Lepton Mass and Mixing Patterns from Residual Modular Symmetries

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PASCOS 2021
IBS, Center for Theoretical Physics of the Universe, via Zoom, 16 June 2021

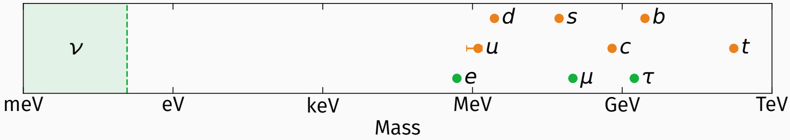
Takeaways/Outline

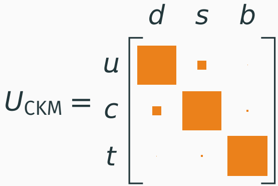
1. Modular symmetry provides a predictive framework for describing flavor parameters
2. Hierarchical mass patterns arise naturally within this framework [Penedo, Petcov, PN, 2102.07488]

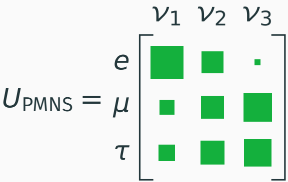
Modular symmetry as a flavor symmetry

Family problem: three generations living together

3 generations $\geq$ 20 flavor observables





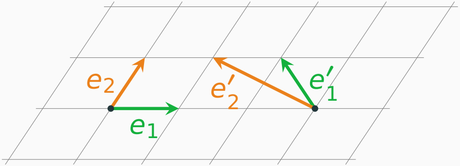


Natural approach: flavor symmetry mixing generations $\phi_i \to \rho_{ij}\,\phi_j$

Modular group

Modulus: $\tau = \dfrac{e_2}{e_1} \in \mathbb{C}$

$$\tau' = \frac{e_2'}{e_1'} = \frac{a\tau + b}{c\tau + d}$$



Modular group: $\Gamma = \mathrm{SL}(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \begin{matrix} a,\, b,\, c,\, d \in \mathbb{Z}\,, \\ ad - bc = 1 \end{matrix} \right\}$

Fields mix and rescale: $\phi_i' = \boxed{(c\tau + d)^{-k}} \; \boxed{\rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij}} \; \phi_j$

weight
$k \in \mathbb{Z}$

representation of Γ
“almost trivial”
$\Gamma/\Gamma(N) \simeq S_n, A_n$

Bottom-up modular invariance

[Feruglio, 1706.08749]

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma : \quad \tau \to \frac{a\tau + b}{c\tau + d} \qquad \boxed{\phi_i \to (c\tau + d)^{-k} \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} \phi_j}$$

$$W \supset \boxed{Y(\tau)}\ \boxed{\phi^1 \phi^2 \dots \phi^n}$$

$$k = k_1 + k_2 + .. + k_n$$

$$\rho \otimes \rho_1 \otimes \rho_2 \otimes \dots \otimes \rho_n \supset \mathbf{1}$$

$$\boxed{(c\tau + d)^k \rho}\ \boxed{(c\tau + d)^{-(k_1 + k_2 + \dots + k_n)} \rho_1 \otimes \rho_2 \otimes \dots \otimes \rho_n} \times \text{itself}$$

$$\boxed{Y_i\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^k \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} Y_j(\tau)}$$

couplings are modular forms

Modular invariant models of lepton flavor

$L \sim (k_L, \rho_L),\ E^c \sim (k_E, \rho_E),\ \ldots$ → mass matrices:

$$M_\nu = g_1 \begin{pmatrix} 2\,Y_1(\tau) & 0 & 0 \\ 0 & \sqrt{3}\,Y_2(\tau) & -Y_1(\tau) \\ 0 & -Y_1(\tau) & \sqrt{3}\,Y_2(\tau) \end{pmatrix}$$

$$+\ g_2 \begin{pmatrix} 0 & -Y_4(\tau) & Y_5(\tau) \\ -Y_4(\tau) & -Y_3(\tau) & 0 \\ Y_5(\tau) & 0 & Y_3(\tau) \end{pmatrix}$$

$$\chi^2\,(\tau, g_1, g_2, \alpha) \to \min$$

$$M_e = \alpha \begin{pmatrix} 0 & Y_5(\tau) & -Y_4(\tau) \\ -Y_5(\tau) & 0 & Y_3(\tau) \\ Y_4(\tau) & -Y_3(\tau) & 0 \end{pmatrix}$$

Flavor patterns are unexplained

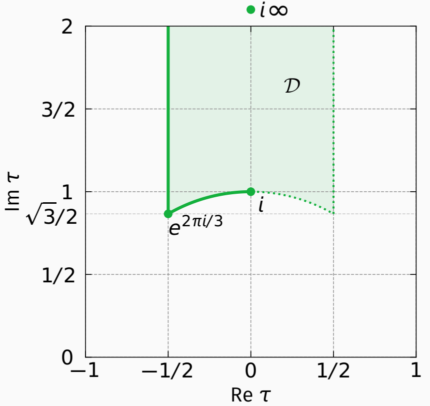
Flavor patterns from residual modular symmetries

Residual symmetries

τ breaks modular symmetry: $\tau \neq \dfrac{a\tau + b}{c\tau + d}$

Symmetric points:

τ_{sym}	inv. under	group
i	$\tau \to -\frac{1}{\tau}$	$\mathbb{Z}_2$
$e^{2\pi i/3}$	$\tau \to -\frac{1}{\tau+1}$	$\mathbb{Z}_3$
$i\infty$	$\tau \to \tau + 1$	$\mathbb{Z}$



Key idea: some flavor observables may vanish as $\tau \to \tau_{\text{sym}}$

Hierarchical mass matrices

$$\epsilon \equiv |\tau - \tau_{\text{sym}}| \qquad\qquad E^c_i\, M_{ij}(\tau)\, L_j$$

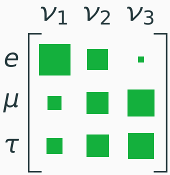
$$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}: \quad M(\gamma\tau) = (c\tau + d)^{k_E + k_L} \rho_E(\gamma)^* M(\tau)\, \rho_L(\gamma)^\dagger$$

$$\left.\begin{array}{ll} L_i \sim \mathbf{3} & \rightsquigarrow \mathbf{1}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_3 \\ E^c_i \sim \mathbf{3}' & \rightsquigarrow \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_4 \end{array}\right\} \Rightarrow M(\tau) \sim \begin{pmatrix} 1 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & \epsilon & \epsilon^3 \end{pmatrix} \Rightarrow \begin{array}{l} m_e \sim \epsilon^4 \\ m_\mu \sim \epsilon \\ m_\tau \sim 1 \end{array}$$

Key results:

- Hierarchical masses are possible
- List of field representations which yield them

Large mixing in the symmetric limit



$$\xrightarrow{\tau \to \tau_{\text{sym}}} \begin{bmatrix} * & * & 0 \\ * & * & * \\ * & * & * \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

1. $\begin{cases} L & \rightsquigarrow 1 \oplus 1 \oplus 1 \\ E^c & \rightsquigarrow 1 \oplus \ldots \end{cases}$
2. $\begin{cases} L & \rightsquigarrow \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}^* \\ E^c & \rightsquigarrow \mathbf{1}^* \oplus \ldots \end{cases}$

[Reyimuaji, Romanino, 1801.10530]

3. $m_e, m_\mu, m_\tau \to 0$
4. $m_1^\nu, m_2^\nu, m_3^\nu \to 0$

Example model

- hierarchical masses
- large mixing
- predictivity

→

$$E^c \sim (\hat{\mathbf{3}}, 4) \qquad \epsilon \simeq \left|\tau - e^{2\pi i/3}\right|$$
$$N^c \sim (\mathbf{3}', 1)$$
$$L \sim (\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}', 2)$$

$$M_e \propto \begin{pmatrix} 1 & \alpha - 2\beta & 2\sqrt{3} i\gamma \\ \sqrt{3}\epsilon & \sqrt{3}(\alpha + 2\beta)\epsilon & 2i\gamma\epsilon \\ \frac{5}{2}\epsilon^2 & \left(\frac{5}{2}\alpha - \beta\right)\epsilon^2 & -\frac{5}{\sqrt{3}} i\gamma\epsilon^2 \end{pmatrix}$$

$$m_e = \mathcal{O}(\epsilon^2)$$
$$m_\mu = \mathcal{O}(\epsilon)$$
$$m_\tau = \mathcal{O}(1)$$

$$M_\nu \propto \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & a \\ 1 & a & 2i\sqrt{\frac{2}{3}} b \end{pmatrix}$$

$\epsilon \simeq 0.02$	$\alpha = 2.45 \pm 0.44$
$a = 1.5 \pm 0.15$	$\beta = 2.14 \pm 0.32$
$b = 2.22 \pm 0.17$	$\gamma = 0.91 \pm 0.05$

$$\Sigma m^\nu \approx 0.059 \text{ eV} \qquad \delta = \pi + \mathcal{O}(10^{-6}) \qquad |\langle m \rangle| = (1.44 \pm 0.33) \text{ meV}$$

Takeaways/Summary

1. Modular symmetry provides a predictive framework for describing flavor parameters: $M(\tau, \alpha_i)$
2. Hierarchical mass patterns arise naturally within this framework: $\tau \approx \tau_{\text{sym}}$

Thank you!