Anomalies and Supersymmetry

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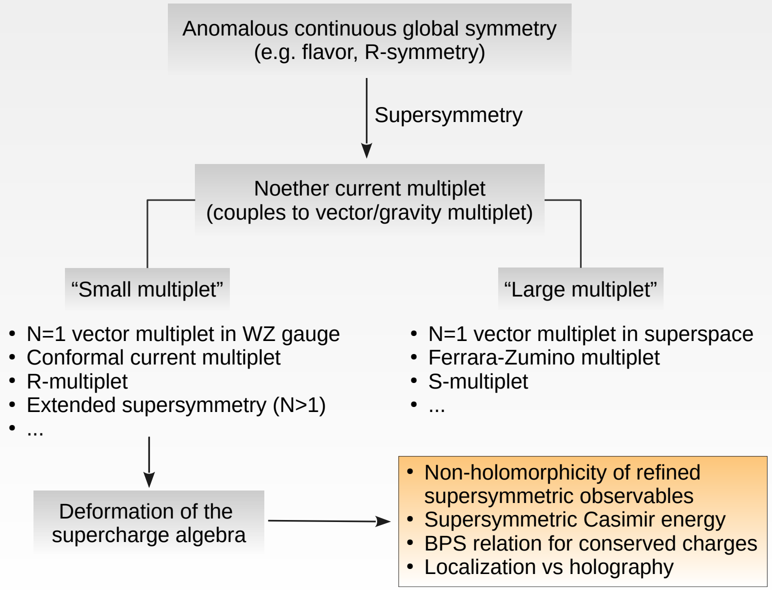
Anomalous continuous global symmetry
(e.g. flavor, R-symmetry)
Supersymmetry
Noether current multiplet
(couples to vector/gravity multiplet)
"Small multiplet"
• N=1 vector multiplet in WZ gauge
• Conformal current multiplet
• R-multiplet
• Extended supersymmetry (N>1)
• ...
"Large multiplet"
• N=1 vector multiplet in superspace
• Ferrara-Zumino multiplet
• S-multiplet
• ...
Deformation of the supercharge algebra
• Non-holomorphicity of refined supersymmetric observables
• Supersymmetric Casimir energy
• BPS relation for conserved charges
• Localization vs holography

References

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Outline

Outline

Noether's theorem

Continuous global symmetries lead to a conserved current

$$\partial_\mu \mathcal{J}^\mu = 0$$

and an associated conserved charge

$$\mathcal{Q} = \int_{\mathcal{C}_t} d^{d-1}x \; \mathcal{J}^t$$

The conserved charges satisfy an algebra

$$\big[\mathcal{Q}_I, \mathcal{Q}_J\big\} = f_{IJ}{}^K \mathcal{Q}_K, \qquad I, J, K = 1, 2, 3, \cdots$$

Background fields

Global symmetries can be studied by turning on background gauge fields A_μ that couple to the Noether currents $\mathcal{J}^\mu$, i.e. (to linear order)

$$Z[A] = e^{iW[A]} = \int \mathcal{D}\phi \exp\left(iS[\phi] + i\int d^d x\; A\cdot\mathcal{J}\right)$$

The full, non-linear, functional $W[A]$ is obtained by gauging the global symmetry and integrating over the matter fields only – not A_μ

$W[A]$ is the generating function of connected current correlators

$$\langle \mathcal{J}^{\mu_1}(x_1)\mathcal{J}^{\mu_2}(x_2)\cdots\mathcal{J}^{\mu_n}(x_n)\rangle = \frac{\delta^n W[A]}{\delta A_{\mu_1}(x_1)\delta A_{\mu_2}(x_2)\cdots\delta A_{\mu_n}(x_n)}$$

't Hooft anomalies

If $W[A]$ cannot not be made invariant under $A \to A + \mathcal{D}\vartheta$ by a choice of local counterterms the theory has a 't Hooft anomaly

$$\delta_\vartheta W[A] = \underbrace{G(\vartheta, A)}_{\text{integrated anomaly}} = \int d^d x\, \vartheta \cdot \underbrace{\mathcal{A}(A)}_{\text{local anomaly}}$$

The gauge dependence of the effective action is equivalent to the non-conservation of the current

$$\delta_\vartheta W[A] = \int d^d x\, \underbrace{\frac{\delta W}{\delta A_\mu^a}}_{\langle \mathcal{J}_a^\mu \rangle} (\mathcal{D}_\mu \vartheta)^a = \int d^d x\, \vartheta \cdot \mathcal{A}(A) \;\;\Rightarrow\;\; \mathcal{D}_\mu \langle \mathcal{J}^\mu \rangle = -\mathcal{A}(A)$$

Wess-Zumino conditions

The anomalies must satisfy the Wess-Zumino (WZ) consistency conditions, whose infinitesimal form is

$$(\delta_{\vartheta_1}\delta_{\vartheta_2} - \delta_{\vartheta_2}\delta_{\vartheta_1})W[A] = \delta_{[\vartheta_1,\vartheta_2]}W[A]$$

or

$$\delta_{\vartheta_1}G(\vartheta_2, A) - \delta_{\vartheta_2}G(\vartheta_1, A) = G([\vartheta_1, \vartheta_2], A)$$

Nontrivial solutions of the WZ conditions are local functionals

$$G(\vartheta, A) = \int d^d x\, \vartheta \cdot \mathcal{A}(A) \neq \delta_\vartheta W_{\mathrm{loc}}[A]$$

for some local functional $W_{\mathrm{loc}}[A]$

Independent solutions are ***consistent anomalies*** – classified by solving a BRST cohomology problem using anomaly descent

BZ polynomial and the covariant current

The anomaly $G(\vartheta, A)$ implies that the consistent current $\mathcal{J}^\mu$ does not transform covariantly under gauge transformations

$$\delta_\vartheta \langle \mathcal{J}^\mu_a \rangle = \delta_\vartheta \Big(\frac{\delta W[A]}{\delta A^a_\mu} \Big) = \delta_\vartheta \Big(\frac{\delta}{\delta A^a_\mu} \Big) W[A] + \frac{\delta G(\vartheta, A)}{\delta A^a_\mu}$$
$$= [\langle \mathcal{J}^\mu \rangle, \vartheta]_a + \frac{\delta G(\vartheta, A)}{\delta A^a_\mu}$$

Bardeen and Zumino (BZ) [Bardeen, Zumino '84] showed that there exists a local polynomial $\mathcal{X}^\mu(A)$ that transforms as

$$\delta_\vartheta \mathcal{X}^\mu_a(A) = [\mathcal{X}^\mu(A), \vartheta]_a - \frac{\delta G(\vartheta, A)}{\delta A^a_\mu}$$

It follows that the sum of $\mathcal{J}^\mu$ and $\mathcal{X}^\mu(A)$ does transform covariantly

$$\langle \mathcal{J}^\mu_{\rm cov} \rangle \equiv \langle \mathcal{J}^\mu \rangle + \mathcal{X}^\mu(A), \qquad \delta_\vartheta \langle \mathcal{J}^\mu_{\rm cov} \rangle = [\langle \mathcal{J}^\mu_{\rm cov} \rangle, \vartheta]$$

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Supersymmetry Ward identity

In supersymmetric theories W satisfies the WZ condition

$$(\delta_\vartheta \delta_\epsilon - \delta_\epsilon \delta_\vartheta) W = 0$$

However, using the general form of the flavor anomaly we determine

$$\delta_\epsilon \delta_\vartheta W = \delta_\epsilon G(\vartheta, A) = -\delta_\vartheta \int d^d x \, \delta_\epsilon A \cdot \mathcal{X}[A] \neq 0$$

Hence, $\delta_\vartheta \delta_\epsilon W \neq 0$. In particular,

$$\delta_\epsilon W = -\int d^d x \, \underbrace{\delta_\epsilon A \cdot \mathcal{X}[A]}_{\text{universal}} + \underbrace{\text{gauge invariant}}_{\text{multiplet dependent}} \neq 0$$

The gauge invariant part is determined by the $\delta_\epsilon \delta_{\epsilon'}$ WZ condition

The supersymmetry Ward identity, therefore, takes the general form

$$\int d^d x \, (-\bar{\epsilon} \, \partial_\mu \langle \mathcal{S}^\mu \rangle + \delta_\epsilon A_\mu \cdot \langle \mathcal{J}^\mu_{\text{cov}} \rangle) = \text{gauge invariant}$$

Example: 4d $\mathcal{N} = 1$ vector multiplet

As a first example to illustrate this structure, let us consider 4d $\mathcal{N} = 1$ theories with an anomalous Abelian flavor symmetry

$$\partial_\mu \langle \mathcal{J}^\mu \rangle = \frac{\kappa}{4} \widetilde{F}^{\mu\nu} F_{\mu\nu}, \qquad \widetilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A_μ and $\mathcal{J}^\mu$ belong to vector multiplets, which in the WZ gauge comprise respectively $(A_\mu, \lambda^\alpha, D)$ and $(\mathcal{J}^\mu, \mathcal{O}^\lambda_\alpha, \mathcal{O}_D)$

The flavor current and background multiplets are related as usual by

$$\langle \mathcal{J}^\mu \rangle = \frac{\delta W}{\delta A_\mu}, \qquad \langle \mathcal{O}^\lambda_\alpha \rangle = \frac{\delta W}{\delta \lambda^\alpha}, \qquad \langle \mathcal{O}_D \rangle = \frac{\delta W}{\delta D}$$

$\mathcal{N} = 1$ supersymmetry Ward identity

The WZ conditions take the form

$$(\delta_\vartheta \delta_{\vartheta'} - \delta_{\vartheta'} \delta_\vartheta) W = 0, \qquad (\delta_\varepsilon \delta_\vartheta - \delta_\vartheta \delta_\varepsilon) W = 0$$

$$(\delta_\varepsilon \delta_{\varepsilon'} - \delta_{\varepsilon'} \delta_\varepsilon) W = (\delta_\xi + \delta_\vartheta) W, \quad \xi^\mu = -2i(\varepsilon \sigma^\mu \bar{\varepsilon} - \varepsilon' \sigma^\mu \bar{\varepsilon}), \quad \vartheta = -\xi^\mu A_\mu$$

Their solution is [Itoyama, Nair, Ren ’85; Guadagnini, Mintchev ’86]

$$\delta_\vartheta W = -\frac{\kappa}{4} \int d^4x \, \vartheta \, \widetilde{F}^{\mu\nu} F_{\mu\nu}, \qquad \kappa = \text{constant}$$

$$\delta_\varepsilon W = \int d^4x \, \Big(- \underbrace{i\varepsilon\sigma_\mu \bar{\lambda}}_{\delta_\varepsilon A_\mu} \, \underbrace{\kappa \, \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}}_{\mathcal{X}^\mu(A)} + \underbrace{3\kappa i \, \varepsilon \lambda \bar{\lambda}^2}_{\text{gauge invariant}} + h.c. \Big) \neq 0$$

This leads to the supersymmetry Ward identity

$$\partial_\mu \langle \mathcal{S}^\mu_\alpha \rangle + i(\sigma_\mu \bar{\lambda})_\alpha \langle \mathcal{J}^\mu_{\text{cov}} \rangle + \big(iD\delta_\alpha{}^\beta + \tfrac{1}{2} (\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta F_{\mu\nu} \big) \langle \mathcal{O}^\lambda_\beta \rangle - (\sigma^\mu \partial_\mu \bar{\lambda})_\alpha \langle \mathcal{O}_D \rangle = 3\kappa i \lambda_\alpha \bar{\lambda}^2$$

General susy non-invariance

The form of the supersymmetry non-invarinace of the effective action

$$\delta_\epsilon W = -\int d^d x \underbrace{\delta_\epsilon A \cdot \mathcal{X}[A]}_{\text{universal}} + \underbrace{\text{gauge invariant}}_{\text{multiplet dependent}} \neq 0$$

in the presence of an anomalous global symmetry is general and extends to local supersymmetry

Examples include, all chiral vector multiplets in even dimensions and several current multiplets (e.g. conformal and R-multiplet in 4D)

This non-invariance is determined by the WZ conditions and may be computed systematically via supersymmetric ***anomaly descent*** or ***anomaly inflow***

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Supersymmetric anomaly inflow

Flavor and gravitational anomalies are in one-to-one correspondence with a Chern-Simons form $\Omega_{d+1}(A,F)$ in one dimension higher

$$\Omega_{d+1}(A,F)=\tfrac{d+2}{2}\int_0^1 dt\,P_{d+2}\Big(A,F_t^{\frac{d}{2}}\Big),\qquad F_t\equiv tF+(t^2-t)A^2$$

which ‘cancels’ the anomaly through inflow

$$\delta_\vartheta\Big(W[A]+N\int_{M_{d+1}}\Omega_{d+1}(A,F)\Big)=0$$

This anomaly inflow mechanism extends to vector and gravity multiplets provided a supersymmetric CS action exists such that

$$\delta_{\{\vartheta,\epsilon,\cdots\}}\big(W[A,\cdots]-S_{\rm CS}[A,\cdots]\big)=0$$

Example: 2d $\mathcal{N} = (p, q)$ flavor anomalies

The maximal vector multiplet with a pure CS action in 3d is the $\mathcal{N} = 3$ multiplet $(A_\mu, \lambda^I, \chi, \sigma^I, D^I)$, $I = 1, 2, 3$

The $\mathcal{N} = 3$ CS action is [Kao, Lee, Lee '95]

$$\mathcal{L}_{\rm CS} = \frac{k}{4\pi}\,\mathrm{tr}\Big(\varepsilon^{\mu\nu\rho}\big(A_\mu\partial_\nu A_\rho - \tfrac{2i}{3}A_\mu A_\nu A_\rho\big) \\ - \bar{\lambda}^I\lambda_I + \bar{\chi}\chi - 2\sigma^I D_I + \tfrac{i}{3}\varepsilon_{IJK}\sigma^I[\sigma^J, \sigma^K]\Big)$$

Varying this action determines the anomalies for the 2d $\mathcal{N} = (p, q)$, $p, q \leq 3$, vector multiplet obtained by dimensional reduction

$$\delta_G(\vartheta)W = \frac{k}{4\pi}\varepsilon^{\hat{\nu}\hat{\rho}}\int d^2x\,\mathrm{tr}\,(\vartheta\,\partial_{\hat{\nu}}A_{\hat{\rho}})$$

$$\delta_Q(\epsilon)W = \frac{k}{4\pi}\int d^2x\,\mathrm{tr}\,\big(\delta_Q(\epsilon)A_{\hat{\nu}}\underbrace{\varepsilon^{\hat{\nu}\hat{\rho}}A_{\hat{\rho}}}_{\mathcal{X}(A)} - \underbrace{2\,\sigma^I(\varepsilon_{IJK}\bar{\epsilon}^J\gamma_*\lambda^K + \bar{\epsilon}_I\gamma_*\chi)}_{\text{gauge invariant}}\big)$$

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Anomalous fermionic operators

How does $\delta_\epsilon W \neq 0$ affect supersymmetric observables?

Vector multiplets:

$$\delta_\epsilon \langle \mathcal{O}_\lambda \rangle = \langle \{ \overline{\mathcal{Q}}[\epsilon], \mathcal{O}_\lambda \} \rangle = \delta_\epsilon \left(\frac{\delta}{\delta \overline{\lambda}} \right) W + \frac{\delta}{\delta \overline{\lambda}} \delta_\epsilon W$$
$$= \epsilon \langle \mathcal{J} \rangle + \underbrace{\frac{\delta}{\delta \overline{\lambda}} \delta_\epsilon W}_{\neq 0}$$

Gravity multiplets:

$$\delta_\epsilon \langle \mathcal{S}^\mu \rangle = \langle \{ \overline{\mathcal{Q}}[\epsilon], \mathcal{S} \} \rangle = \delta_\epsilon \left(\frac{\delta}{\delta \overline{\psi}_\mu} \right) W + \frac{\delta}{\delta \overline{\psi}_\mu} \delta_\epsilon W$$
$$= \epsilon \langle \mathcal{T} + \partial \mathcal{J}_R \rangle + \underbrace{\frac{\delta}{\delta \overline{\psi}_\mu} \delta_\epsilon W}_{\neq 0}$$

Hence, $\langle \mathcal{J} \rangle$ and $\langle \mathcal{T} + \partial \mathcal{J}_R \rangle$ are ***not*** Q-exact!

Non-holomorphic partition functions

Observables such as supersymmetric partition functions naively depend holomorphically on geometric and flavor fugacities

$$Z[\tau, \nu, \cdots]$$

Holomorphicity is violated when $\langle \mathcal{J} \rangle$ and $\langle \mathcal{T} + \partial \mathcal{J}_R \rangle$ are not Q-exact:

Geometric fugacities [IP arXiv:1703.04299]

$$\frac{\partial}{\partial \bar{\tau}} Z[\tau, \nu, \cdots] \sim \int \zeta \langle \mathcal{T} + \partial \mathcal{J}_R \rangle \sim -\int \frac{\delta}{\delta \overline{\psi}_\mu} \delta_\epsilon W \big|_{\epsilon = \zeta} \neq 0$$

Flavor fugacities [IP arXiv:1904.00347; Cosset, Di Pietro, Kim arXiv:1905.05722]

$$\frac{\partial}{\partial \bar{\nu}} Z[\tau, \nu, \cdots] \sim \int \zeta \langle \mathcal{J} \rangle \sim -\int \frac{\delta}{\delta \overline{\lambda}} \delta_\epsilon W \big|_{\epsilon = \zeta} \neq 0$$

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Summary

- In several commonly encountered Noether current multiplets of anomalous global symmetries supersymmetry is violated

$$\delta_\epsilon W = -\int d^d x \, \underbrace{\delta_\epsilon A \cdot \mathcal{X}[A]}_{\text{universal}} + \underbrace{\text{gauge invariant}}_{\text{multiplet dependent}} \neq 0$$

- The supersymmetry violating terms are determined by the WZ consistency conditions in the presence of supersymmetry
- In many cases, these can be solved by a generalized anomaly descent procedure, or a supersymmetric Chern-Simons form
- The fermionic components of global symmetry anomalies deform the supercharge algebra, affecting the dependence of (refined) supersymmetric observables on geometric and flavor fugacities

Thank you for your attention!