

Quantum fluctuations of energy in a hot relativistic gas of fermions

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Motivation:

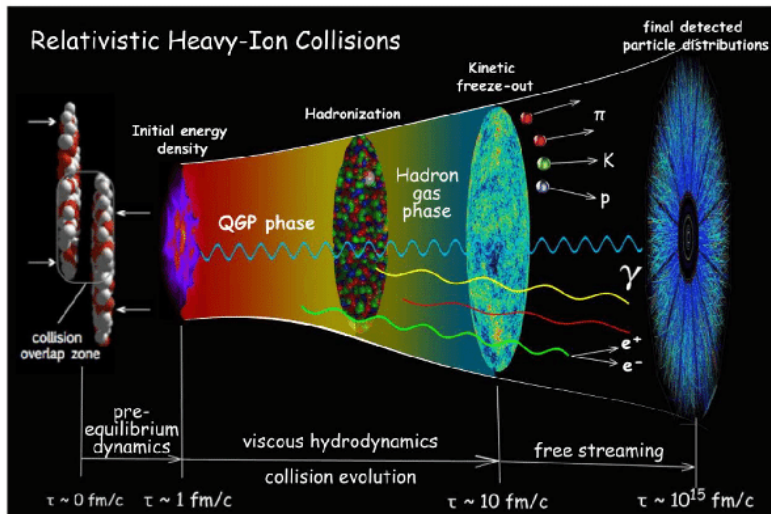


Figure: By Prof. Chun Shen

Motivation:

Fluctuations of various physical quantities play a very important role in all fields of physics, as they reveal the information about

possible phase transitions

formation of structures in the Early Universe

dissipative phenomena

Most common fluctuations we deal with are those arising from quantum uncertainty relation or those present in thermodynamic systems.

Motivation:

Space-time evolution of matter produced in relativistic heavy-ion collision is very well described by **relativistic (dissipative) hydrodynamics**.

One of the concepts used in hydrodynamics are those of **energy density** and **pressure**, both are defined locally – formally, the fluid element has zero size.

Successful hydro models are then used to conclude about the **energy density** attained in the collision processes, usually such values are very large.

Basic concepts and definitions:

A **quantum field operator** for spin- $\frac{1}{2}$ particle has the standard form:

$$\psi(t; \mathbf{x}) = \int_r \int \frac{d^3k}{(2\pi)^3} \frac{1}{2!_k} U_r(k) a_r(k) e^{ik \cdot x} + V_r(k) b_r^\dagger(k) e^{ik \cdot x} ;$$

where $a_r(k)$ and $b_r^\dagger(k)$ are annihilation and creation operators for particles and antiparticles, respectively, satisfying the canonical commutation relations $[a_r(k), a_s^\dagger(k')] = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k')$ and $[b_r(k), b_s^\dagger(k')] = (2\pi)^3 \delta_{rs} \delta^{(3)}(k - k')$, whereas $\epsilon_k = \sqrt{k^2 + m^2}$ is the energy of a particle.

Basic concepts and definitions:

To perform **thermal averaging**, it is sufficient to know the expectation values of the products of two and four creation and/or annihilation operators

$$\langle a_r^\dagger(k) a_s(k^\dagger) \rangle = (2s)^3 \delta_{rs} \delta(k - k^\dagger) f(k);$$

$$\langle a_r^\dagger(k) a_s^\dagger(k^\dagger) a_{r'}(p) a_{s'}(p^\dagger) \rangle = (2s)^6 \delta_{rs'} \delta_{r's} \delta(k - p^\dagger) \delta(k^\dagger - p) f(k) f(k^\dagger);$$

Here $f(k)$ is the Fermi–Dirac distribution function for particles.

Basic concepts and definitions:

We define an operator \hat{T}_a^{00} that represents the energy density of a subsystem S_a placed at the origin of the coordinate system

$$\hat{T}_a^{00} = \frac{1}{(a^3)^3} \int d^3x \hat{T}^{00}(x) \exp\left(-\frac{x^2}{a^2}\right) :$$

In above eq., a smooth Gaussian profile has been used to define the subsystem S_a .

To determine the fluctuation of the energy density of the subsystem S_a , we consider the variance

$$^2(a; m; T) = h: \hat{T}_a^{00} :: \hat{T}_a^{00} : i \quad h: \hat{T}_a^{00} : j^2$$

and the normalized standard deviation

$$n(a; m; T) = \frac{(h: \hat{T}_a^{00} :: \hat{T}_a^{00} : i \quad h: \hat{T}_a^{00} : j^2)^{1/2}}{h: \hat{T}_a^{00} : i}$$

Different forms of energy-momentum tensors:

For any original EM tensor \hat{T} satisfying the continuity equation $\partial \hat{T} = 0$ we can construct a different one by adding the divergence of an antisymmetric object, namely

$$\hat{T}' = \hat{T} + \partial \hat{A}$$

with $\hat{A} = -\hat{A}^T$. By construction, the new tensor is also conserved, i.e., $\partial \hat{T}' = 0$.

We aim here to study the dependence of quantum fluctuations of energy on the so-called pseudo-gauge transformation.

Different forms of energy-momentum tensors:

Canonical energy-momentum tensor:

$$\hat{T}_{\text{Can}} = \frac{i}{2} \dot{\phi} \partial_\mu \phi - \mathcal{H} \delta_{\mu\nu} :$$

Belinfante-Rosenfeld form:

$$\hat{T}_{\text{BR}} = \frac{i}{2} \dot{\phi} \partial_\mu \phi - \frac{i}{16} \partial_\mu \phi \partial_\nu \phi \partial_\lambda \phi \partial_\rho \phi \partial_\sigma \phi \partial_\tau \phi :$$

The de Groot-van Leeuwen-van Weert (GLW) form:

$$\hat{T}_{\text{GLW}} = \frac{1}{4m} \left(\partial_\mu \phi \partial_\nu \phi + (\partial_\mu \phi)(\partial_\nu \phi) + (\partial_\mu \phi)(\partial_\nu \phi) + (\partial_\mu \phi)(\partial_\nu \phi) \right) \delta_{\mu\nu} :$$

Hilgevoord-Wouthuysen form:

$$\hat{T}_{\text{HW}} = \hat{T}_{\text{Can}} + \frac{i}{2m} \partial_\mu \phi \partial_\nu \phi + \frac{ig}{4m} \partial_\mu \phi \partial_\nu \phi - \mathcal{H} \delta_{\mu\nu} :$$

Energy densities and quantum fluctuation expressions:

Energy densities obtained for different pseudo-gauge choices are the same, i.e., $\epsilon_{\text{Can}}(T) = \epsilon_{\text{BR}}(T) = \epsilon_{\text{GLW}}(T) = \epsilon_{\text{HW}}(T)$.

$$\hbar: \hat{T}_{\text{Can};a}^{00} :j = 4 \int \frac{d^3k}{(2\pi)^3} \epsilon_k f(\epsilon_k) \quad \epsilon_{\text{Can}}(T):$$

On the other hand, the fluctuations of \hat{T}_a^{00} are in general different for different pseudo-gauge choices.

Comparison of normalized standard deviation for various pseudo-gauges:

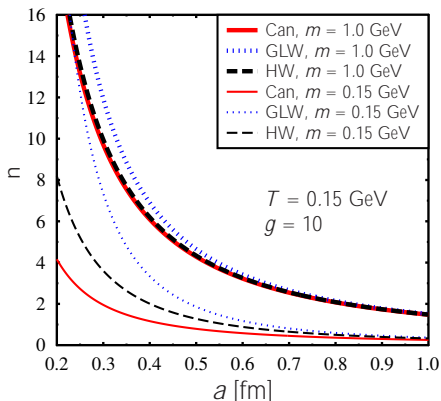


Figure: Comparison of normalized standard deviation for various pseudo-gauges for $T = 0.15$ GeV, $m = 1.0$ GeV (thick lines) and $m = 0.15$ GeV (thin lines).

Comparison of normalized standard deviation for various pseudo-gauges:

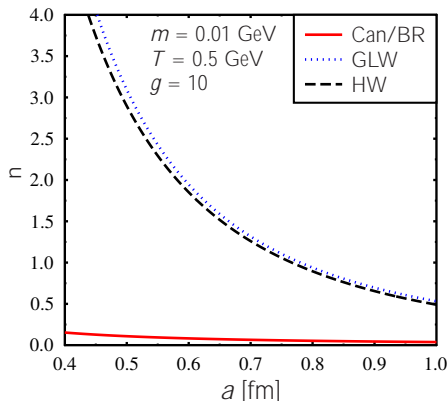


Figure: Comparison of normalized standard deviation for various pseudo-gauges for $T = 0.5$ GeV and $m = 0.01$ GeV.

Variation of the normalized energy fluctuation:

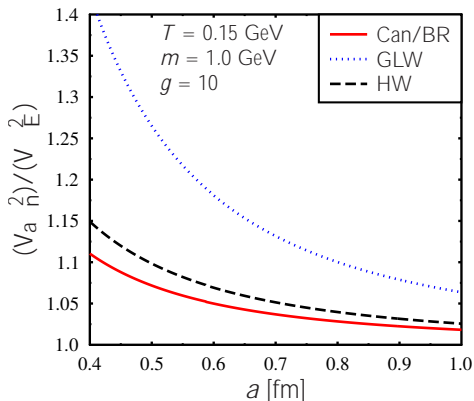


Figure: Variation of the normalized energy fluctuation in the subsystem S_a with the length scale a for $T = 0.15$ GeV and $m = 1.0$ GeV.

Variation of the normalized energy fluctuation:

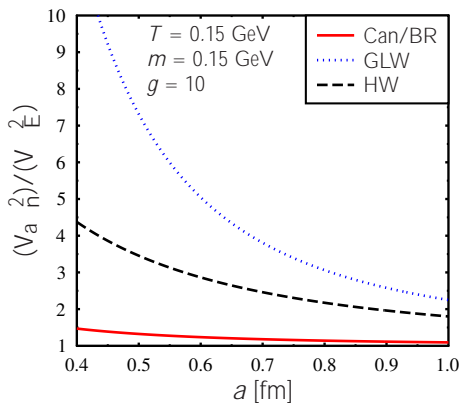


Figure: Same as above but for $T = m = 0.15$ GeV.

Summary:

Expressions for quantum fluctuations of energy density in subsystems of a hot relativistic gas of particles with spin $\frac{1}{2}$ derived.

They depend on the form of the energy-momentum tensor.

For sufficiently large subsystems the results obtained in different pseudo-gauges converge and agree with the canonical-ensemble formula known from statistical physics.

On the practical side, the results of our calculations can be used to determine a scale of coarse graining for which the choice of the pseudo-gauge becomes irrelevant, which may be useful, in particular, in the context of hydrodynamic modeling of high-energy collisions.

