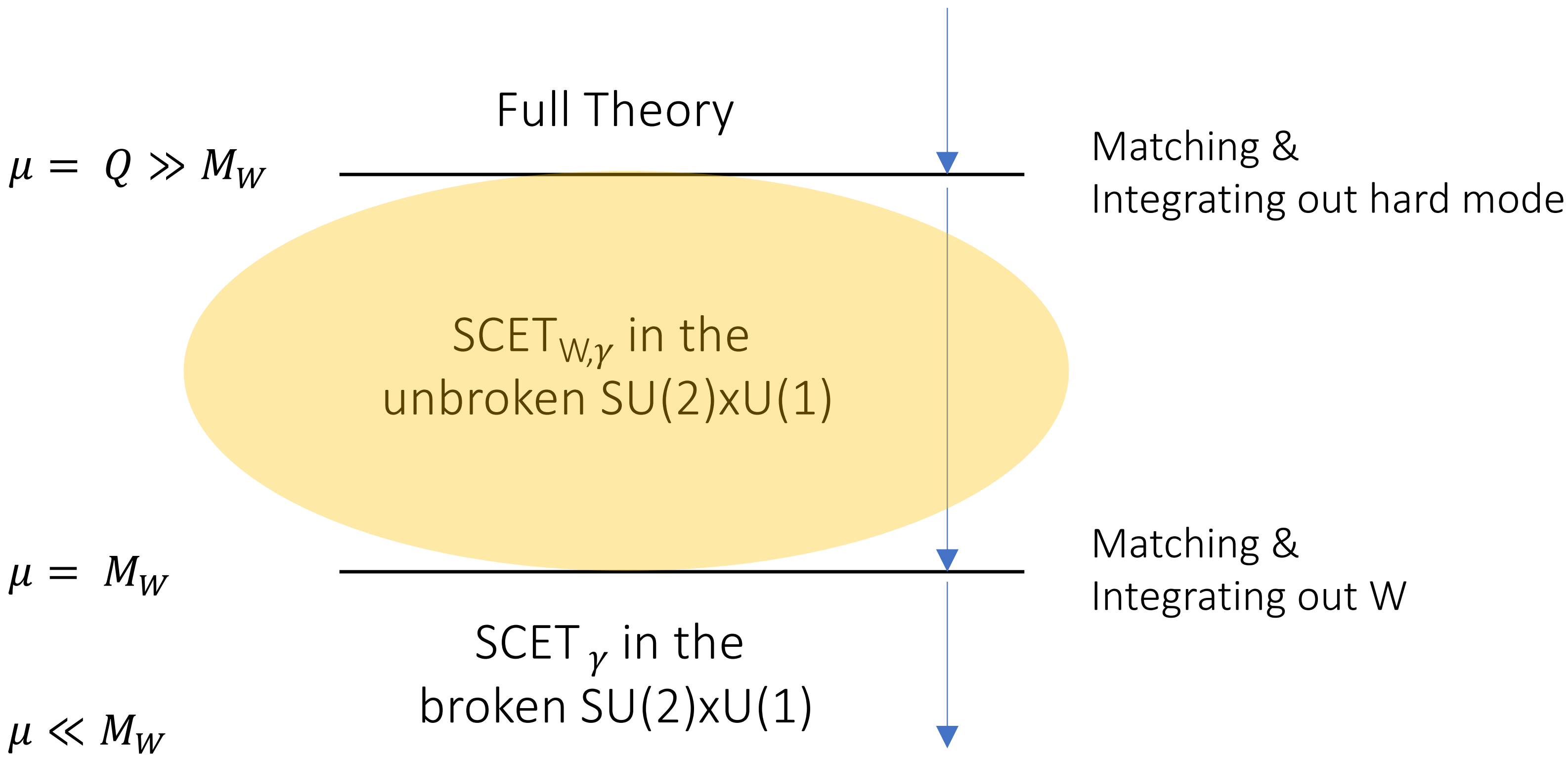
- Scaling for n-collinear & ultrasoft with power counting $\lambda \ll 1$.

$$(p_c^+, p_c^-, p_c^\perp) \sim Q(\lambda^2, 1, \lambda), \qquad (p_{us}^+, p_{us}^-, p_{us}^\perp) \sim Q(\lambda^2, \lambda^2, \lambda^2).$$

- Scale hierarchies.

$$p_h^2 \sim Q^2, \quad p_c^2 \sim Q^2\lambda^2, \quad p_{us}^2 \sim Q^2\lambda^4 \quad \Rightarrow \quad \mathrm{SCET_I}$$

Descending sequence of EFTs



N-jettiness

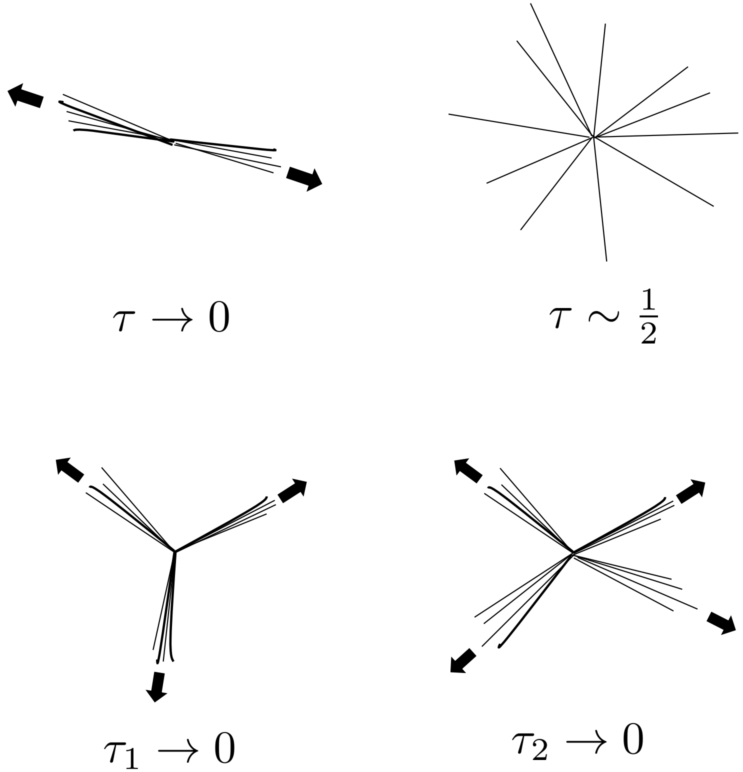
Thrust $\boldsymbol{\tau}$ is defined as

$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|}.$$

N-jettiness $\boldsymbol{\tau_N}$ is defined as

$$\tau_N = \sum_i \tau_N^i = \sum_{i,k} \min\{\frac{2q_i \cdot p_k}{w_i}\},$$

where $q_i^\mu = \frac{1}{2} w_i n_i^\mu$, p_k for the particle in the final state.



N-jettiness is treated as the generalization of the thrust.

By putting $\boldsymbol{\tau_N^i}$ to the ultrasoft scale $Q\lambda^2$, we can study about the N-jettiness in SCET.

Factorization theorem

Cross section is schematically written as

$$
\begin{aligned}
d\sigma \sim &< e_1 e_2 | C_I^* O_I^\dagger | \mu_3 \mu_4 X >< \mu_3 \mu_4 X | C_J O_J | e_1 e_2 > \\
&= C_I^* C_J < e_1 e_2 | \bar{l}_{e_1} T_I \gamma^\nu l_{e_2} \cdot \bar{l}_{\mu_4} T_I \gamma_\nu l_{\mu_3} | \mu_3 \mu_4 X > \\
&\otimes < \mu_3 \mu_4 X | \bar{l}_{e_2} T_J \gamma^\mu l_{e_1} \cdot \bar{l}_{\mu_3} T_J \gamma_\mu l_{\mu_4} | e_1 e_2 >,
\end{aligned}
$$

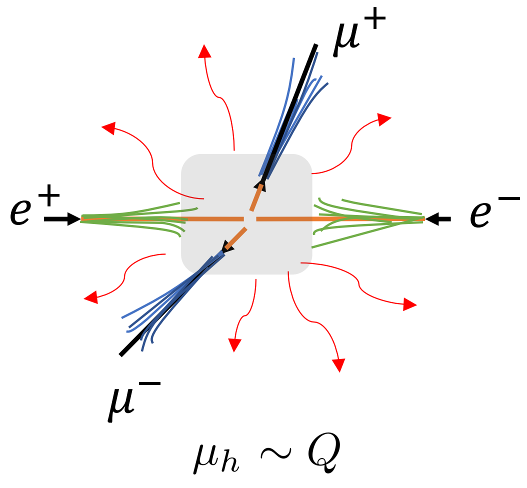
where C_I are the Wilson coefficients. O_I are the four-lepton operators defined as

$$
O_I = \bar{l}_{e_2} T_I \gamma^\mu l_{e_1} \cdot \bar{l}_{\mu_3} T_I \gamma_\mu l_{\mu_4} \quad (I = 1, 2),
$$

$$
\Rightarrow O_1 = \bar{l}_{e_2} t^a \gamma^\mu l_{e_1} \cdot \bar{l}_{\mu_3} t^a \gamma_\mu l_{\mu_4}, O_2 = \bar{l}_{e_2} \gamma^\mu l_{e_1} \cdot \bar{l}_{\mu_3} \gamma_\mu l_{\mu_4}.
$$

In order to decouple soft interactions, perform the field redefinition. $l_{L_i} \to S_i l_{L_i}.$

Factorization theorem - Hard function



Cross section is schematically written as

$$d\sigma \sim < e_1 e_2 | C_I^* O_I^\dagger | \mu_3 \mu_4 X >< \mu_3 \mu_4 X | C_J O_J | e_1 e_2 >$$

$$= C_I^* C_J < e_1 e_2 | \bar{l}_{e_1} S_1^\dagger T_I \gamma^\nu S_2 l_{e_2} \cdot \bar{l}_{\mu_4} S_4^\dagger T_I \gamma_\nu S_3 l_{\mu_3} | \mu_3 \mu_4 X >$$

$$\otimes < \mu_3 \mu_4 X | \bar{l}_{e_2} S_2^\dagger T_J \gamma^\mu S_1 l_{e_1} \cdot \bar{l}_{\mu_3} S_3^\dagger T_J \gamma_\mu S_4 l_{\mu_4} | e_1 e_2 > .$$

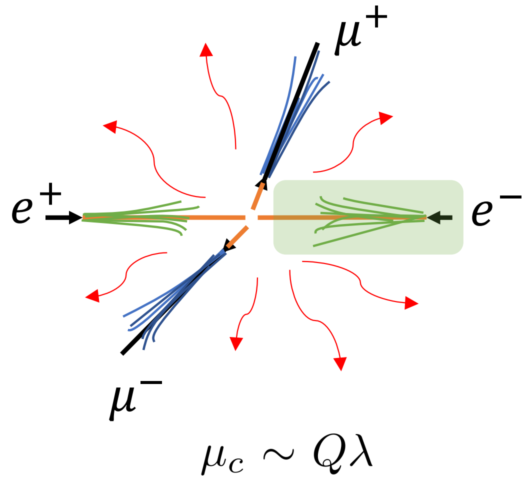
Every field in each mode doesn't interact with the fields in the different modes!!

Factorization theorem is written as

$$\frac{d\sigma}{d\tau_2} \sim H_{JI}(Q) S_{IJ}^{abcd}(\tau_s) \otimes B_1^a(\tau_{e_1}) \otimes B_2^b(\tau_{e_2}) \otimes J_3^c(\tau_{\mu_3}) \otimes J_4^d(\tau_{\mu_4}).$$

Hard function is defined as $H_{JI} = 4\hat{u}^2 C_I^* C_J .$

Factorization theorem – Beam functions



Cross section is schematically written as

$$C_I^* C_J < e_1 e_2 | \bar{l}_{e_1} S_1^\dagger T_I \gamma^\nu S_2 l_{e_2} \cdot \bar{l}_{\mu_4} S_4^\dagger T_I \gamma_\nu S_3 l_{\mu_3} | \mu_3 \mu_4 X >$$
$$\otimes < \mu_3 \mu_4 X | \bar{l}_{e_2} S_2^\dagger T_J \gamma^\mu S_1 l_{e_1} \cdot \bar{l}_{\mu_3} S_3^\dagger T_J \gamma_\mu S_4 l_{\mu_4} | e_1 e_2 >$$

Factorization theorem is written as

$$\frac{d\sigma}{d\tau_2} \sim H_{JI}(Q) S_{IJ}^{abcd}(\tau_s) \otimes B_1^a(\tau_{e_1}) \otimes B_2^b(\tau_{e_2}) \otimes J_3^c(\tau_{\mu_3}) \otimes J_4^d(\tau_{\mu_4}).$$

To combine each l_i into a bilinear, use the following relation.

$$(\bar{l})^i_\alpha (l)^j_\beta = \frac{1}{2N} \delta^{ij} (P_L \not{n})_{\beta\alpha} \bar{l} \frac{\bar{\not{n}}}{2} l + (t^c)^{ji} (P_L \not{n})_{\beta\alpha} \bar{l} \frac{\bar{\not{n}}}{2} t^a l.$$

Beam function is defined as

$$B_1^a(\tau_{e_1}, z_1, \mu) = \frac{1}{w_1} < e_1 | \bar{l}_{e_1} \frac{\bar{\not{n}}_1}{2} T^a \underline{\delta(\tau_{e_1} + n_1 \cdot \mathcal{P})} [\delta(w_1 - \bar{n}_1 \cdot \mathcal{P}) l_{e_1}] | e_1 > .$$

Beam functions vs. Parton distribution functions (PDFs)

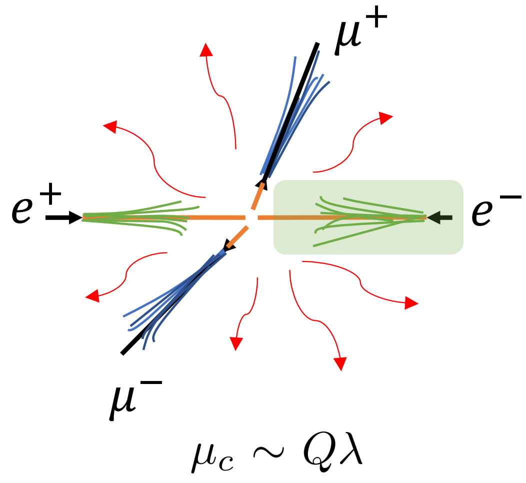
The relation between the BF and the PDF is

$$B_i^a(\tau, z, \mu) = \Sigma_j \mathcal{I}_{ij}(\tau, \tfrac{z}{x}, \mu) \otimes f_j^a(x, \mu).$$

In SCET, the lepton beam function and the electron PDF are defined as

$$B_{l_e}^a(\tau_e, z, \mu) =< e|\bar{l}_e \tfrac{\bar{\not{n}}}{2} T^a \boxed{\delta(w\tau_e + wn \cdot \mathcal{P})} [\delta(w - \bar{n} \cdot \mathcal{P}) l_e]|e >,$$

$$f_e^a(x, \mu) =< e|\bar{l}_e \tfrac{\bar{\not{n}}}{2} T^a [\delta(w - \bar{n} \cdot \mathcal{P}) l_e]|e > .$$



Factorization with a PDF

$$\sigma_{ep \to X} \sim f_a(x, \mu) \otimes \hat{\sigma}_{ea \to X}$$

Factorization with a beam function by inserting $1 = \int d\tau \; \delta(\tau + n \cdot \mathcal{P})$

$$\frac{d\sigma_{ep \to X}}{d\tau} \sim B_a(\tau, z, \mu) \otimes \hat{\sigma}_{ea \to X}$$

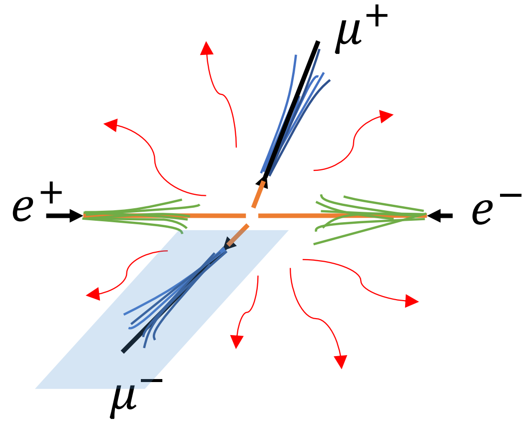
more differential

Factorization theorem – Jet functions

Cross section is schematically written as

$$d\sigma \sim C_I^* C_J < e_1 e_2 | \bar{l}_{e_1} S_1^\dagger T_I \gamma^\nu S_2 l_{e_2} \cdot \bar{l}_{\mu_4} S_4^\dagger T_I \gamma_\nu S_3 l_{\mu_3} | \mu_3 \mu_4 X >$$

$$\otimes < \mu_3 \mu_4 X | \bar{l}_{e_2} S_2^\dagger T_J \gamma^\mu S_1 l_{e_1} \cdot \bar{l}_{\mu_3} S_3^\dagger T_J \gamma_\mu S_4 l_{\mu_4} | e_1 e_2 > .$$



Factorization theorem is written as

$$\frac{d\sigma}{d\tau_2} \sim H_{JI}(Q) S_{IJ}^{abcd}(\tau_s) \otimes B_1^a(\tau_{e_1}) \otimes B_2^b(\tau_{e_2}) \otimes J_3^c(\tau_{\mu_3}) \otimes J_4^d(\tau_{\mu_4}).$$

$$\mu_c \sim Q\lambda$$

Jet function is defined as

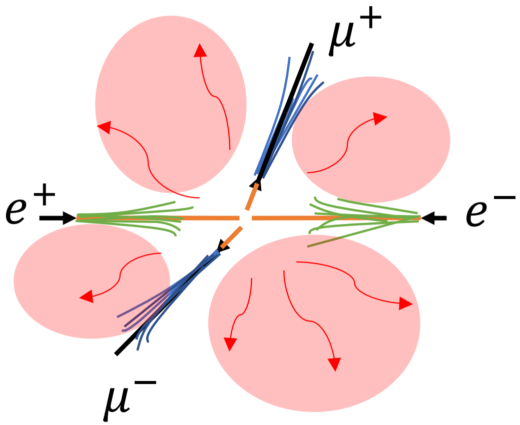
$$J_3^c(\tau_{\mu_3}, \mu) = \frac{2(2\pi)}{w_3} \mathrm{Tr} < 0 | l_{\mu_3} \frac{\bar{\not{n}}_3}{2} T^c \underline{\delta(\tau_{\mu_3} + n_3 \cdot \mathcal{P})} \delta(w_3 + \bar{n}_3 \cdot \mathcal{P}) \delta^{(2)}(p_3^\perp + \mathcal{P}_\perp) l_{\mu_3} | 0 > .$$

Factorization theorem - Soft funtion

Cross section is schematically written as

$$d\sigma \sim C_I^* C_J < e_1 e_2 | \bar{l}_{e_1} S_1^\dagger T_I \gamma^\nu S_2 l_{e_2} \cdot \bar{l}_{\mu_4} S_4^\dagger T_I \gamma_\nu S_3 l_{\mu_3} | \mu_3 \mu_4 X >$$

$$\otimes < \mu_3 \mu_4 X | \bar{l}_{e_2} S_2^\dagger T_J \gamma^\mu S_1 l_{e_1} \cdot \bar{l}_{\mu_3} S_3^\dagger T_J \gamma_\mu S_4 l_{\mu_4} | e_1 e_2 > .$$



$\mu_{us} \sim Q\lambda^2$

Factorization theorem is written as

$$\frac{d\sigma}{d\tau_2} \sim H_{JI}(Q) S_{IJ}^{abcd}(\tau_s) \otimes B_1^a(\tau_{e_1}) \otimes B_2^b(\tau_{e_2}) \otimes J_3^c(\tau_{\mu_3}) \otimes J_4^d(\tau_{\mu_4}).$$

Soft function is defined as

$$S_{IJ}^{abcd}(\tau_s, \mu) = \int d\tau_1 d\tau_2 \delta(\tau_s - \tau_1 - \tau_2)$$
$$\times \langle 0 | \mathrm{Tr} \left\{ \left(S_1^\dagger T_I S_2 T^a \right) \left[\delta\Big(\tau_1 - \sum_{X_s} \min(\{n_i \cdot p_{X_s}\} \Big) \left(S_2^\dagger T_J S_1 T^b \right) \right] \right\}$$
$$\times \mathrm{Tr} \left\{ \left(S_4^\dagger T_I S_3 T^c \right) \left[\delta\Big(\tau_2 - \sum_{X_s} \min(\{n_i \cdot p_{X_s}\} \Big) \left(S_3^\dagger T_J S_4 T^d \right) \right] \right\} | 0 \rangle.$$

Conclusions

- We have reviewed the EFT.
- We have seen the factorization theorem of the 2-jettiness in SU(2), which is valid to all orders of α_2.
- What can be done with the factorization formula:

  - Calculate each factorized function.

  - Sum the large logarithms by solving the renormalization group equation.

  - Extend the theory to SU(2)xU(1).

Backup

The factorization formula for the 2-jettiness is written as

$$\frac{d\sigma}{d\tau_N} = \frac{8}{Q^2}\int d\Phi(\{p_J\})(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\int\frac{dz_1}{z_1}\frac{dz_2}{z_2}\int dt_1 dt_2\int d\tau_S$$
$$\times\sum_{cdef}k_c k_d k_e k_f B_1^d(t_1,z_1,\mu)B_2^c(t_2,z_2,\mu)J_3^e(p_3^2,\mu)J_4^f(p_4^2,\mu)\sum_{IJ}H_{JI}S_{IJ}^{cdef}(\tau_S,\mu)$$
$$\times\,\delta\Big(\tau_N-\frac{t_1}{\omega_1}-\frac{t_2}{\omega_2}-\frac{p_3^2}{\omega_3}-\frac{p_4^2}{\omega_4}-\tau_S\Big),$$

where $d\Phi(\{p_J\})$ represents the phase space for the final-state particles, which is given by

$$d\Phi(\{p_J\}) = \prod_J\frac{d^4p_J}{(2\pi)^3}.$$