

Impact of the interference among the operators in dark matter direct detection Experiments

In collaboration with A. Brenner, G. Herrera, A. Ibarra, S. Kang, S. Scopel
arXiv: 2107.xxxxx

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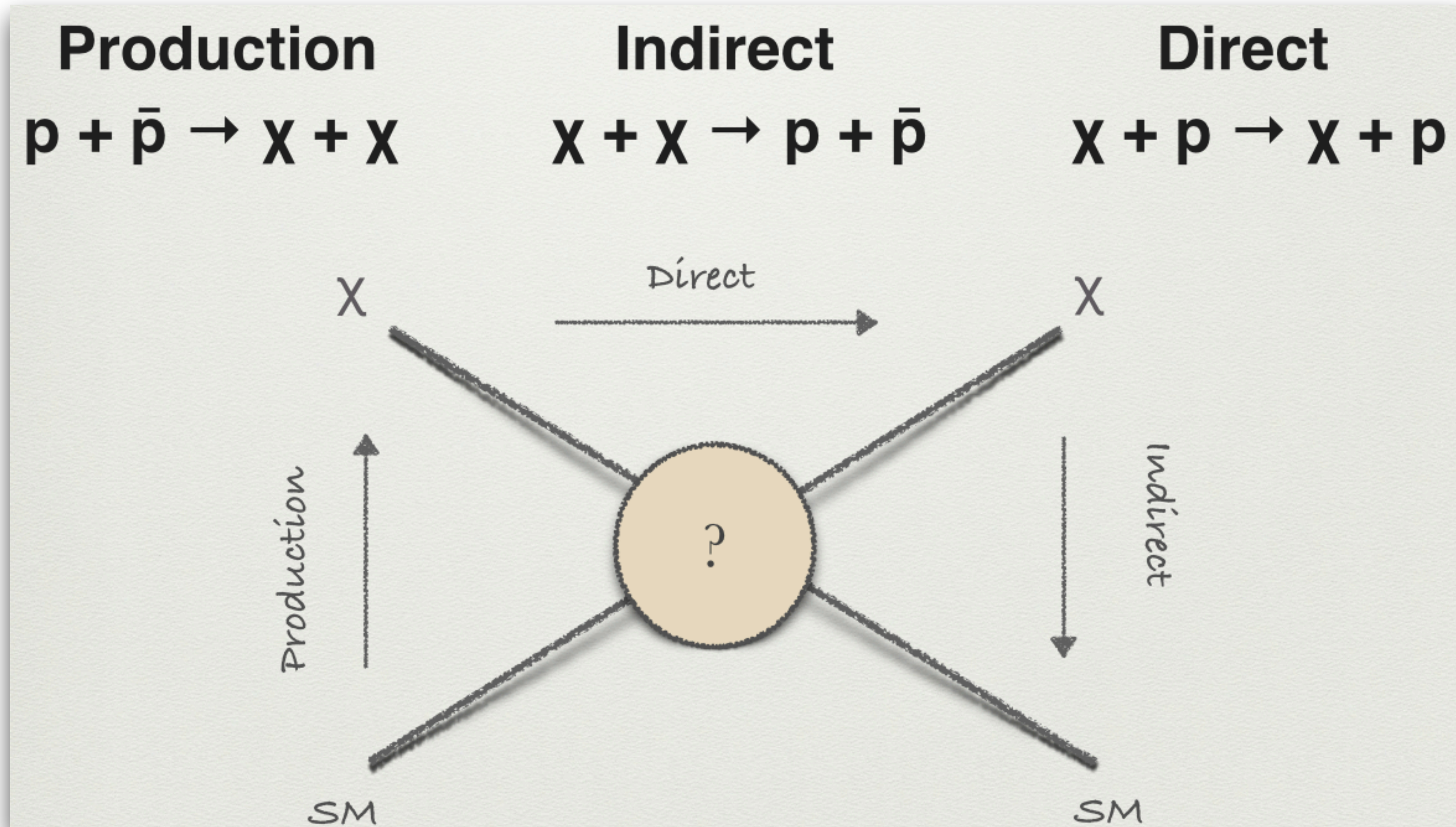


17th June, 2021

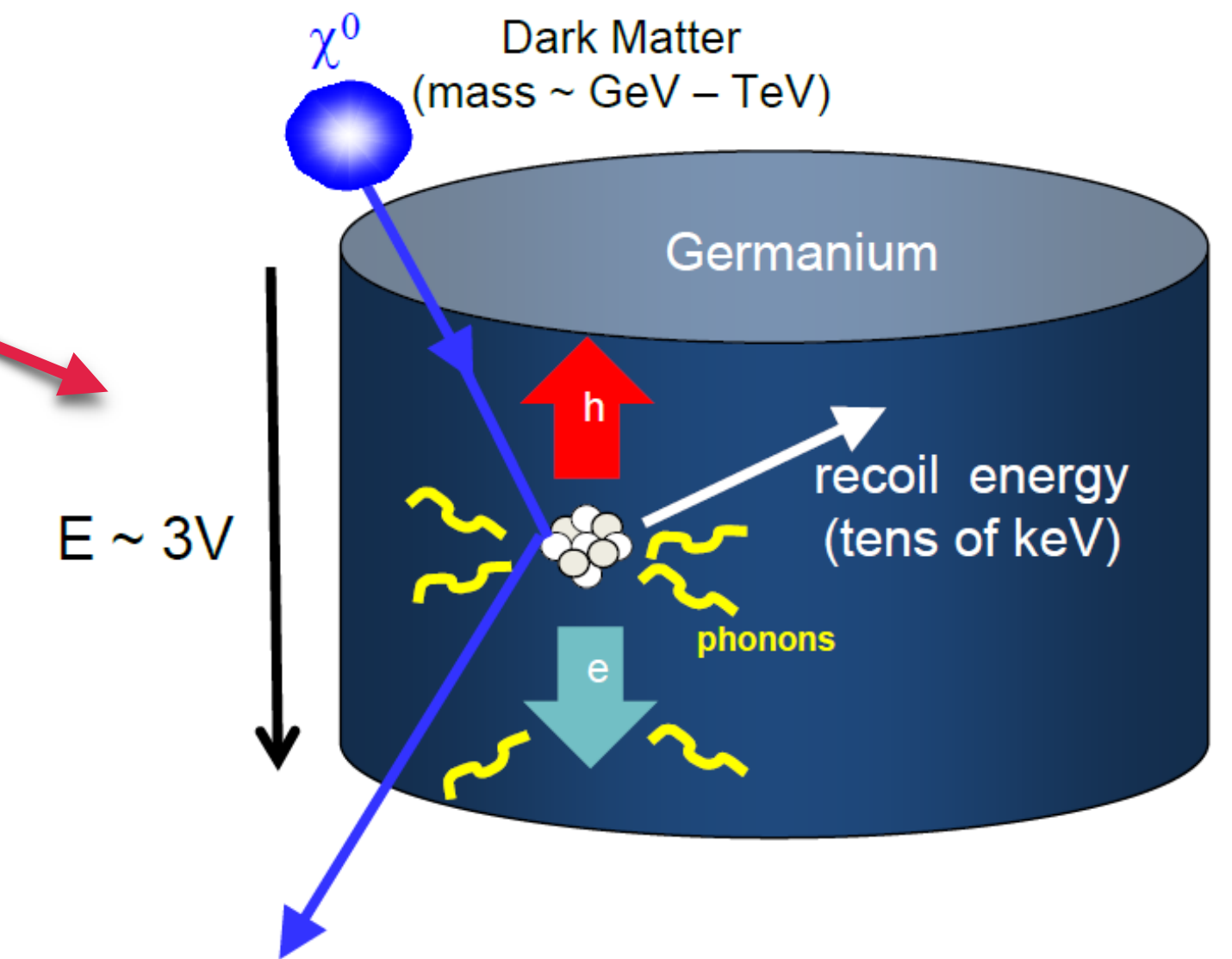
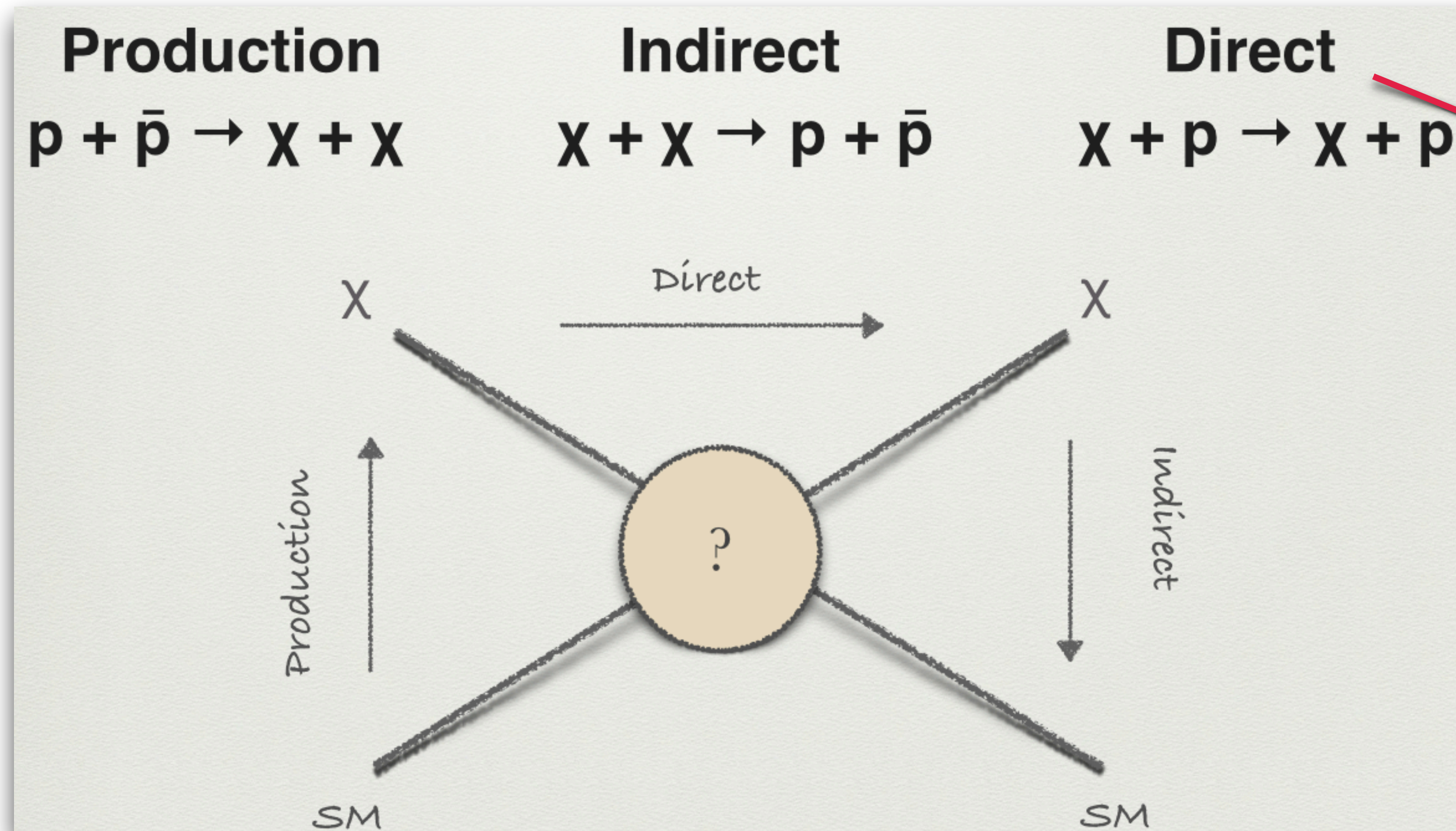
Dark Matter: What we know?

- Weakly Interacting
- Non-baryonic [e.g microlensing searches]
- Mostly cold [e.g clumsiness of structure formation]
- Average mass density [from CMB, $\Omega h^2 = 0.112 \pm 0.006$]
- There is no candidate in the standard model of particle physics

Dark Matter: Possible ways of detection?



Dark Matter: Possible ways of detection?



Dark Matter: What we don't know?

- No direct detection

- Specific properties:

1. Mass

2. Couplings of particles

Particle Physics Uncertainties

3. Density distribution

4. Velocity distribution

Astrophysics Uncertainties

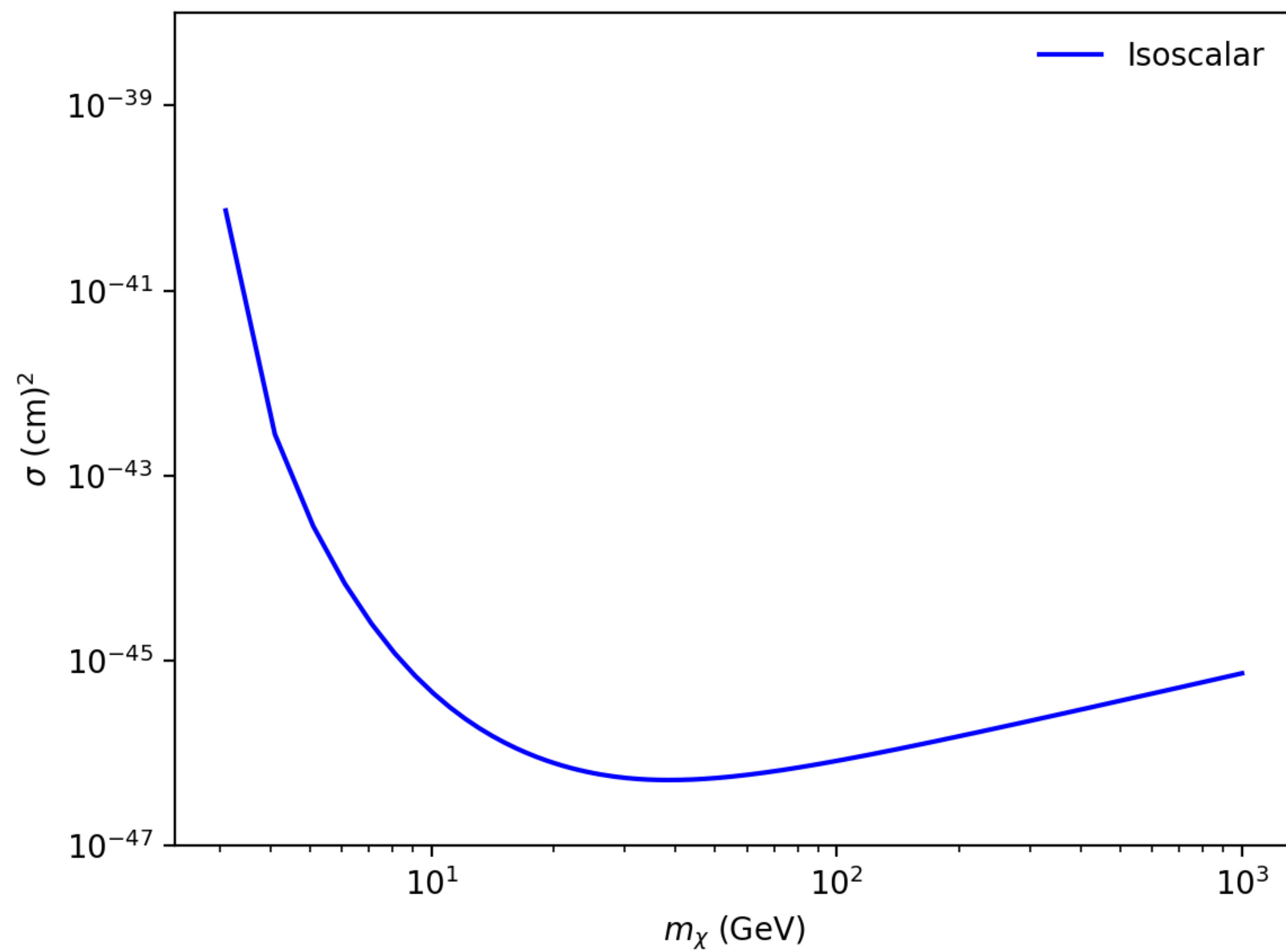
- The expected rate:

$$\frac{dR}{dE_R}(t) = N_T \overbrace{\frac{\rho_\chi}{m_\chi}}^{\text{Astrophysical Uncertainties}} \int_{v_{min}} d^3v_T \overbrace{f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R}}^{\text{Particle Physics Uncertainties}}$$

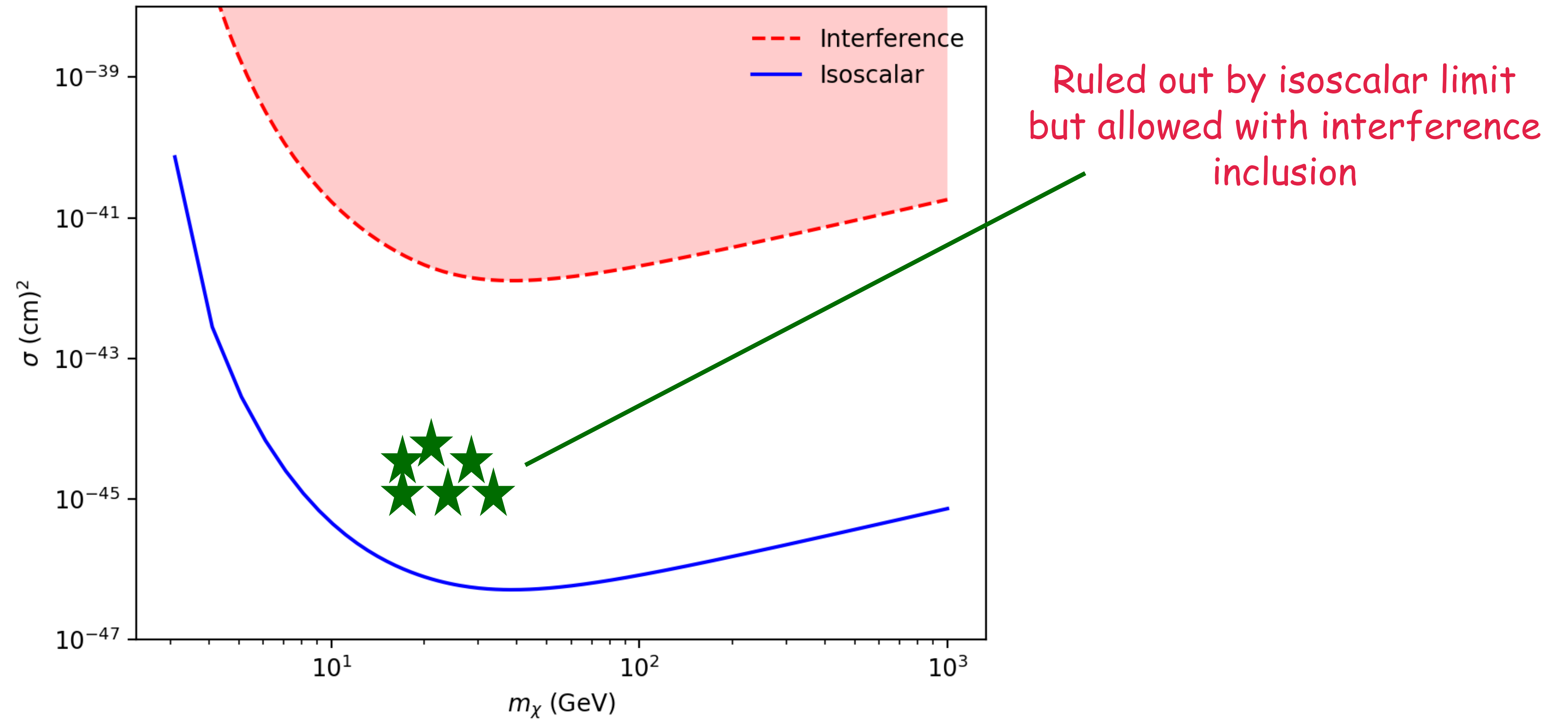
Assumptions:

1. Couplings of DM particles : Spin-Independent iso-scalar interaction ($c_p = c_n$)
2. Velocity distribution: Maxwellian-Boltzmann distribution
3. Density distribution: $\rho_\chi = 0.3 \text{ GeV/cm}^3$

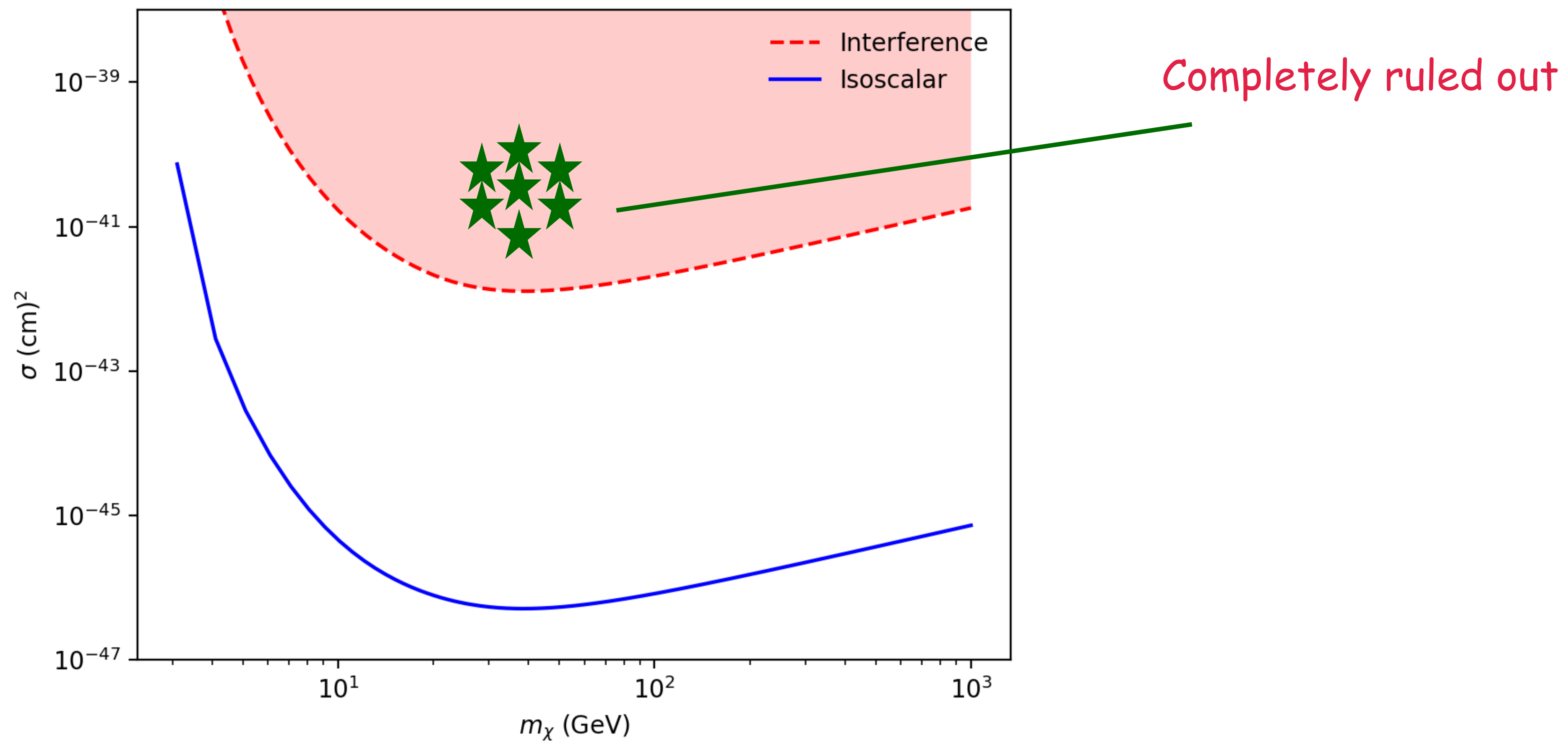
Motivation



Motivation



Motivation



1. couplings of DM particles: Non-Relativistic Effective Field Theory

- Hamiltonian density of DM-nucleus interaction

$$H = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3) \mathcal{O}_j \quad \text{with } c_j^0 = c^p + c^n, c_j^1 = c^p - c^n$$

- All operators are guaranteed to be Hermitian if built by

$$i\frac{\vec{q}}{m_N}, \vec{v}^\perp, \vec{S}_\chi, \vec{S}_N \quad \text{with } \vec{v}^\perp = \vec{v} + \frac{\vec{q}}{2\mu_{\chi N}}$$

Complete set of operators for DM with spin 1/2

$$\mathcal{O}_1 = 1_\chi 1_N; \quad \mathcal{O}_3 = i \vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right); \quad \mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N; \quad \mathcal{O}_5 = i \vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^\perp \right); \quad \mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\vec{S}_N \cdot \frac{\vec{q}}{m_N} \right)$$

$$\mathcal{O}_7 = \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_8 = i \vec{S}_\chi \cdot \vec{v}^\perp; \quad \mathcal{O}_9 = i \vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N} \right); \quad \mathcal{O}_{10} = i \vec{S}_N \cdot \frac{\vec{q}}{m_N}; \quad \mathcal{O}_{11} = i \vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$$

$$\mathcal{O}_{12} = \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}^\perp \right); \quad \mathcal{O}_{13} = i \left(\vec{S}_\chi \cdot \vec{v}^\perp \right) \left(\vec{S}_N \cdot \frac{\vec{q}}{m_N} \right); \quad \mathcal{O}_{14} = i \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\vec{S}_N \cdot \vec{v}^\perp \right);$$

$$\mathcal{O}_{15} = - \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \right) \left(\left(\vec{S}_N \times \vec{v}^\perp \right) \cdot \frac{\vec{q}}{m_N} \right)$$

Anand, Fitzpatrick, Haxton, Phys. Rev. C89, 065501 (2014), 1308.6288

For DM with spin $\geq 1/2$: Gondolo, Kang, Scopel, GT, PRD, to appear, 2008.05120

- Differential cross-section

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v_T^2} \left[\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 \right]$$

- Scattering amplitude

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}_T|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau, \tau'} \overbrace{\sum_k R_k^{\tau\tau'}}^{\text{Dark Matter}} \left[c_j^\tau, (v_T^\perp)^2, \frac{q^2}{m_N^2} \right] \underbrace{W_{Tk}^{\tau\tau'}(y)}_{\text{Nuclear Physics}}$$

Here, $k = M, \Phi'', \tilde{\Phi}', \Sigma', \Sigma'', \Delta, \Phi''M, \Delta\Sigma'$

Impact of operator interference in DM direct detection

- The interaction Hamiltonian

$$H = \sum_{j=1}^{15} (c_j^0 + c_j^1 \tau_3) \mathcal{O}_j$$

- Rate in an experiment

$$R(c) = c_k^T \mathbb{R} c_k \quad \text{—————} \quad c_k = (c_1^0, c_1^1, c_3^0, c_3^1, \dots, c_{15}^0, c_{15}^1)$$

For a spin-1/2 DM

$\mathbb{R} : 28 \times 28$

Catena, Ibarra, Wild, JCAP 1605 (2016), 05, 039

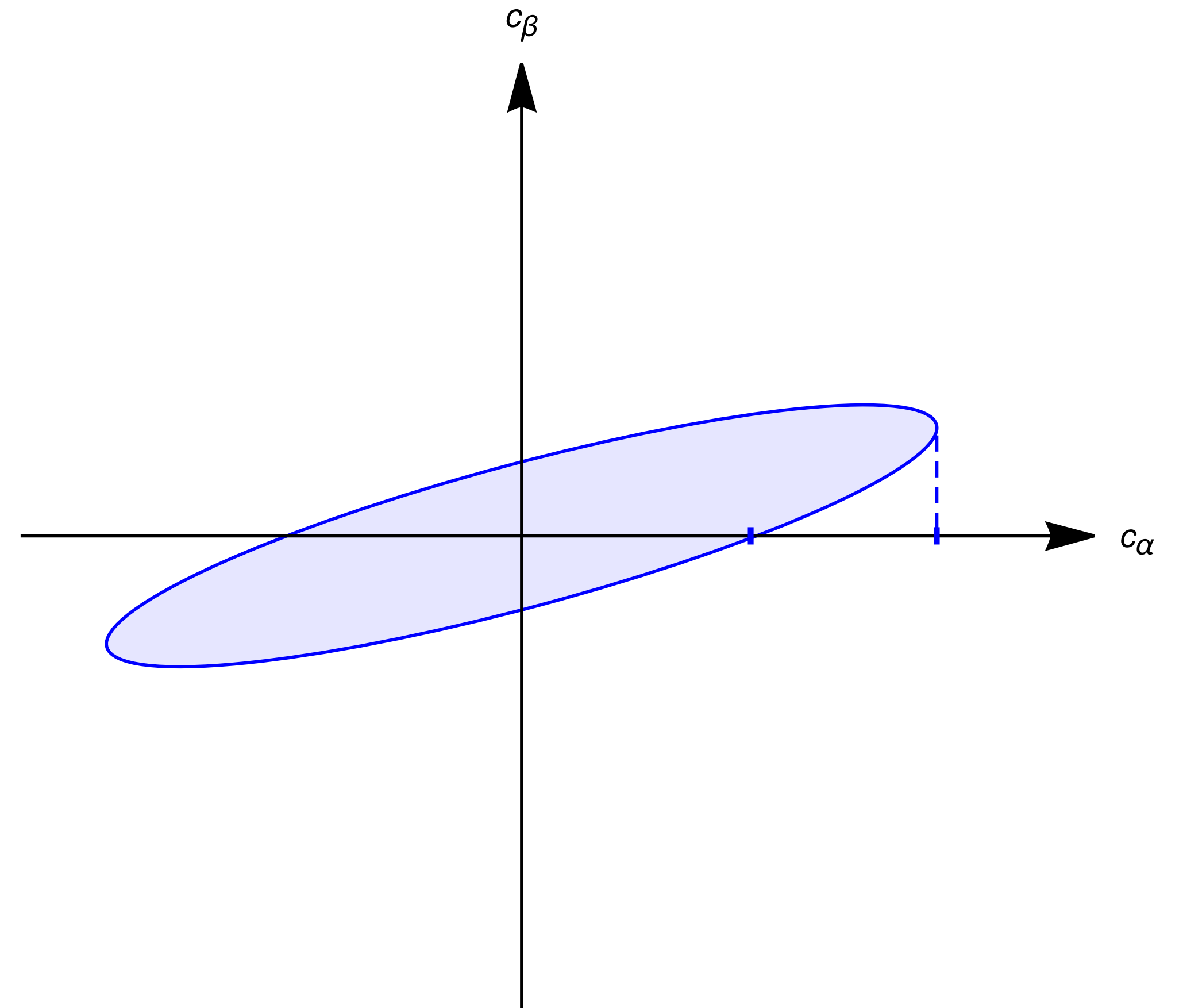
Catena, Ibarra, Rappelt, Wild, JCAP 07 (2018) 028

- $\mathbb{R}$ Matrix depends on: DM mass, density and velocity distribution

Impact of operator interference in DM direct detection

- Rate in an experiment

$$c_k^T \mathbb{R} c_k \leq R^{u.l.}$$



Catena, Ibarra, Wild, JCAP 1605 (2016), 05, 039

Catena, Ibarra, Rappelt, Wild, JCAP 07 (2018) 028

Interference for DM with spin $\geq 1/2$

$$\begin{aligned}
 R_M^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{q}{m_N^2} \right) &= c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{q}{m_N^2} \vec{v}_T^{\perp 2} c_5^\tau c_5^{\tau'} + \vec{v}_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{q}{m_N^2} c_{11}^\tau c_{11}^{\tau'} \right], \\
 R_{\Phi''}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{\vec{q}^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right), \quad \text{Interference terms} \\
 R_{\Phi''M}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi + 1)}{3} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) c_{11}^{\tau'}, \\
 R_{\Phi'}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{j_\chi(j_\chi + 1)}{12} \left[c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'} \right], \\
 R_{\Sigma''}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{\vec{q}^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} \frac{\vec{q}^2}{m_N^2} (c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{\vec{q}^4}{m_N^4} c_6^\tau c_6^{\tau'} + \vec{v}_T^{\perp 2} c_{12}^\tau c_{12}^{\tau'} + \frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'} \right], \quad (38) \\
 R_{\Sigma'}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{1}{8} \left[\frac{\vec{q}^2}{m_N^2} \vec{v}_T^{\perp 2} c_3^\tau c_3^{\tau'} + \vec{v}_T^{\perp 2} c_7^\tau c_7^{\tau'} \right] + \frac{j_\chi(j_\chi + 1)}{12} \left[c_4^\tau c_4^{\tau'} + \frac{\vec{q}^2}{m_N^2} c_9^\tau c_9^{\tau'} + \frac{\vec{v}_T^{\perp 2}}{2} \left(c_{12}^\tau - \frac{\vec{q}^2}{m_N^2} c_{15}^\tau \right) \left(c_{12}^{\tau'} - \frac{\vec{q}^2}{m_N^2} c_{15}^{\tau'} \right) \right. \\
 &\quad \left. + \frac{\vec{q}^2}{2m_N^2} \vec{v}_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'} \right], \\
 R_{\Delta}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{j_\chi(j_\chi + 1)}{3} \left[\frac{\vec{q}^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'} \right], \\
 R_{\Delta\Sigma'}^{\tau\tau'} \left(\vec{v}_T^{\perp 2}, \frac{\vec{q}^2}{m_N^2} \right) &= \frac{j_\chi(j_\chi + 1)}{3} [c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}].
 \end{aligned}$$

Anand, Fitzpatrick, Haxton, Phys. Rev. C89, 065501 (2014), 1308.6288

For DM with spin $\geq 1/2$: Gondolo, Kang, Scopel, GT, PRD, to appear, 2008.05120

Impact of operator interference in DM direct detection: single experiment

- Construct the Lagrangian

$$L = c_\alpha - \lambda(R(c) - R^{u.l.})$$

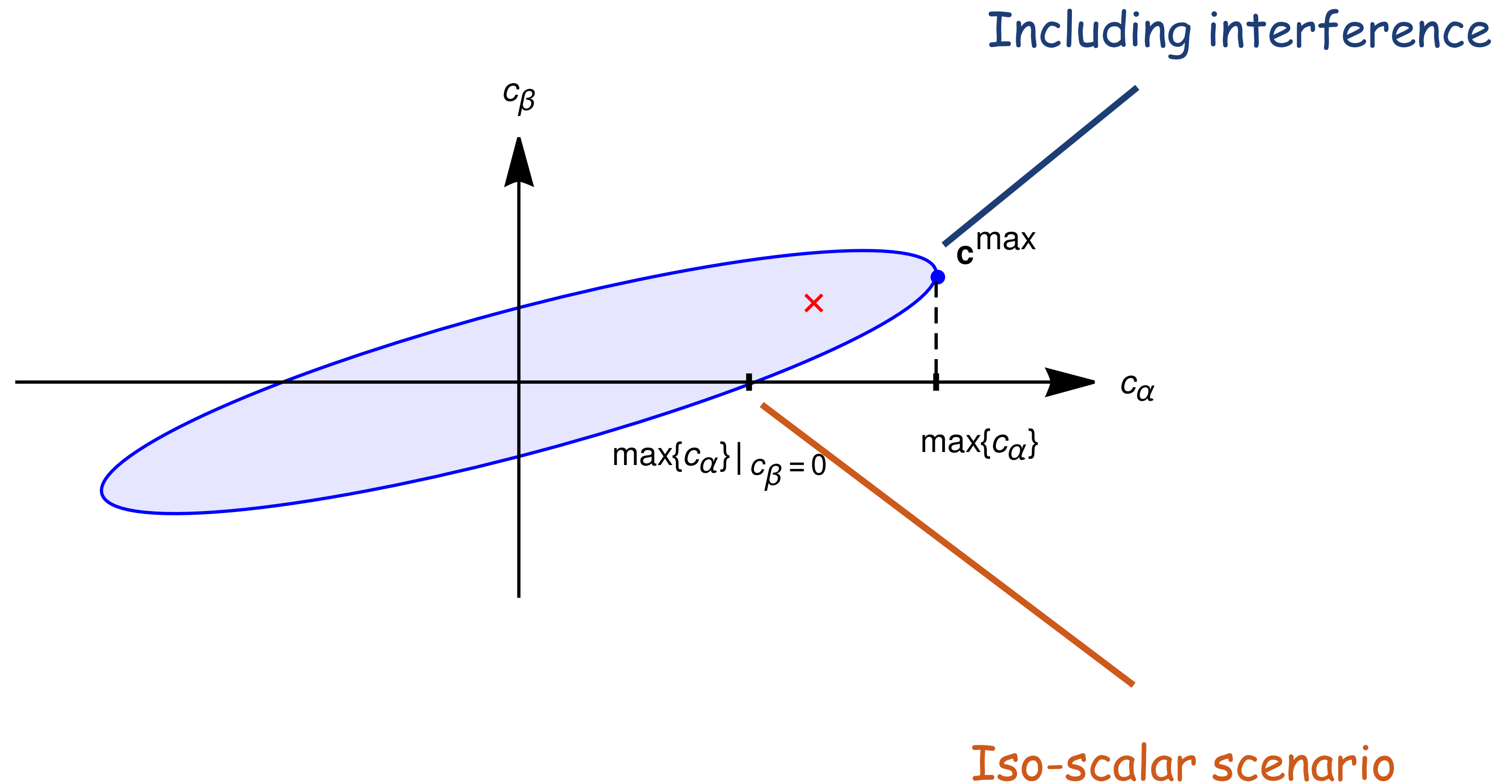
- Maximum coupling

$$c_\alpha^{\max} = \sqrt{(\mathbb{R}^{-1})_{\alpha\alpha} R^{u.l.}}$$

$\mathbb{R} : 28 \times 28$

Using WimPyDD: <https://wimpdd.hepforge.org>

Jeong, Kang, Scopel, GT, arXiv: 2106.06207



Brenner, Ibarra, Rappelt, arXiv: 2011.02929

Impact of operator interference in DM direct detection: combined experiments

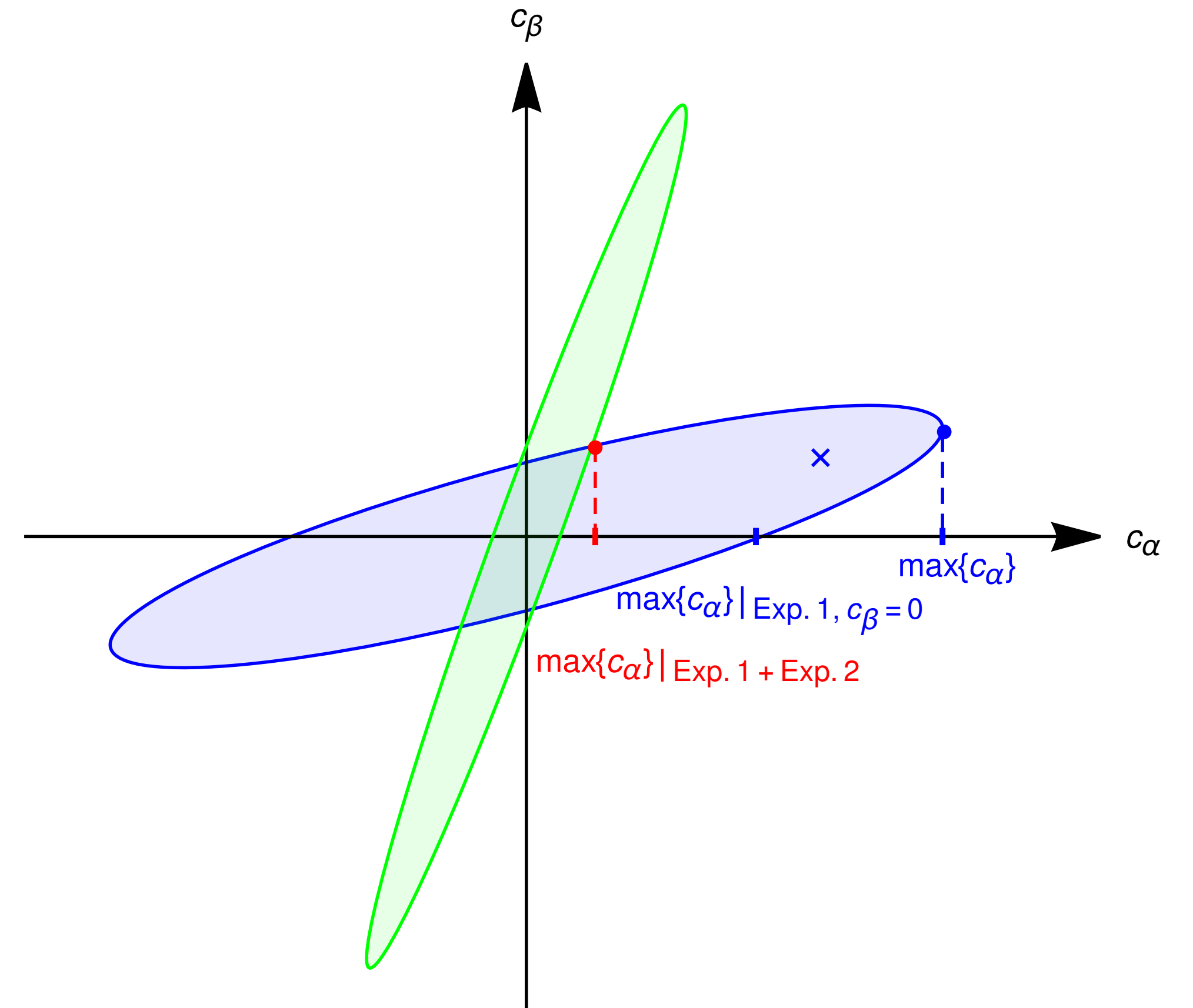
- Construct the Lagrangian

$$L = c_\alpha - \lambda \left(\sum_{\text{exp}}^n \left[\chi_{\text{exp}}^2 - \chi_{\text{exp,min}}^2 \right] - 2.71 \right)$$

$$\chi_{\text{exp}}^2 = -2 \ln \left[\mathcal{L}(m_\chi, N_{\text{exp}}^{\text{sig}}) \right]$$

- Maximum coupling

$$c_\beta^{\text{max}} = \frac{\delta_\alpha}{4\lambda} (X_{\alpha\beta})^{-1}$$



In preparation: Brenner, Herrera, Ibarra, Scopel, Kang, GT, arXiv: 2107.xxxxx

Impact of operator interference in DM direct detection: combined experiments

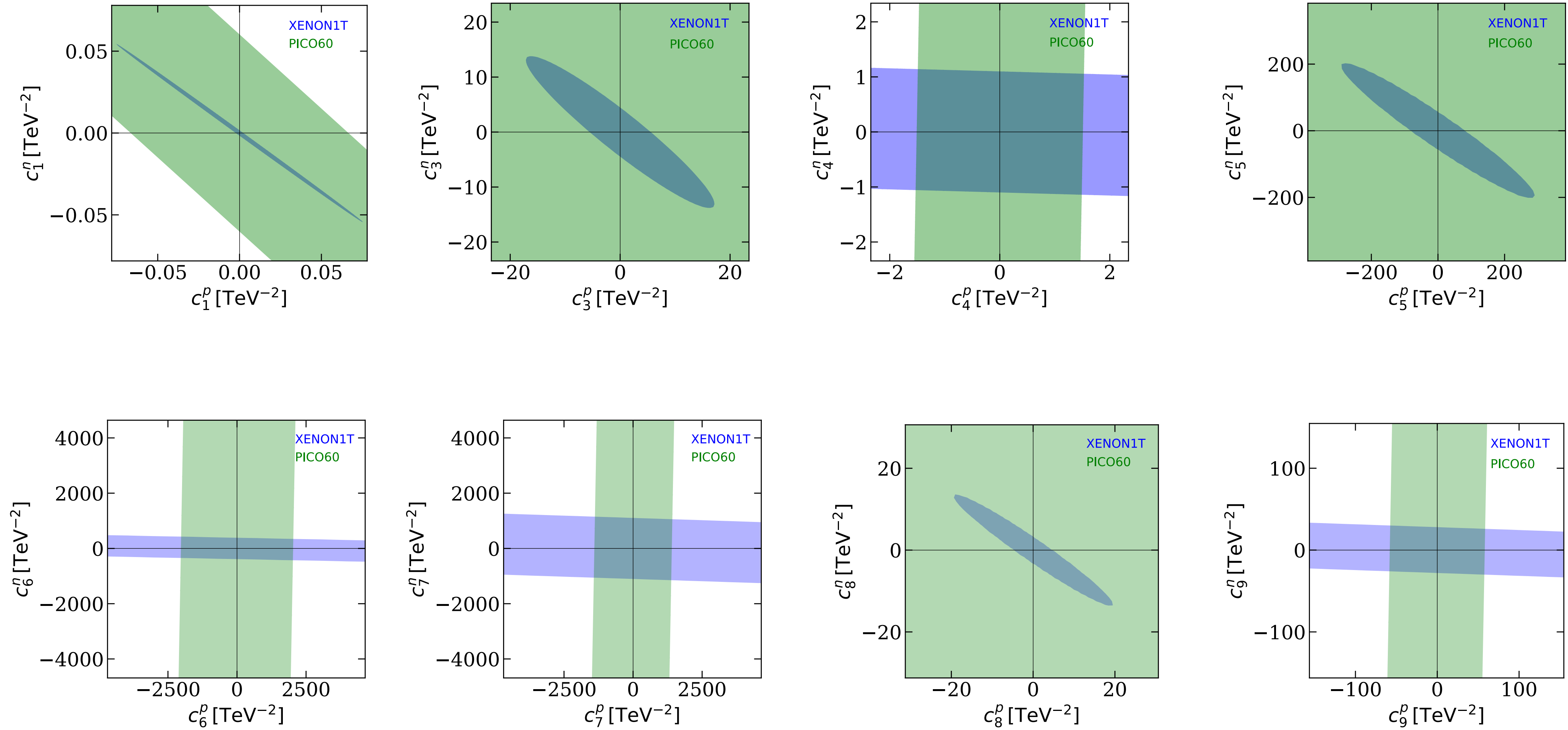
$$X_{\alpha\beta} = \sum_{\text{exp}}^n \left[(\mathbb{R}_{\text{exp}})_{\alpha\beta} \left(1 - \frac{N_{\text{exp}}^{\text{obs}}}{N_{\text{exp}}^{\text{sig}} + N_{\text{exp}}^{\text{back}}} \right) \right]$$

- Expected events in an experiment

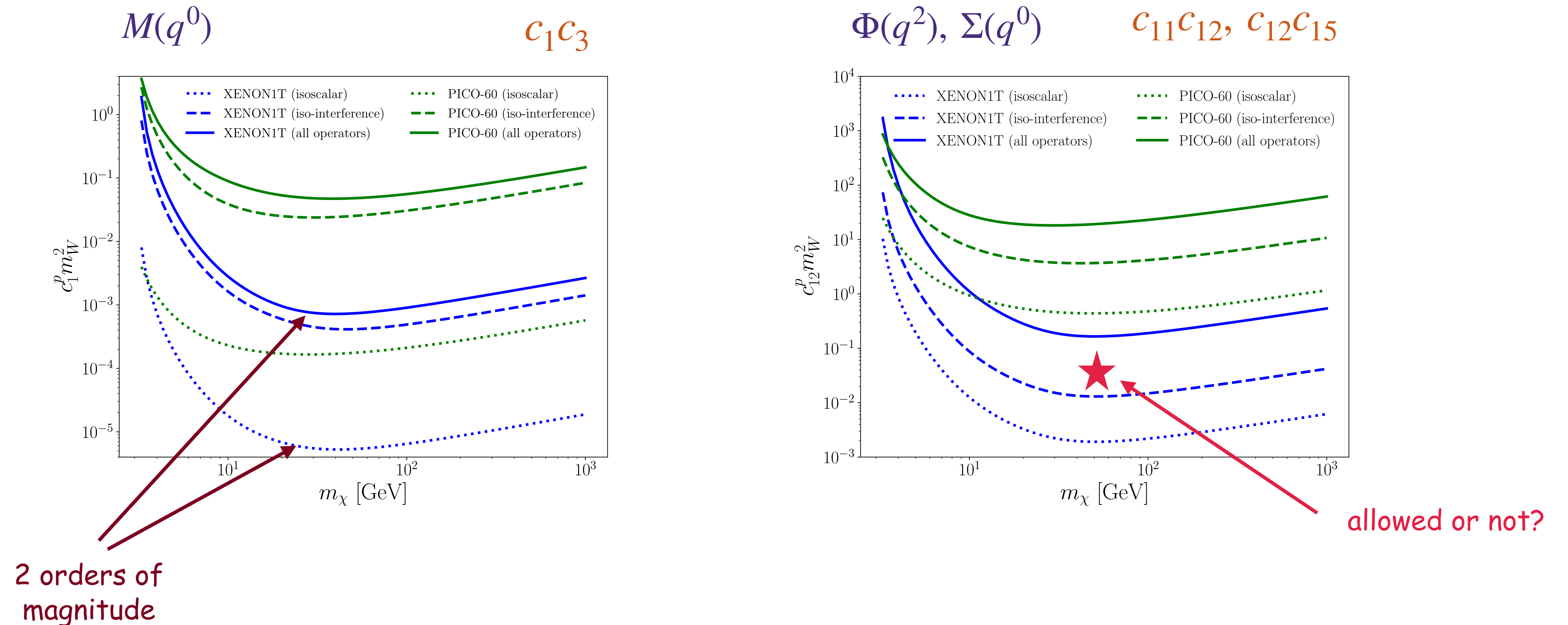
$$\sum_{\text{exp}}^n N_{\text{exp}}^{\text{sig}} = \frac{1}{16\lambda^2} \sum_{\text{exp}}^n \left(X^{-1} \mathbb{R}_{\text{exp}} X^{-1} \right)_{\alpha\alpha}$$

In preparation: Brenner, Herrera, Ibarra, Scopel, Kang, GT, arXiv: 2107.xxxxxx

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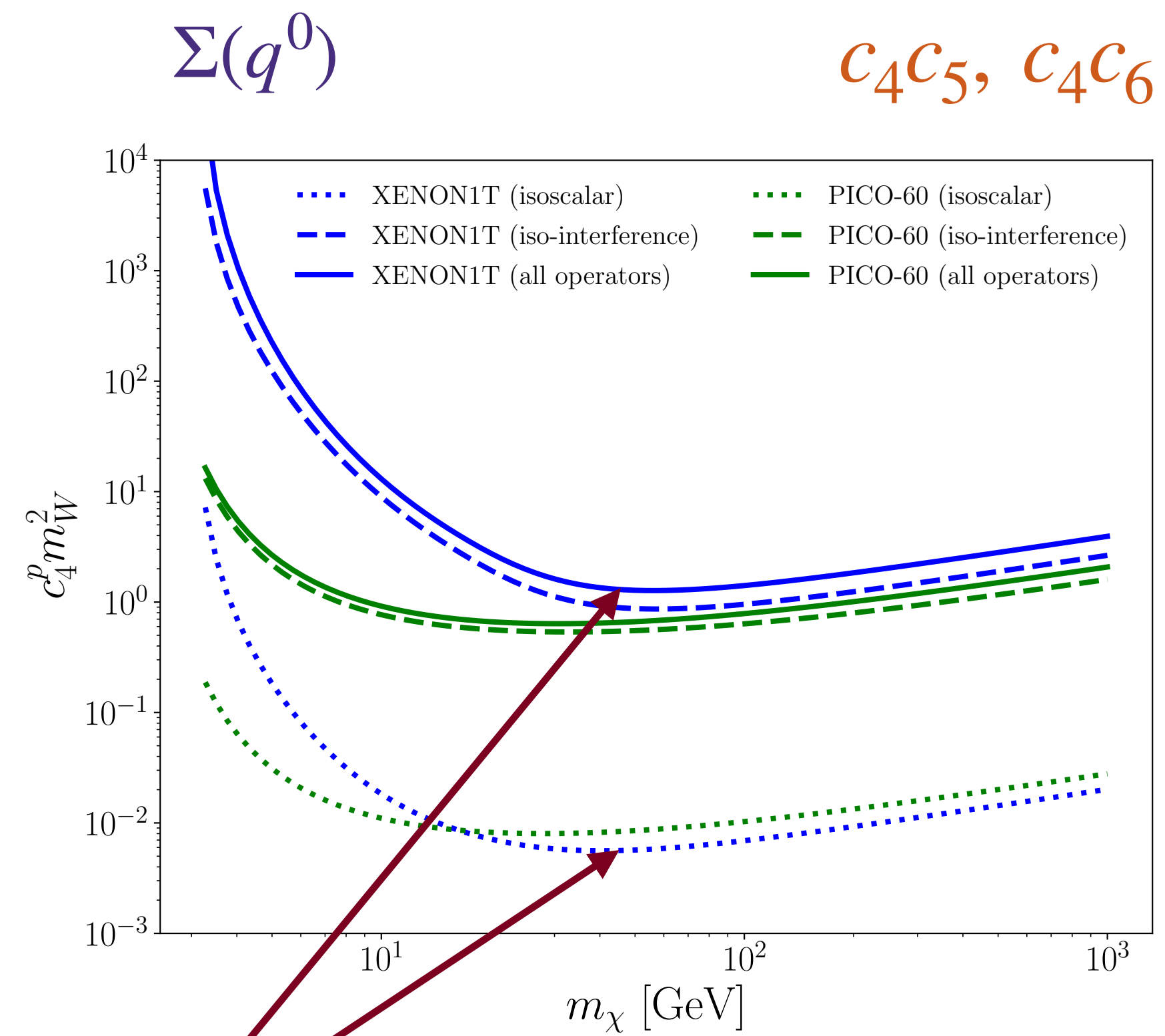


Results: single experiment

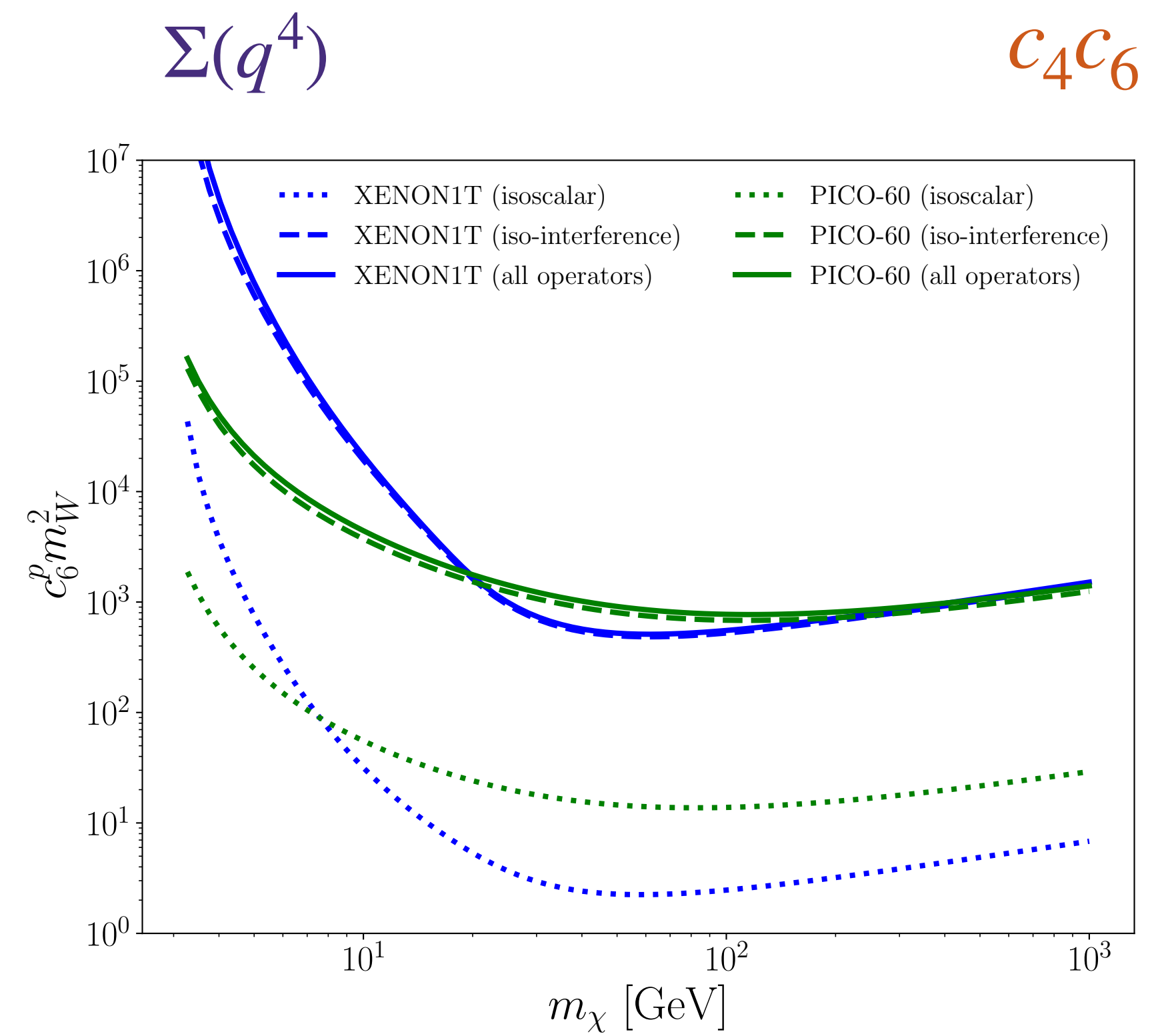


In preparation: Brenner, Herrera, Ibarra, Kang, Scopel, GT, arXiv: 2107.xxxxx

Results: single experiment

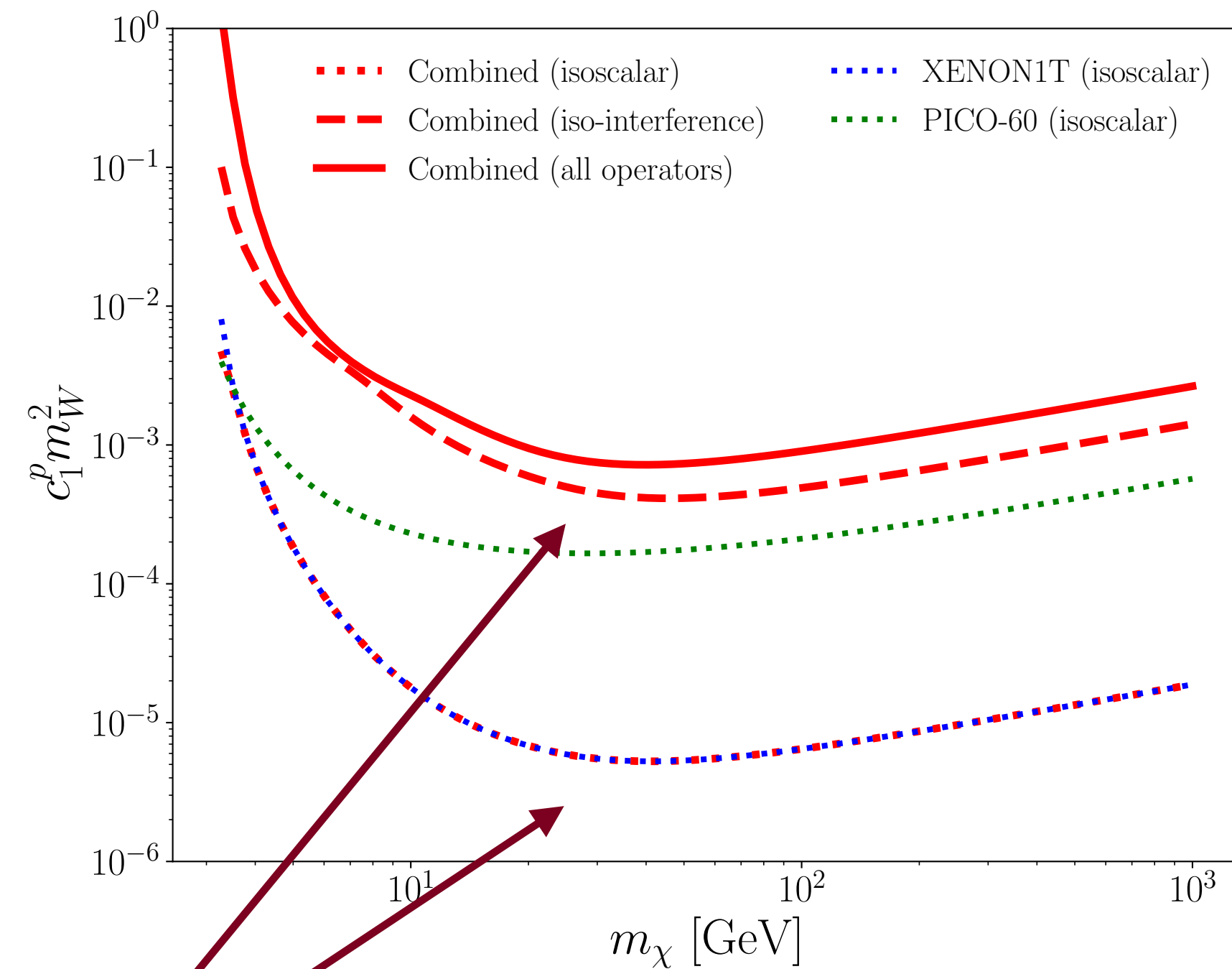


3 orders of
magnitude

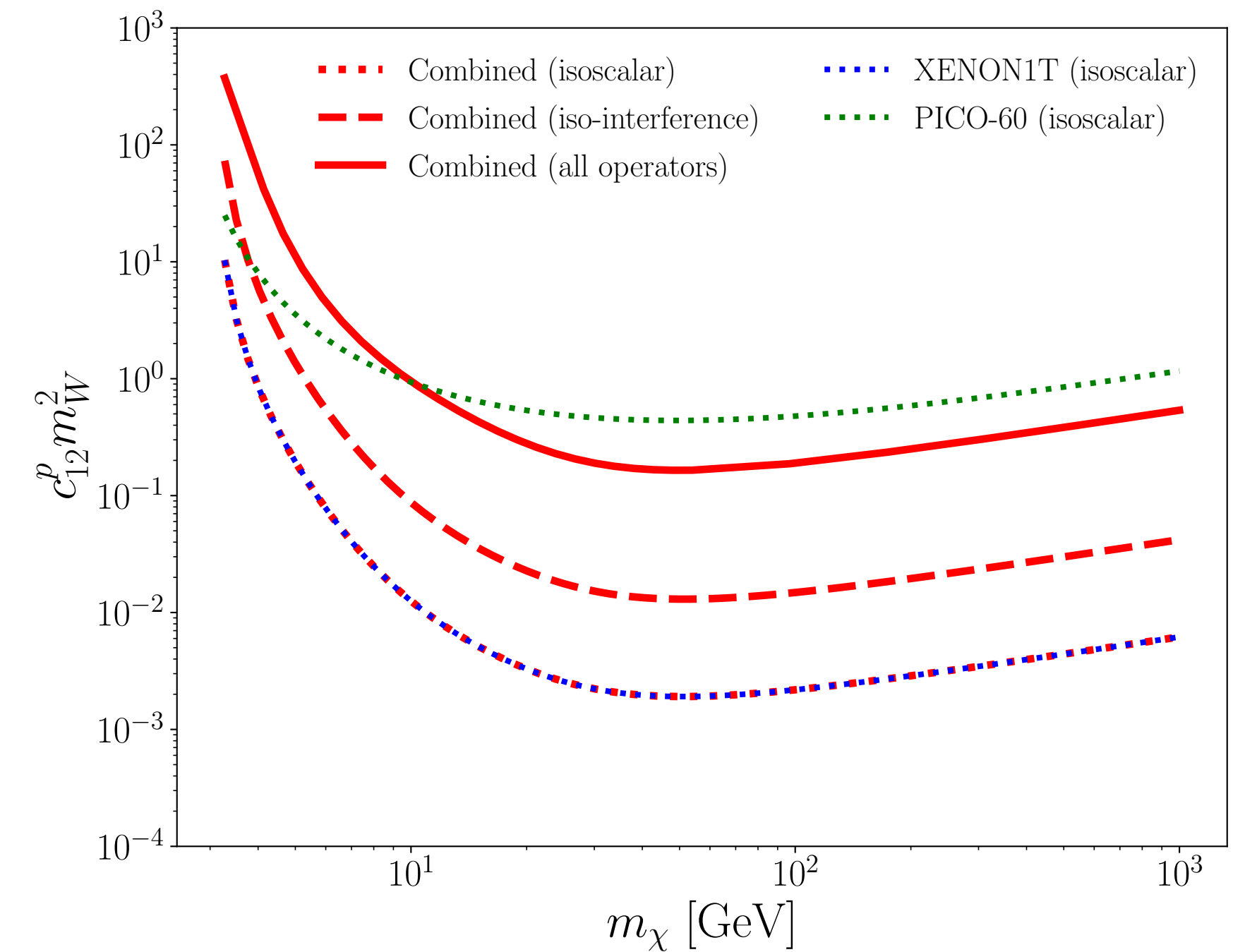


In preparation: Brenner, Herrera, Ibarra, Kang, Scopel, GT, arXiv: 2107.xxxxx

Results: combined experiment



2 orders of
magnitude



In preparation: Brenner, Herrera, Ibarra, Kang, Scopel, GT, arXiv: 2107.xxxxx

Summary

- Consideration of single coupling leads to stringent exclusion limits
- Interference between different coupling strengths is an important effect to take into account
- Extended method developed by Brenner et. al. to direct detection experiments
- Generalised the method for combined experiments
- For some of the coupling strengths, the limit is relaxed by 2 to 3 orders of magnitude
- As a result, parameter space of different models might be actually allowed
- Practical application: useful for model builders (the closest thing you can get to a model-independent bound)

Mapping between DM and nuclear response functions

$\mathbf{c_j}$	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	$\mathbf{c_j}$	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
c_1	$M(q^0)$	-	c_3	$\Phi''(q^4)$	$\Sigma'(q^2)$
c_4	$\Sigma''(q^0), \Sigma'(q^0)$	-	c_5	$\Delta(q^4)$	$M(q^2)$
c_6	$\Sigma''(q^4)$	-	c_7	-	$\Sigma'(q^0)$
c_8	$\Delta(q^2)$	$M(q^0)$	c_9	$\Sigma'(q^2)$	-
c_{10}	$\Sigma''(q^2)$	-	c_{11}	$M(q^2)$	-
c_{12}	$\Phi''(q^2), \tilde{\Phi}'(q^2)$	$\Sigma''(q^0), \Sigma'(q^0)$	c_{13}	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
c_{14}	-	$\Sigma'(q^2)$	c_{15}	$\Phi''(q^6)$	$\Sigma'(q^4)$

Recoil Energy and transfer momentum

$$v_{min}^2 = \frac{q^2}{4\mu_T^2} = \frac{m_T E_R}{2\mu_T^2},$$

$$v_{min}^* = \sqrt{\frac{2\delta}{\mu_{\chi N}}},$$

$$v_{min} = \frac{1}{\sqrt{2m_N E_R}} \left| \frac{m_N E_R}{\mu_{\chi N}} + \delta \right|,$$

DM Velocity distribution

$$f(\vec{v}_T, t) = N \left(\frac{3}{2\pi v_{rms}^2} \right)^{3/2} e^{-\frac{3|\vec{v}_T + \vec{v}_E|^2}{2v_{rms}^2}} \Theta(u_{esc} - |\vec{v}_T + \vec{v}_E(t)|),$$

$$N = \left[\text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2} \right]^{-1},$$

$$|\vec{v}_E(t)| = v_{\odot} + v_{orb} \cos \gamma \cos \left[\frac{2\pi}{T_0} (t - t_0) \right],$$

Interference

for O_1 ,

(1)

$$R_{\text{exp.}} \propto [C_p z + (A-z)C_n]^2$$

$$R_{\text{exp.}} \propto C_p^2 \left[z + (A-z) \frac{C_n}{C_p} \right]^2$$

Compare, $R_{ob} = R_{\text{exp.}}$

$$\Rightarrow C_p^2 = \frac{R_{ob}}{\left[z + (A-z) \frac{C_n}{C_p} \right]^2}$$

$$\text{or } C_p = \frac{\sqrt{R_{ob}}}{\left[z + (A-z) \frac{C_n}{C_p} \right]}$$

$$C_p^{\text{max}}, \text{ when } z = -(A-z) \frac{C_n}{C_p},$$

Similarly for operator interference,

$$\left\{ C_p^{\text{max}} = \frac{\sqrt{R_{ob}}}{[aO_1 + abD_3 + baD_3 + bD_3]} \right\}$$