IBS and KMI Joint workshop 2020

NAGOYA UNIVERSITY

Induced gravitational wave as a cosmological probe of the sound speed during the QCD phase transition

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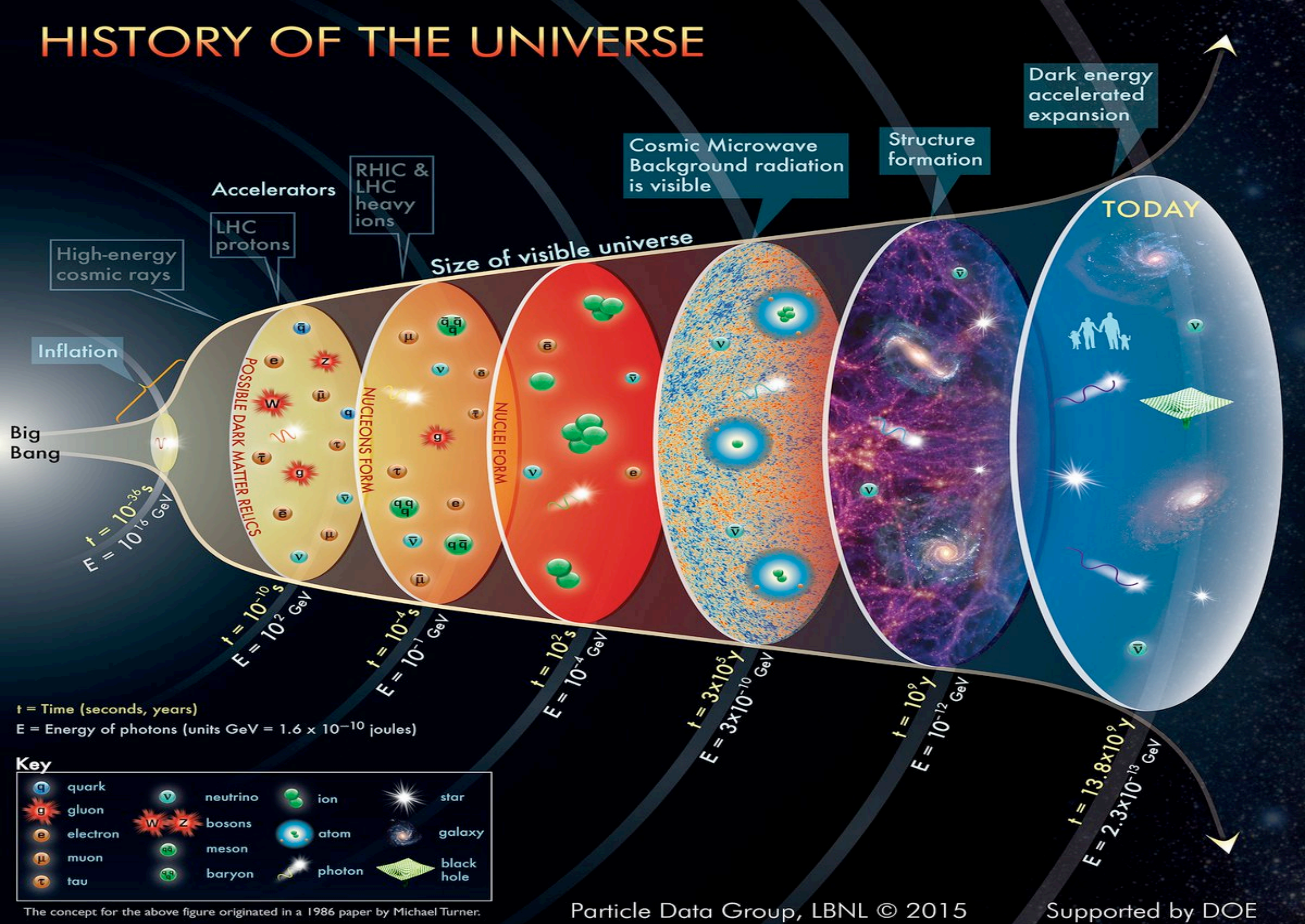
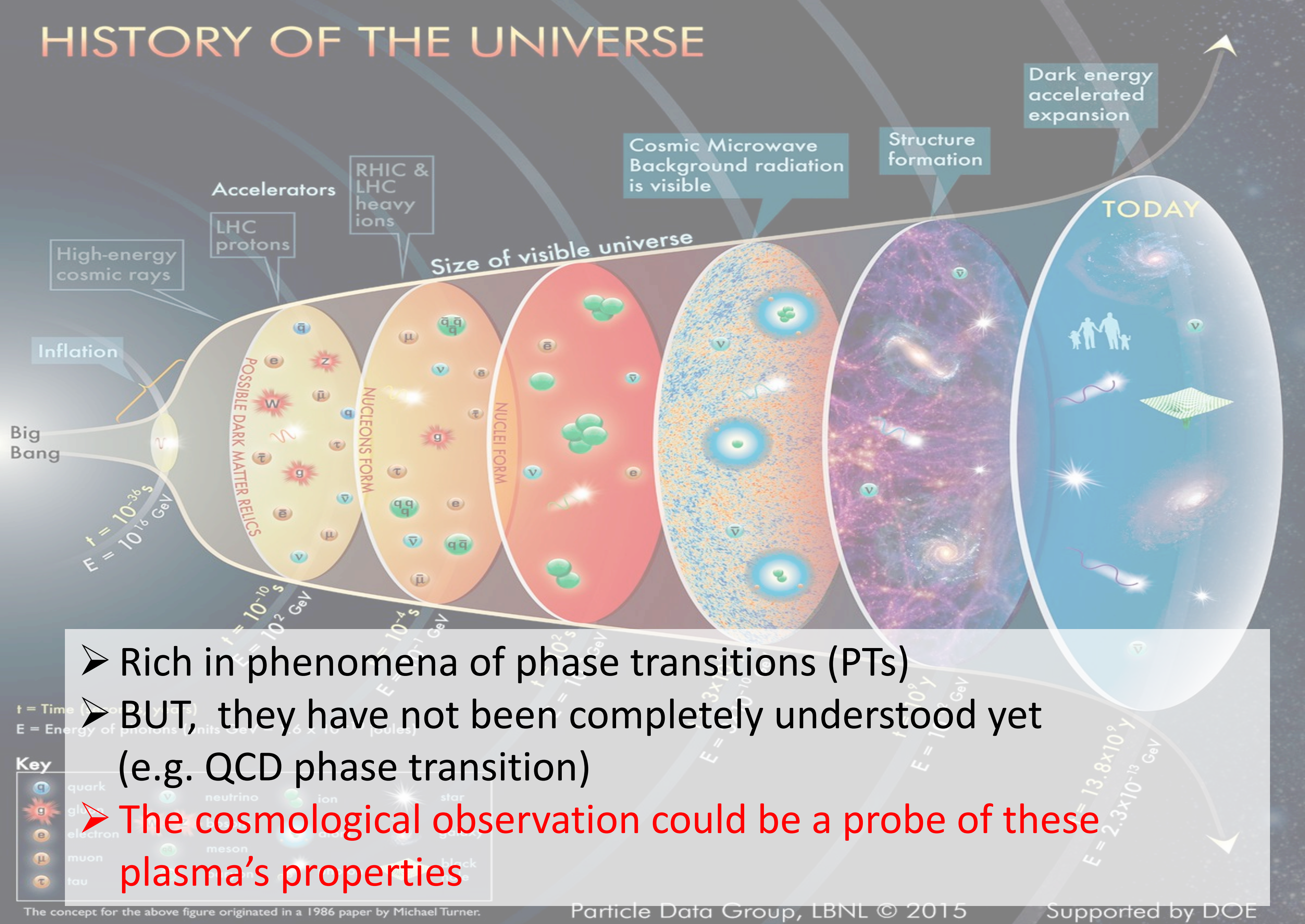
Ikumi Ueda(Nagoya U)

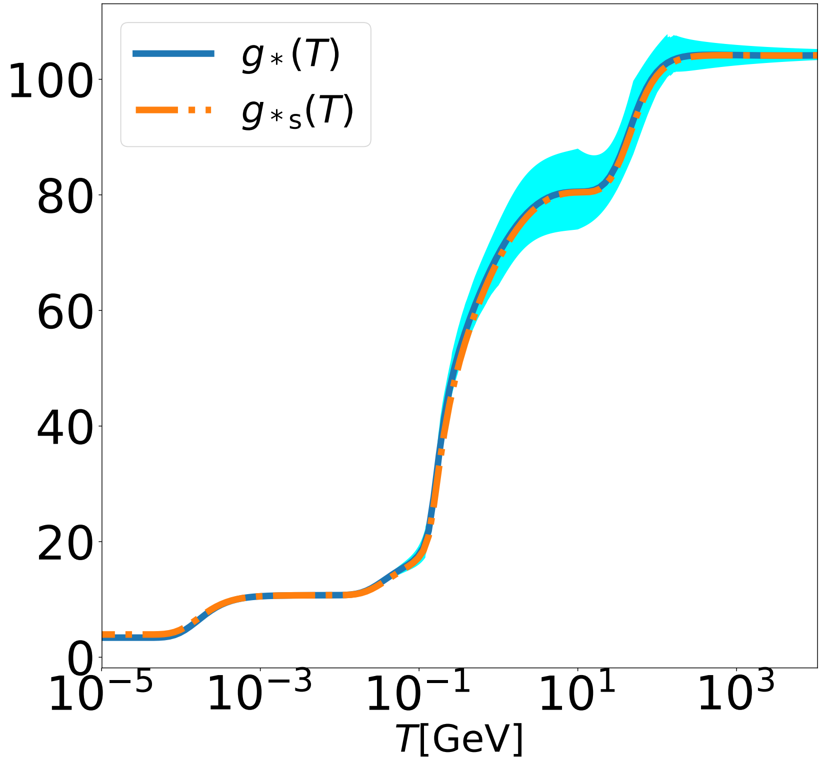
Arxiv:2008.XXXXX


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6×10^{-10} joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE



- Rich in phenomena of phase transitions (PTs)
- BUT, they have not been completely understood yet (e.g. QCD phase transition)
- The cosmological observation could be a probe of these plasma's properties

The effective degrees of freedom (DoF)



Saikawa&Shirai (2018)

- The number of kinds of relativisitic particles
- g_* is the effective DoF for the energy density $\rho(T)$

$$g_*(T) = \frac{30}{\pi^2 T^4}\rho(T)$$

- g_{*s} is the effective DoF for the entropy density $s(T)$

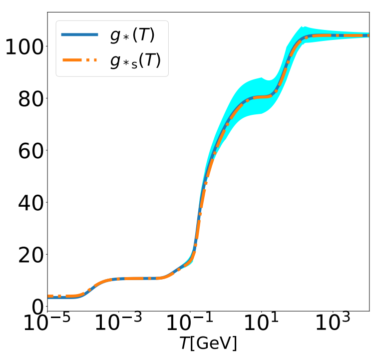
$$g_{*s}(T) = \frac{45}{2\pi^2 T^3}s(T)$$

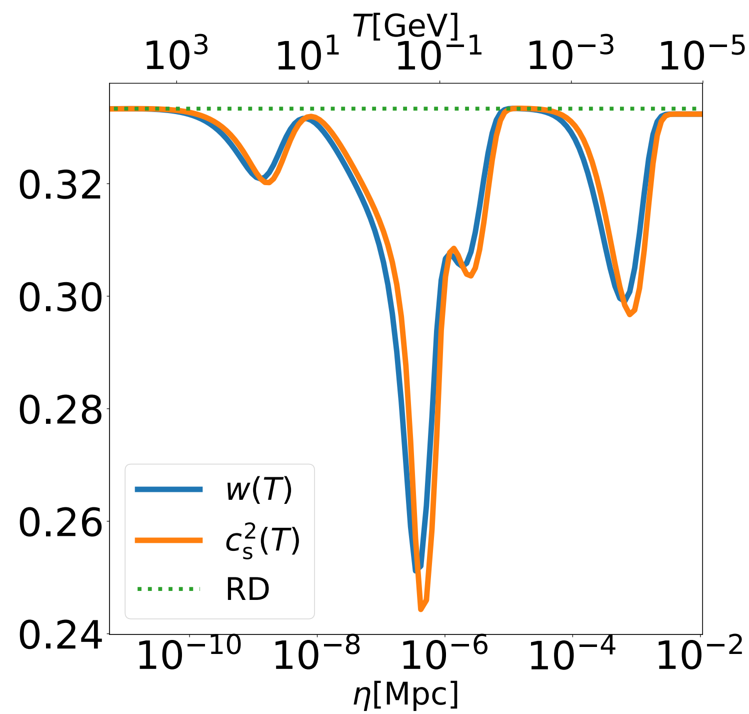
Equation of State parameter & sound speed

$$w \equiv p/\rho \qquad c_s^2 \equiv \partial p/\partial \rho$$

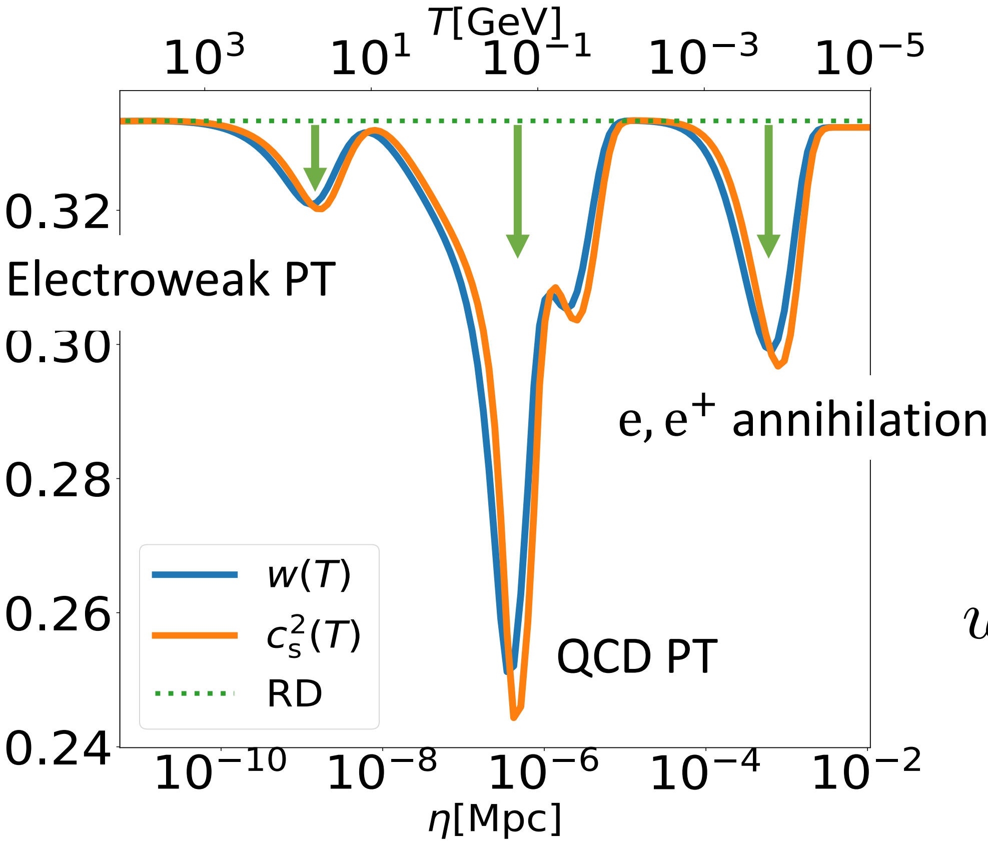
$$w(T) = \frac{4g_{*s}(T)}{3g_*(T)} - 1$$

$$c_s^2(T) = \frac{4\left(g'_{*s}(T)T + 4g_{*s}(T)\right)}{3\left(g'_*(T)T + 4g_*(T)\right)} - 1$$





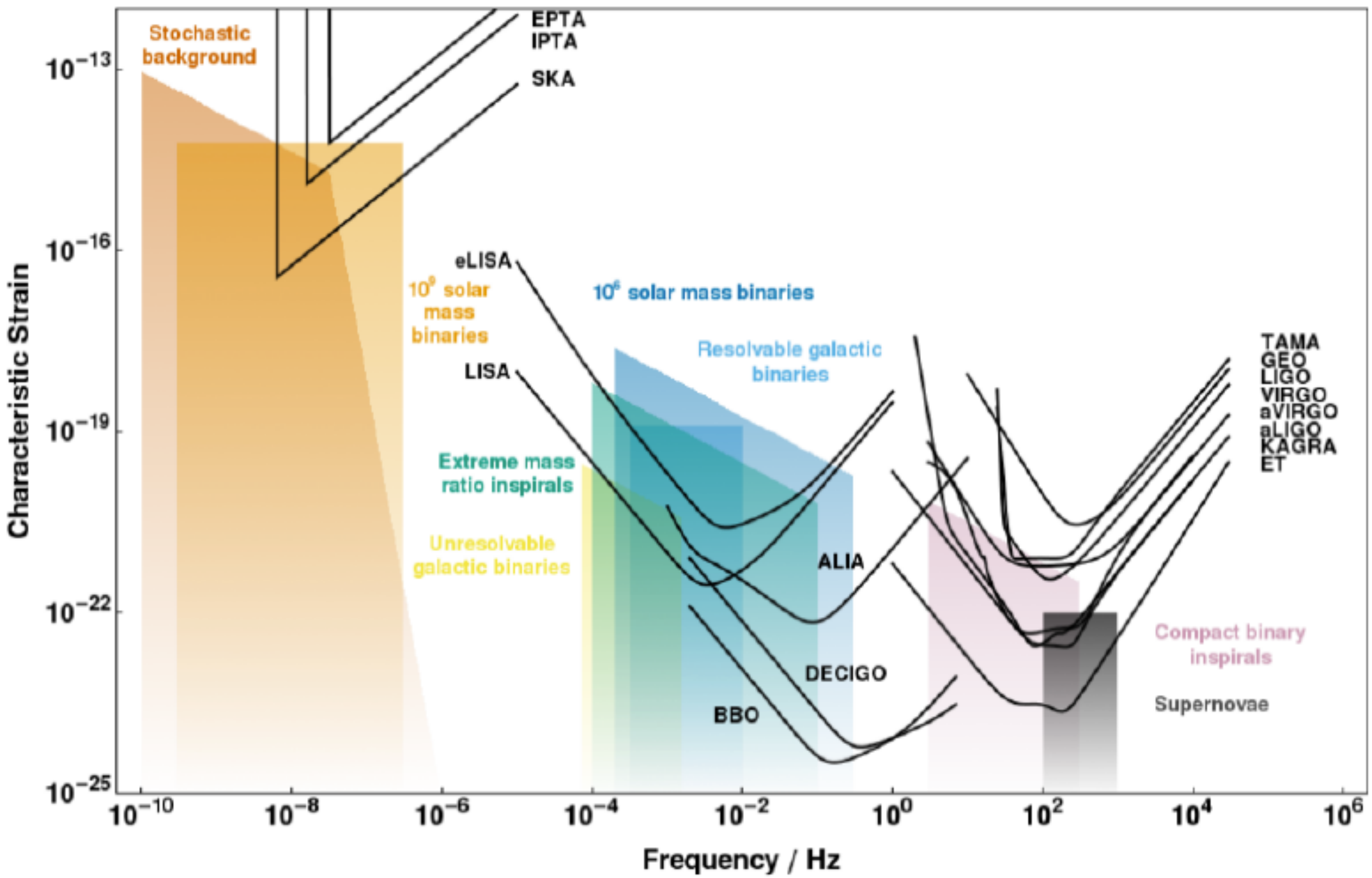
The point of EoSp & sound speed during PTs



Decrease of the number of relativistic particles in PTs

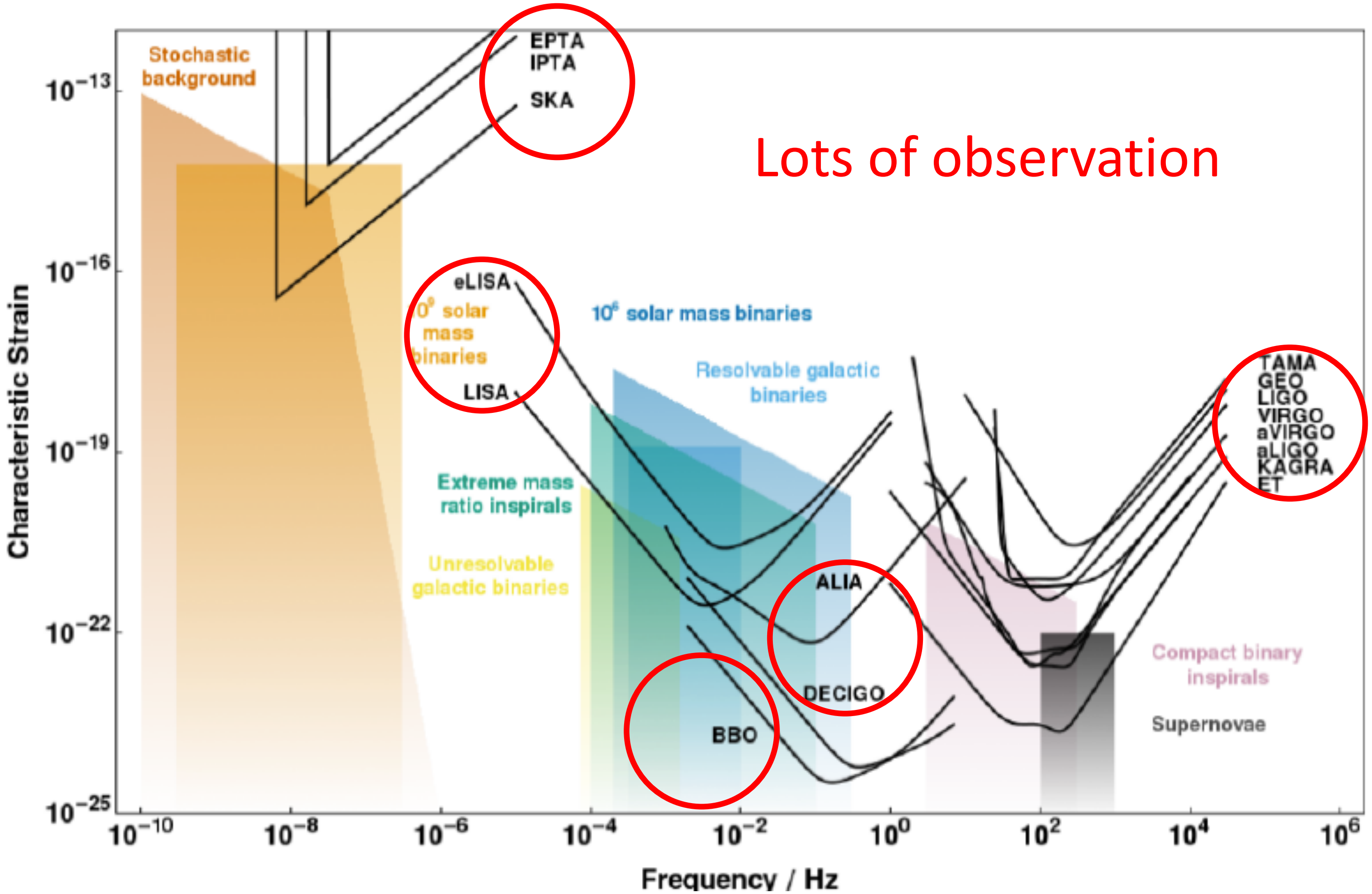
w, c_s^2 become lower than 1/3

Gravitational wave (GW)



https://smirshekari.files.wordpress.com/2014/04/gw_sensitivity_almost_all_detectors.png

Gravitational wave (GW)

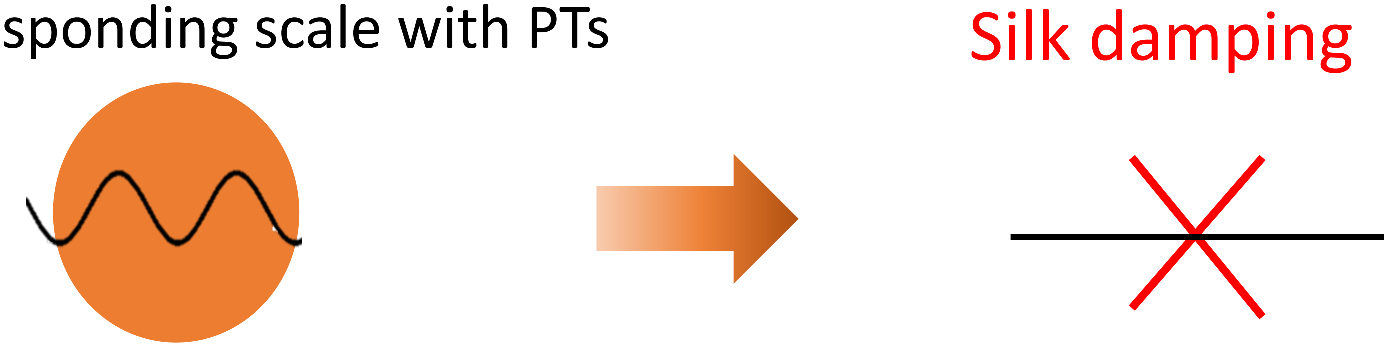


← Wide range covering of frequency →

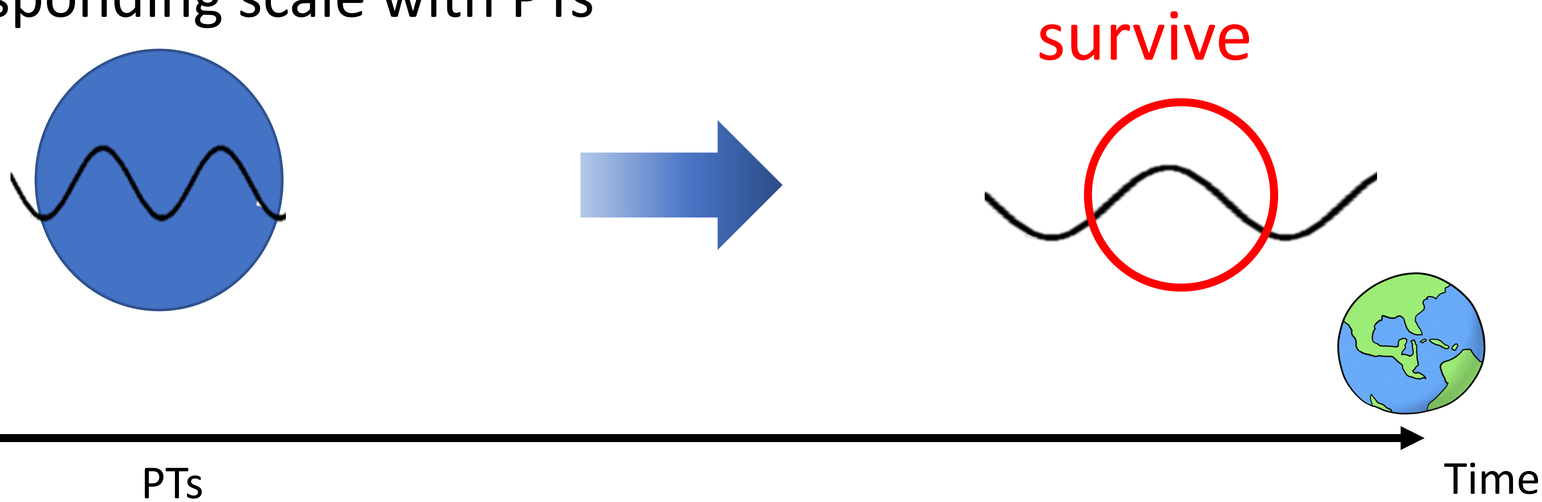
…nost_all_detectors.png

Another reason why we chose GW

Scalar perturbation
in corresponding scale with PTs

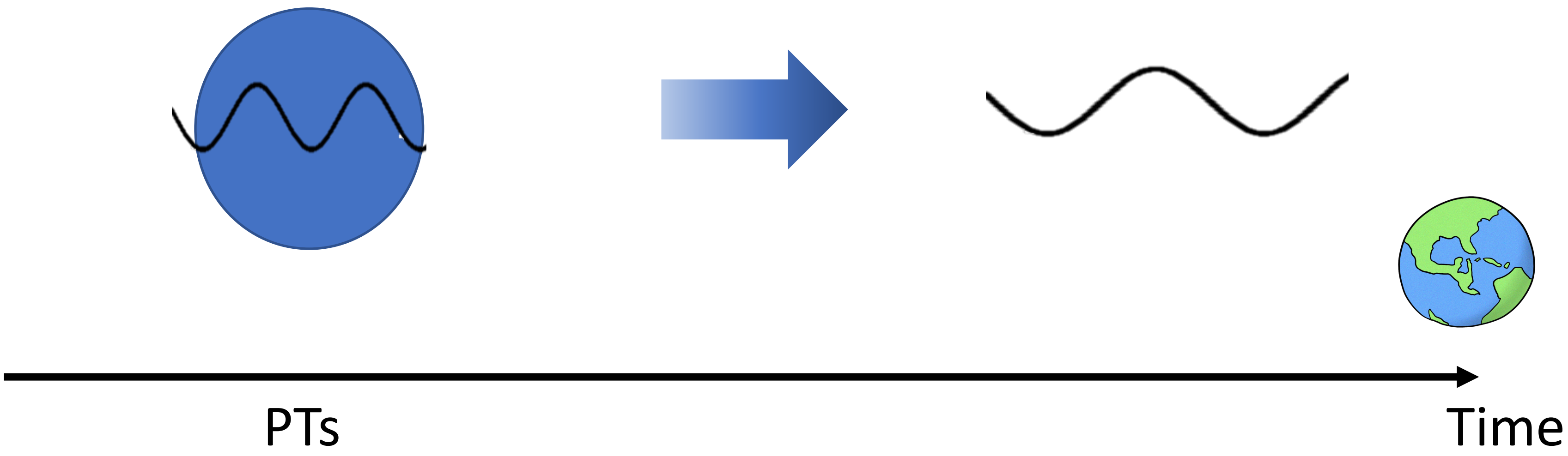


Tensor perturbation
in corresponding scale with PTs

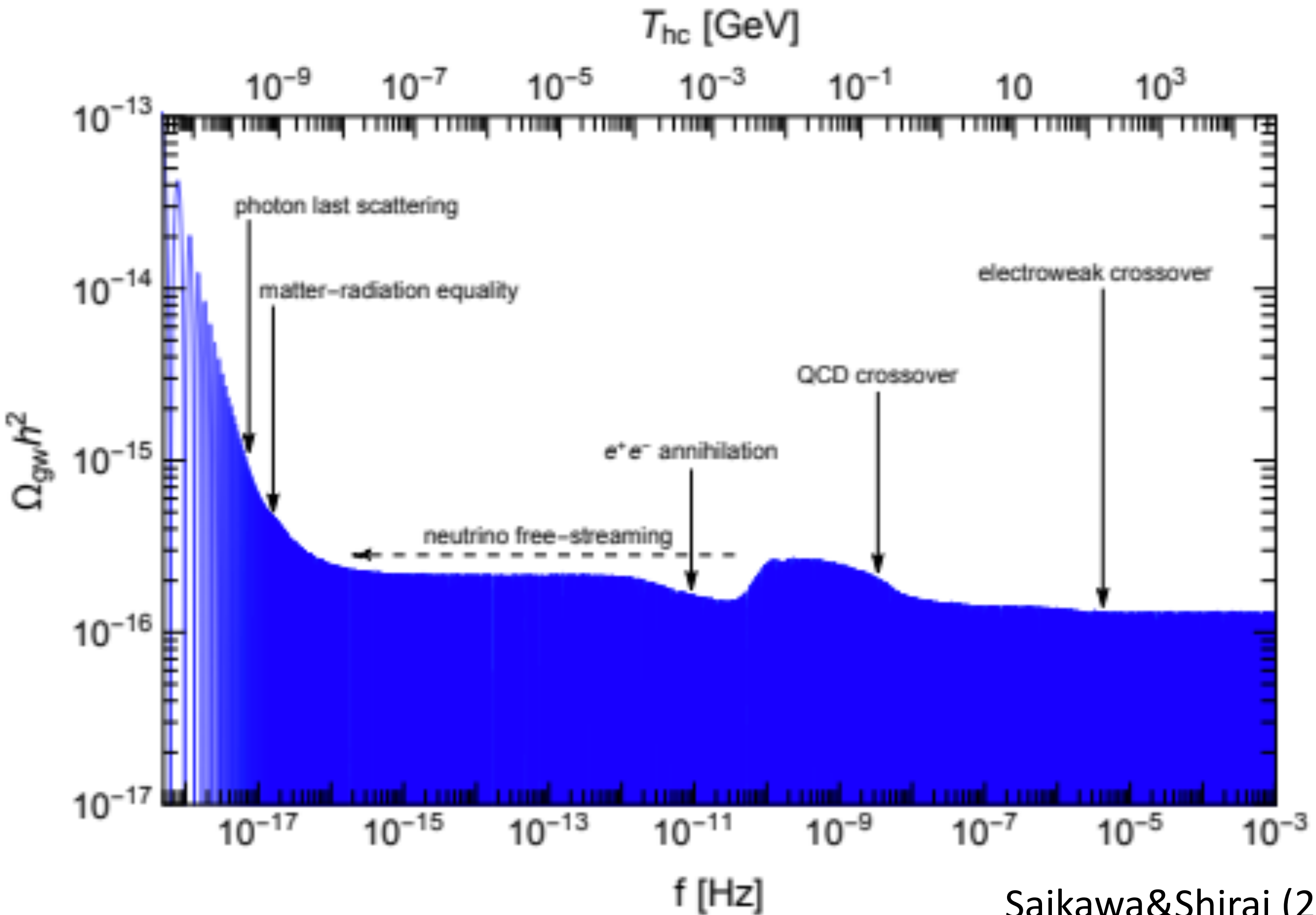


Linear Stochastic GW

Primordial tensor perturbation
coming from Inflation

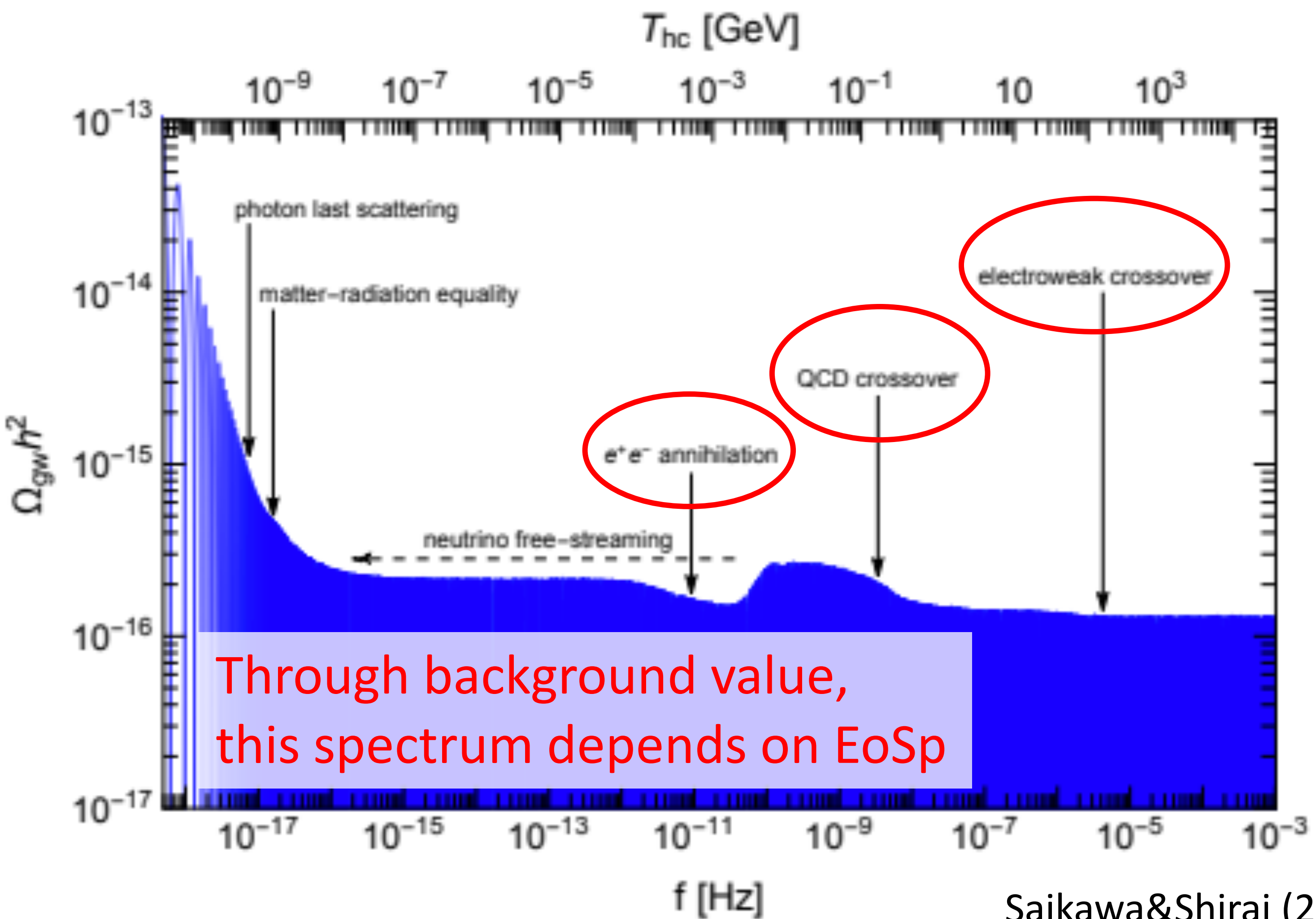


Linear Stochastic GW



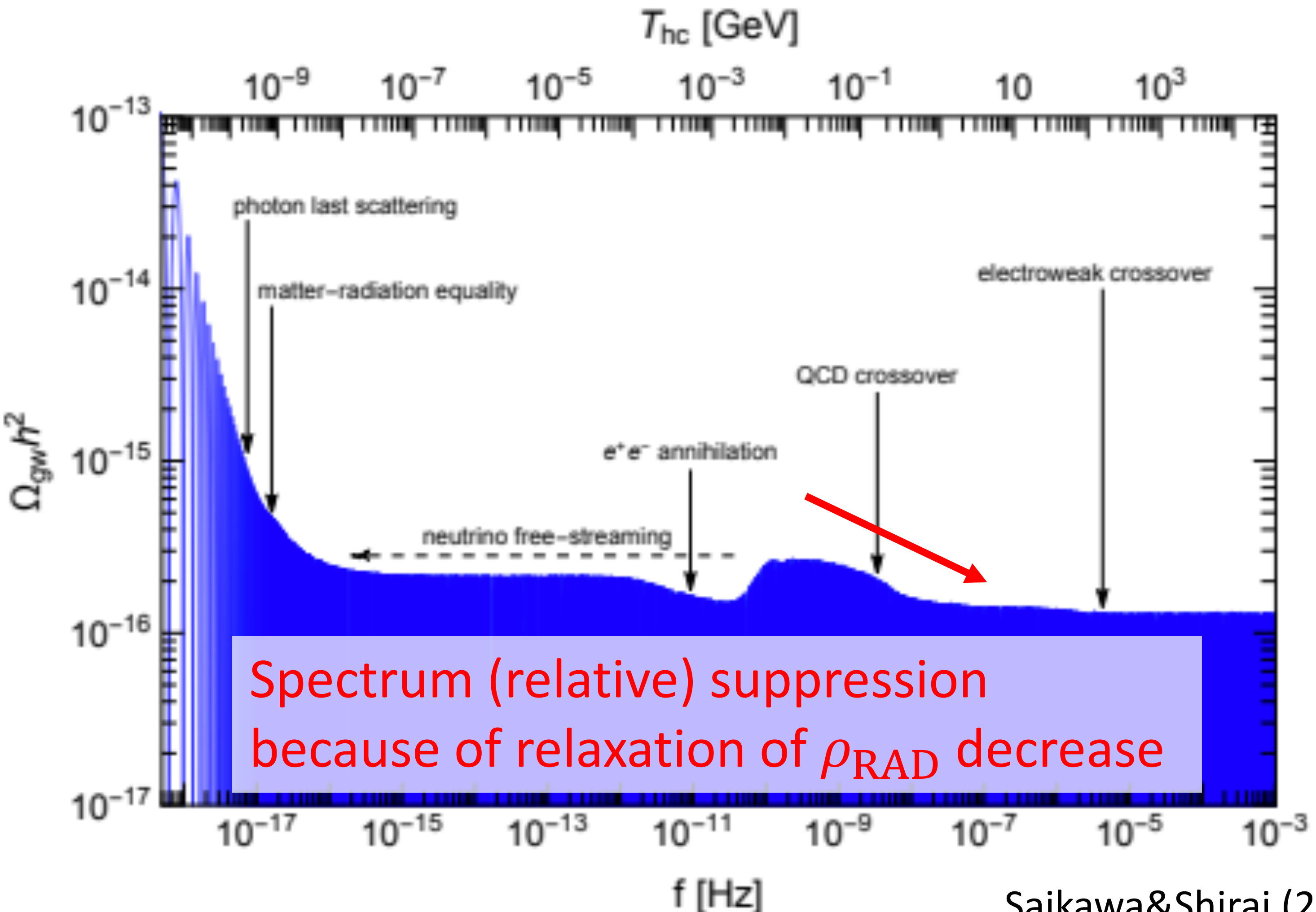
Saikawa&Shirai (2018)

Point ①- Linear Stochastic GW



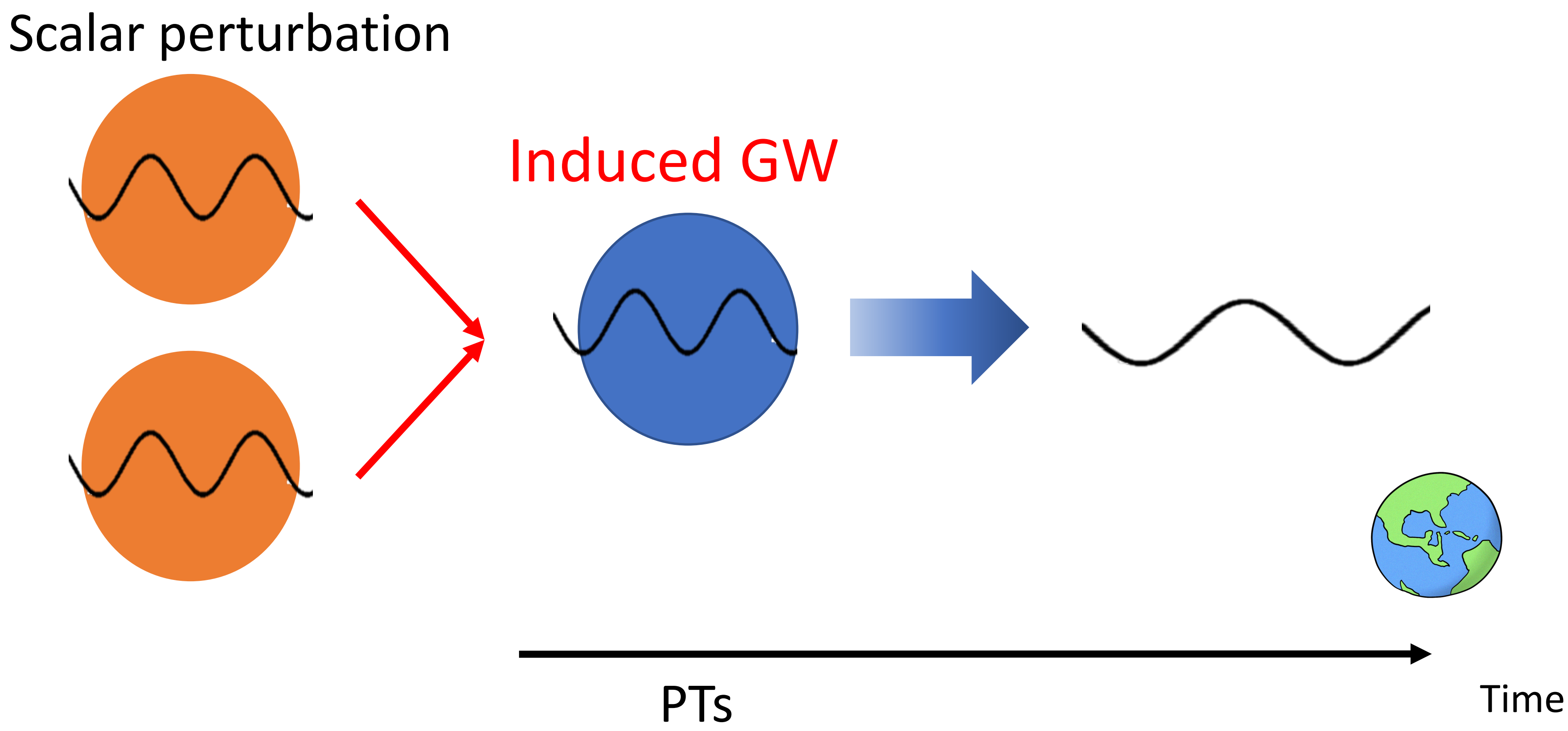
Saikawa&Shirai (2018)

Point ② - Linear Stochastic GW

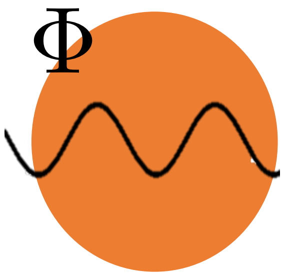


Saikawa&Shirai (2018)

The other type of GW during PTs

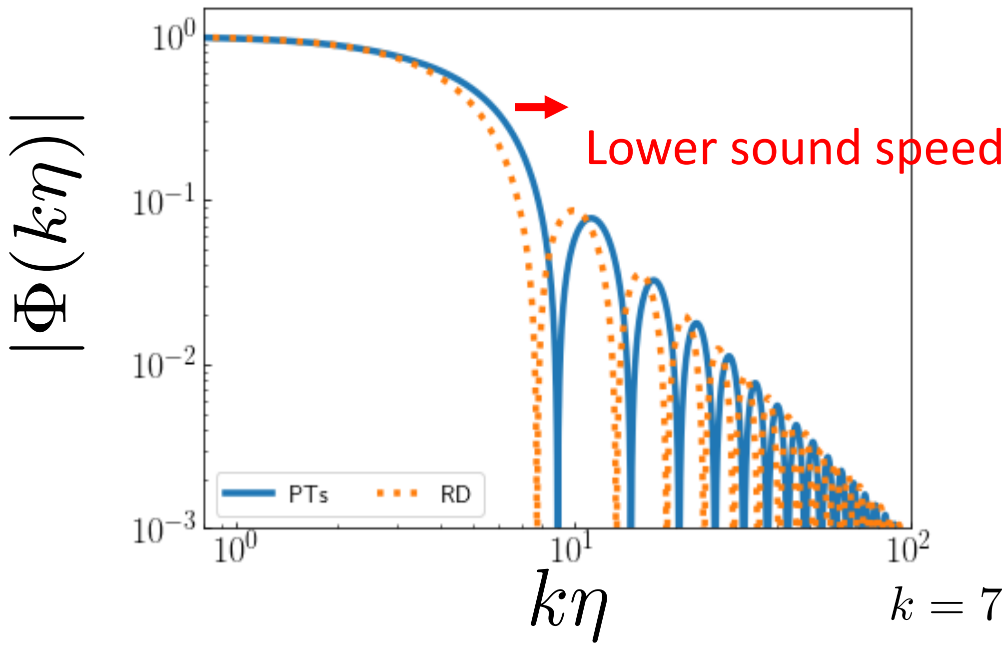


Scalar perturbation as a source of induced GW



- In a nutshell, it's like density perturbation
- It depends on not only EoSp but also sound speed because of Jeans scale

$$\Phi''_{\mathbf{k}}(\eta) + 3\mathcal{H}(1+c_s^2)\Phi'_{\mathbf{k}}(\eta) + \left[c_s^2 k^2 + 3\mathcal{H}^2(c_s^2 - w)\right]\Phi_{\mathbf{k}}(\eta) = 0$$



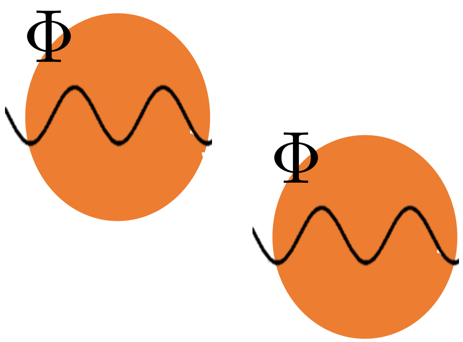
$k = 7.8 \times 10^6 [\mathrm{Mpc}^{-1}]$

The flow of calculation of induced GW

$$\Box h_{\mathbf{k}}(\eta) = 4\mathcal{S}_{\mathbf{k}}\left(\Phi_{\mathbf{k}}\right) \qquad \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}\frac{d}{d\eta} + k^2\right)$$

η : conformal time $\mathcal{H}$: Hubble paramter

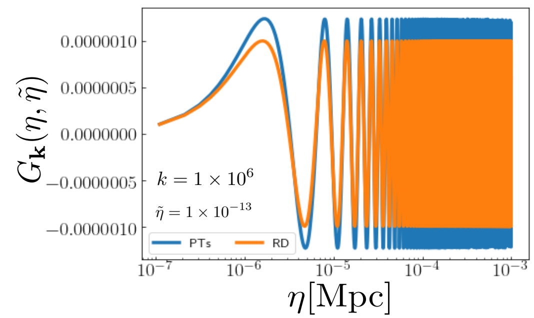
Source func.



\+

GW mode func.

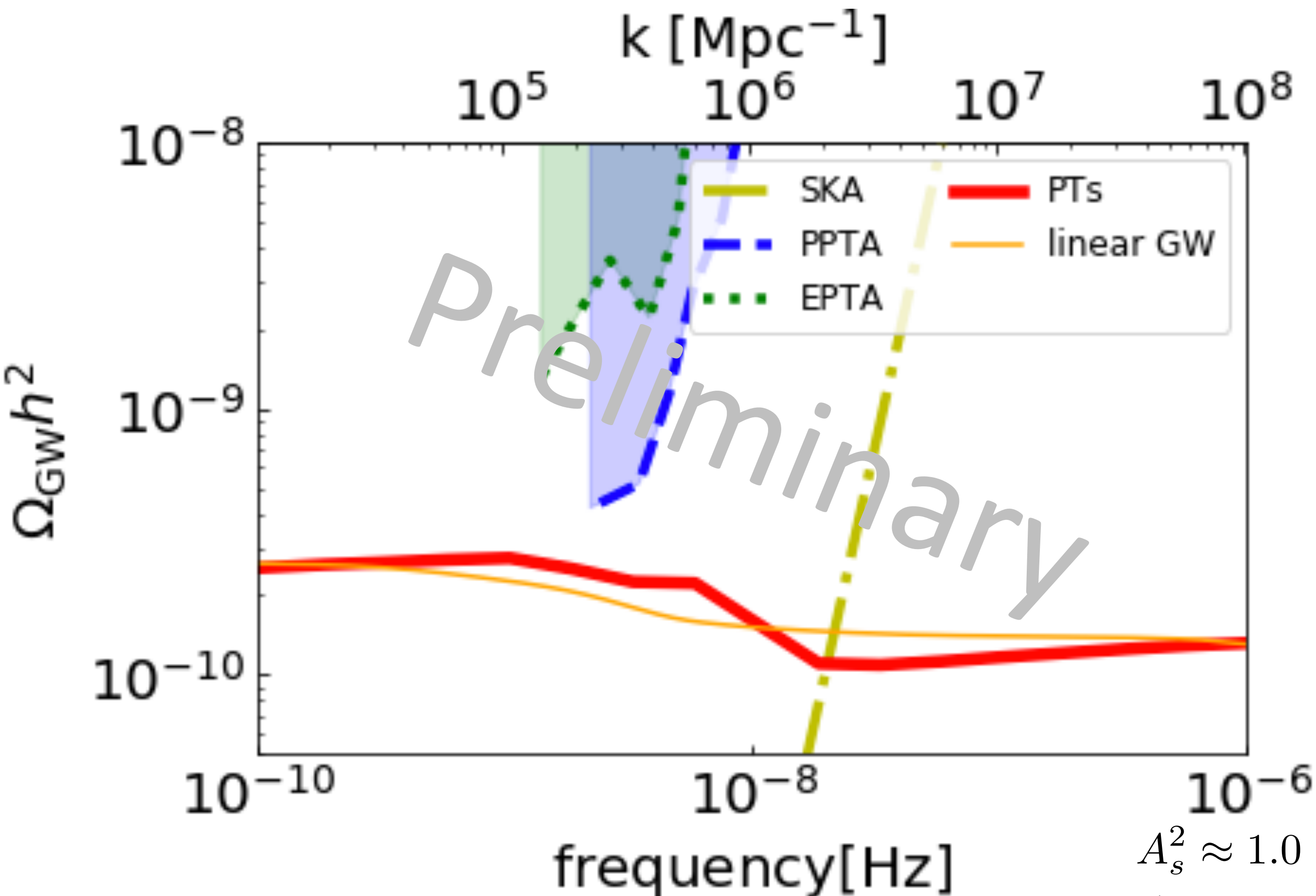
$$G''_{\mathbf{k}}(\eta,\tilde{\eta}) + \left(k^2 - \frac{1-3\omega}{2}\mathcal{H}^2\right) G_{\mathbf{k}}(\eta,\tilde{\eta}) = \delta(\eta - \tilde{\eta})$$



➡ $$\mathcal{P}_h(k,\eta) \propto \int_{|k_1-k_2|\le k\le k_1+k_2} \mathrm{d}k_1 \mathrm{d}k_2 \left(\int d\tilde{\eta}\; G_{\mathbf{k}}(\eta,\tilde{\eta})\mathcal{S}(\Phi_{\mathrm{k}_1}(\tilde{\eta}), \Phi_{\mathrm{k}_2}(\tilde{\eta}))\right)^2$$

Spectrum of induced GW

$$\Omega_{\rm GW}(k,\eta_0) = \frac{1}{24}\left(\frac{k}{\mathcal{H}_0}\right)^2 \overline{\mathcal{P}_h(k,\eta_0)}$$

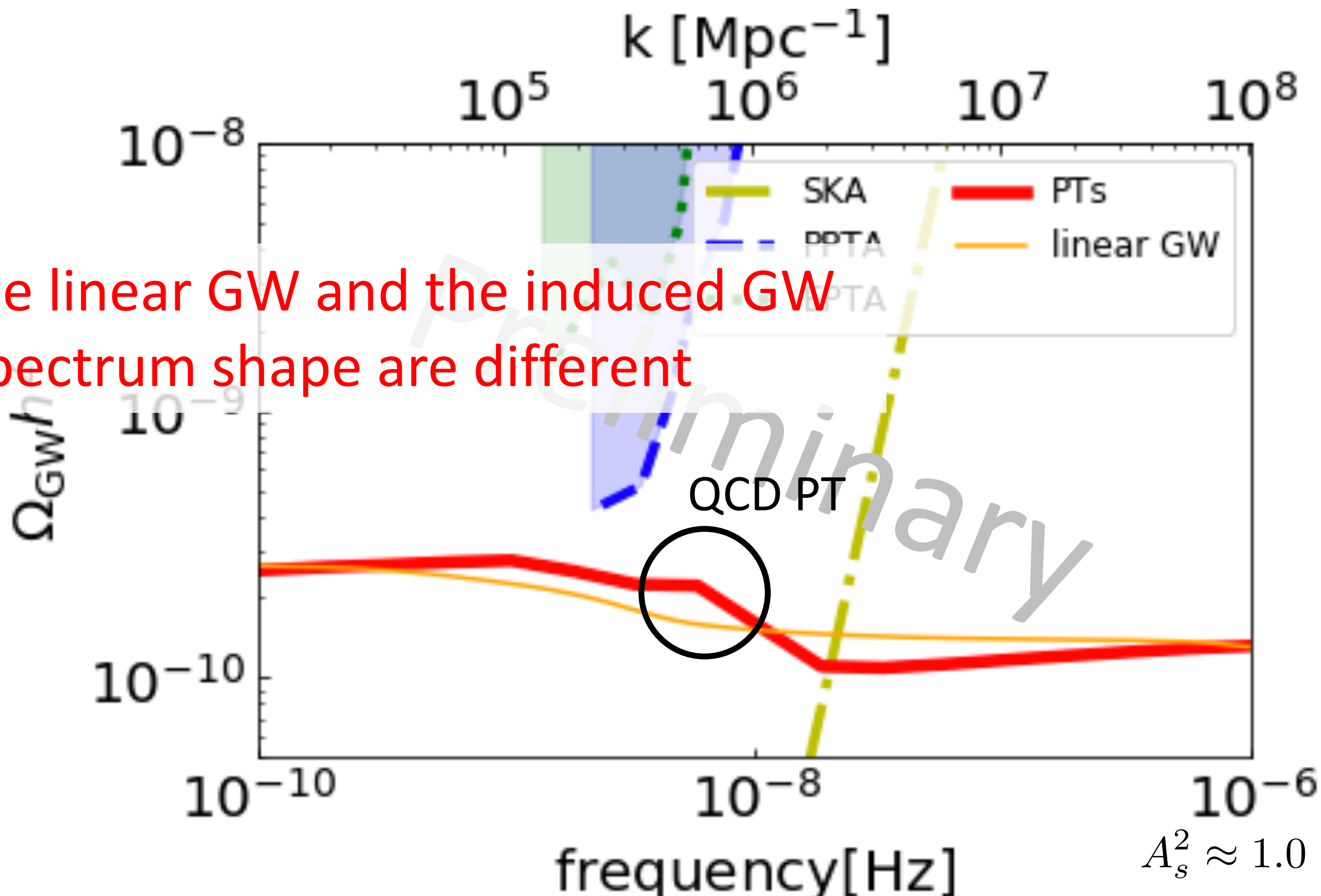


$A_s^2 \approx 1.0\times10^{-5}$

$A_{\rm h} \approx 2.0\times10^{-4}$

Point ① - Spectrum of induced GW

$$\Omega_{\rm GW}(k,\eta_0) = \frac{1}{24}\left(\frac{k}{\mathcal{H}_0}\right)^2 \overline{\mathcal{P}_h(k,\eta_0)}$$



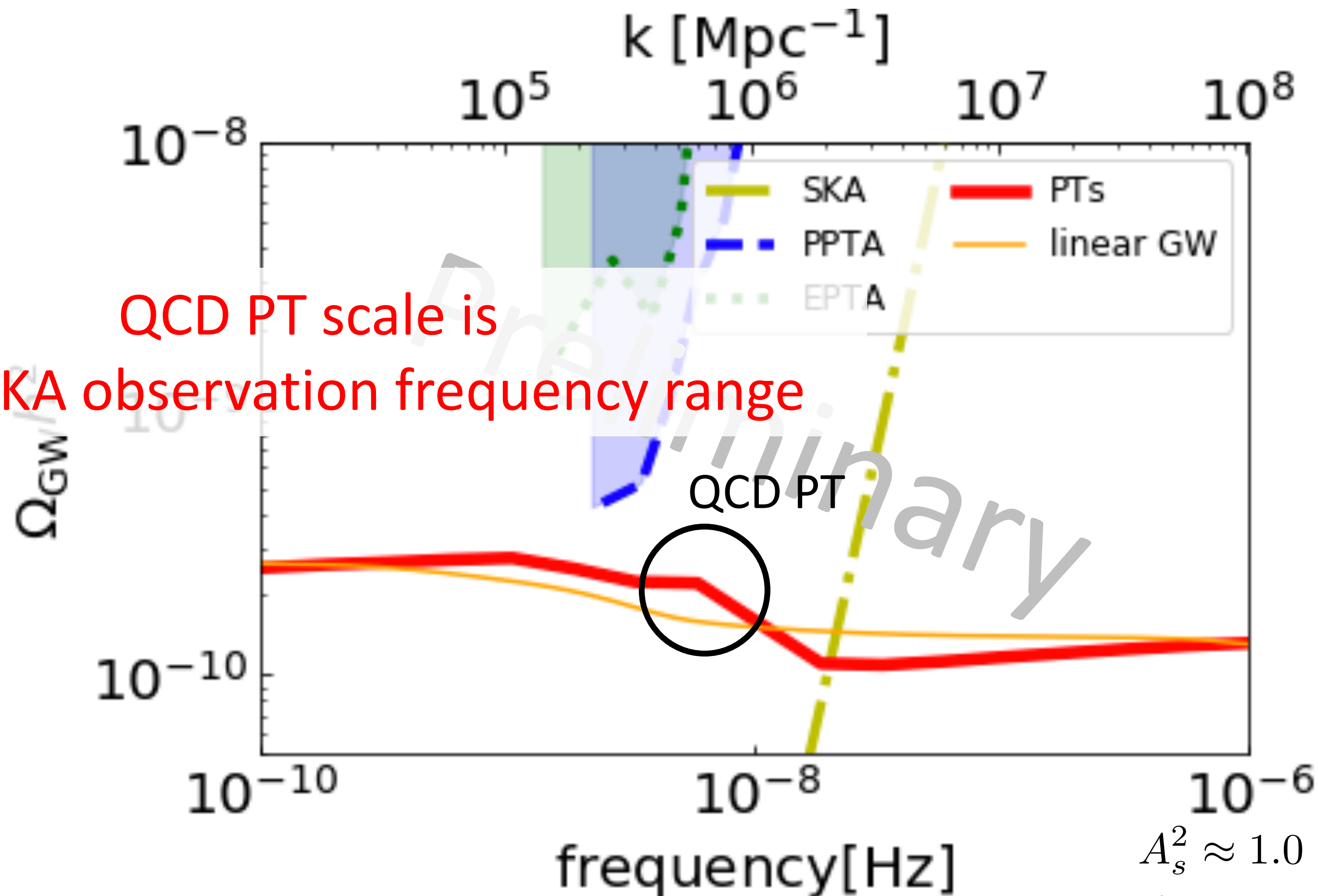
Both the linear GW and the induced GW spectrum shape are different

$A_s^2 \approx 1.0 \times 10^{-5}$

$A_{\rm h} \approx 2.0 \times 10^{-4}$

Point ② - Spectrum of induced GW

$$\Omega_{\rm GW}(k,\eta_0) = \frac{1}{24}\left(\frac{k}{\mathcal{H}_0}\right)^2 \overline{\mathcal{P}_h(k,\eta_0)}$$



QCD PT scale is
in the SKA observation frequency range

$A_s^2 \approx 1.0 \times 10^{-5}$

$A_{\rm h} \approx 2.0 \times 10^{-4}$

Summary & Take-home message

- It is rich in phenomena of phase transitions in the Universe
- BUT, they have not been completely understood yet (e.g. QCD phase transition)
- We calculated the GW spectrum induced from scalar perturbations. (induced GW)

Through the observation of GWs such as by SKA, we could obtain the EoSp and the sound speed during corresponding PTs with the frequency.

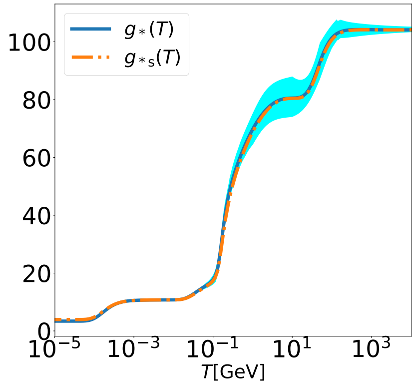
BACK-UP SLIDES

Induced GW during PTs

$$\Box h_{\mathbf{k}}(\eta) = 4\mathcal{S}_{\mathbf{k}}\left(\eta, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right) \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}\frac{d}{d\eta} + k^2\right)$$

Plasma's properties

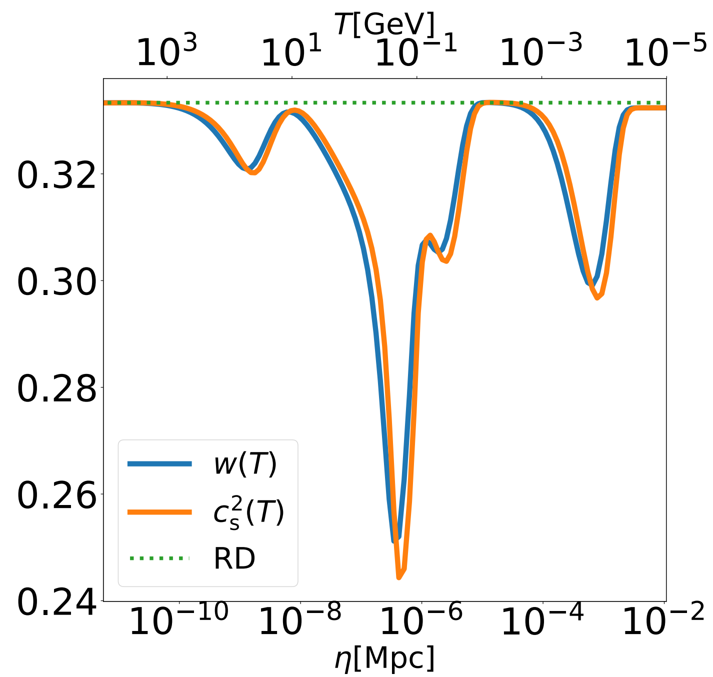
The effective degrees of freedom



$$w(T) = \frac{4g_{*s}(T)}{3g_*(T)} - 1$$

$$c_s^2(T) = \frac{4\left(g'_{*s}(T)T + 4g_{*s}(T)\right)}{3\left(g'_*(T)T + 4g_*(T)\right)} - 1$$

The Equation of State parameter (EoSp) & sound speed



Physical value of Background

✓Assume the entropy conservation's law

$$a(T) = \left(\frac{g_{*\mathrm{s},0}}{g_{*\mathrm{s}}(T)}\right)^{\frac{1}{3}} \frac{T_0}{T}$$

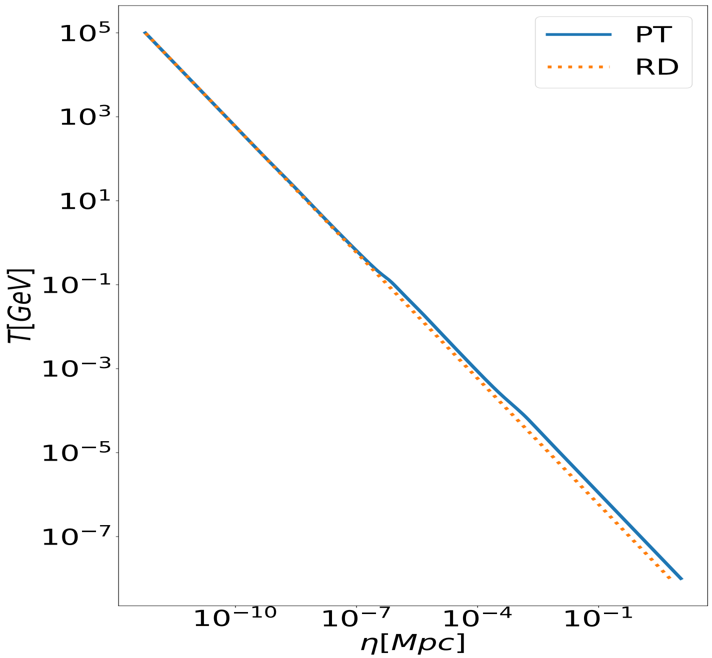
✓ Friedmann equation

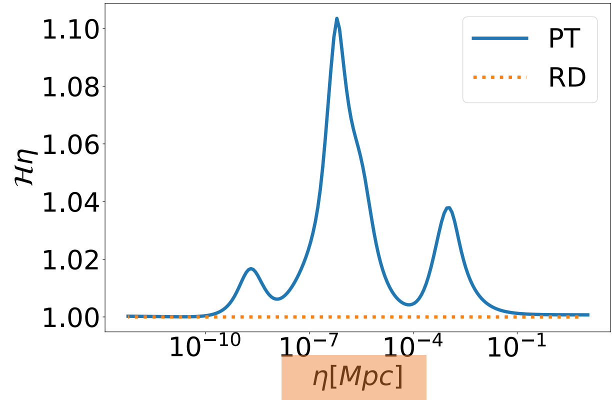
$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho \qquad \frac{d\rho}{d\eta} = -3(1+w)\mathcal{H}\rho$$

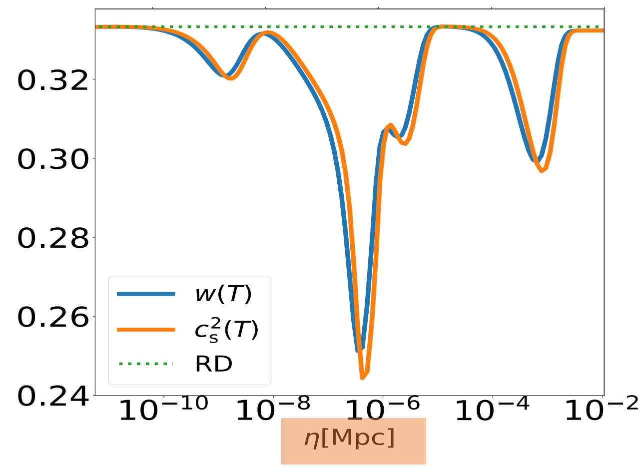
$$\frac{d\eta}{dT} = -\left(\frac{90}{8\pi^3 G}\right)^{\frac{1}{2}} \left(\frac{g_{*\mathrm{s}}}{g_{*\mathrm{s},0}}\right)^{\frac{1}{3}} g_*^{-\frac{3}{2}} T_0^{-1} \frac{\frac{dg_*}{dT} T + 4g_*}{3(1+w)T^2}$$

Physical value of Background

The relation between Temp. vs. Conformal time







Calculation for the induced GW

$$\Box h_{\mathbf{k}}(\eta) = 4\mathcal{S}_{\mathbf{k}}\left(\eta, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right) \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}\frac{d}{d\eta} + k^2\right)$$

$$h_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta} G_{\mathbf{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}_{\mathbf{k}}\left(\tilde{\eta}, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right)\right]$$

Calculation for the induced GW

$$\Box h_{\mathbf{k}}(\eta) = 4\mathcal{S}_{\mathbf{k}}\left(\eta, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right) \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}\frac{d}{d\eta} + k^2\right)$$

$$h_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta}\, G_{\mathbf{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}_{\mathbf{k}}\left(\tilde{\eta}, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right)\right]$$

Methods of Green func.

Source term
induced from scalar pert.

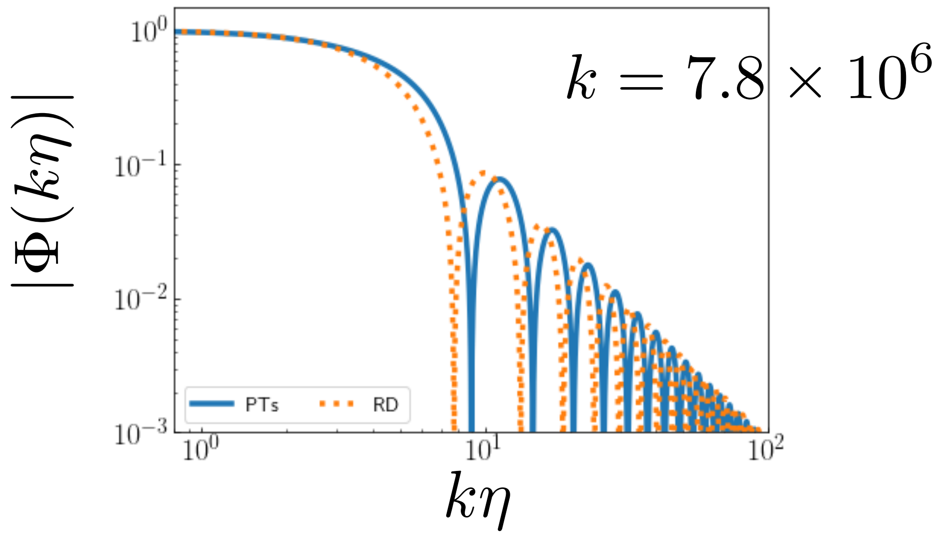
Source term induced from scalar pert.

$$h_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta} G_{\mathbf{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}_{\mathbf{k}} \left(\tilde{\eta}, \mathbf{k}; \omega, c_s^2 \right) \right]$$

$$\mathcal{S}_{\mathbf{k}}(\eta) = \int \frac{d^3\tilde{\mathbf{k}}}{(2\pi)^3} e_{ij}(\mathbf{k}) \tilde{k}^i \tilde{k}^j \Big\{ 2\Phi_{\tilde{k}}(\tilde{k}\eta) \Phi_{|\mathbf{k}-\mathbf{k}'|}(|\mathbf{k}-\tilde{\boldsymbol{k}}|\eta)$$

$$+ \frac{4}{3(1+\omega(\eta))} \left(\Phi_{\tilde{k}}(\tilde{k}\eta) + \tilde{k} \frac{\Phi'_{\tilde{k}}(\tilde{k}\eta)}{\mathcal{H}} \right) \left(\Phi_{|\mathbf{k}-\tilde{\mathbf{k}}|}(|\mathbf{k}-\tilde{\mathbf{k}}|\eta) + |\mathbf{k}-\tilde{\mathbf{k}}| \frac{\Phi'_{|\mathbf{k}-\tilde{\mathbf{k}}|}(|\mathbf{k}-\tilde{\mathbf{k}}|\eta)}{\mathcal{H}} \right) \Big\} \psi_{\tilde{k}} \psi_{|\mathbf{k}-\tilde{\mathbf{k}}|}$$

$$\Phi''_{\mathbf{k}}(\eta) + 3\mathcal{H}(1+c_s^2)\Phi'_{\mathbf{k}}(\eta) + \left[c_s^2 k^2 + 3\mathcal{H}^2(c_s^2 - w) \right] \Phi_{\mathbf{k}}(\eta) = 0$$



The definition of the value related GW

$$h_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta}\, G_{\mathbf{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}_{\mathbf{k}} \left(\tilde{\eta}, \mathbf{k}; \omega, c_{\mathrm{s}}^2 \right) \right]$$

$$\langle h_{\mathbf{k}}(\eta) h_{\mathbf{K}}(\eta) \rangle \equiv (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{K}) \times \frac{2\pi^2}{k^3} \mathcal{P}_h(k, \eta)$$

$$\Omega_{\mathrm{GW}}(k, \eta_0) = \frac{1}{24} \left(\frac{k}{\mathcal{H}_0} \right)^2 \overline{\mathcal{P}_h(k, \eta_0)}$$

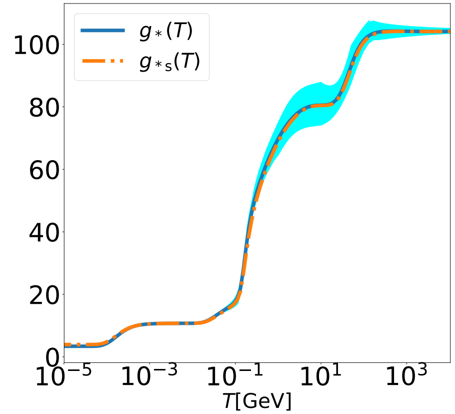
Definition of Parameters

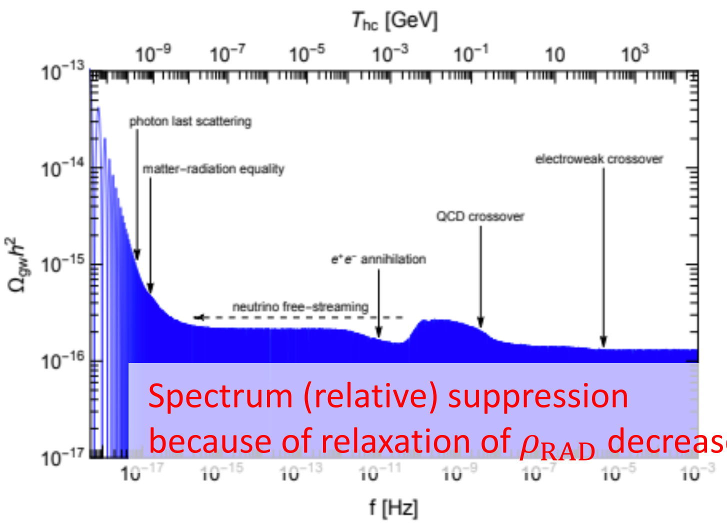
$$\rho(T) = \frac{\pi^2}{30} T^4 g_*(T)$$

$$s(T) = \frac{2\pi^2}{45} T^3 g_{*s}(T)$$

$$a(T) = \left(\frac{g_{*\mathrm{s},0}}{g_{*\mathrm{s}}(T)}\right)^{\frac{1}{3}} \frac{T_0}{T}$$

$$\rho \propto g_* g_{*s}^{-4/3} a^{-4}$$





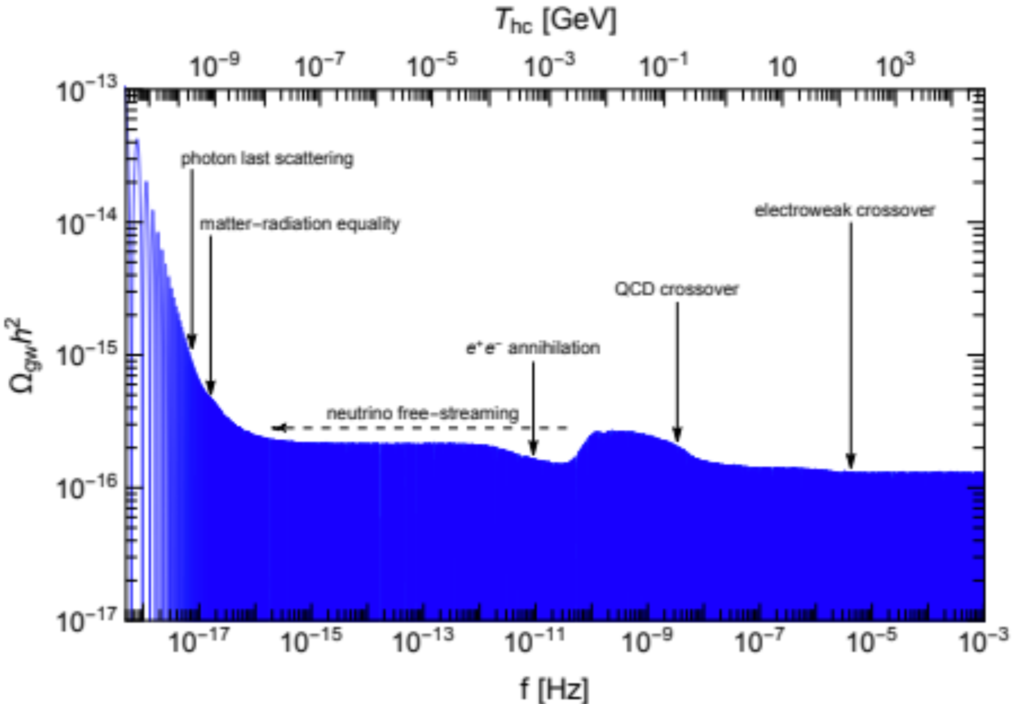
Spectrum (relative) suppression
because of relaxation of ρ_{RAD} decrease

Linear Stochastic GW

$$\Box h_{\mathrm{k}}(\eta) = 0 \qquad \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}(\eta; w)\frac{d}{d\eta} + k^2\right)$$

η : conformal time

$\mathcal{H}$: Hubble paramter



Saikawa&Shirai (2018)

Calulation for BG

$$\frac{d\rho}{d\eta} = -3(1+w)\mathcal{H}\rho \quad \Rightarrow \quad \frac{d\rho}{d\eta} = \frac{\pi^2}{30}\left(\frac{dg_*}{dT}T + 4g_*\right)T'T^3$$

$$\mathcal{H}^2 = \frac{8\pi G}{3}a^2\rho \qquad a(T) = \left(\frac{g_{*s,0}}{g_{*s}(T)}\right)^{\frac{1}{3}}\frac{T_0}{T}$$

$$-3(1+w)\mathcal{H}\rho = -\frac{\pi^2(1+w)}{10}\left(\frac{8\pi^3 G}{90}\right)^{\frac{1}{2}}\left(\frac{g_{*s,0}}{g_{*s}}\right)^{\frac{1}{3}} g_*^{\frac{3}{2}} T_0 T^5$$

$$\frac{d\eta}{dT} = -\left(\frac{90}{8\pi^3 G}\right)^{\frac{1}{2}}\left(\frac{g_{*s}}{g_{*s,0}}\right)^{\frac{1}{3}} g_*^{-\frac{3}{2}} T_0^{-1}\frac{\frac{dg_*}{dT}T + 4g_*}{3(1+w)T^2}$$

Calculation for Induced GW

$$\Box h_{\mathrm{k}}(\eta) = \mathcal{S}\left(\eta, k; \omega, c_{\mathrm{s}}^2\right) \quad \left(\Box = \frac{d^2}{d\eta^2} + 2\mathcal{H}\frac{d}{d\eta} + k^2\right)$$

$$h_{\boldsymbol{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta} G_{\boldsymbol{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}\left(\tilde{\eta}, \boldsymbol{k}; \omega, c_{\mathrm{s}}^2\right)\right]$$

$$\mathcal{S}_{\mathbf{k}}(\eta) = \int \frac{\mathrm{d}^3\tilde{\mathbf{k}}}{(2\pi)^3} e_{ij}(\mathbf{k}) \tilde{k}^i \tilde{k}^j \left\{ 2\Phi_{\tilde{k}}(\tilde{k}\eta) \Phi_{|\mathbf{k}-\mathbf{k}'|}(|\mathbf{k} - \tilde{\boldsymbol{k}}|\eta) \right.$$

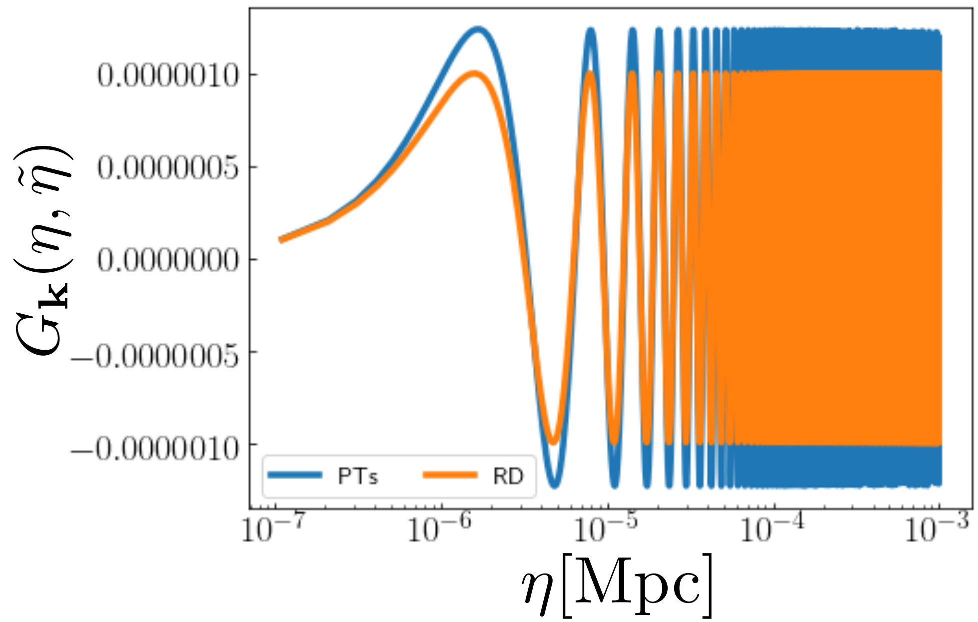
$$\left. + \frac{4}{3(1+\omega(\eta))} \left(\Phi_{\tilde{k}}(\tilde{k}\eta) + \tilde{k}\frac{\Phi'_{\tilde{k}}(\tilde{k}\eta)}{\mathcal{H}}\right) \left(\Phi_{|\mathbf{k}-\tilde{\mathbf{k}}|}(|\mathbf{k} - \tilde{\mathbf{k}}|\eta) + |\mathbf{k} - \tilde{\mathbf{k}}| \frac{\Phi'_{|\mathbf{k}-\tilde{\mathbf{k}}|}(|\mathbf{k} - \tilde{\mathbf{k}}|\eta)}{\mathcal{H}}\right) \right\} \psi_{\tilde{k}} \psi_{|\mathbf{k}-\tilde{\mathbf{k}}|}$$

Methods of Green func.

$$\Box h_{\mathbf{k}}(\eta) = 4\mathcal{S}_{\mathbf{k}}\left(\eta, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right) \qquad G_{\mathbf{k}}(\eta, \tilde{\eta}) \equiv a(\eta) h_{\mathbf{k}}$$

$$\Rightarrow \quad G''_{\mathbf{k}}(\eta, \tilde{\eta}) + \left(k^2 - \frac{1-3\omega}{2}\mathcal{H}^2\right) G_{\mathbf{k}}(\eta, \tilde{\eta}) = \delta(\eta - \tilde{\eta})$$

$k = 1 \times 10^6 \quad \tilde{\eta} = 1 \times 10^{-13}$



The general solution is made by the two homogeneous solutions g_1 and g_2

$$G_{\mathbf{k}}(\eta, \tilde{\eta}) = \frac{g_1(\eta) g_2(\tilde{\eta}) - g_1(\tilde{\eta}) g_2(\eta)}{g'_1(\tilde{\eta}) g_2(\tilde{\eta}) - g_1(\tilde{\eta}) g'_2(\tilde{\eta})}$$

Initial condition of Green func.

$$G''_{\mathbf{k}}(\eta,\tilde{\eta}) + \left(k^2 - \frac{1-3\omega}{2}\mathcal{H}^2\right) G_{\mathbf{k}}(\eta,\tilde{\eta}) = \delta(\eta-\tilde{\eta})$$

Initial condition

For (g1, g1’) -> (1.0 , 0.0)

For (g2, g2’) -> (0.0, k)

$$G_{\mathbf{k}}(\eta,\tilde{\eta}) = \frac{g_1(\eta)g_2(\tilde{\eta}) - g_1(\tilde{\eta})g_2(\eta)}{g_1'(\tilde{\eta})g_2(\tilde{\eta}) - g_1(\tilde{\eta})g_2'(\tilde{\eta})}$$

Expression for the induced tensor perturbation

$$h_{\mathbf{k}}(\eta) = \frac{1}{a(\eta)} \int d\tilde{\eta} G_{\mathbf{k}}(\eta, \tilde{\eta}) \left[a(\tilde{\eta}) \mathcal{S}_{\mathbf{k}}\left(\tilde{\eta}, \mathbf{k}; \omega, c_{\mathrm{s}}^2\right)\right]$$

$$\mathcal{P}_h(k,\eta) = \frac{64}{81 a^2(\eta)} \int_{|k_1 - k_2| \le k \le k_1 + k_2} \mathrm{d}\log \mathrm{k}_1 \mathrm{d}\log \mathrm{k}_2 \mathrm{I}^2(\mathrm{k}, \mathrm{k}_1, \mathrm{k}_2, \eta) \frac{(\mathrm{k}_1^2 - (\mathrm{k}^2 - \mathrm{k}_2^2 + \mathrm{k}_1^2)^2/(4\mathrm{k}^2))^2}{\mathrm{k}_1 \mathrm{k}_2 \mathrm{k}^2} \mathcal{P}_\zeta(\mathrm{k}_1) \mathcal{P}_\zeta(\mathrm{k}_2)$$

$$\Omega_{\mathrm{GW}}(k,\eta_0) h^2 = \left(\frac{a_{\mathrm{c}} \mathcal{H}_{\mathrm{c}}}{a_0 \mathcal{H}_0}\right)^2 \Omega_{\mathrm{GW}}(k, \eta_{\mathrm{c}}) h^2 \simeq \left(\frac{g_{*\mathrm{s},0}}{g_{*\mathrm{s,c}}}\right)^{\frac{4}{3}} \frac{g_{*,\mathrm{c}}}{g_{*,0}} \Omega_{\mathrm{r}0} h^2 \frac{1}{24} \left(\frac{k}{\mathcal{H}_{\mathrm{c}}}\right)^2 \overline{\mathcal{P}_h(k, \eta_{\mathrm{c}})}$$

Previous work

Primordial black holes with an accurate QCD equation of state

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Making use of definitive new lattice computations of the Standard Model thermodynamics during the quantum chromodynamic (QCD) phase transition, we calculate the enhancement in the mass distribution of primordial black holes (PBHs) due to the softening of the equation of state. We find that the enhancement peaks at approximately $0.7M_\odot$, with the formation rate increasing by at least two orders of magnitude due to the softening of the equation of state at this time, with a range of approximately $0.3M_\odot < M < 1.4M_\odot$ at full width half-maximum. PBH formation is increased by a smaller amount for PBHs with masses spanning a large range, $10^{-3}M_\odot < M_{\rm PBH} < 10^3 M_\odot$, which includes the masses of the BHs that LIGO detected. The most significant source of uncertainty in the number of PBHs formed is now due to unknowns in the formation process, rather than from the phase transition. A near scale-invariant density power spectrum tuned to generate a population with mass and merger rate consistent with that detected by LIGO should also produce a much larger energy density of PBHs with solar mass. The existence of BHs below the Chandresekhar mass limit would be a smoking gun for a primordial origin and they could arguably constitute a significant fraction of the cold dark matter density. They also pose a challenge to inflationary model building which seek to produce the LIGO BHs without overproducing lighter PBHs.

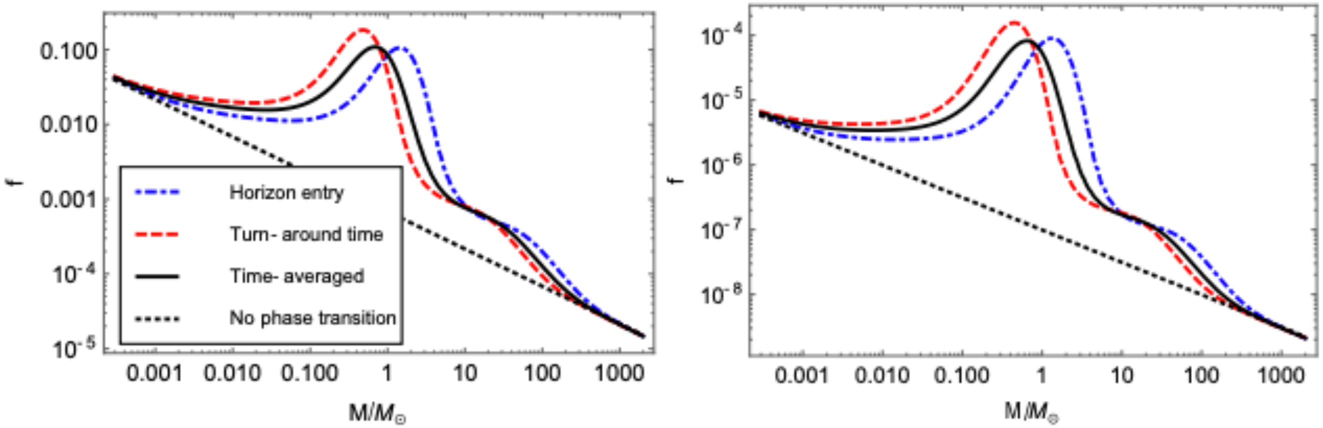

FIG. 3: The mass distribution f of PBHs forming during the phase transition is shown for a scale-invariant density power spectrum, with different ways of treating the equation of state parameter during the black hole formation process. The different lines correspond to using the equation of state at horizon entry, turn-around and a time-averaged value. The straight dashed black line represents the mass function f of PBHs if there is no phase transition (taking the critical density perturbation $\delta_c = 0.453$). The variance of the density contrast at horizon crossing is taken to be $\sigma^2 = 0.004$ for the left plot and $\sigma^2 = 0.003$ for the right. Using the time-averaged value of δ_c, for $\sigma^2 = 0.004$, the peak occurs at $0.69M_{\odot}$ and the range at half-maximum is $0.30M_{\odot} < M < 1.4M_{\odot}$, whilst for $\sigma^2 = 0.003$, the peak (which becomes sharper for smaller values of σ^2) occurs at $0.65M_{\odot}$ and the range at half-maximum is $0.32M_{\odot} < M < 1.2M_{\odot}$.

$$\mathcal{P}_h(k,\eta) \propto \int_{|k_1-k_2|\leq k\leq k_1+k_2} \mathrm{dlog}\, k_1 \mathrm{dlog}\, k_2 \left(\int d\tilde{\eta}\; G_{\mathbf{k}}(\eta,\tilde{\eta})\mathrm{F}(\Phi_{\mathbf{k_1}}(\tilde{\eta}),\Phi_{\mathbf{k_2}}(\tilde{\eta}))\right)^2$$