Based on 2104.09529
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$T\bar{T}$ Deformed Fermionic Theories Revisited

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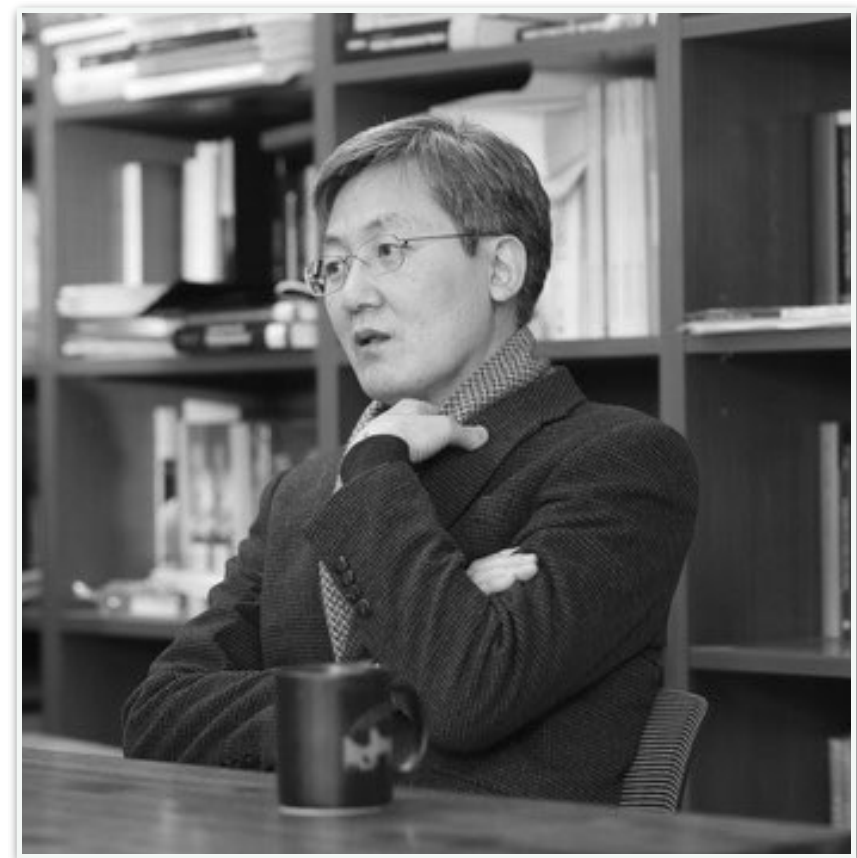
$T\bar{T}$-deformed Fermionic Theories Revisited

Kyungsun Lee, Piljin. Yi and JY
arXiv: 2104.09529

Kyungsun Lee
GIST



Piljin Yi
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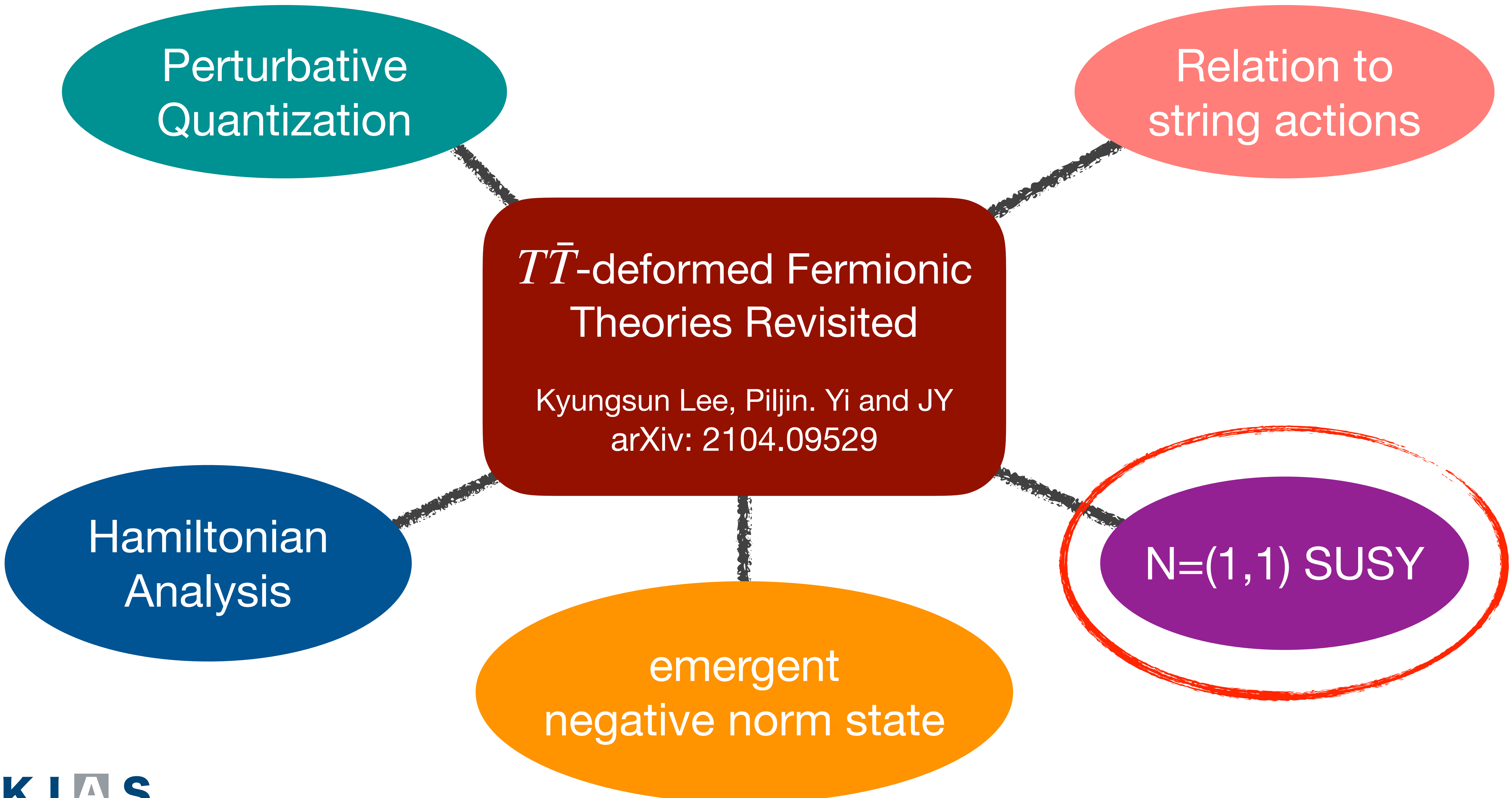
Topics of Our Paper



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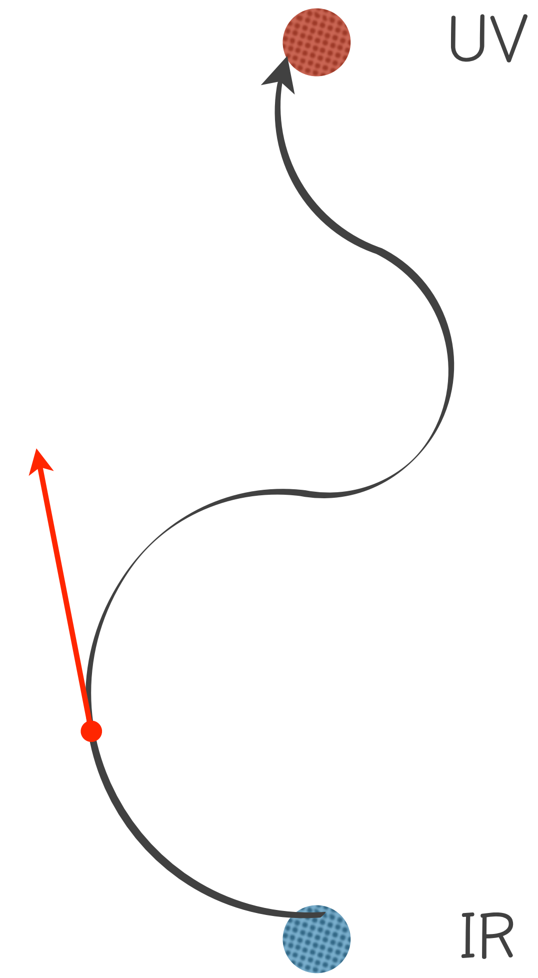
$T\overline{T}$ Deformation

✴ Flow equation for $T\overline{T}$ deformation

$$\partial_\lambda \mathscr{L} = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$$



Features of $T\bar{T}$ Deformation 1

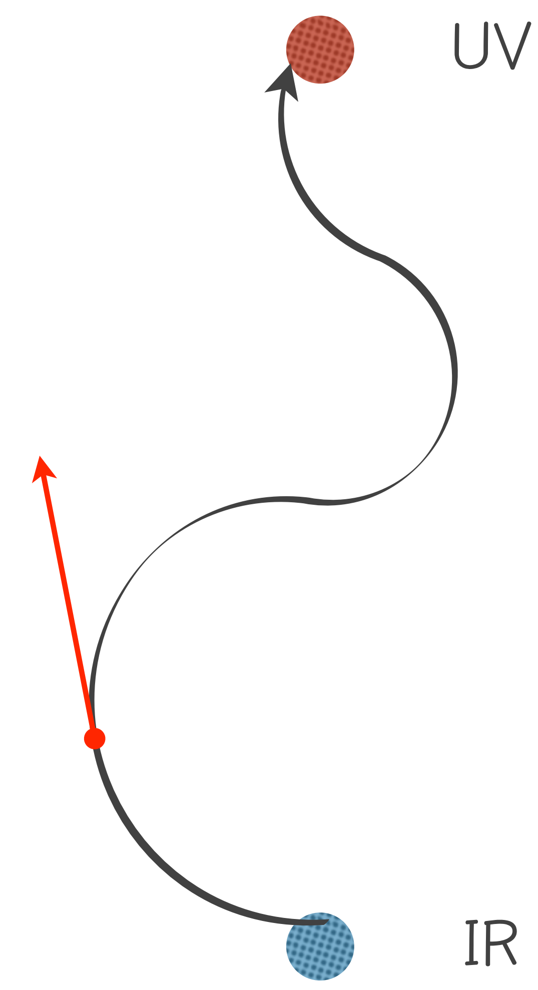
✳ Universal formula for $T\bar{T}$ Deformed Spectrum

$$E_n(L,\lambda) = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}E_n+\frac{4\lambda^2}{L^2}P_n^2}-1\right]$$

$$P_n(L,\lambda) = P_n(L)$$

E_n : Undeformed Energy

P_n : Undeformed Momentum



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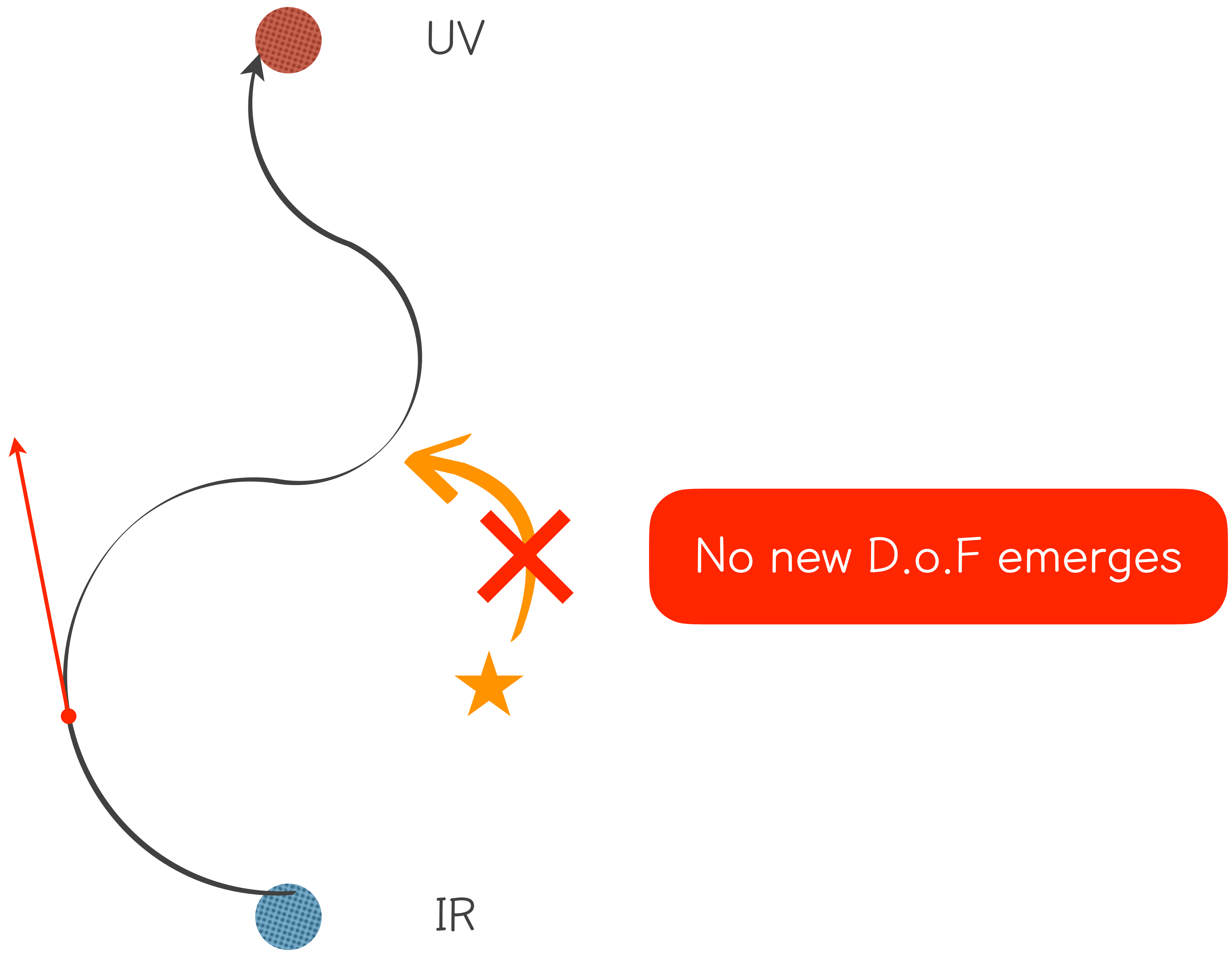
Features of $T\overline{T}$ Deformation 2



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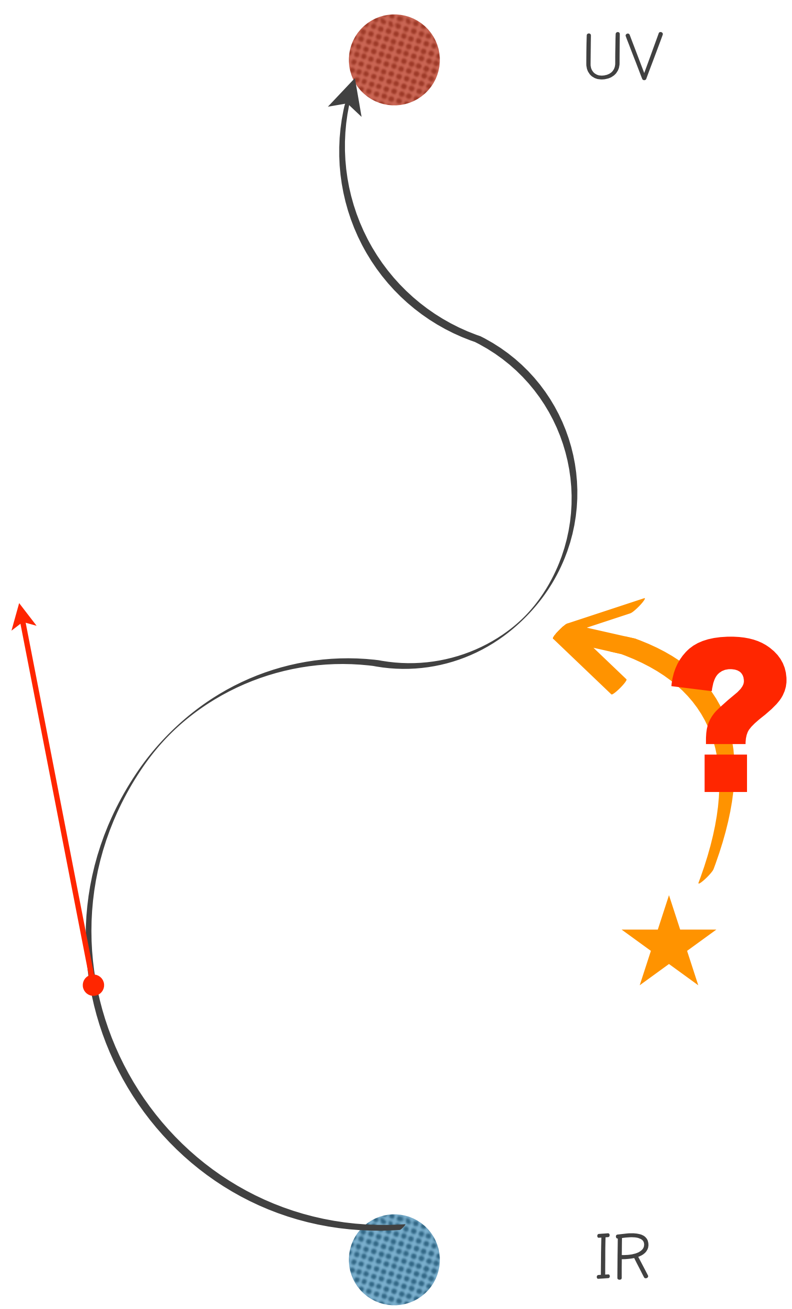
Features of $T\bar{T}$ Deformation 2



$$\partial_\lambda \mathscr{L} = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$$

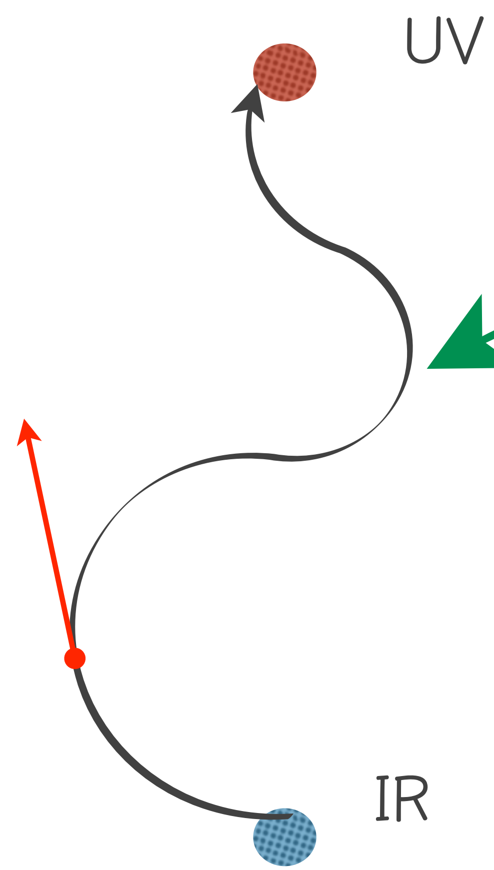
$T_{\mu\nu}$ from Noether procedure

"Usually" NO emergent D.o.F

$T_{\mu\nu}$ from Metric variation

"Usually" emergent D.o.F with negative norm

$T\bar{T}$ Deformation of $\mathcal{N} = (1,1)$ SUSY



$\mathcal{N} = (1,1)$ SUSY

Does $T\bar{T}$ deformation preserve $\mathcal{N} = (1,1)$ SUSY?

$$\partial_\lambda \mathcal{L} = \boxed{\frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}}$$

: Deformation operator is not supersymmetric

Bose-Fermi Degeneracy

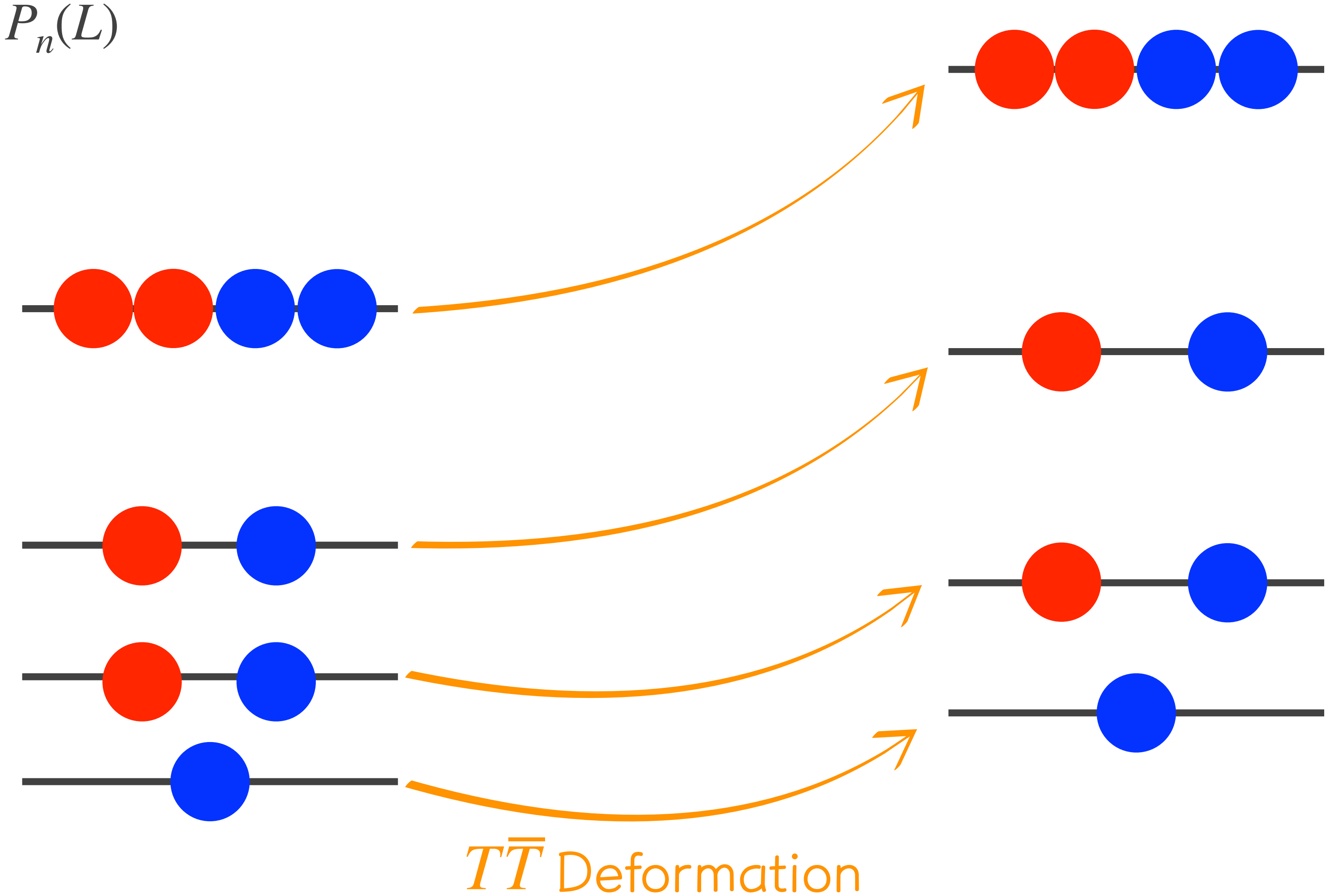
Deformed spectrum both boson and fermion

$$E_n(L,\lambda) = \frac{L}{2\lambda}\left[\sqrt{1+\frac{4\lambda}{L}E_n+\frac{4\lambda^2}{L^2}P_n^2}-1\right]$$

$$P_n(L,\lambda) = P_n(L)$$

E_n : Undeformed Energy

P_n : Undeformed Momentum



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Is there local expression for supercharge Q?

What is the deformed supersymmetry transformation?

$$\delta_+\phi = -\frac{1}{2}\psi_+ \quad + \quad ?$$

SUSY $T\bar{T}$ Deformation?

- SUSY $T\bar{T}$ deformation by superfield $\mathscr{T}_\alpha^{\mu}$
  - ✓ $\mathscr{T}_{++\,+} = S_{++\,+} - \theta^+ T_{++\,++} + \cdots$ energy-momentum tensor
  - ✓ $\partial_\lambda \mathscr{L} = -\int d^2\theta \left(\mathscr{T}_{++\,++} \mathscr{T}_{=\,=} + \mathscr{T}_{++\,=} \mathscr{T}_{=\,++} \right)$
- Emergent extra degrees of freedom related to negative norm states
  - ✓ Deformed Lagrangian: $\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- + \cdots \boxed{+ \lambda\psi_+\dot{\psi}_+\psi_-\dot{\psi}_-}$
  - ✓ Doubling of Fermi degrees of freedom

$T\bar{T}$ Deformation of $\mathcal{N} = (1,1)$ SUSY Model

- Undeformed Lagrangian
  - ✓ $\mathscr{L}_0 = 2\partial_{++}\phi\partial_{=}\phi + i\psi_+\partial_{=}\psi_+ + i\psi_-\partial_{++}\psi_-$
- Solve the flow equation

  $T^{\mu}{}_{\nu}$ from Noether procedure

  - ✓ $\partial_\lambda \mathscr{L} = \frac{1}{2}\epsilon_{\mu\nu}\epsilon^{\rho\sigma}T^{\mu}{}_{\rho}T^{\nu}{}_{\sigma}$
  - ✓ Solution: $\mathscr{L} = -\frac{1}{2\lambda}\left[\sqrt{1 - 8\lambda\partial_{++}\phi\partial_{=}\phi} - 1\right] + \cdots$
- Conjugate momenta
  - ✓ $\pi = \frac{\delta\mathscr{L}}{\delta\dot{\phi}} \qquad \pi_\pm = \frac{\delta\mathscr{L}}{\delta\dot{\psi}_\pm}$

Example: Dirac Bracket of Free Fermion

- Example: Free Fermion $\mathscr{L} = \frac{i}{2}\psi_+\dot{\psi}_+ + \frac{i}{2}\psi_-\dot{\psi}_- - \frac{i}{2}\psi_+\psi_+' + \frac{i}{2}\psi_-\psi_-'$

Linear in $\dot{\psi}_\pm$

$$\pi_\pm = \frac{\delta \mathscr{L}}{\delta \dot{\psi}_\pm} = \frac{i}{2}\psi_\pm$$: 2nd class constraint

Dirac bracket: $\{\psi_+(x), \psi_+(y)\}_D = \delta(x-y)$

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Dirac Bracket

- $T\overline{T}$ deformation of $\mathcal{N} = (1,1)$ model

  2nd class constraint:

$$\pi_+ - \frac{i}{4}\psi_+\left(1 - 2\lambda\pi\phi' + \sqrt{(1+2\lambda\pi^2)(1+2\lambda\phi'^2)}\right) - \lambda\left[\frac{1+\lambda(\pi^2+\phi'^2)}{4\sqrt{(1+2\lambda\pi^2)(1+2\lambda\phi'^2)}} + \frac{1}{4}\right]\psi_+\psi_-\psi_-' = 0$$

- Dirac brackets

$$\{\phi(x), \pi(y)\}_D = \delta(x-y) \qquad \{\phi(x), \phi(y)\}_D = \{\pi(x), \pi(y)\}_D = 0$$

  : same

$$i\{\psi_+(x), \psi_+(y)\}_D = \frac{2\lambda\pi\phi' - 1 + \sqrt{(1+2\lambda\pi^2)(1+2\lambda\phi'^2)}}{\lambda(\pi+\phi')^2}\delta(x-y) + \cdots$$

Supercharges

- Supercharges

$$Q^1_+ = \int dx\, \psi_+(\pi + \phi') \qquad\qquad Q^1_- = \int dx\, \psi_-(\pi - \phi')$$

$$\dot{\phi} = \frac{\pi\sqrt{1+2\lambda\phi'^2}}{\sqrt{1+2\lambda\pi^2}} + \cdots \neq \pi$$

$$\{Q^1_+, Q^1_-\}_D = \{Q^1_\pm, H\}_D = \{Q^1_\pm, P\}_D = 0$$

- Hamiltonian and momentum

$$H = \frac{i}{4}\{Q^1_+, Q^1_+\}_D + \frac{i}{4}\{Q^1_-, Q^1_-\}_D$$

$$P = \frac{i}{4}\{Q^1_+, Q^1_+\}_D - \frac{i}{4}\{Q^1_-, Q^1_-\}_D$$

Global Symmetry

- Global symmetry

Shift scalar field $\phi(x) \longrightarrow \phi(x) + a$ ⇒ $\mathbb{P}^2 = \frac{2\pi}{L}\int dx\ \pi$

Shift fermion $\psi_\pm(x) \longrightarrow \psi_\pm(x) + \eta_\pm$ ⇒ $Q^2_\pm = -\frac{8\pi i}{L}\int dx\ \pi_\pm$

$a\,,\ \eta_\pm$: constants

- Commute with Hamiltonian and momentum

$$\{Q^2_\pm, H\}_D = \{\mathbb{P}^2, H\}_D = \{Q^2_\pm, P\}_D = \{\mathbb{P}^2, P\}_D = 0$$

SUSY and Global Symmetry Algebra

SUSY

$$\boxed{\begin{aligned}\{Q^1_\pm, Q^1_\pm\}_D &= -2i(H \pm P)\\ \{Q^1_+, Q^1_-\}_D &= 0\end{aligned}}$$

Global

$$\boxed{\begin{aligned}\{Q^2_\pm, Q^2_\pm\}_D &= -\frac{16\pi^2 i}{L} - \frac{16\pi^2 i\lambda}{L^2}(H \mp P)\\ \{Q^2_+, Q^2_-\}_D &= 0\end{aligned}}$$

$$\{Q^1_\pm, Q^2_\pm\}_D = -2i\left(\mathbb{P}^2 \pm \frac{4\pi^2}{L^2}\mathbb{W}^2\right)$$

$$\{Q^1_\pm, Q^2_\mp\}_D = 0$$

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Green-Schwarz Action

- $\mathcal{N} = 2$ Green-Schwarz Action for 3D target space

$$\mathcal{L}_{GS} = -\frac{1}{2}\gamma^{\alpha\beta}\Pi^{\mu}_{\alpha}\Pi^{\nu}_{\beta}G_{\mu\nu} - i\epsilon^{\alpha\beta}\partial_\alpha X^\mu(\bar{\Psi}_+\Gamma^\nu\partial_\beta\Psi_+ - \bar{\Psi}_-\Gamma^\nu\partial_\beta\Psi_-)G_{\mu\nu} - \epsilon^{\alpha\beta}(\bar{\Psi}_+\Gamma^\mu\partial_\alpha\Psi_+)(\bar{\Psi}_-\Gamma^\nu\partial_\beta\Psi_-)G_{\mu\nu}$$

- ✓ $\Pi^{\mu}_{\alpha} \equiv \partial_\alpha X^\mu + i\bar{\Psi}_+\Gamma^\mu\partial_\alpha\Psi_+ + i\bar{\Psi}_-\Gamma^\mu\partial_\alpha\Psi_-$
- ✓ WZ term: topological
- ✓ spacetime supersymmetry
- ✓ $\Psi_\pm$: two component Majorana spinor

"Dictionary"

- Shifted Light-cone coordinates and target space metric

$$X^+ \equiv \left(\frac{1}{2} - \Lambda\right) X^1 + \left(\frac{1}{2} + \Lambda\right) X^0 \qquad X^- \equiv X^1 - X^0$$

$T\overline{T}$ Deformation parameter Λ

$$ds^2 = 2\Lambda (dX^-)^2 + 2dX^+ dX^- + (dX^2)^2$$

- 3D target coordinates and spinor

Light-cone gauge

$X^+ = t$: worldsheet time in $T\overline{T}$

X^-

$X^2 = \phi$: scalar field in $T\overline{T}$

fermion in $T\overline{T}$

$$\Psi_\pm = \frac{1}{2}\begin{pmatrix} \psi_\pm \\ 0 \end{pmatrix}$$

gauge fixing of kappa symmetry

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Solving Constraints

- Light-cone gauge : $X^{+} = t$

  Charge p_{+} for translation of target coordinate X^{+} = Hamiltonian of $T\overline{T}$

- Discrete Light-cone quantization : X^{-} is compactified

  non-trivial topological charge for winding mode

$$\mathbb{W}^{-} = \oint d\sigma\, \partial_{\sigma} X^{-} = - mR$$

quantized

- Identification of topological charge and momentum in $T\overline{T}$

$$P = -\frac{2\pi}{L} p_{-} \mathbb{W}^{-}$$

level-matching condition

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3D Target Space SUSY

- $\mathcal{N} = 2$ SUSY of 3D target space

$Q^1_\pm$: $\mathcal{N} = (1,1)$ supercharge

$Q^2_\pm$: fermionic global charge

$$\{Q^\alpha_a, Q^\beta_b\}_D = -2i\delta_{ab}(\Gamma^\mu C)^{\alpha\beta}\mathbb{P}_\mu - \frac{2i}{2\pi\ell_s^2}\sigma^3_{ab}A^{\alpha\beta}$$

topological charge from WZ term

Hamiltonian

$$\Gamma^\mu C\mathbb{P}_\mu = \begin{pmatrix} H & \mathbb{P}^2 \\ \mathbb{P}^2 & \frac{8\pi^2}{L} + 2\Lambda H \end{pmatrix} \qquad A = \Gamma_\mu C \oint d\sigma\, \partial_\sigma X^\mu = \frac{L^2}{4\pi^2}\begin{pmatrix} P & \frac{4\pi^2}{L^2}\mathbb{W}^2 \\ \frac{4\pi^2}{L^2}\mathbb{W}^2 & -2\Lambda P \end{pmatrix}$$

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: bosonic global charge

3D Target Space SUSY

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: momentum

3D Target Space SUSY

- $\mathcal{N} = 2$ SUSY of 3D target space

$Q^1_\pm$: $\mathcal{N} = (1,1)$ supercharge

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topological charge from WZ term

$$\Gamma^\mu C\mathbb{P}_\mu = \begin{pmatrix} H & \mathbb{P}^2 \\ \mathbb{P}^2 & \frac{8\pi^2}{L} + 2\Lambda H \end{pmatrix} \qquad A = \Gamma_\mu C \oint d\sigma\, \partial_\sigma X^\mu = \frac{L^2}{4\pi^2}\begin{pmatrix} P & \frac{4\pi^2}{L^2}\mathbb{W}^2 \\ \frac{4\pi^2}{L^2}\mathbb{W}^2 & -2\Lambda P \end{pmatrix}$$

: topological charge for compactified boson

Comments

✴ Partially broken rigid SUSY

$$\{Q^2_\pm, Q^2_\pm\}_D = -\frac{16\pi^2 i}{L} - \frac{16\pi^2 i\lambda}{L^2}(H \mp P)$$

✓ Due to topological charge

✓ fermionic global symmetry in $T\overline{T}$ deformation

✴ BPS States

✓ BPS states from the point of view of 3D $\mathcal{N} = 2$ SUSY

✓ Protected along $T\overline{T}$ deformation

$$E = \frac{L}{2\lambda}\left[\sqrt{1 + \frac{4\lambda}{L}\left|P - \frac{1}{L}p_m p_w\right| + \frac{2\lambda}{L^2}\left(\frac{L^2}{4\pi^2}p_m^2 + \frac{4\pi^2}{L^2}p_w^2\right) + \frac{4\lambda^2}{L^2}P^2} - 1\right]$$

Thank You