ENTANGLEMENT ENTROPY AT CRITICAL POINTS IN THE MULTIVERSE

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"Entanglement entropy at critical points of classical evolution in oscillatory and exotic singularity multiverse models"
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OUTLINE

1) CANONICAL QUANTUM GRAVITY
2) "THIRD" QUANTIZATION
3) INVARIANT VACUUM
4) ENTANGLEMENT ENTROPY:

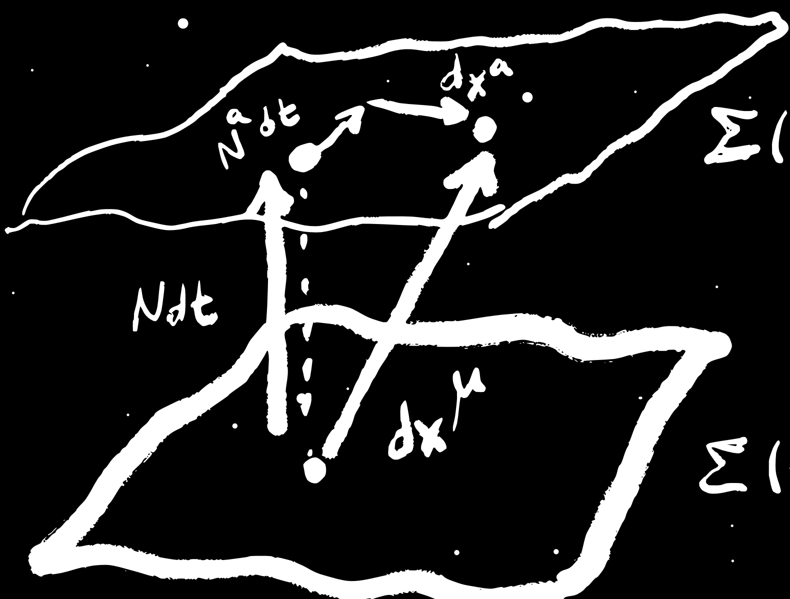
- TWO INTERACTING UNIVERSES -

5) CONCLUSIONS AND PROSPECTIVE

1 CANONICAL QUANTUM GRAVITY

- DIRAC (1958) → ADM FORMALISM (1959)

  → DEWITT (1967)



$\Sigma(t+dt) \Rightarrow \{g_{\mu\nu}\} \to \{N_a, h_{ab}\}$

Foliation of the spacetime (ADM)

$\Sigma(t)$

$$\Rightarrow \boxed{\tilde{p}^{ab} = -i\frac{\partial}{\partial h_{ab}}, \quad p_\phi = -i\frac{\partial}{\partial \phi}}$$

$\Rightarrow$ $\boxed{\mathcal{H}\Psi = 0}$ "Hamiltonian Constraint"

$$\left.\begin{array}{l}\Psi \equiv \text{wave-function of the Universe} \\ \mathcal{H} \equiv \text{quantized Hamiltonian}\end{array}\right\}$$

$\Rightarrow$ $\boxed{\text{FLRW}}$ Homogeneity & Isotropy : $\{h_{ab}\} \rightarrow \{a\}$

Minisuperspace !!

\+ Matter Fields $\{\phi_i\}$

2 "THIRD" QUANTIZATION

- Analogy : $\begin{cases} QM \rightarrow QFT \\ \Psi \rightarrow \text{Particles on a field of Universes} \end{cases}$

$\rightsquigarrow$ Ψ complex scalar field

$\rightsquigarrow$ Pairs of Universes

$\equiv$ particle + Antiparticles

$|0\rangle$

$[\Psi = \bar{\Psi}^{+}]$

$\rightsquigarrow$ $|0\rangle \equiv$ No time / no space.

3 INVARIANT VACUUM

$\rightsquigarrow$ $\boxed{N=2} \Rightarrow dN=0 \Leftrightarrow$ $\boxed{\text{Invariant Representation}}$

$\rightsquigarrow$ Lewis & Riesenfeld (1969)

$$\Rightarrow |\bar{\Psi}\rangle_{inv}^{t=0} \equiv |00\rangle_{inv} \sim \sum_{ij} a_{ij}\, |ij\rangle_{varying}$$

$\rightsquigarrow$ Then, the <u>entanglement entropy</u>:

$$\rho_{red}^{(i)} = \langle j | \rho_{0,inv} | j \rangle \Rightarrow \boxed{S_{ent}^{(i)} = -\mathrm{Tr}\left[\rho_i \ln \rho_i\right]}$$

4 ENTANGLEMENT ENTROPY

[Pair of Universes]

- $\{a, \phi\}$ with $V(\phi) = 0$.
- Wheeler-de Witt Eq.: $[\alpha = \ln(a)]$

$$\Big[\partial^2/\partial\alpha^2 \underbrace{- \frac{\partial^2}{\partial\phi^2} + f(\Lambda, \tilde{K}, \alpha \ldots)}_{\omega^2(a, \phi)} \Big] \Psi(\alpha, \phi) = 0$$

$\Lambda \equiv$ cosmological constant

$\tilde{K} \equiv$ curvature index

$$\Rightarrow \Psi(\alpha, \phi) = \int_K dK \, A(K) \, e^{\pm iK\phi} \, \varphi_K(\alpha)$$

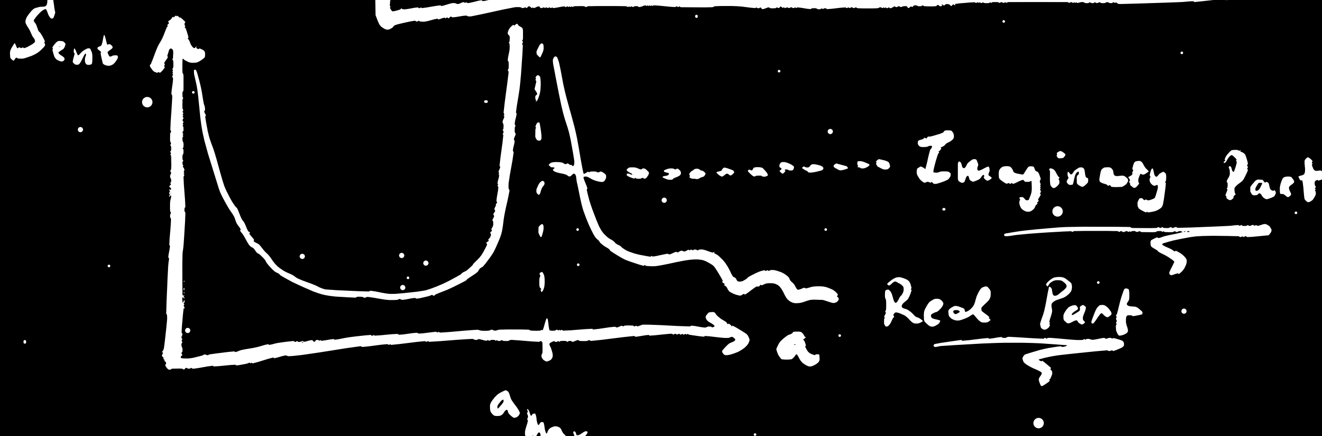
$\rightsquigarrow$ Symmetries : $\boxed{A(K) = A(-K)}$

$\Rightarrow$ Exact $\boxed{S_{ent}}$ for any model.

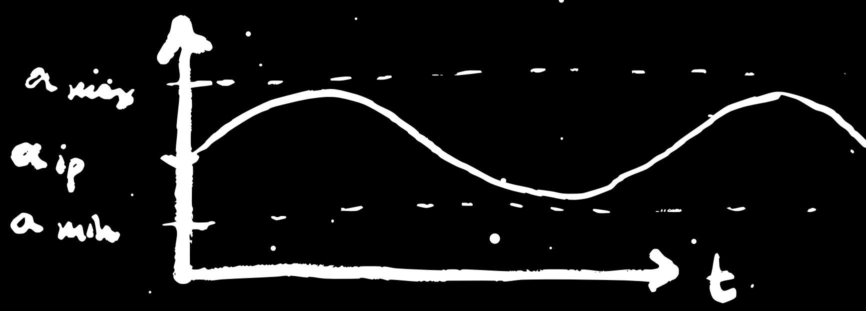
<u>But</u>: $\lim_{\omega \to 0} [S_{ent}] \to \infty$. (Analytically)

$\rightsquigarrow$ Kiefer (1988): Maximum Expansion $\equiv$ Quantum Point

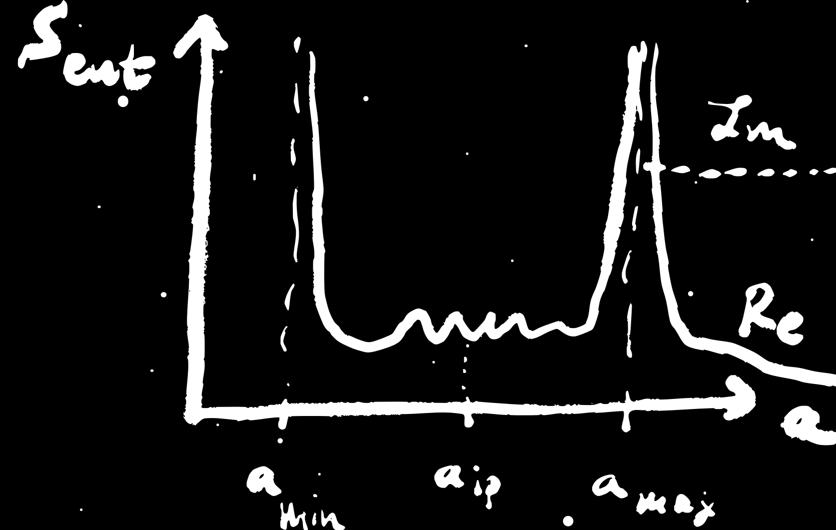
$\Rightarrow$ We found: $\boxed{S_{ent} \equiv \text{Test of classically allowed and forbidden region}}$



- Interesting Model: <u>Oscillating Universe</u>



$a(t) \sim \sin(t)$



H = 0

$\Rightarrow$ $S_{ent} \to \infty$ at critical points (max, min)

$\Rightarrow$ $S_{ent} \not\to \infty$ at inflection points (i.p.)

$H \neq 0$

- We found Renyi and Tsallis entropies to behave similarly.
- It was previously suggested that the Temperature of Entanglement

$$T(a) = \frac{\omega(a)}{2 \ln[\coth(r)]}, \quad r = \operatorname{arctgh}\left(\frac{|\beta|}{|\alpha|}\right)$$

Bogoliubov coefficients

was a good measure of the quantumness of the system.

$\Rightarrow$ We found it not to be always true for some exotic singularities

- Finally, a relation between S_{ent} and H around critical points:

We tried:

$$S_{ent} = \begin{cases} c_0 + \frac{c_1}{H} + \frac{c_2}{H^2} + \frac{c_3}{H^3} + \cdots \\ c_0 + c_1 \ln(H) \end{cases}$$

FITS

for $A(K) = \delta(K-1)$, $\tilde{K} = 1$.

$\Rightarrow$ PERFECT FIT : $\boxed{S_{ent} = c_0 - \ln(H)} \sim I(H)$

Shannon Information

around maxima and minima of the classical evolution.

5 CONCLUSIONS AND PROSPECTIVE

- Exact Entropy of Entanglement for Pair of Universes
- $A(K)$, if ψ represents a Universe of a pair, fulfils $A(K) = A(-K)$.
- It diverges at critical points where $H=0$.

  $\Rightarrow$ Kiefer (1988) confirmed.

- Temperature of Entanglement sometimes fails as a measure of the quantumness.
- $S_{ent} \sim -\ln(H)$ around critical points where $H=0$.

⇒ Third Quantization : Quite Natural

Imprints of a unique pair ?

⇒ Actual Research

⇒ More complex scenario where :

- $V(\phi) \neq 0$, and very massive (to behave as infleton)
- There is not only a pair, but a whole interacting multiverse.

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