Cosmology of Linear Higgs-Sigma Models with Conformal Invariance

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Motivation

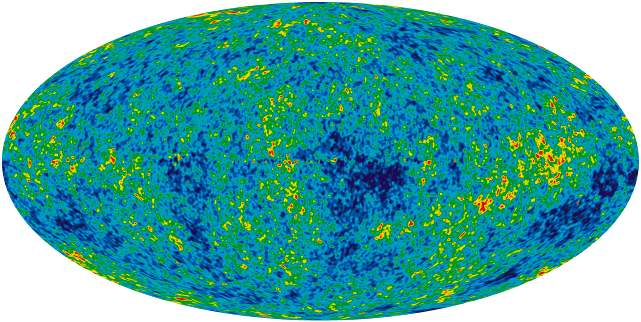


Figure: WMAP data. 13.77 billion year old temperature fluctuations corresponding to the seeds responsible for structure formation.

- Standard Big Bang Cosmology can't explain the correlation in primordial density perturbations.

Inflation

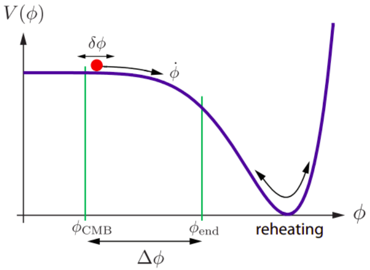


Figure: Inflaton slowly rolls down a potential during inflation. [Baumann - 0907.5424]

- Inflation provides a solution to the horizon problem.
- It must last enough to provide enough seeds for structure formation (50-60 e-folds).
- After inflation the energy of the inflaton is transferred to the SM particles during reheating.

Higgs Inflation

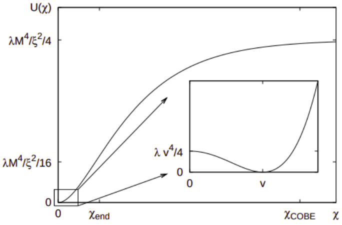


Figure: Potential for Higgs inflation.

- SM Higgs is identified with the inflaton [Bezrukov, Shaposhnikov - 0710.3755].

$$\mathcal{L} = \sqrt{-\bar{g}}\Big[-\frac{1}{2}(1+\xi\bar{\phi}_i^2)\bar{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}_i\partial_\nu\bar{\phi}_i - \frac{\lambda}{4}(\bar{\phi}_i^2)^2\Big].$$

- The non-minimal coupling must be at least $\xi \approx 10^4$ leading to unitarity violation.
- A new scalar field can solve the unitarity problems. [Giudice, Lee - 1010.1417]

Linear Sigma Models

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi^i\right)^2 + \frac{1}{2}\mu^2\left(\phi^i\right)^2 - \frac{\lambda}{4}\left[\left(\phi^i\right)^2\right]^2$$

- The lagrangian is invariant under $\phi^i \longrightarrow R^{ij}\phi^j$
- the potential is minimized by $\left(\phi_0^i\right)^2 = \frac{\mu^2}{\lambda}$
- The lenght is fix but the direction is arbitrary.
- reparametrization $\phi^i(x) = \left(\pi^k(x), v + \sigma(x)\right)$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi^k\right)^2 + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \frac{1}{2}\left(2\mu^2\right)\sigma^2 \\ - \sqrt{\lambda}\mu\sigma^3 - \sqrt{\lambda}\mu\left(\pi^k\right)^2\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}\left(\pi^k\right)^2\sigma^2 - \frac{\lambda}{4}\left[\left(\pi^k\right)^2\right]^2$$

- we obtain a massive σ field and N-1 massless π^k.
- O(N) is broken to O(N-1)

Non-linear Sigma Models: Chiral Lagrangian

- consider the constraint $(\pi)^2 + \sigma^2 = f_\pi^2$
- then $\sigma = \pm\sqrt{f_\pi^2 - \pi^2}$
- and the kinetical term $(\partial_\mu \pi)^2 + (\partial_\mu \sigma)^2 = (\partial_\mu \pi)^2 + \frac{\pi^2(\partial_\mu \pi)^2}{f_\pi^2 - \pi^2}$
- Comparing to Higgs inflation in the Einstein frame:

$$\mathcal{L}_{kin} \quad = \quad -\frac{1}{2(1+\xi\phi^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi^2\phi_i\phi_j/M_P^2}{1+\xi\phi^2/M_P}\right)\partial_\mu\phi_i\partial^\mu\phi_j$$

Higgs Inflation as a NLSM

$$\mathcal{L} = \sqrt{-\bar{g}}\Big[-\frac{1}{2}(1+\xi\bar{\phi}_i^2)\bar{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\bar{\phi}_i\partial_\nu\bar{\phi}_i - \frac{\lambda}{4}(\bar{\phi}_i^2)^2\Big].$$

Conformal transformation $\bar{g}_{\mu\nu} = e^{2\varphi}g_{\mu\nu}$

$$\begin{aligned}\hat{R} &= e^{-2\varphi}R - 6e^{-3\varphi}\Box e^{\varphi},\\ \sqrt{-\hat{g}} &= e^{4\varphi}\sqrt{-g}\end{aligned}$$

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}e^{2\varphi}(1+\xi\hat{\phi}_i^2)R + 3(1+\xi\hat{\phi}_i^2)e^{\varphi}\Box e^{\varphi} + \frac{1}{2}e^{2\varphi}(\partial_\mu\hat{\phi}_i)^2 - \frac{\lambda}{4}e^{4\varphi}(\hat{\phi}_i^2)^2\right].$$

- Field redefinitions $\phi_i = e^{\varphi}\hat{\phi}_i$ and $\Phi = \sqrt{6}\,e^{\varphi}$

$$\sigma = \frac{1}{2}\left(\sqrt{\phi^2 - 12\Big(\xi + \frac{1}{6}\Big)\phi_i^2} - \phi\right).$$

- contraint equation

$$\mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\Big(\frac{1}{6}\phi^2 - \frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\Big)R - \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2\Big].$$

- we eliminated the non-minimal coupling but introduced a **non-canonical kinetic term** $(\partial_\mu\sigma)^2$

Conformally invariant Lagrangian

- Fix the gauge $\phi = \sqrt{6}$

$$\mathcal{L} = \sqrt{-g}\Big[-\frac{1}{2}\Big(1 - \frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\lambda}{4}(\phi_i^2)^2\Big]$$

$$f(\sigma, \phi_i) \equiv \left(\sigma + \frac{\sqrt{6}}{2}\right)^2 + 3\Big(\xi + \frac{1}{6}\Big)\phi_i^2 - \frac{3}{2} = 0.$$

- Higgs Inflation can be regarded as a Non Linear Sigma Model with a constraint equation.[Ema, Mukaida, van de Vis - 2002.11739]
- Include the constraint equation as a Lagrange multiplier.

$$\begin{aligned} \mathcal{L} \quad = \quad & \sqrt{-g}\Big\{ -\frac{1}{2}\Big(1 - \frac{1}{6}\phi_i^2 - \frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu \phi_i)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 \\ & -\frac{\lambda}{4}(\phi_i^2)^2 - \frac{\kappa}{4}\left[\left(\sigma + \frac{\sqrt{6}}{2}\right)^2 + 3\Big(\xi + \frac{1}{6}\Big)\phi_i^2 - \frac{3}{2}\right]^2\Big\}. \end{aligned}$$

- If σ is dynamical this is a UV completion.

Starobinsky model as a linear sigma model

- R^2 + Higgs inflation

$$\frac{\mathcal{L}_{R2}}{\sqrt{-g}} = -\frac{1}{2}R\Big(1-\frac{1}{6}\phi_i^2-\frac{1}{6}\sigma^2\Big)+\frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\phi_i)^2-\alpha\chi^2-\frac{\lambda}{4}\phi_i^4,$$

$$\chi = \frac{1}{4\alpha}\left[\frac{1}{2}-\frac{1}{3}\Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2-\Big(\xi+\frac{1}{6}\Big)\phi_i^2\right].$$

- We get the same non-linear sigma model for Higgs inflation as for Starobinsky.

$$U(\sigma,\phi_i) = \alpha\chi^2 = \frac{1}{16\alpha}\left[\frac{1}{2}-\frac{1}{3}\Big(\sigma+\frac{\sqrt{6}}{2}\Big)^2-\Big(\xi+\frac{1}{6}\Big)\phi_i^2\right]^2$$

Higher curvature terms

$$\mathcal{L}_{\rm gen} = \sqrt{-\hat{g}}\Big[-\frac{1}{2}(1+\xi\hat{\phi}_i^2)\hat{R} + \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\phi}_i\partial_\nu\hat{\phi}_i - \frac{\lambda}{4}(\hat{\phi}_i^2)^2 + \sum_k \frac{2(-1)^{k+1}\alpha_k}{k+1}\,\hat{R}^{k+1}\Big]$$

$$U(\sigma,\phi_i) = \sum_k \Big(\frac{2^{k+2}k}{k+1}\Big)\alpha_k(\Omega(\sigma))^{2k-2}(y(\sigma,\phi_i))^{k+1}$$

- decoupling condition for σ, $\frac{\partial U}{\partial \sigma} = 0$

$$\sum_k 4\alpha_k\, 2^k \Omega^{2k-2}\, y^k = \frac{1}{2} - \frac{1}{3}\Big(\sigma + \frac{\sqrt{6}}{2}\Big)^2 - \Big(\xi + \frac{1}{6}\Big)\phi_i^2.$$

$$\begin{aligned} 0 &= \sum_k 2^{k+2}k\alpha_k(\Omega(\sigma))^{2k-2}(y(\sigma,\phi_i))^k \\ &\quad + \sum_k \Big(\frac{2^{k+2}k}{k+1}\Big)\alpha_k(2k-2)(\Omega(\sigma))^{2k-3}(y(\sigma,\phi_i))^{k+1}. \end{aligned}$$

- If $k \geq 1$, there always exists an extremum for $y = 0$.

Inflation in linear sigma models

$$\mathcal{L} = \sqrt{-g}\Big\{ -\frac{1}{2}\Big(1 - \frac{1}{6}h^2 - \frac{1}{6}\sigma^2\Big)R + \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\lambda}{4}h^4 - U(\sigma, h)\Big\}$$

This Lagrangian is renormalizable as for the non-gravitational part concerns.

$$U(\sigma, h) = \frac{\kappa_1}{4}\ \Big[\sigma(\sigma + \sqrt{6}) + 3\Big(\xi + \frac{1}{6}\Big)h^2\Big]^2 .$$

Going to the Einstein frame $g_{\mu\nu} = g_{E,\mu\nu}/\Omega'^2$, $\Omega'^2 = 1 - \frac{1}{6}h^2 - \frac{1}{6}\sigma^2$

$$\mathcal{L}_E \quad = \quad \sqrt{-g_E}\Big\{ -\frac{1}{2}\,R(g_E) + \frac{3}{4\Omega'^4}(\partial_\mu \Omega'^2)^2 + \frac{1}{2\Omega'^2}(\partial_\mu h)^2 + \frac{1}{2\Omega'^2}(\partial_\mu \sigma)^2 - V(\sigma, h)\Big\}$$

$$V(\sigma, h) = \frac{1}{\left(1 - \frac{1}{6}h^2 - \frac{1}{6}\sigma^2\right)^2}\Big[\frac{1}{4}\kappa_1\Big(\sigma(\sigma + \sqrt{6}) + 3\Big(\xi + \frac{1}{6}\Big)h^2\Big)^2 + \frac{1}{4}\lambda h^4\Big]$$

Effective Einstein frame potential

- Integrating out the Higgs field

$$h^2 = \frac{\kappa_1 \sigma(\sigma+\sqrt{6})\big(\sigma - 3\big(\xi+\frac{1}{6}\big)(\sigma-\sqrt{6})\big)}{\lambda(\sigma-\sqrt{6}) - 3\kappa_1\big(\xi+\frac{1}{6}\big)\big(\sigma-3\big(\xi+\frac{1}{6}\big)(\sigma-\sqrt{6})\big)}.$$

Using this constraint:

$$V_{\text{eff}}(\sigma) = 9\lambda\,\kappa_1\sigma^2\left[\lambda(\sigma-\sqrt{6})^2 + \kappa_1\left(\sigma - 3\left(\xi+\frac{1}{6}\right)(\sigma-\sqrt{6})\right)^2\right]^{-1}.$$

And in terms of the canonical field:

$$\sigma = -\sqrt{6}\tanh\left(\frac{\chi}{\sqrt{6}}\right)$$

The final effective potential is

$$V_{\text{eff}}(\chi) = \frac{9\kappa_1}{4}\left(1-e^{-2\chi/\sqrt{6}}\right)^2\left[1+\frac{\kappa_1}{4\lambda}\left(6\xi+e^{-2\chi/\sqrt{6}}\right)^2\right]^{-1}$$

Inflationary predictions

- From the effective Einstein potential we can get the small roll parameters

$$
\begin{aligned}
\epsilon &= \frac{1}{3}\frac{(2\lambda+3\kappa_1\xi(1+6\xi))^2}{(\lambda+9\kappa_1\xi^2)^2}\, e^{-4\chi/\sqrt{6}} \\
\eta &= -\frac{2}{3}\cdot\frac{2\lambda+3\kappa_1\xi(1+6\xi)}{\lambda+9\kappa_1\xi^2}\, e^{-2\chi/\sqrt{6}} \\
&\qquad +\frac{2\kappa_1}{3}\cdot\frac{(-\lambda+12\lambda\xi+18\kappa_1\xi^2(1+6\xi))}{(\lambda+9\kappa_1\xi^2)^2}\, e^{-4\chi/\sqrt{6}}
\end{aligned}
$$

- We can rewrite them in terms of the number of e-foldings.

$$
N = \frac{3}{2}\cdot\frac{\lambda+9\kappa_1\xi^2}{2\lambda+3\kappa_1\xi(1+6\xi)}\left(e^{2\chi_*/\sqrt{6}} - e^{2\chi_e/\sqrt{6}}\right)
$$

Inflationary predictions

$$n_s = 1 - \frac{2}{N} - \frac{9}{2N^2} + \frac{3\kappa_1}{N^2}\frac{(-\lambda + 12\lambda\xi + 18\kappa_1\xi^2(1+6\xi))}{(2\lambda + 3\kappa_1\xi(1+6\xi))^2}$$

$$r = 16\epsilon_* = \frac{12}{N^2}$$

- Terms $1/N^2$ are different from Starobinsky or pure sigma inflation due to the Higgs quartic coupling.
- but $\lambda \approx \kappa_1\xi^2 \leq 1$ the extra terms don't contribute much.
- A more precise measurement of n_s in the future CMB experiments could see the difference due to the $1/N^2$ terms.

Reheating temperature is larger than in previous cases [S.M Choi, H.M.L - 1601.05979], [D.Y. Cheong, S.C. Park, H.M. Lee 2002.07981]

$$T_{RH} = \left(\frac{90}{\pi^2 g^*}\right)^{1/4}\sqrt{M_P\Gamma_\sigma} = 4\times 10^9 \sim 4\times 10^{14} GeV$$

Dark Energy

- If we include a R^{p+1} term the sigma field potential becomes

$$V(\chi) \quad = \quad \frac{9\kappa_n}{4^n} \cdot e^{-2\left(1-\frac{1}{p}\right)\chi/\sqrt{6}}\left(1-e^{-2\chi/\sqrt{6}}\right)^{1+\frac{1}{p}} \equiv V_0(\chi).$$

- for p=1 we recover Starobinsky inflation
- for $p > 1$ or $p < 0$ the inflaton potential is exponentially suppressed, so it is quintessence-like for Dark Energy

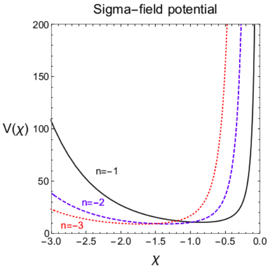


Figure: Potential for $n = -1, -2, -3$ in the case of decoupled Higgs.

Attractor behaviour

$$V = V_0 \, e^{-c(\chi)\chi},$$

Friedmann equation together with the scalar field equation

$$\begin{aligned} H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left(\Omega_{m0} \left(\frac{a_0}{a}\right)^3 + \Omega_{r0} \left(\frac{a_0}{a}\right)^4 + \Omega_{\chi 0} \cdot \frac{\rho_\chi}{\rho_{\chi,0}} \right), \\ 0 &= \ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial \chi} \end{aligned}$$

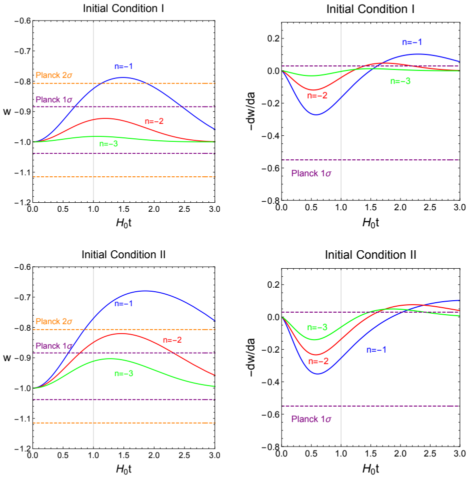
can be recasted as [S. Tsujikawa-1304.1961]

$$\begin{aligned} w' &= (w-1)\left[3(1+w) - c\sqrt{3(1+w)\Omega_\chi}\right], \\ \Omega'_\chi &= -3(w - w_m)\Omega_\chi(1 - \Omega_\chi), \\ c' &= -\sqrt{3(1+w)\Omega_\chi}\,(\Gamma - 1)c^2 \end{aligned}$$

with

$$\Gamma = V \, \frac{d^2 V}{d\chi^2} \left(\frac{dV}{d\chi}\right)^{-2}.$$

Attractor Behaviour



Initial conditions I: $\phi_i = -2M_P, \dot{\phi}_i = 0$

Initial conditions II: $\phi_i = -2.5M_P, \dot{\phi}_i = 0$

Conclusions

- General linear sigma models with conformal invariance are UV completions of Higgs inflation.
- A particular family of models coming from R^2 leads to successful inflation.
- We compared the predictions to those in the literature.
- Higher curvature terms R^{p+1} can provide Dark Energy with tracker behavior.
- Predictions for the time-varying equation of state can be consistent with observations within 1σ. [Plank 18 -1807.06209]
- Future work:
  - Generalize corrections for inflation predictions.