E-string on Spheres

Based on arXiv:2103.09149, 2010.10446, 2002.12897
in collaboration with Sara Pasquetti, Shlomo S. Razamat, Evyatar Sabag, Matteo Sacchi

Chiung Hwang (DAMTP, Univ. of Cambridge)

PASCOS 2021, 18 June 2021

Mysterious E_6

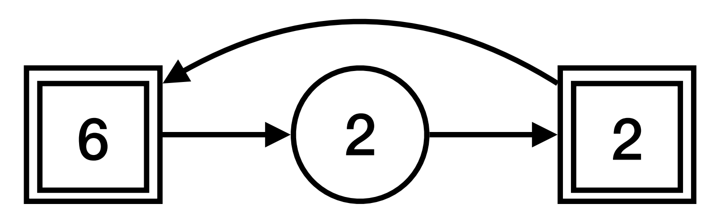
SU(2) theory with 8 chirals

- Let us consider a simple $\mathcal{N}=1$ supersymmetric gauge theory in 4d
- The $SU(2)$ theory with 8 fundamental chirals and 12 gauge singlets with the superpotential

$$\mathscr{W} = MQ\tilde{Q}$$

- It preserves the global symmetry

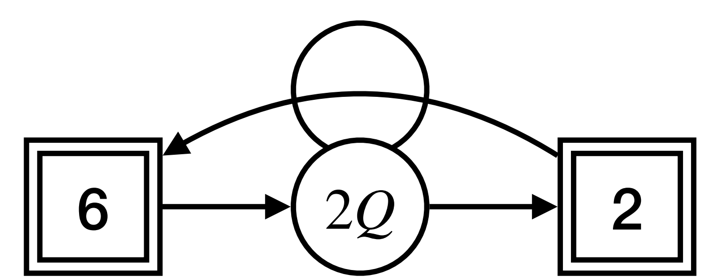
$$SU(6) \times SU(2) \times U(1)$$



- Surprisingly, this is enhanced in the IR to [Razamat-Sela-Zafrir]

$$E_6 \times U(1)$$

- A higher rank generalization with the $USp(2Q)$ gauge group [CH-Pasquetti-Sacchi]



Why?

What we will see

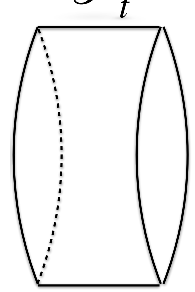
What we have seen

- Intro: Mysterious E_6
- E-string Compactification
- $FE^{\sigma}_{\rho}[USp(2Q)]$ Family
- Compactification on Spheres
- Back to E_6
- Remarks

E-string Compactification

Compactification on tubes

- M5-branes probing M9-wall -> a 6d CFT, which we call the E-string theory
- Put the E-string theory on a tube, or a sphere with two punctures and flux $\mathcal{F}_t$

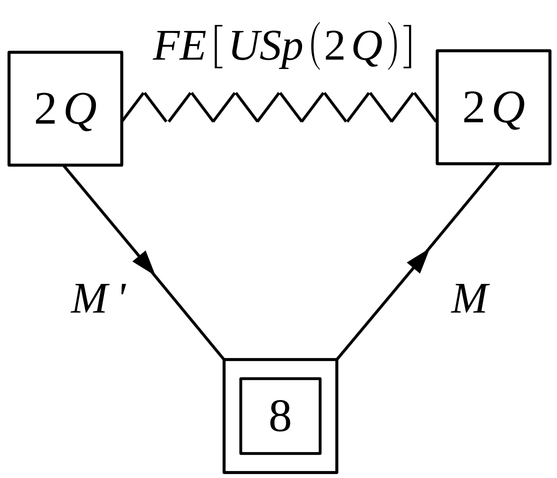


The basic tube

$$\mathcal{F}_t = \left(n_t\,;n_1,\ldots,n_8\right) = \left(0\,;\frac{1}{2},\ldots,\frac{1}{2}\right)$$

$$U(1)_t \times U(1)^8 \subset U(1)_t \times SO(16) \subset SU(2)_L \times E_8$$

- The 4d $\mathcal{N}=1$ $FE[USp(2Q)]$ theory coupled to 8+8 fundamental chirals
  [Kim-Razamat-Vafa-Zafrir, Pasquetti-Razamat-Sacchi-Zafrir]



$$USp(2Q)_x \times USp(2Q)_y \times U(1)_t \times U(1)_c \times SU(8)_u \times U(1)_f$$

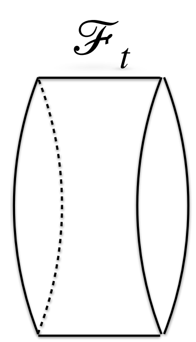
$$\longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times SU(2)_t \times U(1)_c \times E_7 \times U(1)_f$$

E-string Compactification

Compactification on tubes

- M5-branes probing M9-wall -> a 6d CFT, which we call the E-string theory

- Put the E-string theory on a tube, or a sphere with two punctures and flux $\mathscr{F}_t$

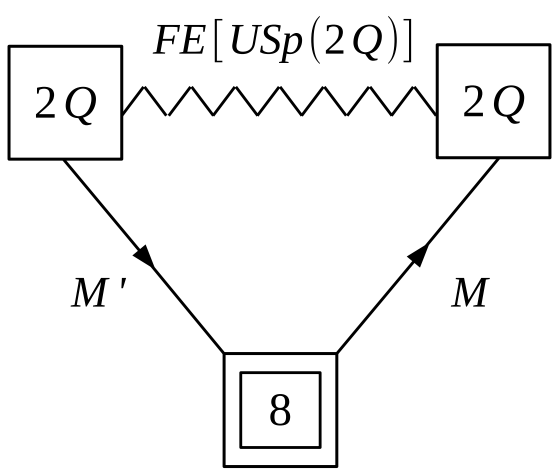


The basic tube

$$\mathscr{F}_t = (n_t; n_1, \ldots, n_8) = \left(0; \frac{1}{2}, \ldots, \frac{1}{2}\right)$$

$$U(1)_t \times U(1)^8 \subset U(1)_t \times SO(16) \subset SU(2)_L \times E_8$$

- The 4d $\mathcal{N} = 1$ $FE[USp(2Q)]$ theory coupled to 8+8 fundamental chirals
  [Kim-Razamat-Vafa-Zafrir, Pasquetti-Razamat-Sacchi-Zafrir]



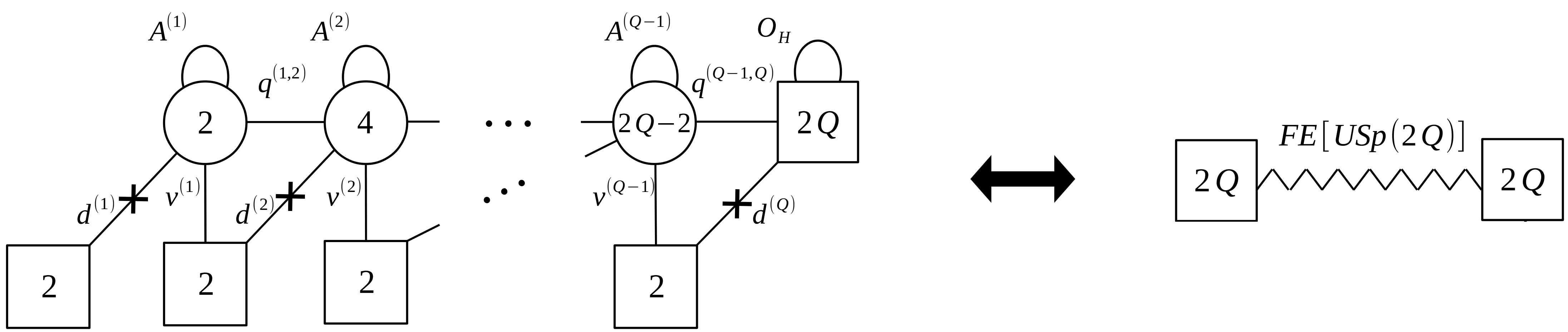
$$USp(2Q)_x \times USp(2Q)_y \times U(1)_t \times U(1)_c \times SU(8)_u \times U(1)_f$$

$$\longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times SU(2)_t \times U(1)_c \times E_7 \times U(1)_f$$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

What is $FE[USp(2Q)]$

- The Lagrangian description of $FE[USp(2Q)]$ [Pasquetti-Razamat-Sacchi-Zafrir]



- The manifest global symmetry is

$$SU(2)^Q \times USp(2Q)_y \times U(1)_c \times U(1)_t \quad \longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times U(1)_c \times U(1)_t$$

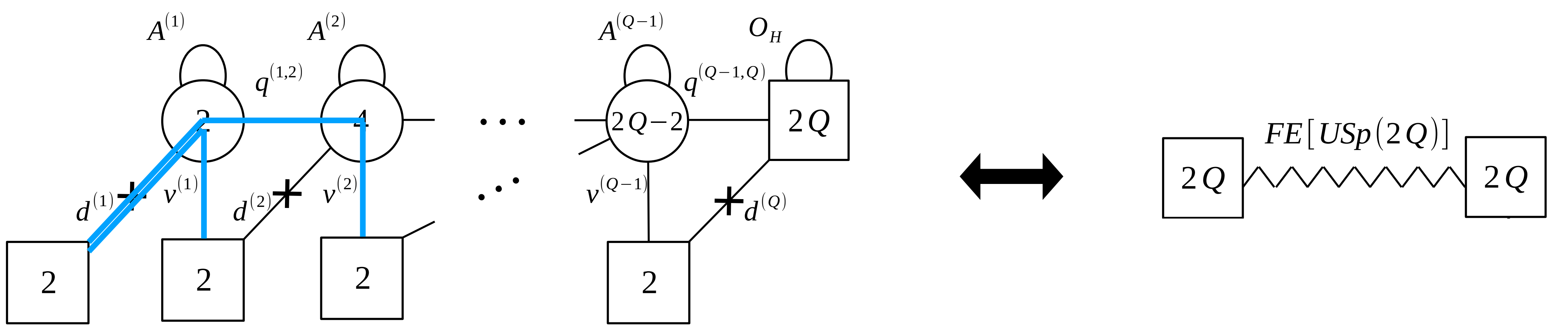
- Gauge invariant operators $\qquad C \qquad \Pi \qquad O_H$

- Self-duality $\qquad USp(2Q)_x \quad \longleftrightarrow \quad USp(2Q)_y$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

What is $FE[USp(2Q)]$

- The Lagrangian description of $FE[USp(2Q)]$ [Pasquetti-Razamat-Sacchi-Zafrir]



- The manifest global symmetry is

$$SU(2)^Q \times USp(2Q)_y \times U(1)_c \times U(1)_t \quad \longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times U(1)_c \times U(1)_t$$

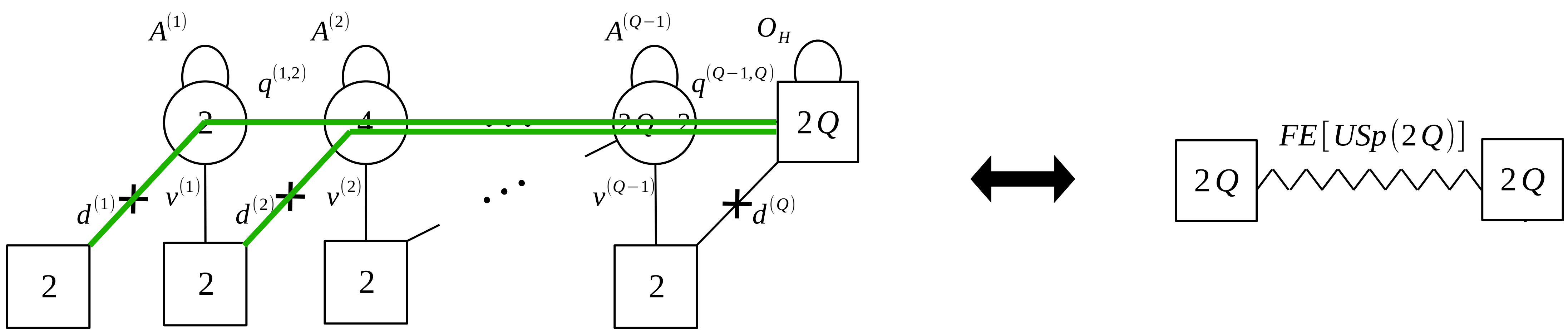
- Gauge invariant operators $\quad C \quad \Pi \quad O_H$

- Self-duality $\quad USp(2Q)_x \longleftrightarrow USp(2Q)_y$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

What is $FE[USp(2Q)]$

- The Lagrangian description of $FE[USp(2Q)]$ [Pasquetti-Razamat-Sacchi-Zafrir]



- The manifest global symmetry is

$$SU(2)^Q \times USp(2Q)_y \times U(1)_c \times U(1)_t \quad \longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times U(1)_c \times U(1)_t$$

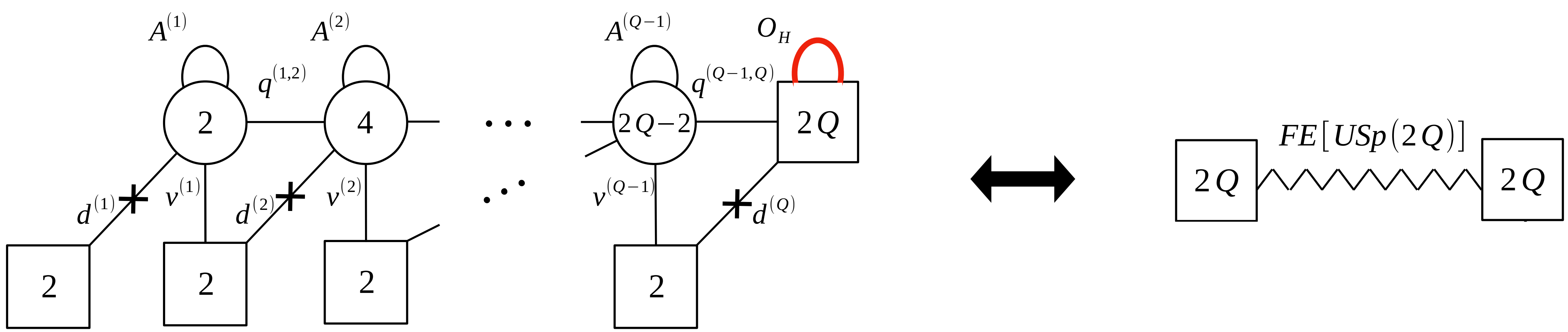
- Gauge invariant operators $\quad C \quad \Pi \quad O_H$

- Self-duality $\quad USp(2Q)_x \longleftrightarrow USp(2Q)_y$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

What is $FE[USp(2Q)]$

- The Lagrangian description of $FE[USp(2Q)]$ [Pasquetti-Razamat-Sacchi-Zafrir]



- The manifest global symmetry is

$$SU(2)^Q \times USp(2Q)_y \times U(1)_c \times U(1)_t \quad \longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times U(1)_c \times U(1)_t$$

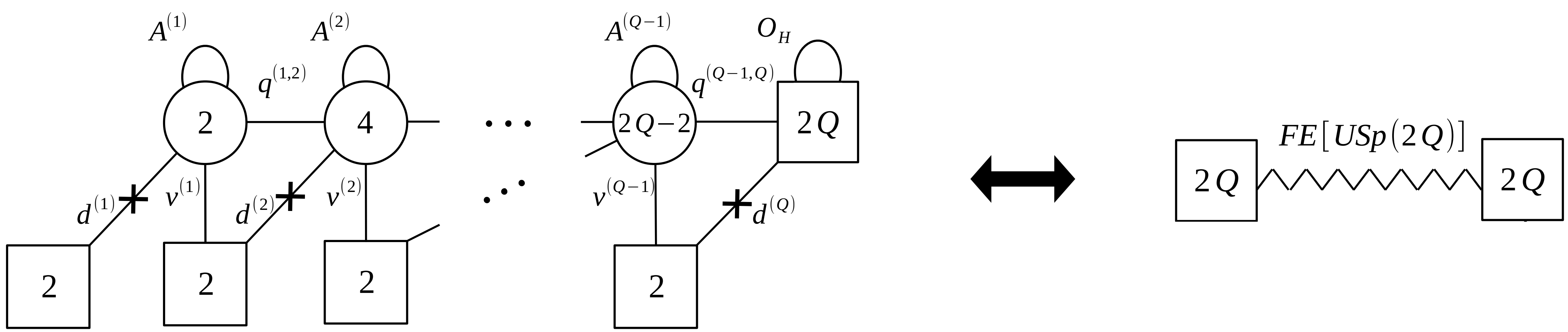
- Gauge invariant operators $\quad C \quad \Pi \quad O_H$

- Self-duality $\quad USp(2Q)_x \longleftrightarrow USp(2Q)_y$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

What is $FE[USp(2Q)]$

- The Lagrangian description of $FE[USp(2Q)]$ [Pasquetti-Razamat-Sacchi-Zafrir]



- The manifest global symmetry is

$$SU(2)^Q \times USp(2Q)_y \times U(1)_c \times U(1)_t \quad \longrightarrow \quad USp(2Q)_x \times USp(2Q)_y \times U(1)_c \times U(1)_t$$

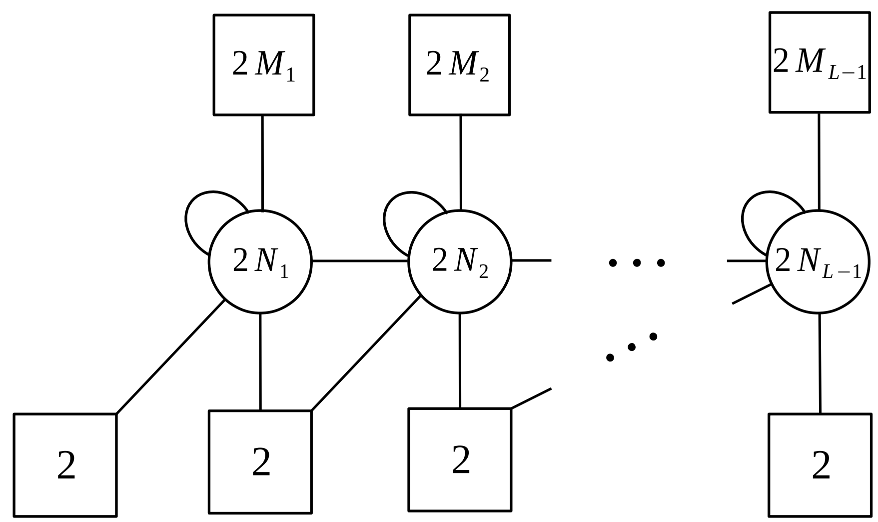
- Gauge invariant operators $\quad C \quad \Pi \quad O_H$

- Self-duality $\quad USp(2Q)_x \longleftrightarrow USp(2Q)_y$

$FE^{\sigma}_{\rho}[USp(2Q)]$ Family

$FE^{\sigma}_{\rho}[USp(2Q)]$ and the 4d mirror-like duality

- More general theories can be obtained by giving vev to O_H and mass to C, (or vice-versa,) which are labeled by two partitions $\rho = [\rho_1, \ldots, \rho_N] = \left[N^{l_N}, \ldots, 1^{l_1}\right]$ & $\sigma = [\sigma_1, \ldots, \sigma_N] = \left[N^{k_N}, \ldots, 1^{k_1}\right]$ [CH-Pasquetti-Sacchi]



$$M_{L-i} = k_i\,,$$
$$N_{L-i} = \sum_{j=i+1}^{L} \rho_j - \sum_{j=i+1}^{N} (j-i)k_j$$

- The global symmetry is broken as

$$USp(2N)_x \times USp(2N)_y \quad \longrightarrow \quad \prod_{m=1}^{N} USp(2k_m)_{x^{(m)}} \times \prod_{n=1}^{N} USp(2l_n)_{y^{(n)}}$$

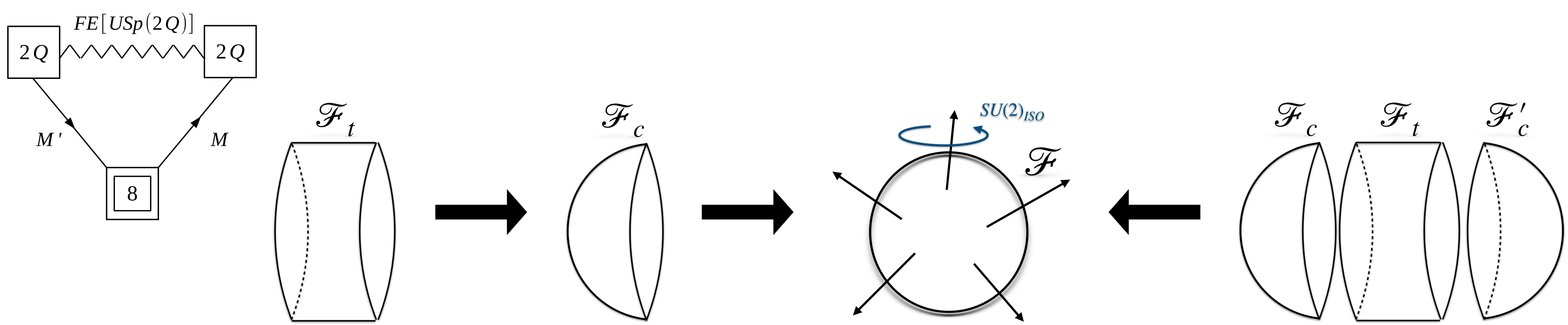
- The self-duality of $FE[USp(2Q)]$ exchanging $USp(2Q)_x \leftrightarrow USp(2Q)_y$ now corresponds to $\rho \leftrightarrow \sigma$

- A new 4d duality reminiscent of the 3d mirror symmetry

Compactification on Spheres

Closing punctures & gluing

- One can obtain a sphere either by closing punctures or gluing caps and tubes [CH-Razamat-Sabag-Sacchi]



- To obtain a cap theory, we need to break a $USp(2Q)$ puncture symmetry

- First turn on vev of O_H taking $\rho = [1^N]$ & $\sigma = [N]$, which breaks the puncture symmetries into

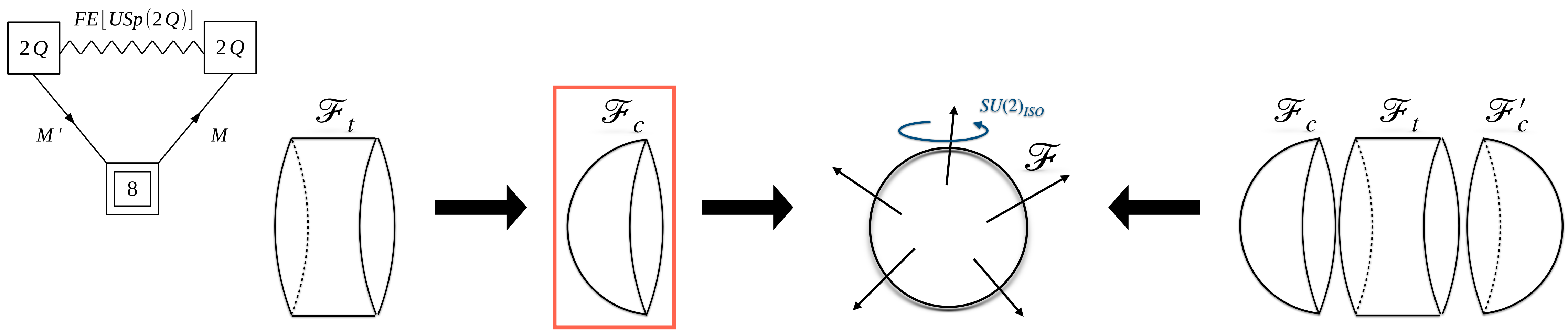
$$SU(2)_x \times USp(2Q)_y$$

- Further break $SU(2)_x$ by giving vev to M so that the $USp(2Q)_x$ puncture symmetry is completely broken

Compactification on Spheres

Closing punctures & gluing

- One can obtain a sphere either by closing punctures or gluing caps and tubes [CH-Razamat-Sabag-Sacchi]



- To obtain a cap theory, we need to break a $USp(2Q)$ puncture symmetry

- First turn on vev of O_H taking $\rho = [1^N]$ & $\sigma = [N]$, which breaks the puncture symmetries into

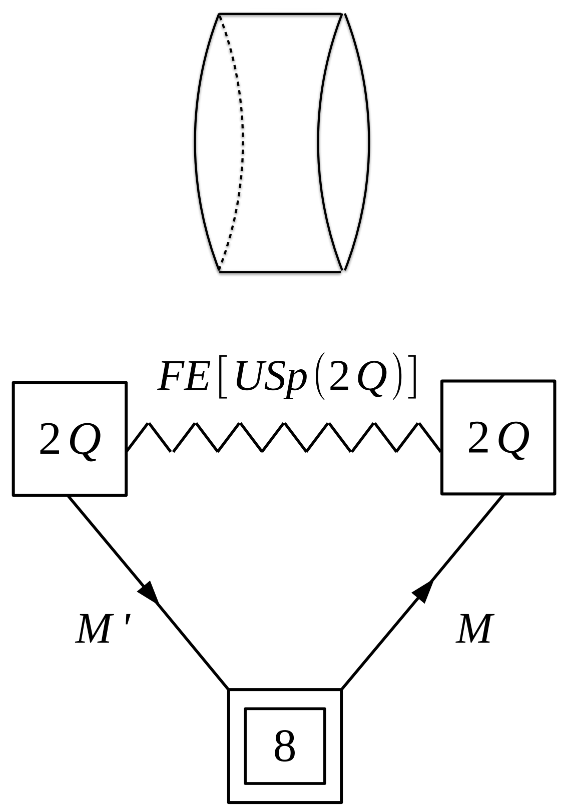
$$SU(2)_x \times USp(2Q)_y$$

- Further break $SU(2)_x$ by giving vev to M so that the $USp(2Q)_x$ puncture symmetry is completely broken

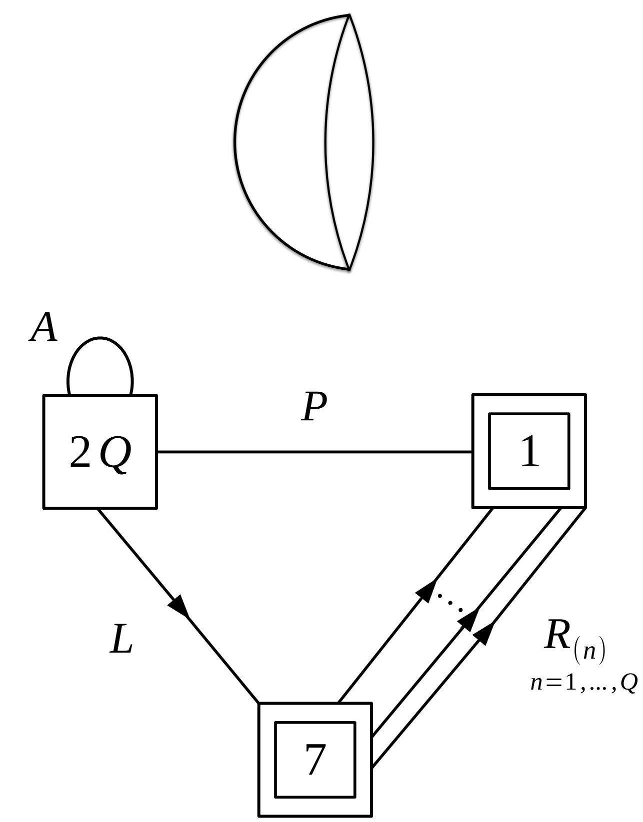
Compactification on Spheres

The basic cap theory

The basic tube



The basic cap



$$\mathscr{W}_{tube} = \mathscr{W}_{FE[USp(2Q)]} + \sum_{a=1}^{8} \mathrm{Tr}_x \mathrm{Tr}_y M^a \Pi M'_a$$

$$\mathscr{F}_t = \left(0\,; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$$

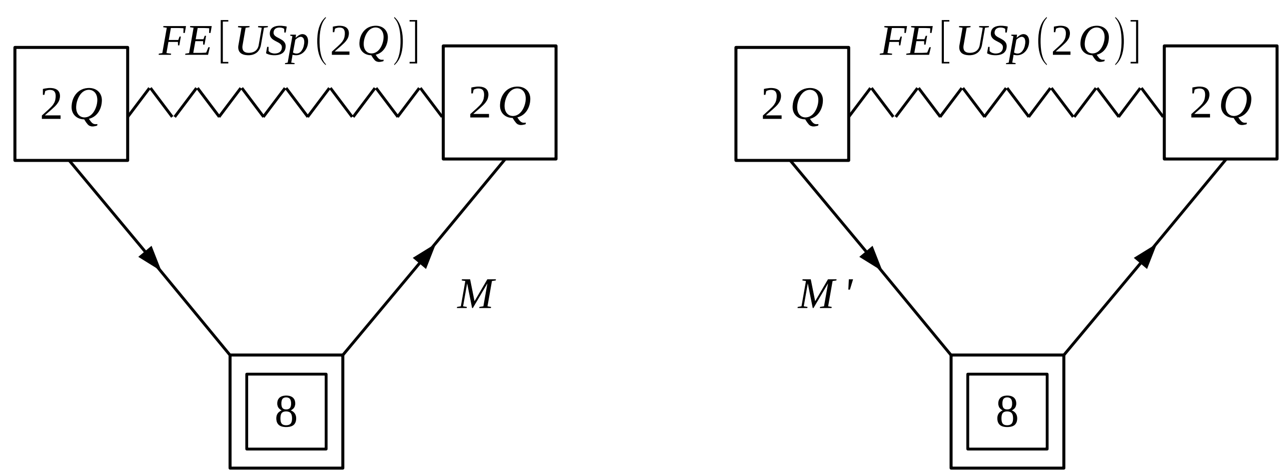
$$\mathscr{W}_{cap} = \sum_{n=1}^{Q} \sum_{a=1}^{7} \mathrm{Tr}_y \left(R^{(n)a} A^{Q-n} P L_a\right) + \sum_{n=1}^{Q} \mathrm{Tr}_y \left(b_n A^{Q-n} P K\right) + K L_8$$

$$\mathscr{F}_c = \left(-\frac{1}{2}\,; 1,1,1,1,1,1,1,-1\right)$$

Compactification on Spheres

Φ-gluing & S-gluing

- Need to specify how to glue punctures, i.e., how to identify/couple two copies of $USp(2Q)$ and their charged fields



- Φ-gluing: coupling M and M' to a new field Φ (therefore, they are identified)

$$\Delta W_a = \Phi_a \cdot (M_a - M'_a)$$

- S-gluing: coupling M and M' to each other (therefore, they become massive)

$$\Delta W_a = M_a \cdot M'_a$$

- The resulting theory corresponds to flux

$$n'_a = \begin{cases} n_a^{(1)} + n_a^{(2)} , & a \in \Phi , \\ n_a^{(1)} - n_a^{(2)} , & a \in S , \end{cases}$$

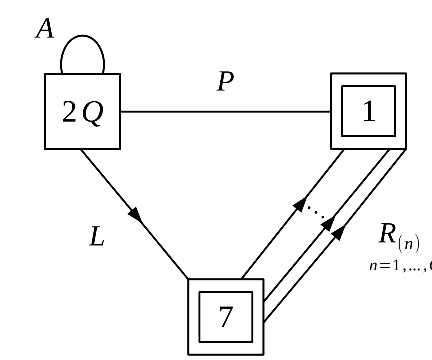
Back to E_6

Gluing two basic caps

- Let us glue two basic caps by S-gluing $L_{1,2}$ and Φ-gluing the rest including the antisym A

$$n'_a = \begin{cases} n_a^{(1)} + n_a^{(2)}, & a \in \Phi, \\ n_a^{(1)} - n_a^{(2)}, & a \in S, \end{cases}$$

$$\mathcal{F}_c = \left(-\frac{1}{2}; 1,1,1,1,1,1,1,-1\right) \quad \Longrightarrow \quad \mathcal{F} = (-1; 0,0,2,2,2,2,2,-2)$$



- The resulting theory is a $USp(2Q)$ theory with 8 fund & 1 antisym chirals, and $16Q$ extra gauge singlets preserving

$$SU(6)_w \times SU(2)_d \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$

- Matters in the representation (of the nonabelian factors)

$$(\mathbf{2Q}, \overline{\mathbf{6}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{2Q}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{asym}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \times Q \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) \times Q$$

- Symmetry enhancement in the IR

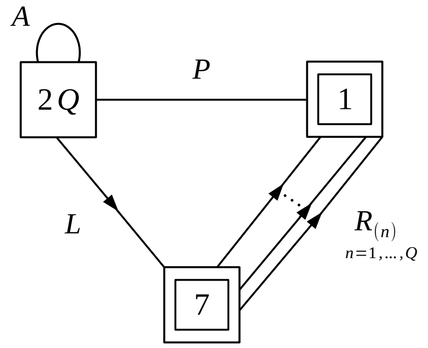
$$E_6 \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$

Back to E_6
Gluing two basic caps

- Let us glue two basic caps by S-gluing $L_{1,2}$ and Φ-gluing the rest including the antisym A

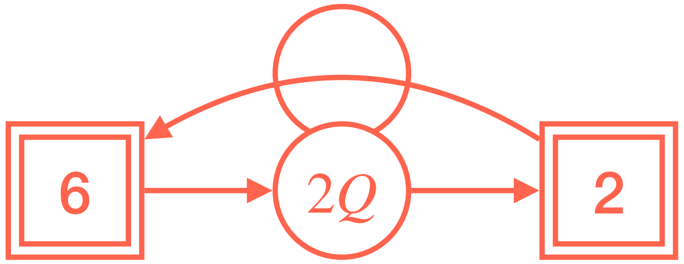
$$n'_a = \begin{cases} n_a^{(1)} + n_a^{(2)}, & a \in \Phi, \\ n_a^{(1)} - n_a^{(2)}, & a \in S, \end{cases}$$

$$\mathscr{F}_c = \left(-\frac{1}{2}; 1,1,1,1,1,1,1,-1\right) \quad \Longrightarrow \quad \mathscr{F} = (-1; 0,0,2,2,2,2,2,-2)$$



- The resulting theory is a $USp(2Q)$ theory with 8 fund & 1 antisym chirals, and $16Q$ extra gauge singlets preserving

$$SU(6)_w \times SU(2)_d \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$



- Matters in the representation (of the nonabelian factors)

$$(\mathbf{2Q}, \bar{\mathbf{6}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{2Q}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{asym}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \times Q \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) \times Q$$

- Symmetry enhancement in the IR

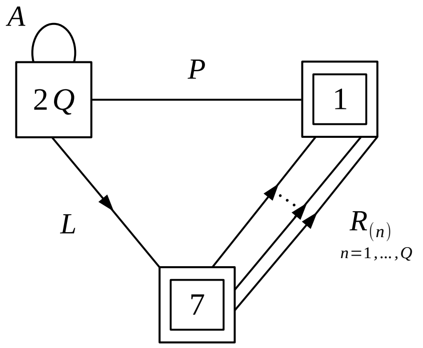
$$E_6 \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$

Back to E_6
Gluing two basic caps

- Let us glue two basic caps by S-gluing $L_{1,2}$ and Φ-gluing the rest including the antisym A

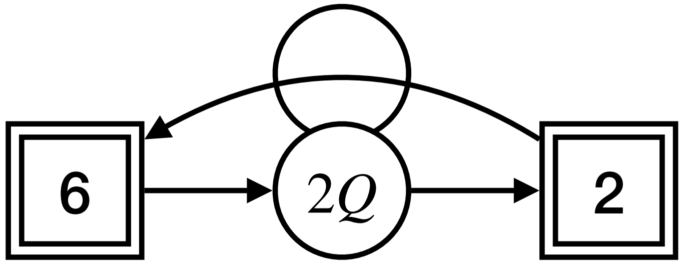
$$n'_a = \begin{cases} n_a^{(1)} + n_a^{(2)}, & a \in \Phi, \\ n_a^{(1)} - n_a^{(2)}, & a \in S, \end{cases}$$

$$\mathcal{F}_c = \left(-\frac{1}{2}\,;1,1,1,1,1,1,1,-1\right) \quad \Rightarrow \quad \mathcal{F} = (-1\,;0,0,2,2,2,2,2,-2)$$



- The resulting theory is a $USp(2Q)$ theory with 8 fund & 1 antisym chirals, and $16Q$ extra gauge singlets preserving

$$SU(6)_w \times SU(2)_d \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$



- Matters in the representation (of the nonabelian factors)

$$(\mathbf{2Q}, \overline{\mathbf{6}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{2Q}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{asym}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \times Q \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) \times Q$$

- Symmetry enhancement in the IR

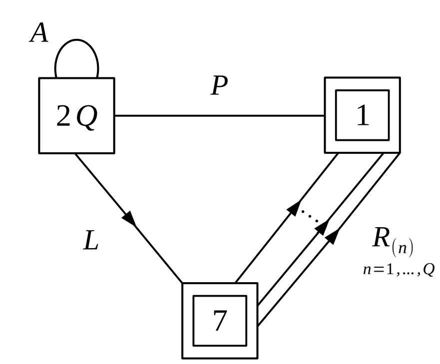
$$E_6 \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$

Back to E_6

Gluing two basic caps

- Let us glue two basic caps by S-gluing $L_{1,2}$ and Φ-gluing the rest including the antisym A

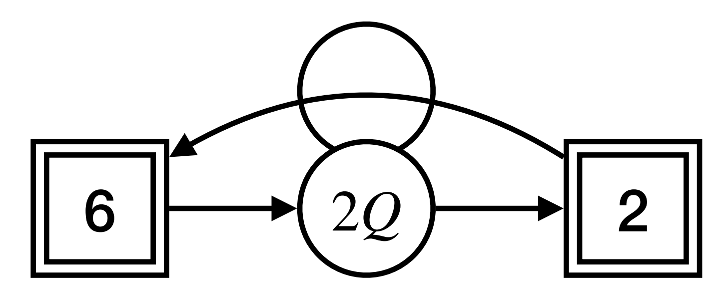
$$n'_a = \begin{cases} n_a^{(1)} + n_a^{(2)}, & a \in \Phi, \\ n_a^{(1)} - n_a^{(2)}, & a \in S, \end{cases}$$



$$\mathcal{F}_c = \left(-\frac{1}{2}\,; 1,1,1,1,1,1,1, -1\right) \quad \longrightarrow \quad \mathcal{F} = (-1\,; 0,0,2,2,2,2,2, -2)$$

- The resulting theory is a $USp(2Q)$ theory with 8 fund & 1 antisym chirals, and $16Q$ extra gauge singlets preserving

$$SU(6)_w \times SU(2)_d \times SU(2)_v \times U(1)_b \times U(1)_t \times SU(2)_f$$



- Matters in the representation (of the nonabelian factors)

$$(\mathbf{2Q}, \overline{\mathbf{6}}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{2Q}, \mathbf{1}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{asym}, \mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{6}, \mathbf{2}, \mathbf{1}, \mathbf{1}) \times Q \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{2}, \mathbf{2}) \times Q$$

- Symmetry enhancement in the IR

$$\subset E_8 \times SU(2)_L$$

$$\boxed{E_6 \times SU(2)_v \times U(1)_b \times U(1)_t} \times SU(2)_f$$

And More

Remarks

- So far we have seen that one can understand the surprising E_6 symmetry enhancement of a simple 4d $SU(2)$ theory with 8 chirals (and its generalization to higher rank theories) from the 6d perspective
- This is only a particular example and there are more other interesting examples using different ways of gluing
- The same E_8 flux can be written in different ways in the $SO(16)$ basis, which may lead to different looking 4d theories
- Those should flow to the same IR fixed point of the same 6d theory compactified on a sphere with the same flux -> the 6d origin of lower dimensional dualities
- Extra $SU(2)_f$ symmetry not descending from that of the E-string theory -> identified with the $SU(2)$ isometry of the sphere
- $U(1)_f \subset SU(2)_f$ is preserved by tube theories as well but used to be overlooked because it becomes anomalous when tubes are glued to form a torus
- Our sphere models are concrete examples manifesting such a symmetry stemming from the isometry of a compactifying surface