

The Causal Structure of Superfluid Dark Matter

Mark Hertzberg, Tufts University
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Based on [arXiv.2105.02241](https://arxiv.org/abs/2105.02241), with

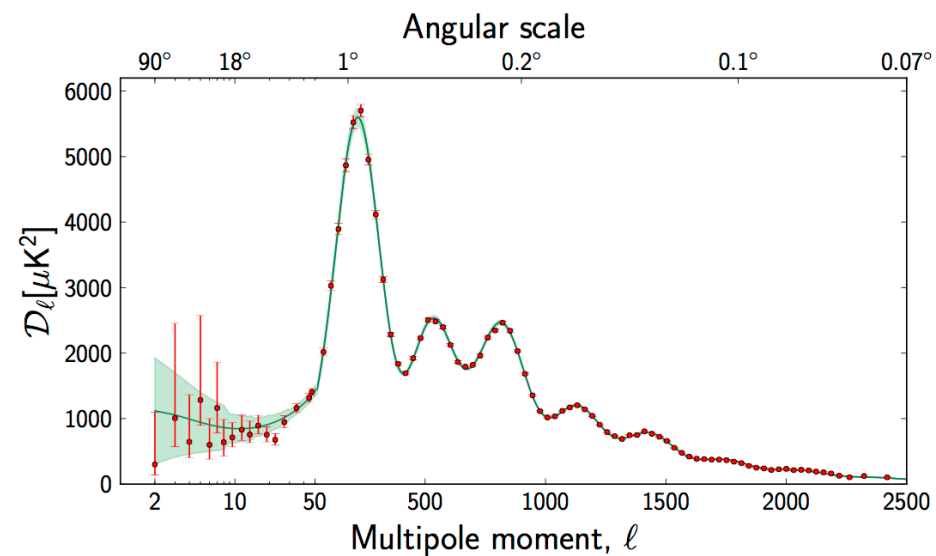
Jacob Litterer



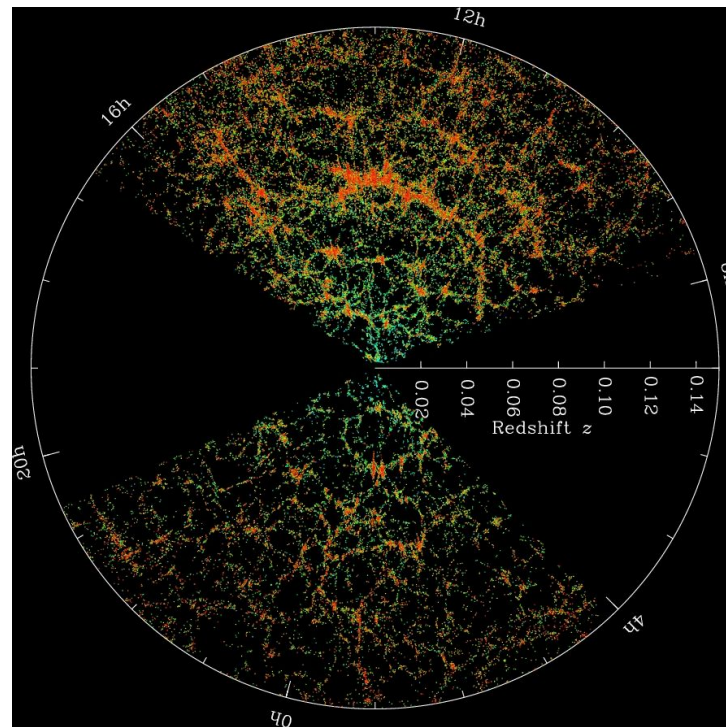
Neil Shah



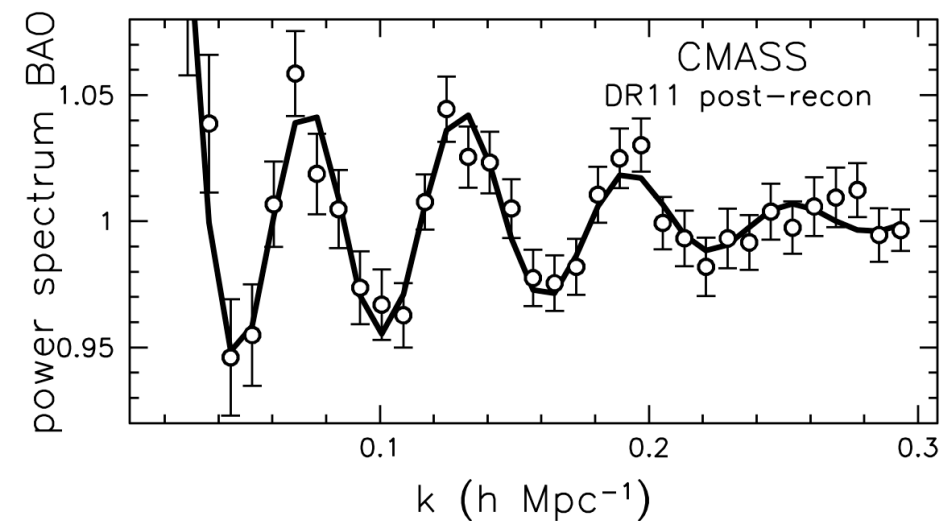
Tremendous Success of Λ CDM on Large Scales



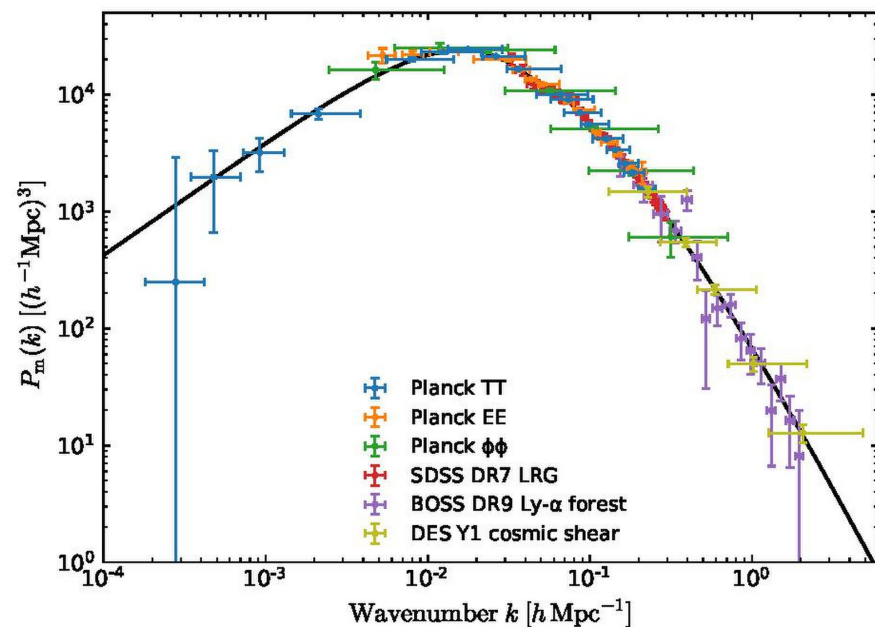
CMB (Planck)



Large Scale Structure (SDSS)



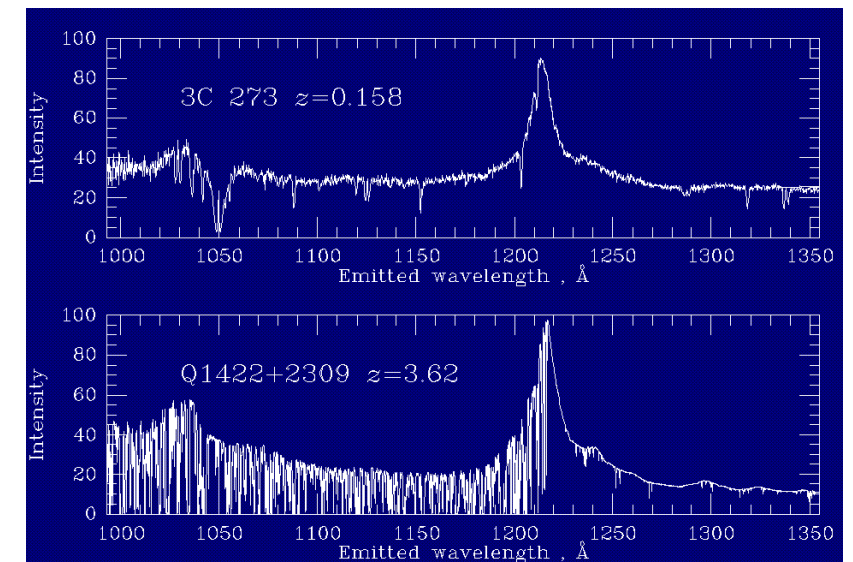
BAO (BOSS)



Concordance (Planck)



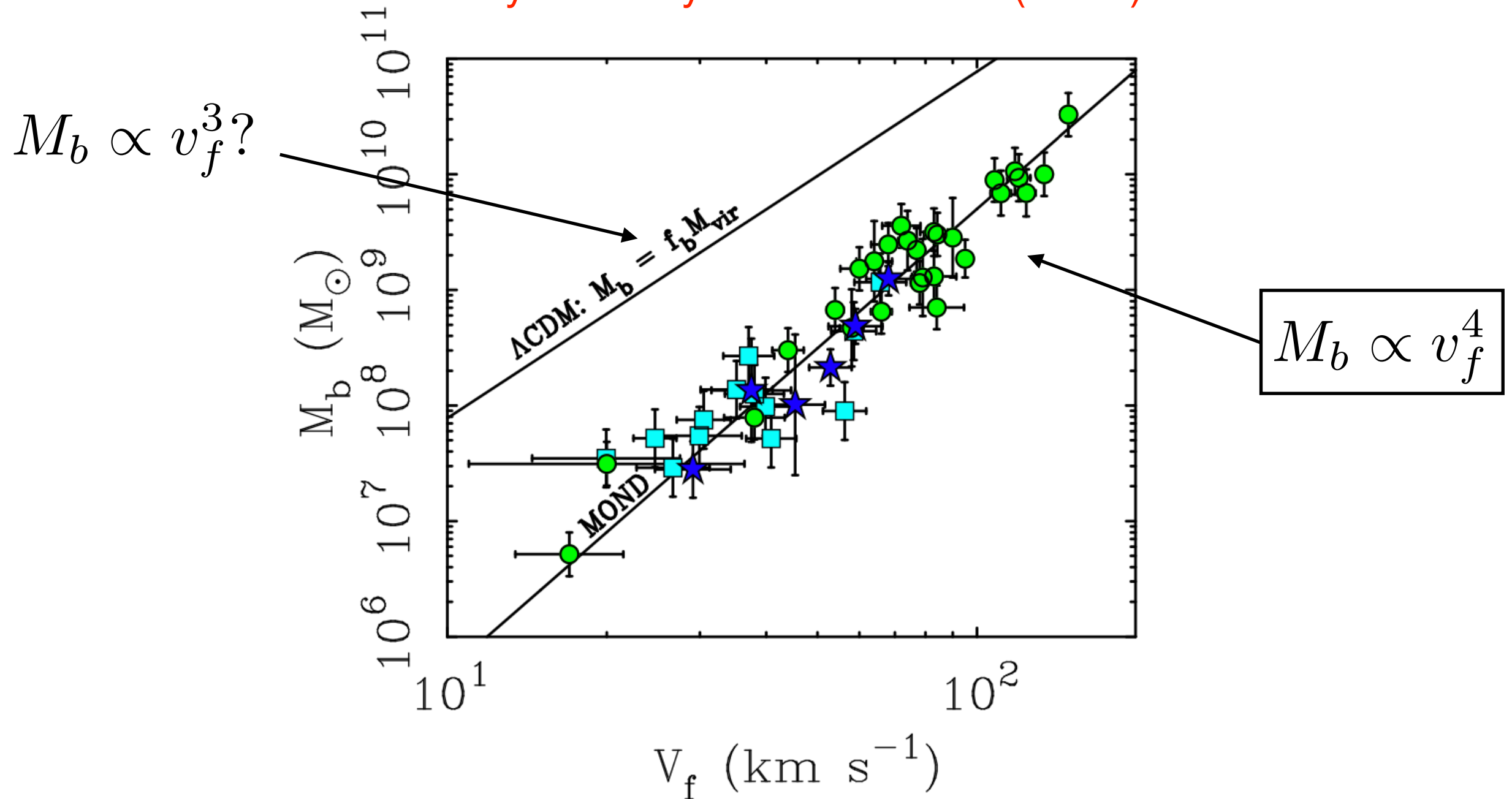
Galaxy Clustering (Hubble)



Lyman Alpha Forest (Keck)

Possible Difficulties with CDM on Galactic Scales?

Baryonic Tully-Fisher Relation (BTFR)



(from McGaugh 2011)

Modify Gravity on Galactic Scales (MOND)?

$$a \propto \frac{M_{enc}}{R^2} \quad \text{If instead:} \quad \frac{v^2}{R} = a \propto \sqrt{\frac{M_{enc}}{R^2}} \implies \boxed{M_b \propto v_f^4}$$

(Milgrom)

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(Milgrom)

Implementing this is very difficult:

The unique, causal, Lorentz invariant, theory of massless spin 2 particles, at large distances, is general relativity

(Feynman, Weinberg, Deser,...)

So one needs to add new degrees of freedom, namely scalars, to mediate a new long range (peculiar) interaction

Simplistic Attempt

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

$$S = - \int d^4x \sqrt{-g} \left[F(X, \varphi) - \tilde{\beta} \varphi T_B + \frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{SM} \right]$$

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Introduce function F with 2 different asymptotic regimes:

Low densities/large scales

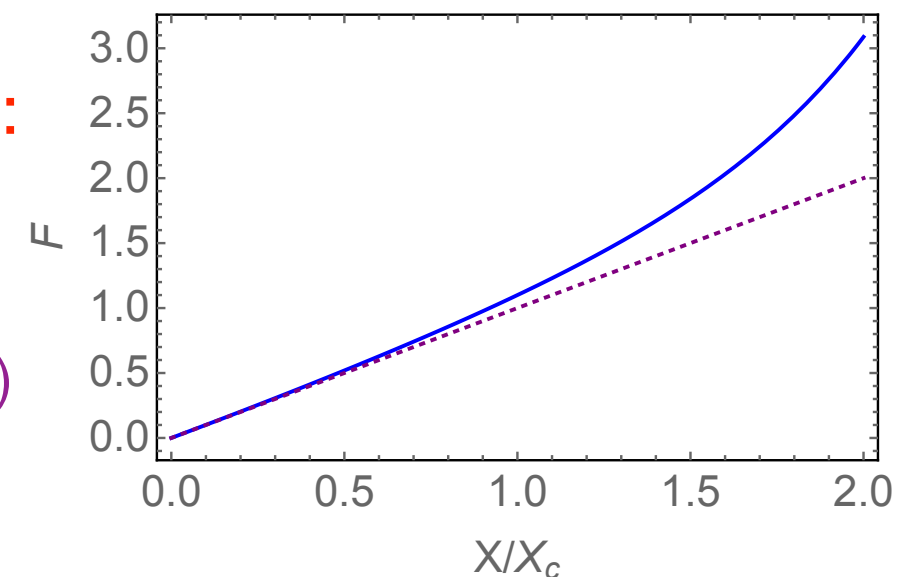
$$F = X \quad (\text{canonical})$$

(3/2 scaling)

High densities/galactic scales

$$F = \tilde{\alpha} X \sqrt{|X|}$$

Mediates a MOND-like force



Example:

$$F = X (1 + \tilde{\alpha}^4 X^2)^{1/4}$$

Large φ , can stay within regime of EFT

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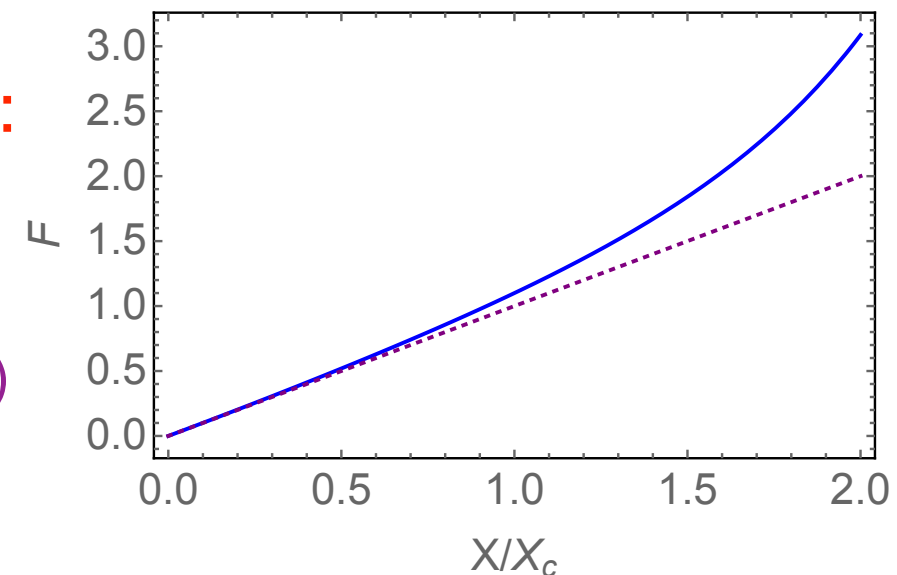
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Mediates a MOND-like force



$$-\frac{3\tilde{\alpha}}{2^{3/2}} \nabla \cdot (\nabla \varphi |\nabla \varphi|) = \tilde{\beta} T_B \quad \mathbf{a} \propto -\text{sign}(\tilde{\alpha}) \sqrt{\frac{M_{enc}}{R^2}} \hat{r}$$

Simplistic Attempt

Two Problems

$$F = \tilde{\alpha} X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are **superluminal** (related details later)

Simplistic Attempt

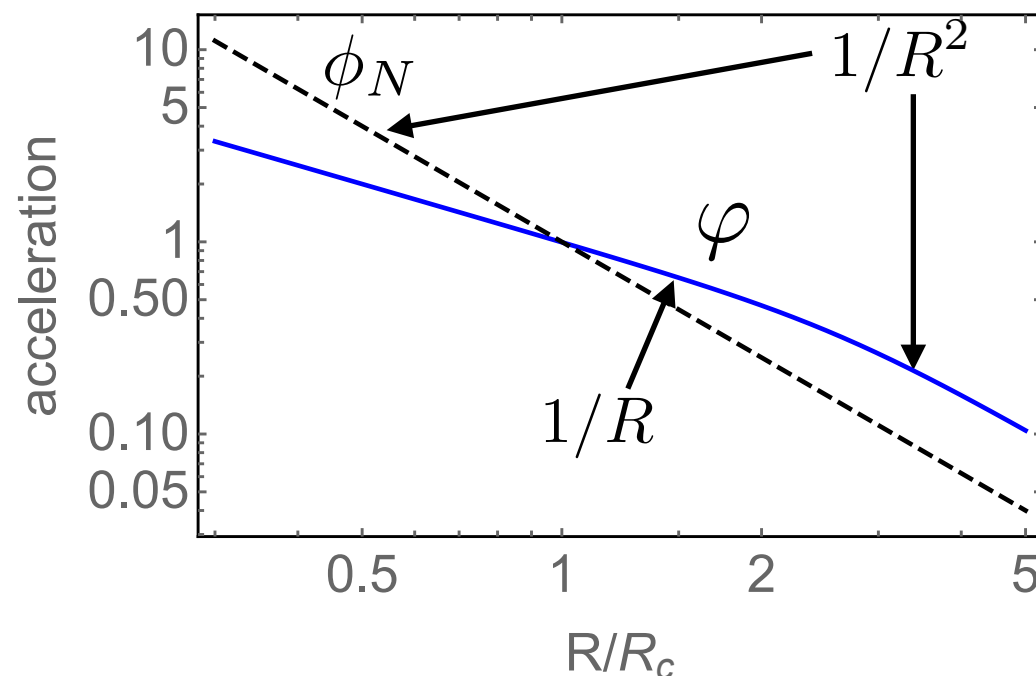
Two Problems

$$F = \tilde{\alpha} X \sqrt{|X|}$$

Theoretical: High energy perturbations on top of the MONDian solution are **superluminal** (related details later)

$$F = X$$

Phenomenological: Although the scalar becomes canonical at large scales, it introduces another $1/r^2$ force. So it is **difficult to consistently obtain the desired galactic and large scale behaviors**



Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter
 $U(1)$ Symmetry

Φ

$$X = g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi^*$$

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Clever idea: Use Spontaneous Symmetry Breaking

Complex Scalar Dark Matter
 $U(1)$ Symmetry

Φ

$$X = g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi^*$$

Example:

$$F = \frac{1}{2} (X + m^2 |\Phi|^2) + \frac{\Lambda^4}{6(\Lambda_c^2 + |\Phi|^2)^6} (X + m^2 |\Phi|^2)^3$$

Reproduces CDM on large scales

Allows for phase transition to superfluid
at galactic densities

$$\Phi = \rho e^{i(\theta + mt)}$$

Goldstone θ can act as long-ranged force mediator

Sophisticated Attempt - SuperFluid Dark Matter (SFDM)

Slowly varying phase θ and modulus ρ around superfluid condensate $\Phi = \rho e^{i(\theta + mt)}$

$$X + m^2 |\Phi|^2 = (\nabla \rho)^2 - 2m \rho^2 Y \quad \text{with} \quad Y \equiv \dot{\theta} - m \phi_N - \frac{(\nabla \theta)^2}{2m}$$

At tree-level, can integrate out heavy modulus (Higgs mode) $\rho^2 = \Lambda \sqrt{2m|Y|}$

Find low energy effective action for Goldstone is
$$F_{\text{eff}} = - \frac{2\Lambda(2m)^{3/2}}{3} Y \sqrt{|Y|} \quad (3/2 \text{ scaling})$$

By coupling to baryons, can mediate MOND-like force — reproduce BTFR, and CDM on large scales

Analysis of High Energy Perturbations ε_j

Decompose into components $\Phi = (\phi_1 + i \phi_2)/\sqrt{2}$

Expand around superfluid $\phi_j = \phi_j^b + \varepsilon_j \quad (j = 1, 2) \quad \left(F' \equiv \frac{\partial F}{\partial X} \right)$

Linear equation of motion for
high energy perturbations
$$\sum_{j=1}^2 [F' \eta^{\mu\nu} \delta^{ij} + F'' \partial^\mu \phi_i^b \partial^\nu \phi_j^b] \partial_\mu \partial_\nu \varepsilon_j = 0$$

Diagonalize to obtain Higgs normal mode perturbations
and associated effective metric
$$\psi = \partial^\mu \phi_1^b \partial_\mu \varepsilon_1 + \partial^\mu \phi_2^b \partial_\mu \varepsilon_2$$

$$G_\phi^{\mu\nu} \partial_\mu \partial_\nu \psi = 0$$

$$G_\phi^{\mu\nu} = F' g^{\mu\nu} + F'' (\partial^\mu \phi_1^b \partial^\nu \phi_1^b + \partial^\mu \phi_2^b \partial^\nu \phi_2^b)$$

Causal Propagation?

Obtain **eigenvalues** of effective metric $G_{\phi}^{\mu\nu} = F' g^{\mu\nu} + F'' (\partial^{\mu} \phi_1^b \partial^{\nu} \phi_1^b + \partial^{\mu} \phi_2^b \partial^{\nu} \phi_2^b)$

Conditions for **hyperbolicity**

$$(A) \quad A \equiv F' > 0$$

$$(B) \quad B \equiv F' + 2X F'' > 0$$

(Aharanov, Komar, Susskind;
Wald; Adams, Arkani-Hamed,
Dubovsky, Nicolis, Rattazzi;
Bruneton,...)

Condition for **subluminality**

$$(C) \quad C \equiv -F'' \geq 0$$

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Evaluate in SFDM model

$$A > 0$$

$$B = \frac{4 m^3 \Lambda^4 Y}{\rho^8}$$

$$C = \frac{2 m \Lambda^4 Y}{\rho^{10}}$$

MOND regime

$$Y \approx -\frac{(\nabla\theta)^2}{2m}$$

$$\implies \overset{\text{(ghost-like)}}{\boxed{B < 0}} \quad \text{and} \quad C < 0$$

Causal Propagation? - General Analysis

Obtain **eigenvalues** of effective metric $G_{\phi}^{\mu\nu} = F' g^{\mu\nu} + F'' (\partial^{\mu} \phi_1^b \partial^{\nu} \phi_1^b + \partial^{\mu} \phi_2^b \partial^{\nu} \phi_2^b)$

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Most general form

$$F = (X + m^2 |\Phi|^2) \sum_{n=0} g_n \frac{\Lambda^{2n} (X + m^2 |\Phi|^2)^n}{(\Lambda_c^2 + |\Phi|^2)^{3n}}$$

We **proved** that when g_n allow **MOND regime** $(\tilde{\alpha} > 0)$ $\implies$ $B < 0$ ^(ghost-like) and $C < 0$

Conclusions

Superfluid Dark Matter is a novel way (only known?) to obtain success of CDM on large scales and success of MOND on galactic scales

We studied a general class of models, and proved that high energy perturbations always violate hyperbolicity — ghost like behavior — in MOND regime

Intermediate regions can exhibit forms of superluminality.
There are problems in related models too.

Open question: is there ANY other model free of these theoretical problems?
Alternatively, can one rigorously show that CDM reproduces BTFR, etc?