Flow-induced soft hair and the modification to the Hawking temperature

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Outline

- Motivation: Information loss paradox
- Soft hair on Schwarzschild black hole   Hawking, Perry, Strominger (2016)
  - The conserved charge of BMS symmetry
- Soft hair on Vaidya black hole   Chu, Koyama (2018)
- Two extensions to time-dependent supertranslation
- Hawking temperature of a dynamical black hole
- Conclusion

Information loss paradox

- Classical black hole → Information is hidden forever
- Black hole thermodynamics → Hawking radiation
- No hair theorem → M, Q, J are enough to describe Hawking radiation
- Unitarity →Entropy must come form somewhere (Entanglement)
- Pure thermal radiation→ Information loss paradox
- We need more DOFs (secret channel) for BH?
- Evade no-hair theorem → Soft hair on black hole
  - How does the soft hair store black hole information?
- Soft hair induced by a continuous flow in Vaidya spacetime

BMS symmetry

- Bondi gauge $g_{rr} = g_{rA} = 0,\ \det(g_{AB}) = \det(r^2 \gamma_{AB})$
- Ignoring boost, rotation & uniform translation, the most general coordinate transformation preserving the asymptotically flat condition is

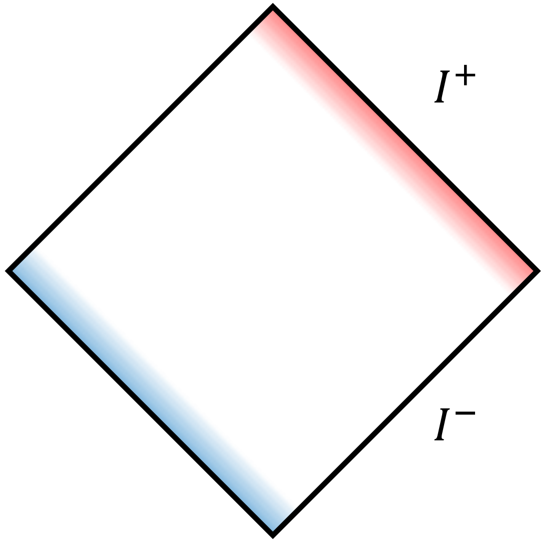
$$\delta v = f(\Theta^A), \delta r = -\frac{1}{2} D^2 f + O(r^{-1}), \delta \Theta^A = \frac{1}{r} D^A f + O(r^{-2})$$

- The generating vector field for BMS symmetry

$$\zeta_f = f \partial_v - \frac{1}{2} D^2 f \partial_r + \frac{1}{r} D^A f \partial_A$$

- The generator of BMS transformation

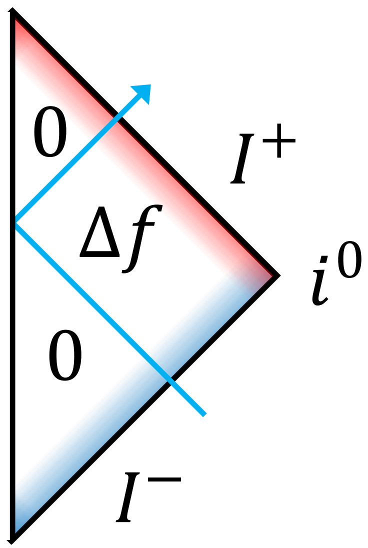
$$Q^-[f] \equiv \int_{I^-} d^2\Theta f(\Theta) \int_{-\infty}^{\infty} dv \frac{1}{32\pi G} N_{AB} N^{AB} + T^M_{vv}$$

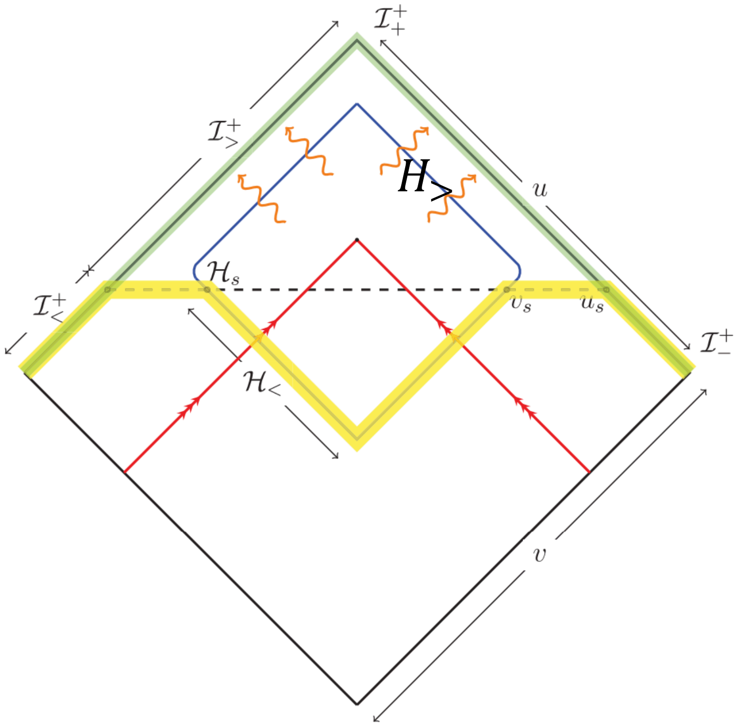


Bondi, Metzner, Sachs (1962)

Scattering Process with Black Hole

$$Q_f^{I^+} = Q_f^{I^-}$$





$$Q_f^{I^+} \approx Q_f^{H_>} + Q_f^{I^+_<}$$

$$Q_f^{H_<} \rightarrow Q_f^{H_>}$$

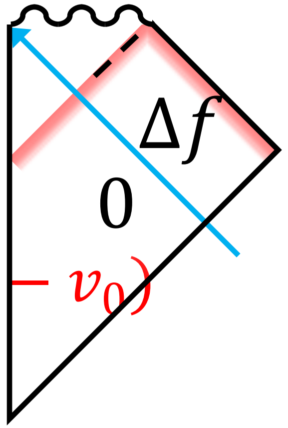
Unknown process…

$$Q_f^{I^-} = Q_f^{H_<} + Q_f^{I^+_<}$$

Hawking, Perry, Strominger (2016)

Implant Soft hair on black hole

- Supertranslated Schwarzschild geometry
- $ds^2 = -\left(V - \frac{MD^2 f}{r^2}\right) dv^2 + 2dvdr - D_A\left(2Vf + D^2 f\right)dvd\Theta^A + \left(r^2\gamma_{AB} + 2rD_AD_Bf - r\gamma_{AB}D^2 f\right)d\Theta^A d\Theta^B, \;\; V = 1 - 2M/r$
- How to implant soft hair? $\rightarrow$ Send a shock-wave from I^-
- Metric transforms as $g_{\mu\nu} \rightarrow g_{\mu\nu} + \theta(v - v_0)\mathcal{L}_f g_{\mu\nu}$
- The constraint equation $\partial_v m = \frac{1}{4} D^A D^B N_{AB} + \left(\mu + \frac{1}{4} D^2 (D^2 + 2) f\right) \delta(v - v_0)$
- The perturbation of the metric is $h_{\mu\nu} = \theta(v - v_0) \left[\mathcal{L}_f g_{\mu\nu} + \frac{2\mu}{r} \delta_\mu^v \delta_\nu^v\right]$
- Supertranslation ⊂ diffeomorphism → observable only on the null shell
- Extension of time dependency?



Hawking, Perry, Strominger (2016)

Super-translated Vaidya metric

- Consider an accreting black hole
- Vaidya metric: $-Vdv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B, \quad V = 1 - 2M(v)/\mathrm{r}$
- The associate energy-momentum tensor $T_{vv} = \frac{M'(v)}{4\pi r^2}$
- Super-translated Vaidya metric:
- $ds^2 = -\left(V - \frac{2M'f}{r} - \frac{MD^2f}{r^2}\right)dv^2 + 2dvdr - D_A\left(2Vf + D^2f\right)dvd\Theta^A + \left(r^2\gamma_{AB} + 2rD_AD_Bf - r\gamma_{AB}D^2f\right)d\Theta^A d\Theta^B$

Time-dependent supertranslation

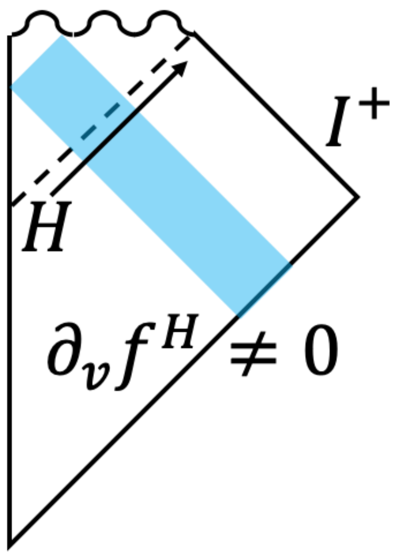
- A coordinate transformation on spacetime: tST transformation
- $g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_{f(v)} g_{\mu\nu}, \quad T_{\mu\nu} \to T + \mathcal{L}_{f(v)} T_{\mu\nu}$
- The tSTed Vaidya spacetime
- $ds^2 = -\left(V - \frac{2M'f}{r} - \frac{MD^2 f}{r^2} + (D^2 + 2V)f'\right) dv^2 + 2(1 + f')dvdr - D_A(D^2 f + 2Vf + 2rf')dvd\Theta^A + (r^2\gamma_{AB} + 2rD_A D_B f - r\gamma_{AB} D^2 f)d\Theta^A d\Theta^B$
- $\mathcal{L}_{f(v)} T_{vv} = \frac{M'f'}{2\pi r^2} + \frac{M''f}{4\pi r^2} + \frac{M'D^2 f}{4\pi r^3}, \quad \mathcal{L}_{f(v)} T_{vA} = \frac{M'D_A f}{4\pi r^2}$
- Recover BMS transformation when $f' = 0$
- What's the effect of tST transformation on physical observable?

Doppler and Unruh effect

- tST transformation can be recast as a dressing process on a scalar fields and the Fourier basis up to $O(f)$ near the horizon
$$\phi(v) = \int_0^\infty \frac{dp}{\sqrt{2\pi p}}\left(a_p e^{-ipv} + a_p^\dagger e^{ipv}\right), \phi(u) = \int_0^\infty \frac{dp}{\sqrt{2\pi p}}\left(b_p e^{-ipu} + b_p^\dagger e^{ipu}\right)$$
- Consider almost radially outgoing modes in the Eikonal limit $q = \omega du + Y_A d\Theta^A + O(f^2)$
- We obtain the dressing : $e^{i\omega f}$: $\hat{b}_\omega \approx b_\omega$, where $b_\omega = b_q, q_u = \omega, f = f(\partial_\omega)$
- Imposing the adiabatic condition $Mf''(u) \ll f'(u)$, the modified Hawking spectrum is
$$\langle g(q) b_q^\dagger b_q \rangle = \langle g(q) \hat{b}_q^\dagger \hat{b}_q \rangle|_{q\to(1+f')q, \kappa\to\kappa-f''}$$
- tST is a coordinate transformation on I preserves all physical observable up to a non-trivial Bogoliubov transformation
- Notice that the dressing of incoming modes is irrelevant to the Hawking radiation. The black hole information requires another channel to restore.

Flow-induced supertranslation

- Can supertranslation has non-trivial effect on Hawking radiation?
- Consider an anisotropic null flow into the black hole
- The metric is transformed by a flow-induced supertranslation (FST), which is not a coordinate transformation.
- $g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_f g_{\mu\nu}|_{f\to f(v)}$
- $T_{\mu\nu} \to T_{\mu\nu}[g_{\mu\nu} \to g_{\mu\nu} + \mathcal{L}_f g_{\mu\nu}|_{f\to f(v)}]$
- FST is the only form for sub-leading part of the initial condition at I^- that leads to a stable asymptote (Christodoulou, Klainerman)



Linear response relation

- BMS transformation: $T_{\mu\nu} \rightarrow T_{\mu\nu} + \mathcal{L}_f T_{\mu\nu}$
- Shock-wave induced BMS transformation(special case of FST) :
- $T_{\mu\nu} \rightarrow T_{\mu\nu} + \mathcal{L}_{\theta(v-v_0)f} T_{\mu\nu} + T^S_{\mu\nu}$
- FST transformation:
- $T_{\mu\nu} \rightarrow T_{\mu\nu} + \mathcal{L}_{f(v)} T_{\mu\nu} + \int_{v_0=-\infty}^{v_0=v} T^S_{\mu\nu}(v_0)$ $\quad T^{FST}_{\mu\nu}$

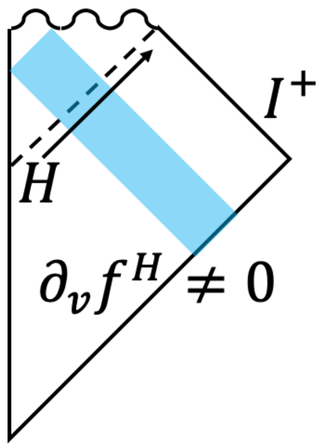
  $T^{tST}_{\mu\nu}$
- $T^{FST}_{\mu\nu} - T^{tST}_{\mu\nu} = \int_{v_0=-\infty}^{v_0=v} T^S_{\mu\nu}(v_0)\,, T^S_{\mu\nu} \equiv (T^{FST}_{\mu\nu} - T^{tST}_{\mu\nu})\big|_{f(v)\rightarrow f'(v_0)\theta(v-v_0)}$
- Dynamically generate lineariz conserved charge of BMS symmetry

Exact conserved charge

- The linear response relation between f and T at $O(f)$ is satisfied under a specific tST transformation → A linear conserved charge of BMS symmetry.
- How about at $O(f^n)$? → In fact, T has the form $T_1(f^{'}) + \partial_t T_2(f^{'})$ → Composed by local quantity $f^{'}$ and a boundary term!
- There exist a conserved charge along the direction v → $\nabla^{\mu}(T_{\mu\nu} n_r^{\nu}) = 0$

where $n_r^{\nu}\partial_{\nu} = \partial_r$ is the null congruence of v.

- n_r is an asymptotic Killing vector field→ the combined transformation of tST and FST generates BMS charge at I^{-} dynamically!



Exact supertranslated black hole

- With the combined transformation of tST and FST, we find the appropriate coordinate to study the interaction between Hawking radiation of dynamical black hole and flow-induced soft hair
$$ds^2 = g_{\mu\nu} + \mathcal{L}_f g_{\mu\nu}\Big|_{f\to f(v)} - \mathcal{L}_{f(v)} g_{\mu\nu}$$
$$= -\left((1-2f')(1-2M/r) - D^2 f'\right)dv^2 + 2(1-f')dvdr - 4rD_A f' dv d\Theta^A + r^2\gamma_{AB} d\Theta^A d\Theta^B$$
- In this coordinate, we found the surface gravity is modified as
$$\kappa_{\mathcal{H}} = \frac{1}{4M}\left(1 + \frac{D^2\psi}{M} - f' + \chi'\right) - f'' + \chi'' + O(f^2)$$
- The lensing potential $\psi(v) = -\frac{1}{4M}\int_0^\infty e^{-\int_v^{v_1}\frac{1}{4M(v_2)}dv_2} D^2 f'(v_1) dv_1 + O(f^2)$
- The anisotropy of the temperature is hard to be eliminated by a coordinate transformation! → Observable soft hair?

Observing soft hair

- To verify the observability of soft hair, we also consider the additional lensing effect on Hawking radiation intensity due to the same anisotropic flow

$$\Delta I_H = \frac{1}{4M}\left(-(D^2+1)f(v) + \frac{\int_v^\infty dv_1 D^2 f'(v_1)}{\omega\left(\frac{v_1 - v}{4M}\right)}\right) + O(f^2, M'), \omega^{-1}(x) = x + lnx$$

and compare the form to $\psi(v) = -\frac{1}{4M}\int_0^\infty e^{-\int_v^{v_1}\frac{1}{4M(v_2)}dv_2} D^2 f'(v_1) dv_1 + O(f^2)$

- The different modification to two observables can isolate the dressing effect and won't be simultaneously compensated by a coordinate transformation!

Conclusions

- The effect of dressing on the Hawking temperature is equivalent to the Doppler effect and Unruh effect generated by a tST transformation.
- We generalized the shock-wave induced supertranslation to an anisotropic null flow (FST) and preserve the asymptotic fall-off conditions.
- The FST transformation together with a suitable choice of tST transformation can dynamically generate linear BMS charge (soft hair) on black hole horizon.
- The flow-induced soft hair can be observed by the Hawking temperature and Hawking radiation intensity that extract the macroscopic information of Hawking radiation.

Thank you for your attention