Gravitational Wave Production

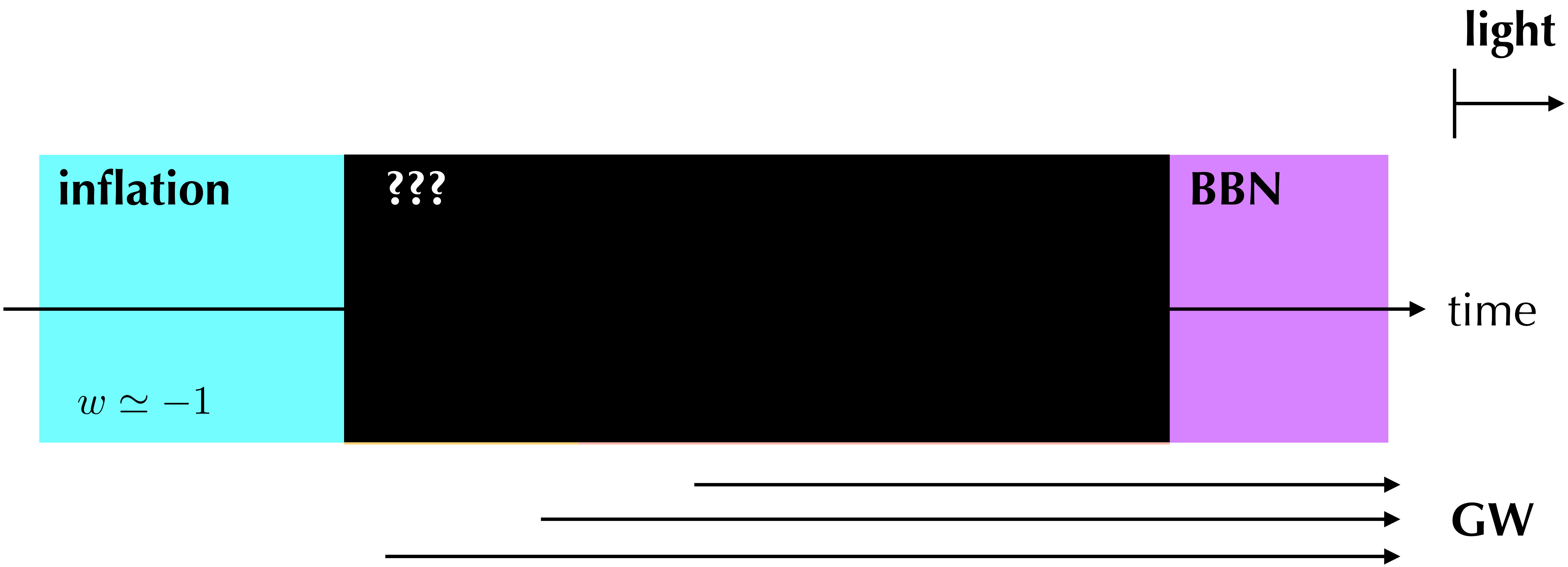
right after Primordial Black Hole Evaporation

Takahiro Terada
(CTPU, IBS)

Based on: Inomata, Kawasaki, Mukaida, TT, Yanagida, PRD101 (2020) 123533, 2003.10455 [astro-ph.CO].

GW to probe Early Universe

GW to probe Early Universe



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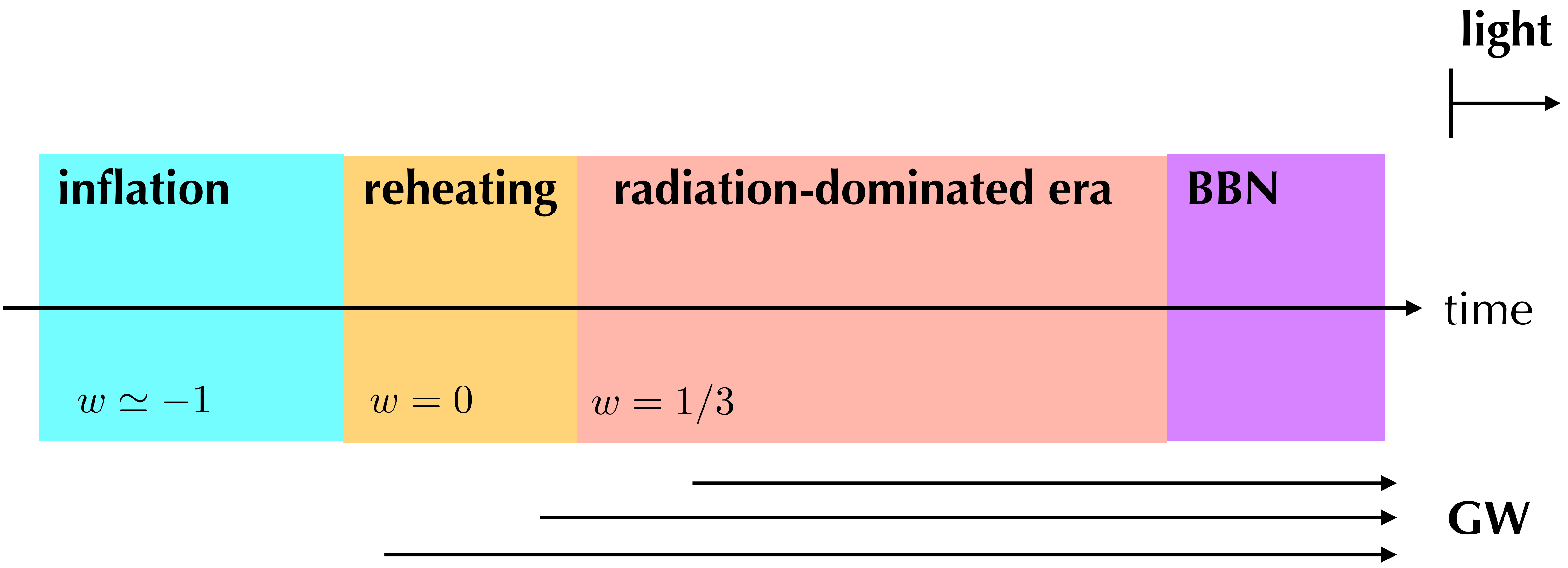

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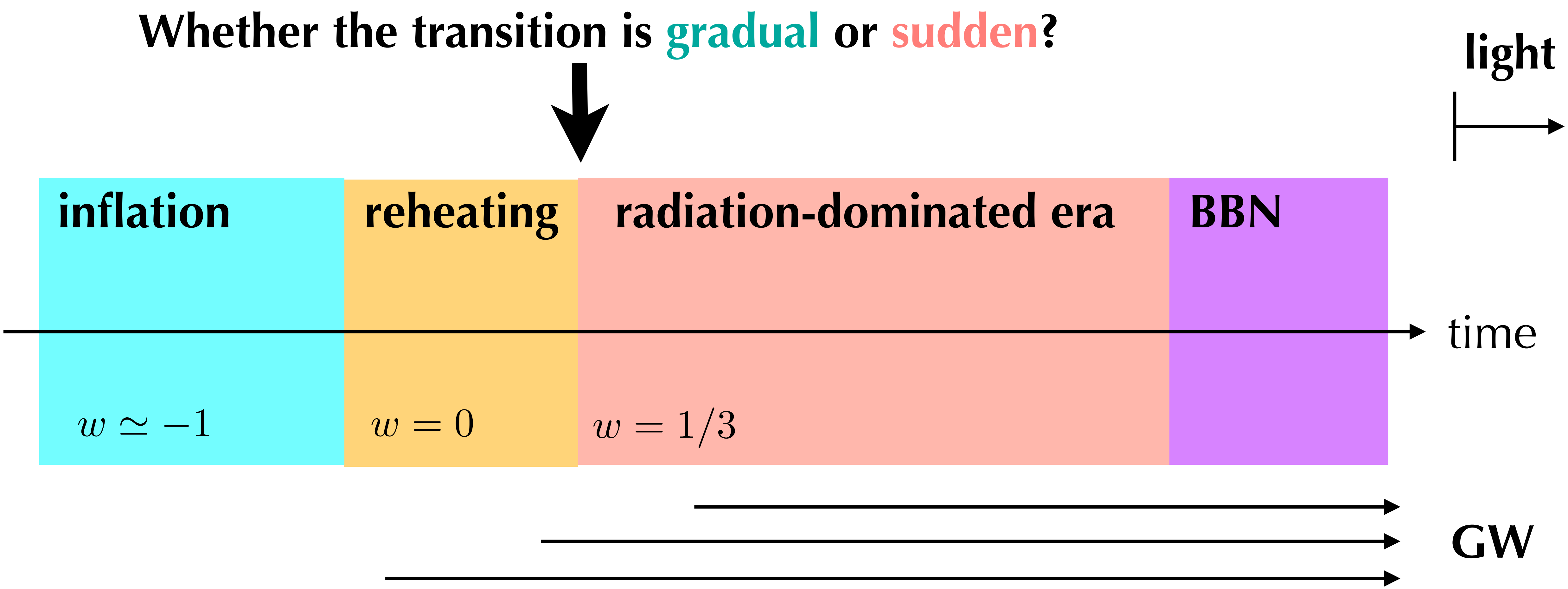

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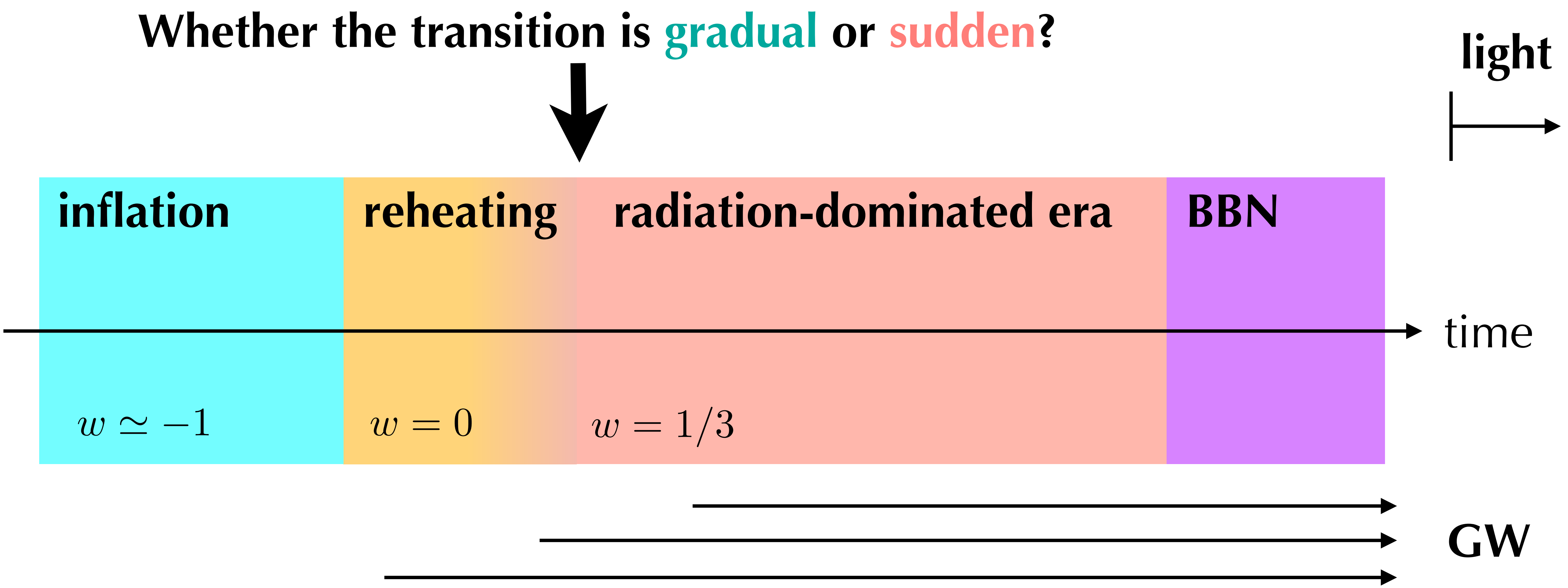

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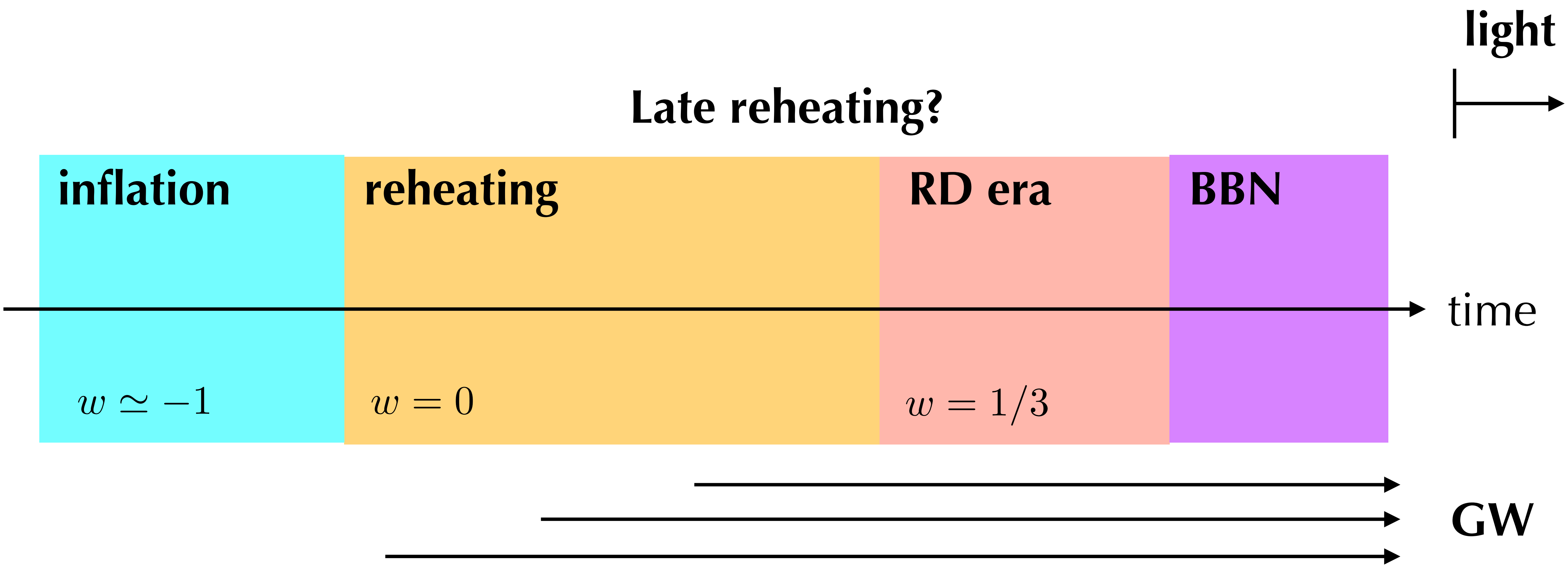

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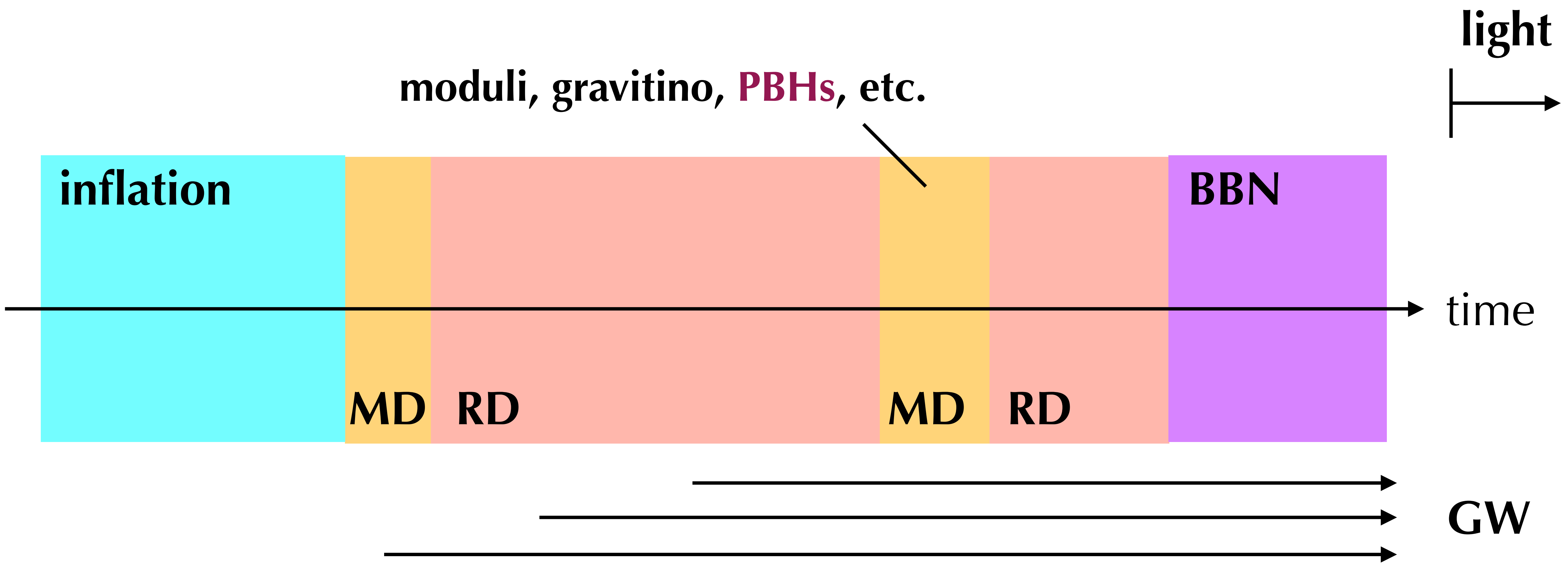

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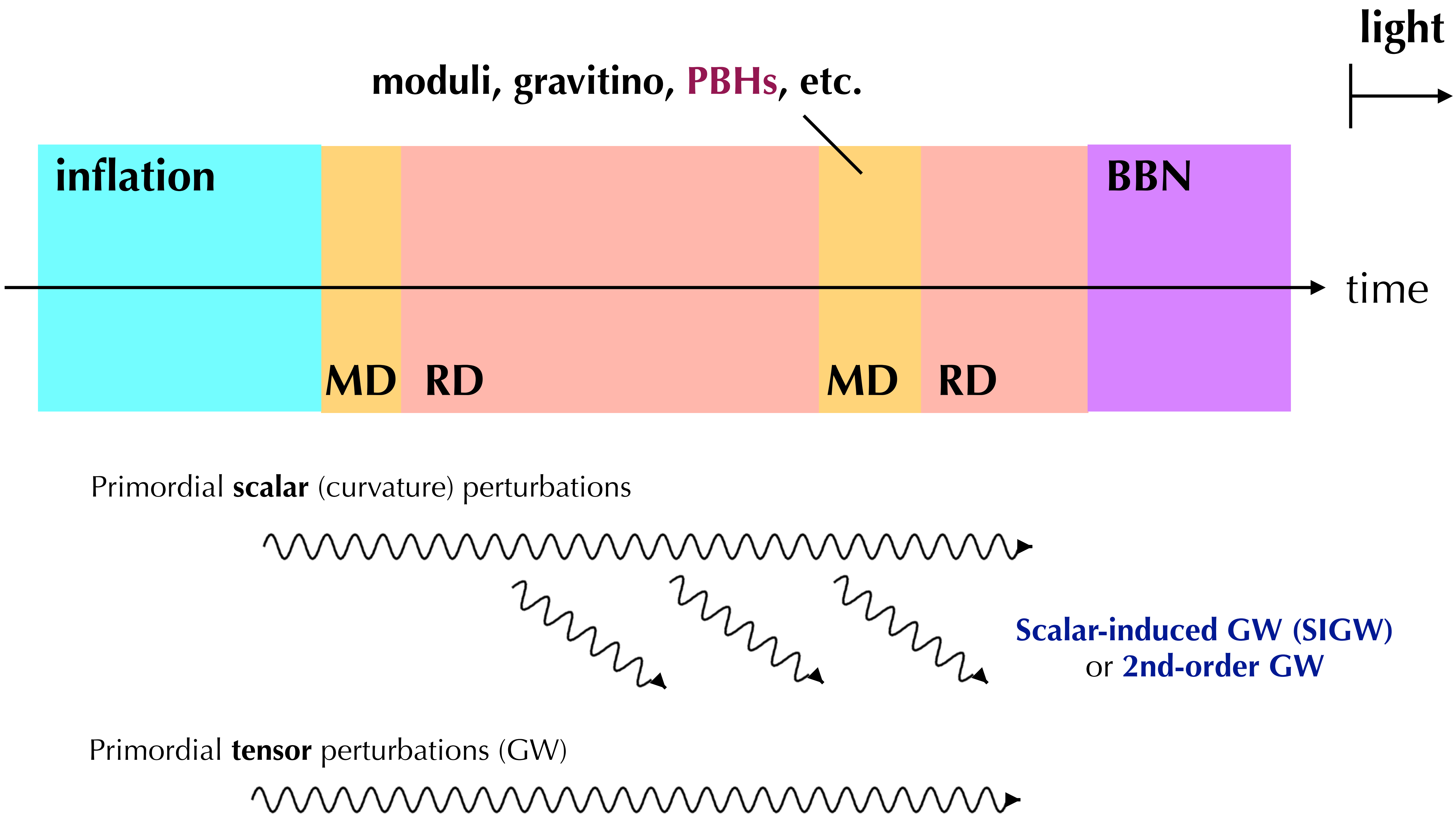

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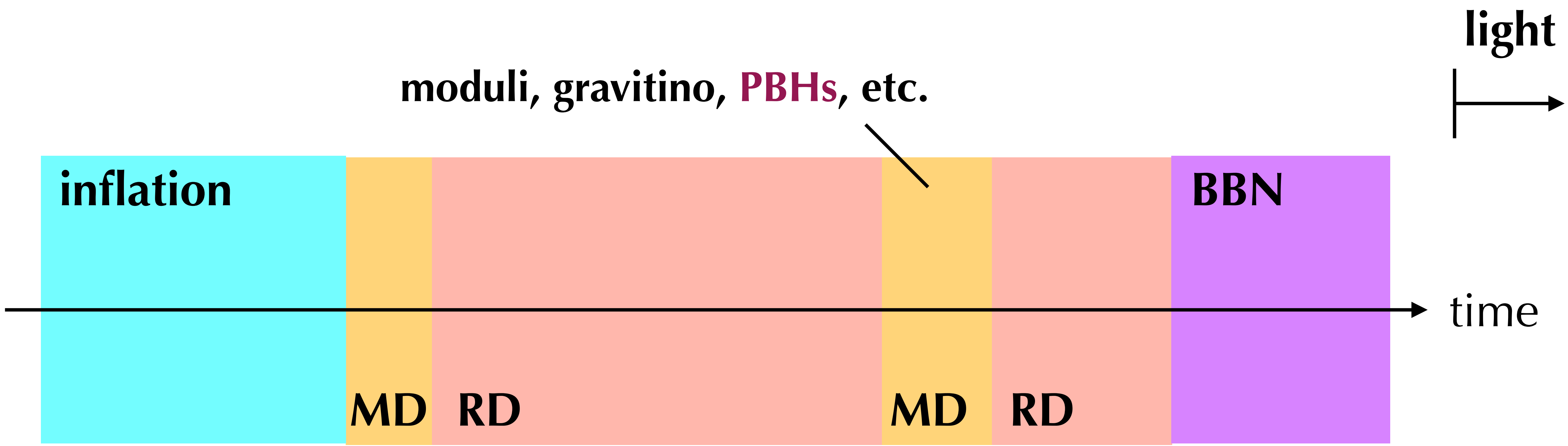


Scalar-induced GW (SIGW)
or **2nd-order GW**

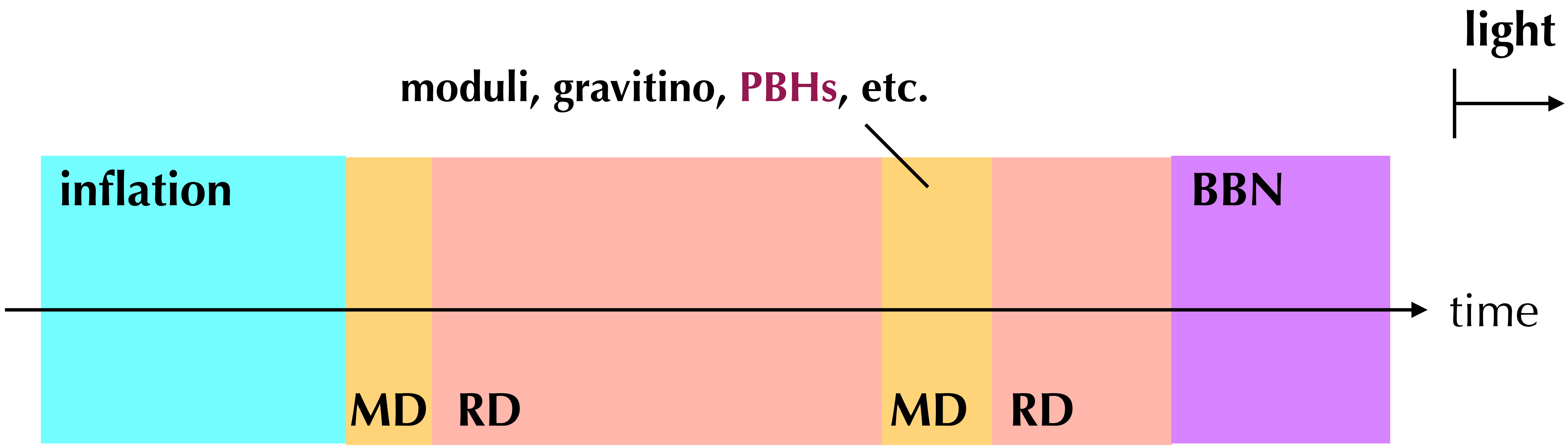
[Ananda, Clarkson, Wands, gr-qc/0612013]
[Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290]

$$\mathrm{d}s^2 = -a^2(1+2\Phi)\mathrm{d}\eta^2 + a^2\left((1-2\Phi)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j$$

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k}) q_i q_j \left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k-q}} + \frac{4}{3(1+w)}\left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k-q}} + \Phi_{\mathbf{k-q}}\right)\right)$$

GW to probe Early Universe



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(2nd-order tensor) = (1st-order scalar)^2

Primordial Black Hole

General motivation

- Dark Matter $10^{-16} \lesssim M/M_\odot \lesssim 10^{-11}$
- Binary Black Holes (LIGO/Virgo) $10^{1} \lesssim M/M_\odot \lesssim 10^{2}$
- Supermassive Black Holes, etc. $10^{4} \lesssim M/M_\odot \lesssim 10^{5}$

$(1\,M_\odot = 2.0 \times 10^{30}\,\mathrm{kg})$

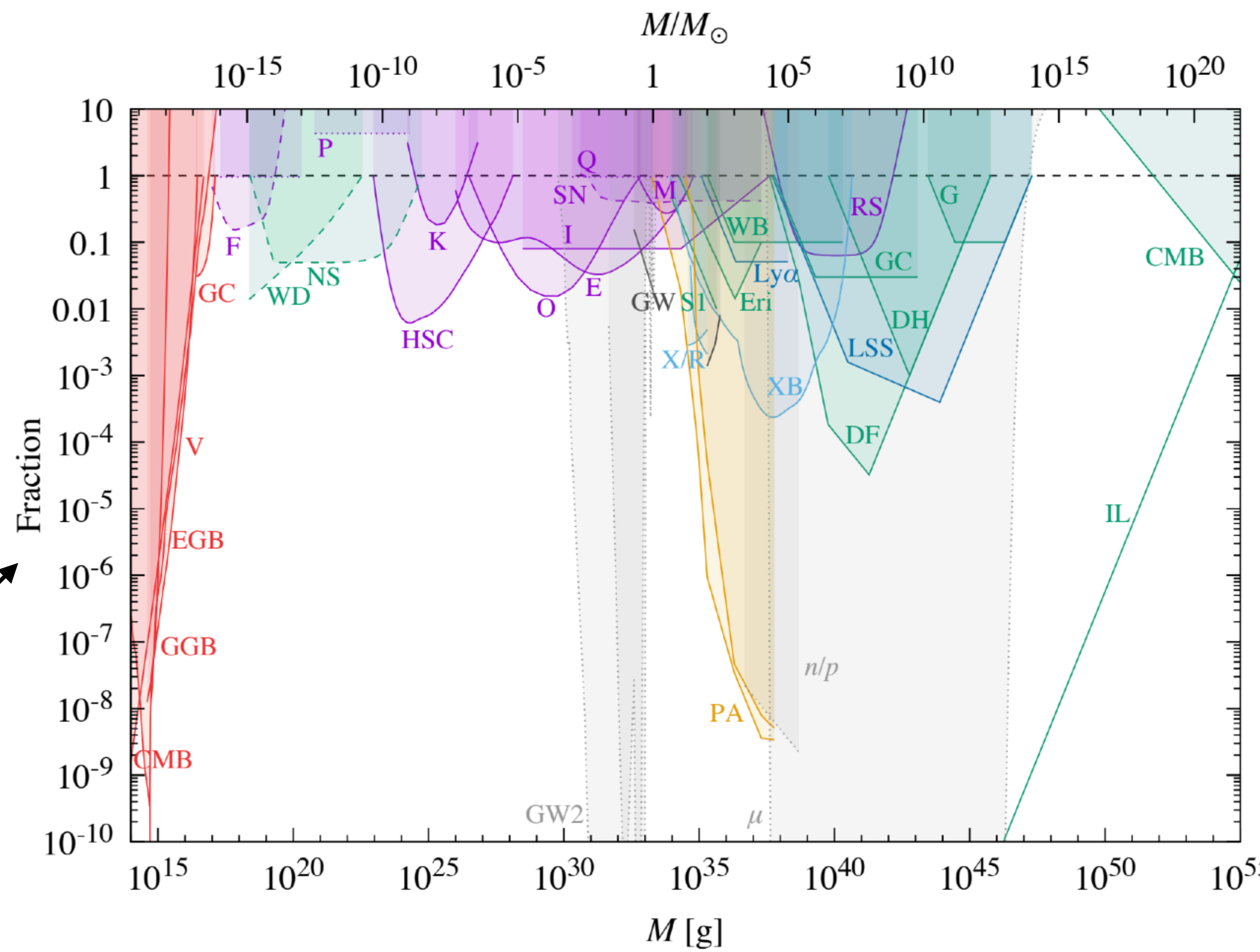
Production mechanism

- Phase transition (topological defects, soft EoS)
- Enhanced curvature perturbations, etc.

Change of the mass

- Accretion
- Mergers
- Hawking radiation $\tau_{\mathrm{BH}} \approx \tau_{\mathrm{Universe}} \times \left(\frac{M}{10^{15}\mathrm{g}}\right)^3$

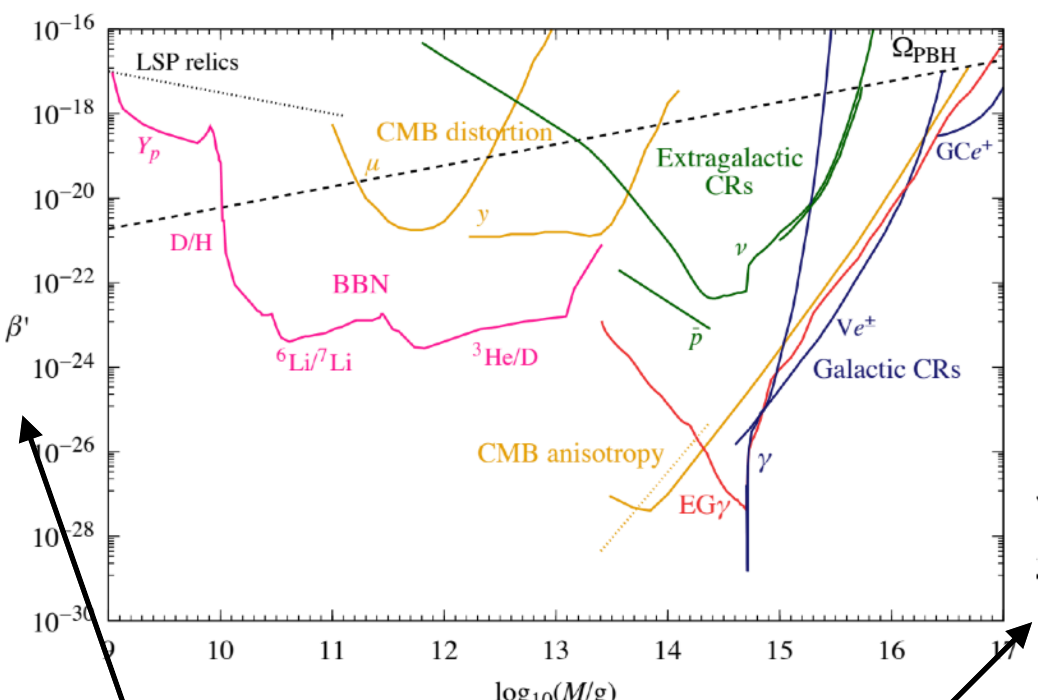
Constraints on PBHs

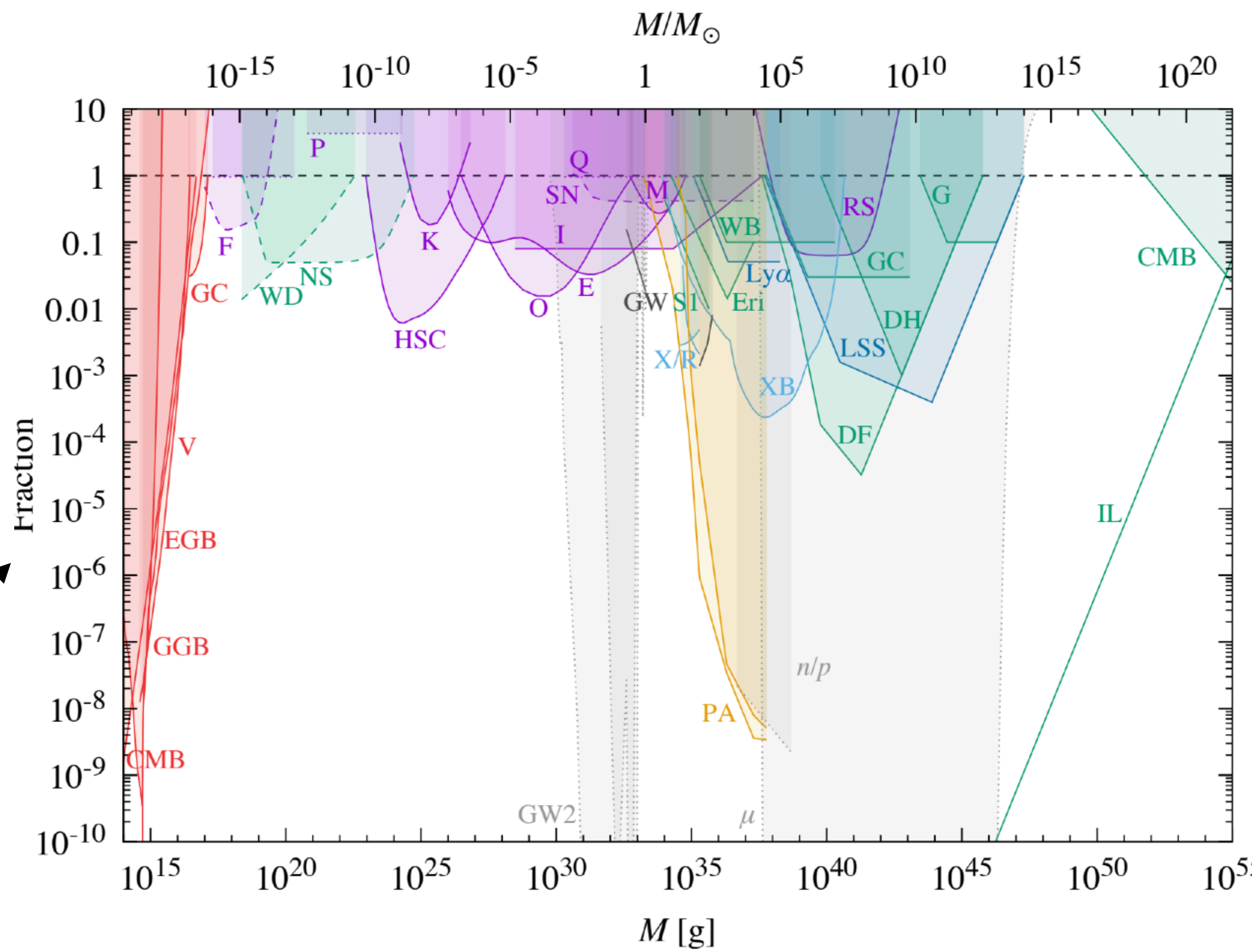


$$f(M) \equiv \left. \frac{\rho_{\rm PBH}(M)}{\rho_{\rm DM}} \right|_{\rm now}$$

[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

Constraints on PBHs



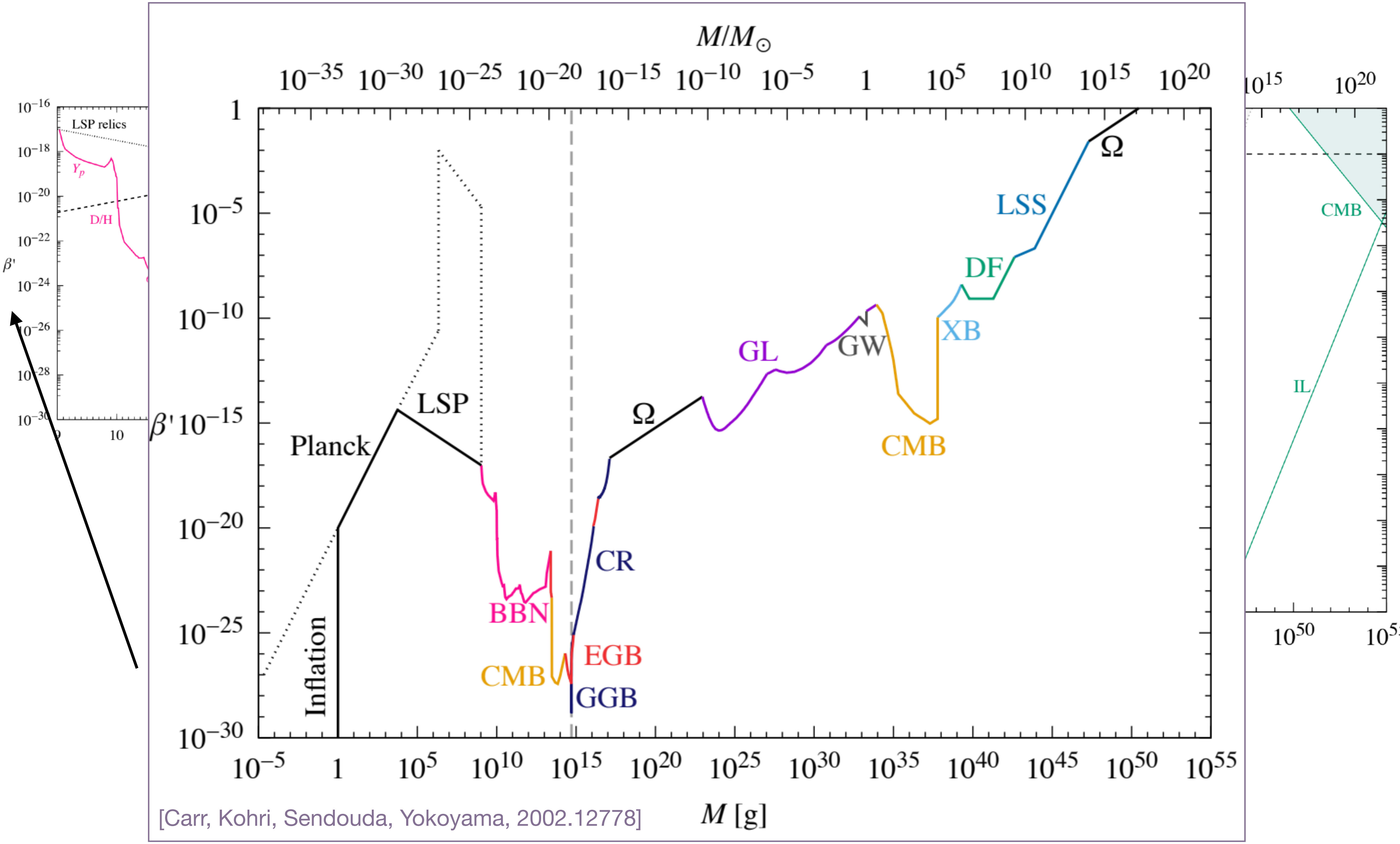


$$f(M) \equiv \left.\frac{\rho_{\rm PBH}(M)}{\rho_{\rm DM}}\right|_{\rm now}$$

$$\beta(M) \equiv \left.\frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}}\right|_{\rm formation}$$

[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

Constraints on PBHs



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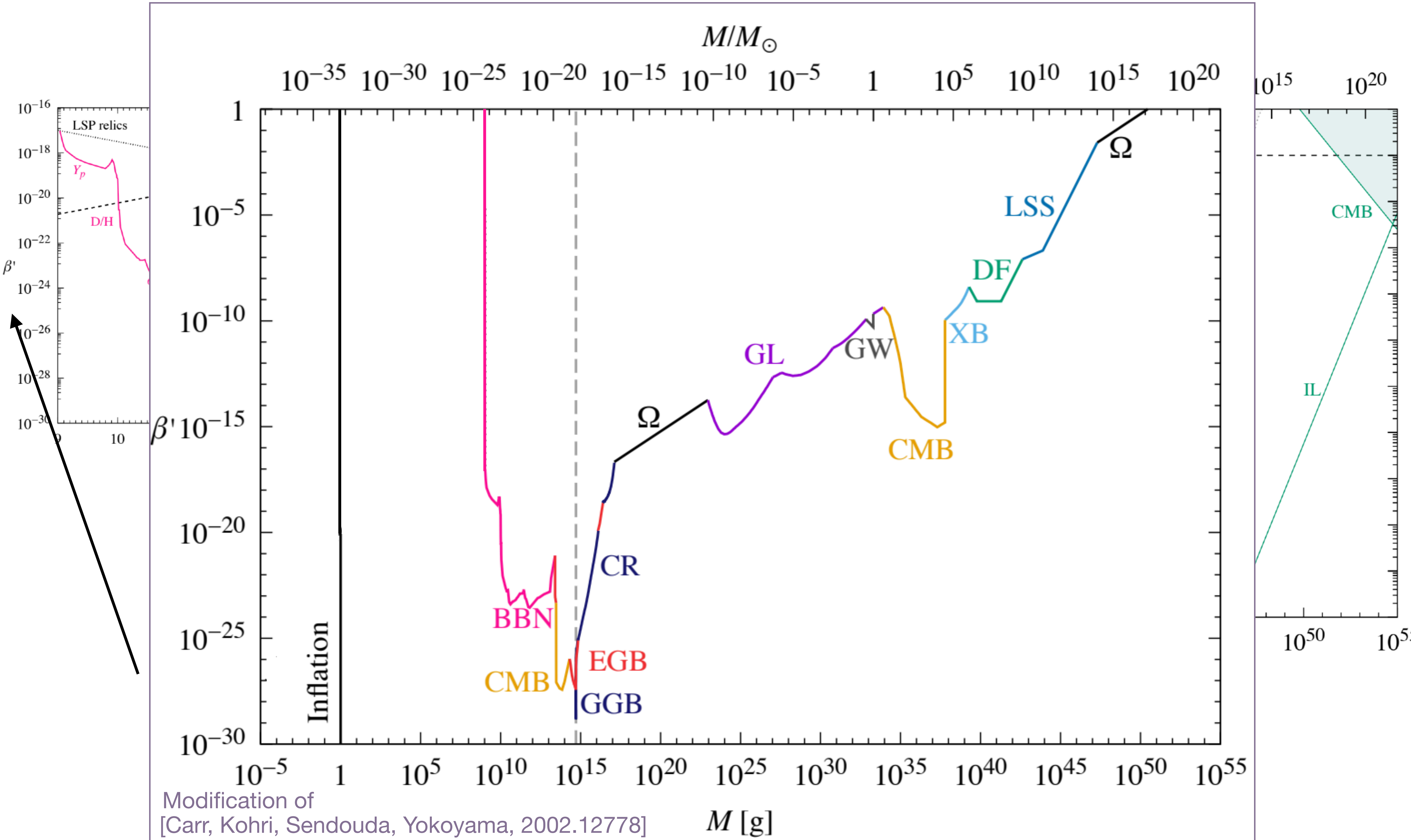

Modification of
[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

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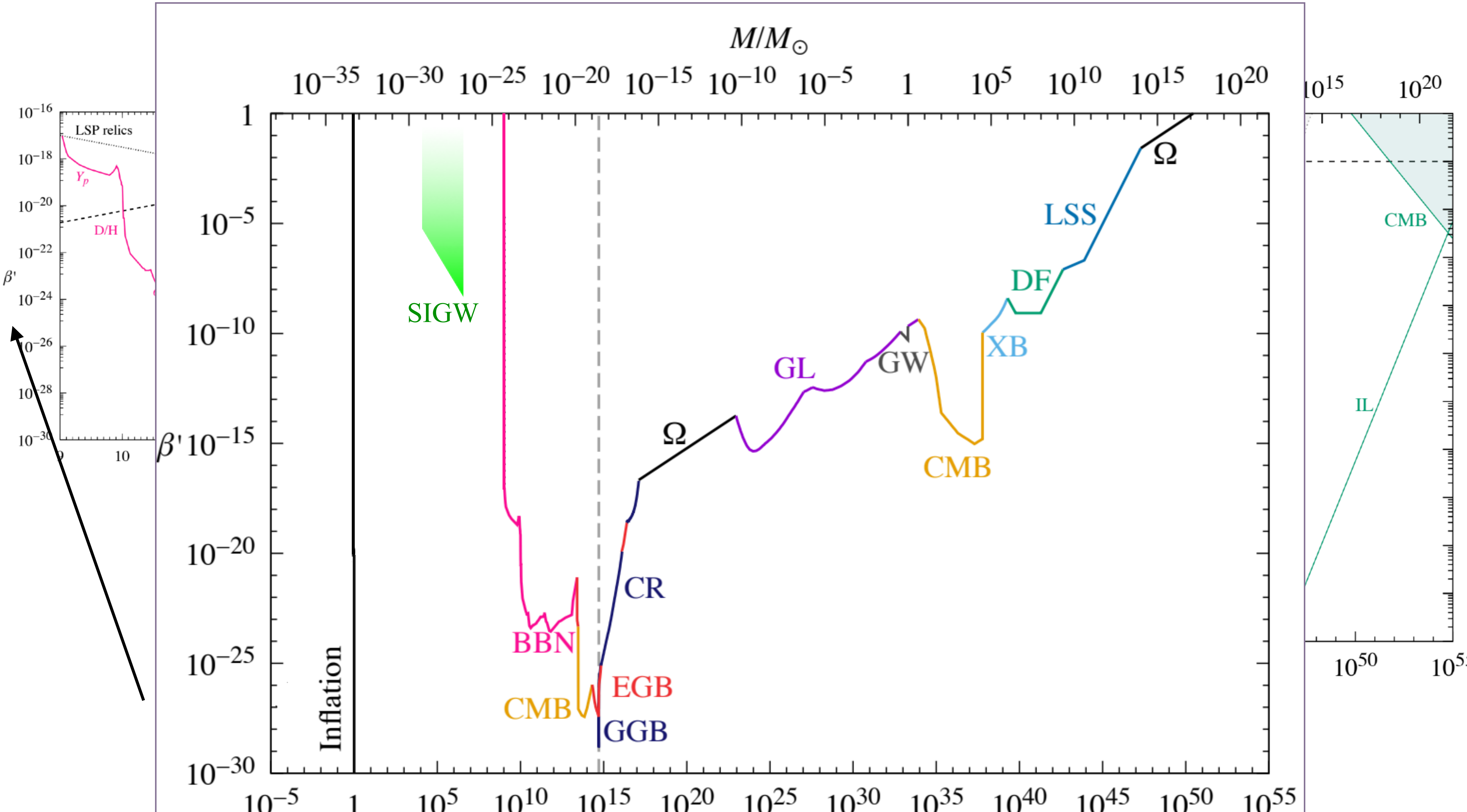

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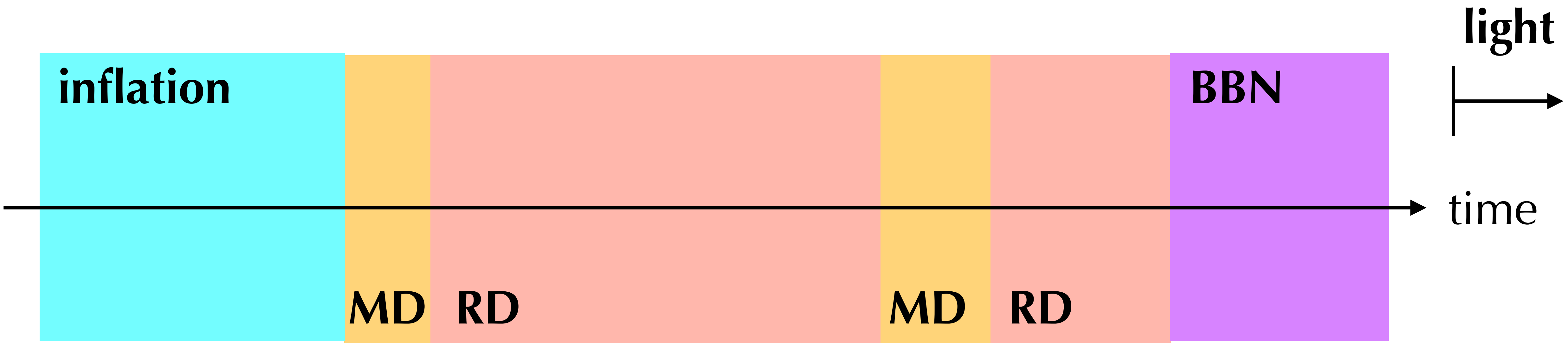
PBH dominating scenario



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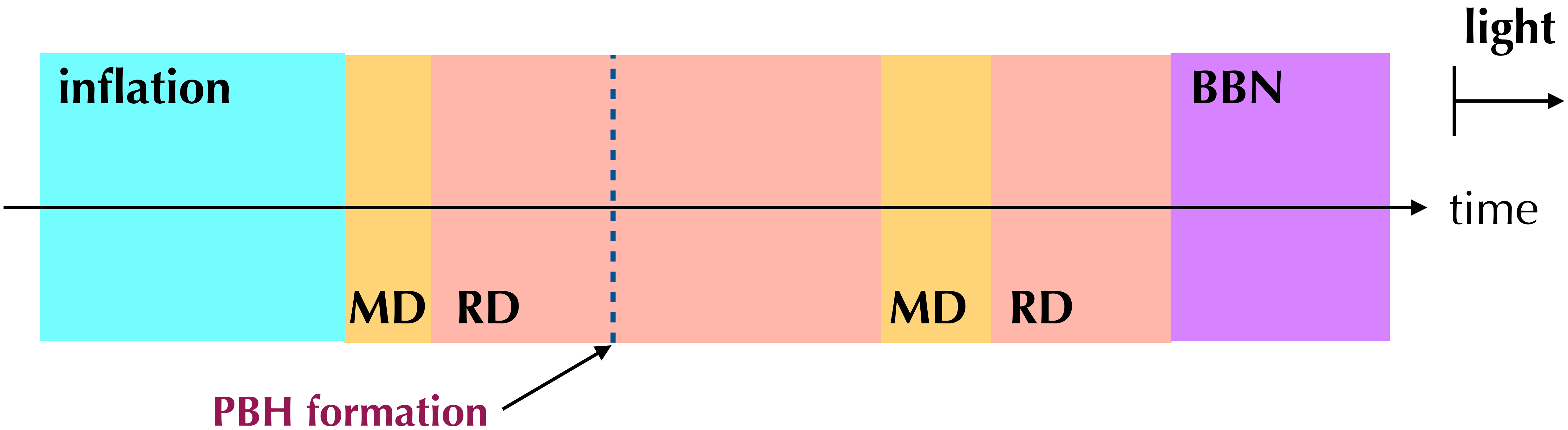
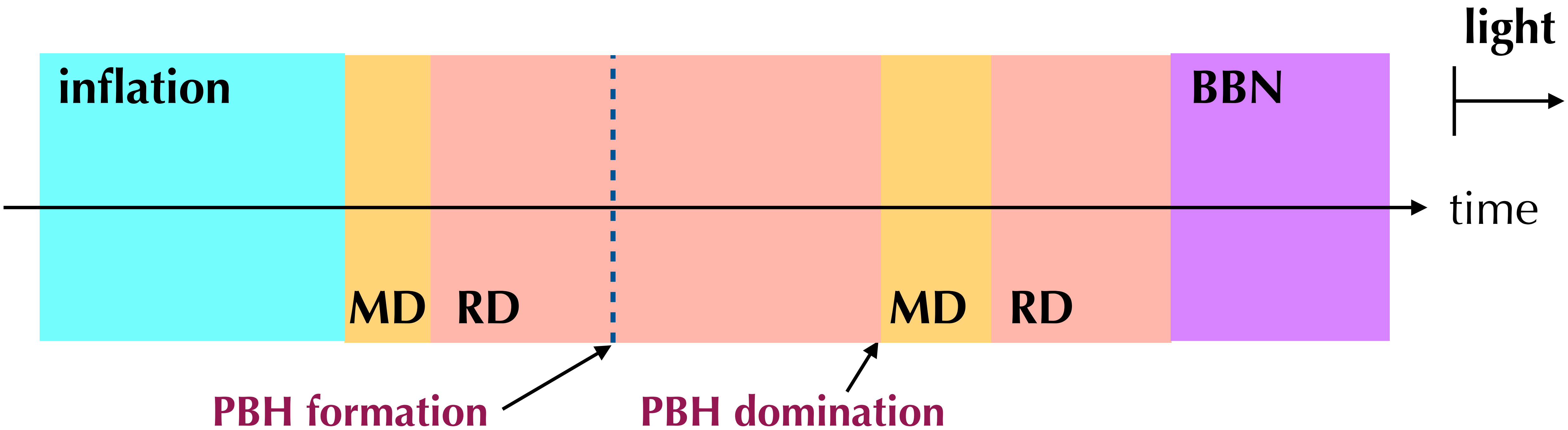


$$T_{\rm form} \simeq 4.3 \times 10^{13}\,{\rm GeV} \left(\frac{M_{\rm PBH,form}}{10^4\,{\rm g}}\right)^{-1/2}$$

PBH dominating scenario



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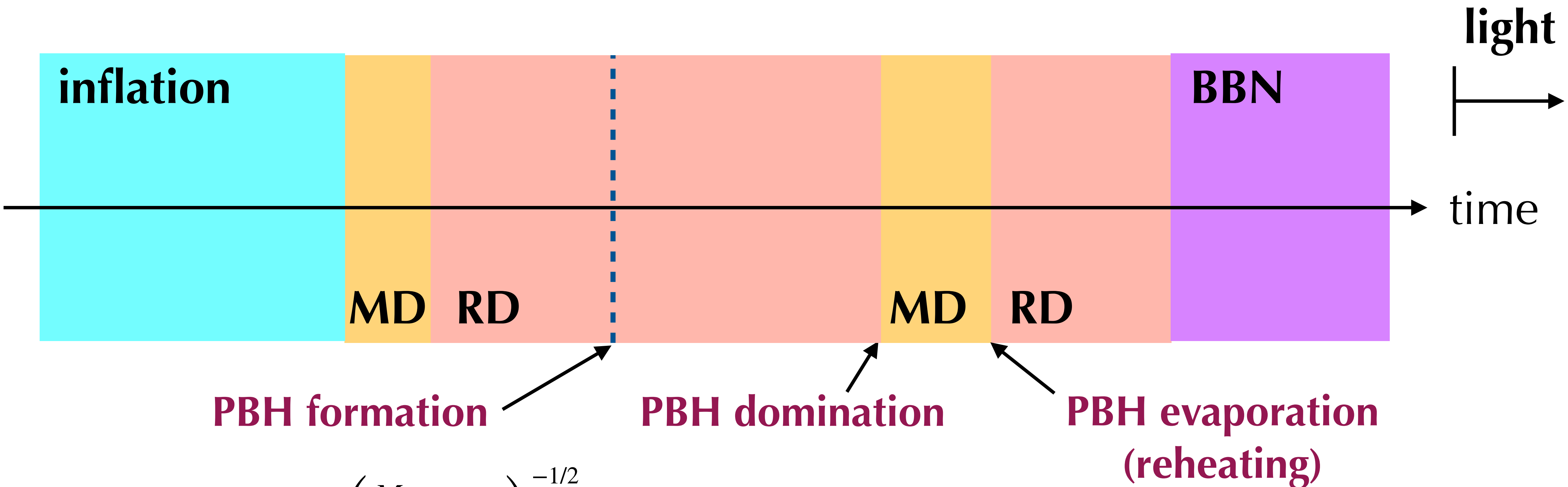

$$T_{\rm form} \simeq 4.3 \times 10^{13}\,{\rm GeV} \left(\frac{M_{\rm PBH,form}}{10^4\,{\rm g}} \right)^{-1/2}$$

$$T_{\rm R} \simeq 2.8 \times 10^{4}\,{\rm GeV} \left(\frac{M_{\rm PBH,form}}{10^4\,{\rm g}} \right)^{-3/2}$$

PBH dominating scenario

PBH formation

$$T_{\rm form} \simeq 4.3 \times 10^{13}\,{\rm GeV}\left(\frac{M_{\rm PBH,form}}{10^4\,{\rm g}}\right)^{-1/2}$$

PBH domination happens if

$$\beta > \beta_{\rm min} \equiv \frac{T_{\rm R}}{T_{\rm form}}$$

PBH evaporation (reheating)

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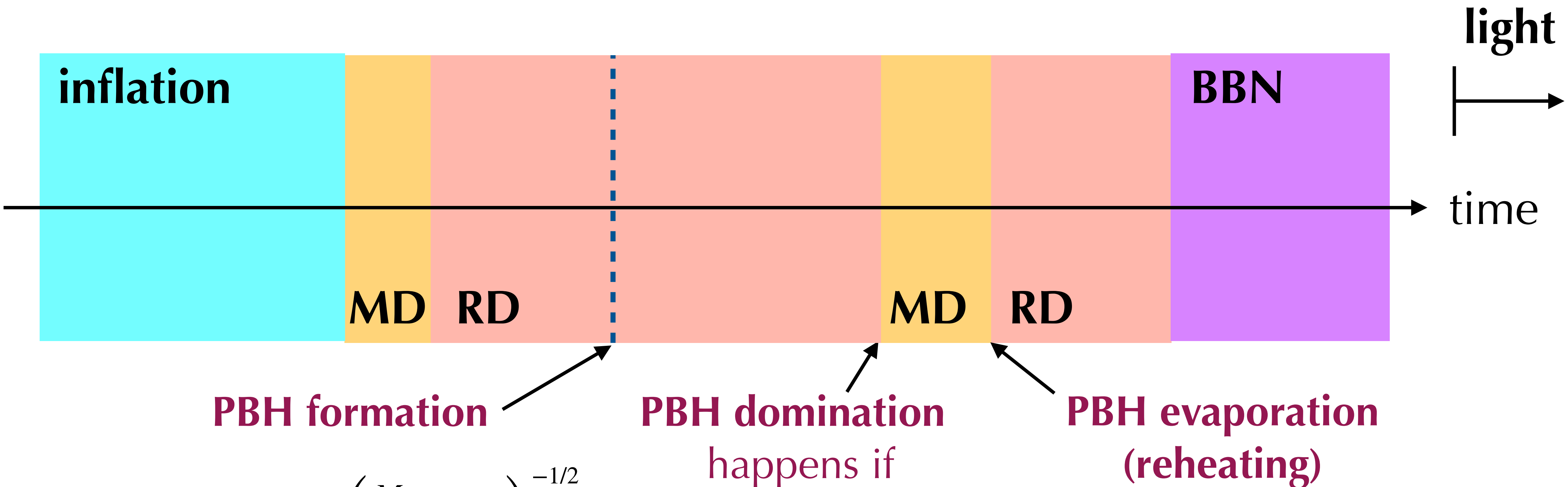
PBH dominating scenario



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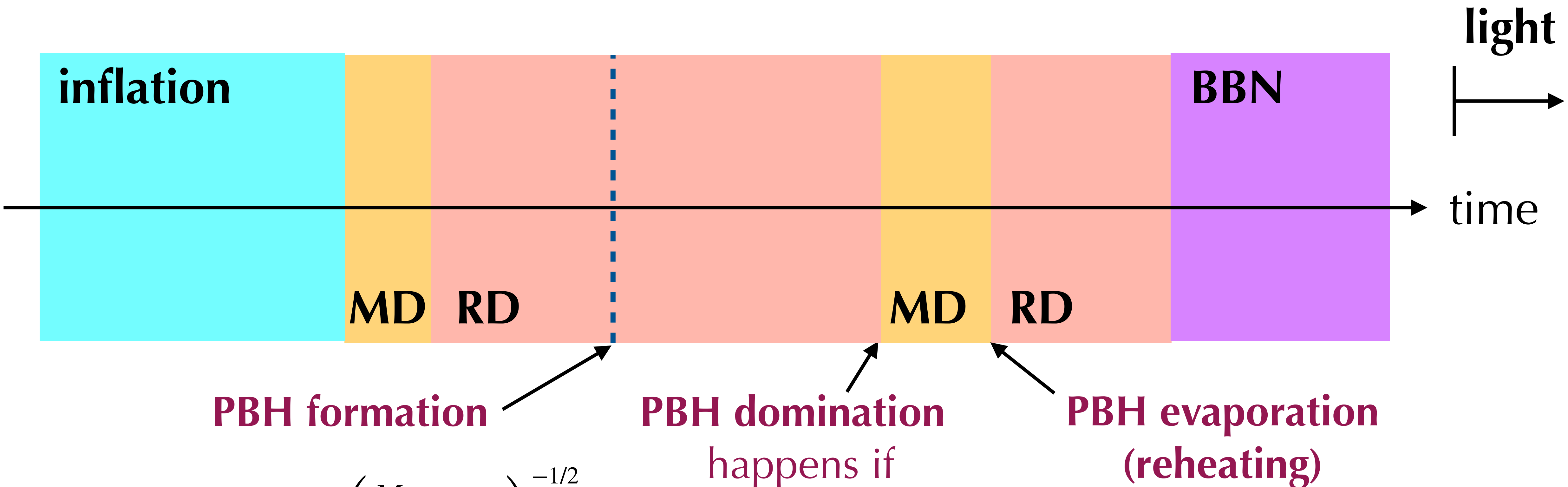

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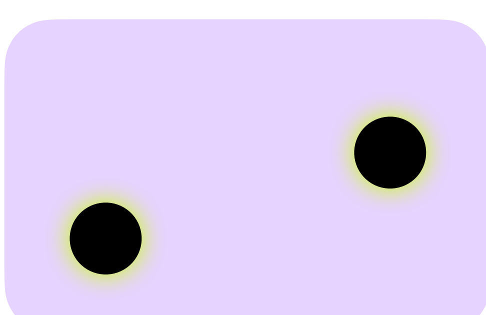

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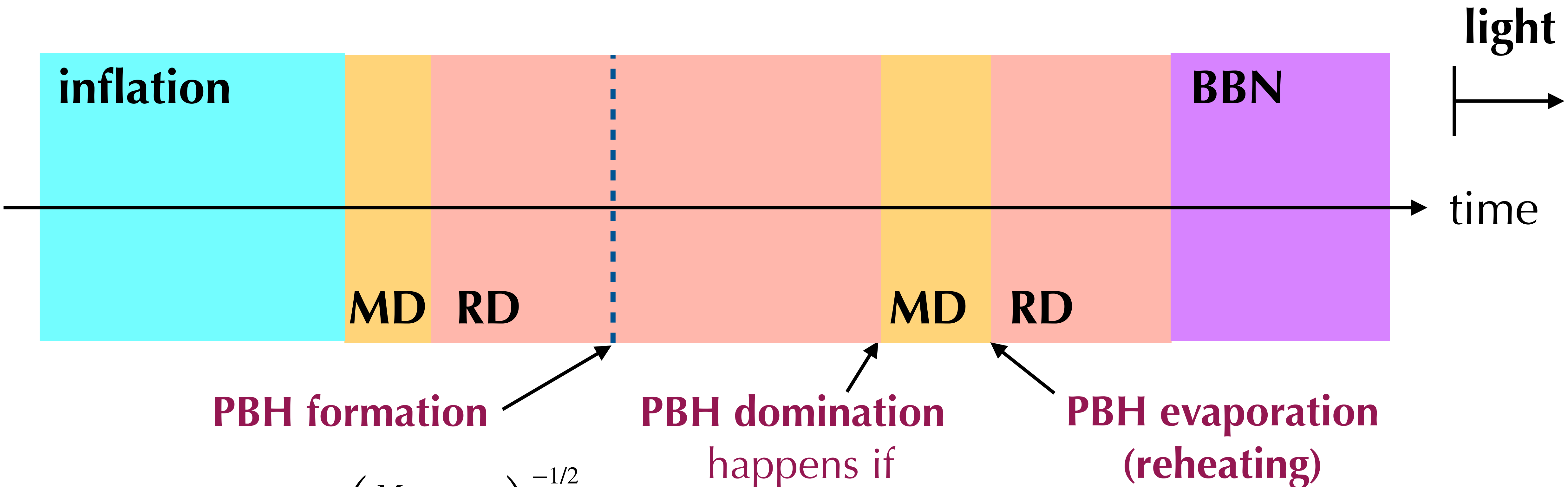

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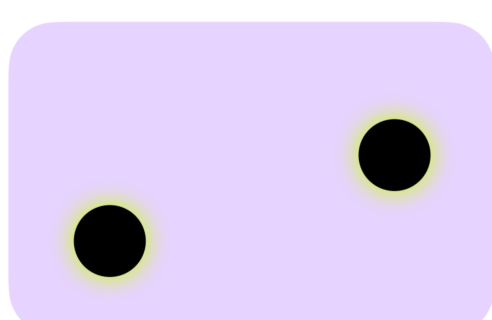

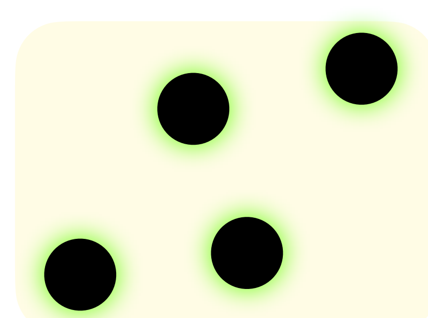

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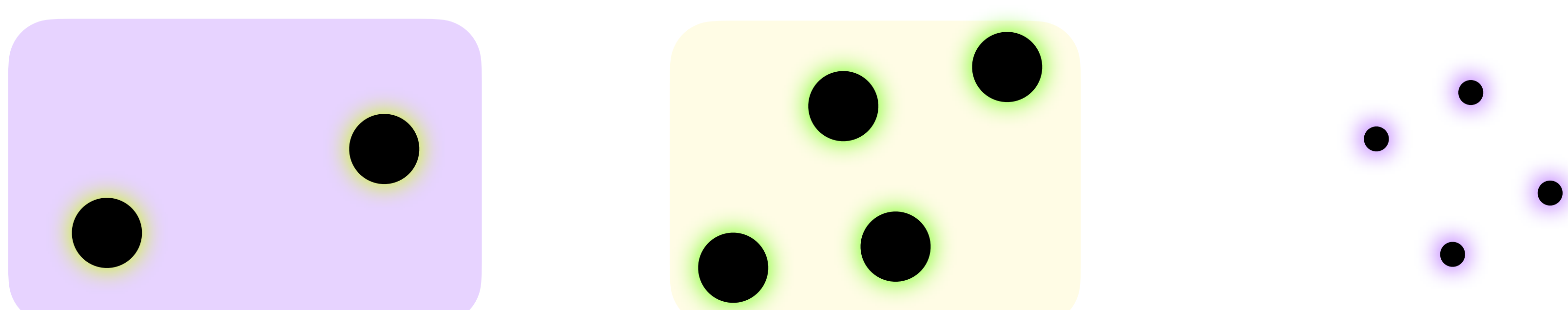

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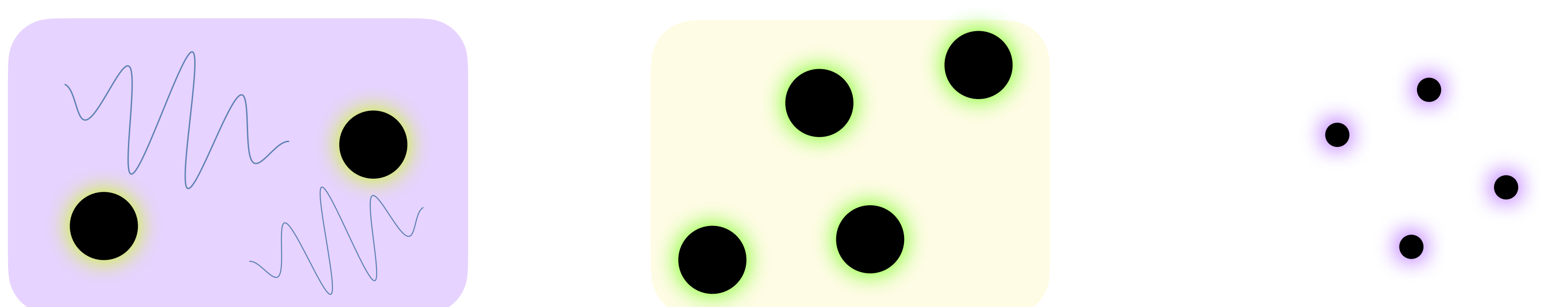

SIGWs

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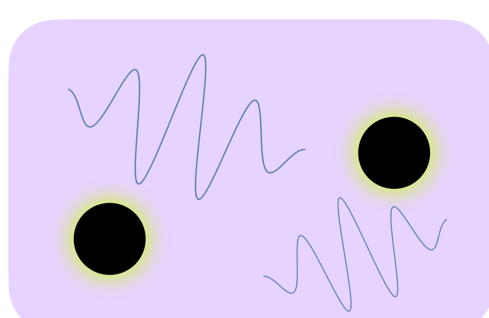

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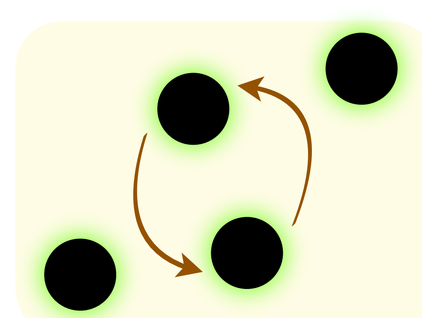
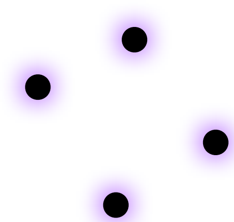


GWs from merger



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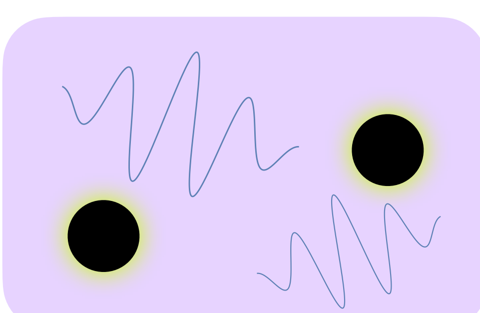
GWs from Hawking rad.



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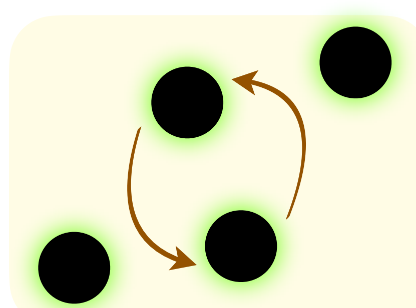

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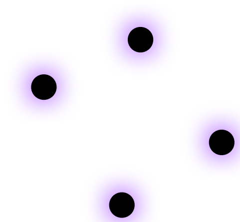

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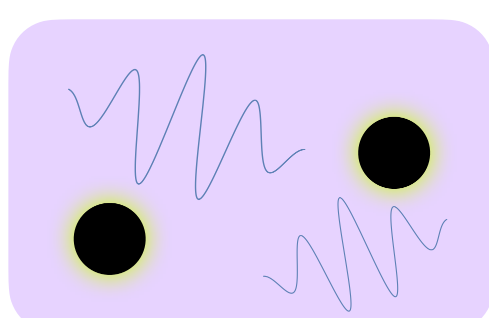

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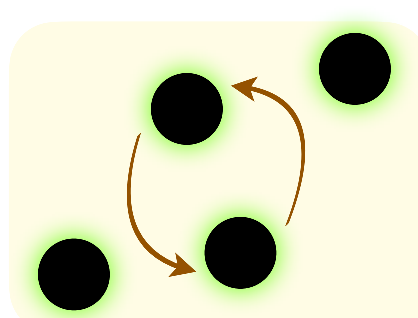

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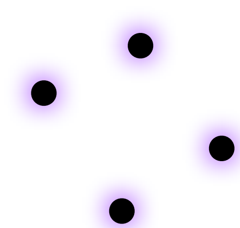

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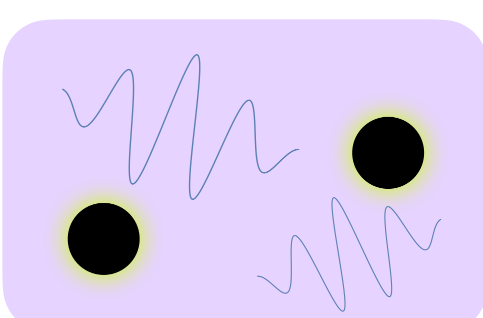

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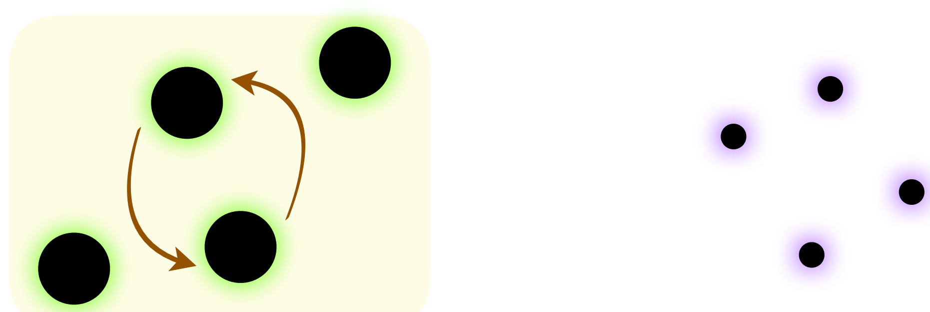

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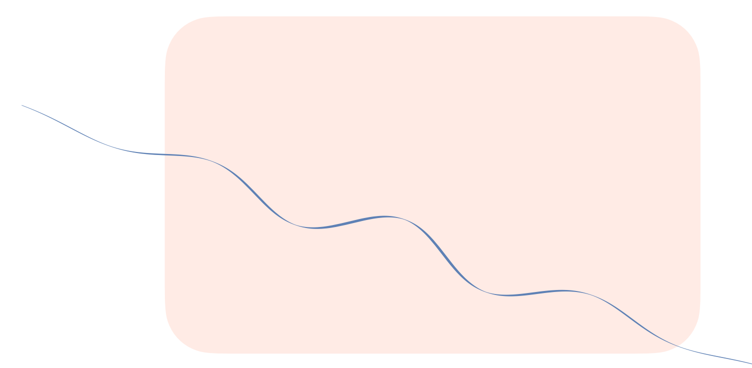


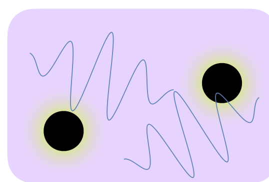
SIGWs

The PBH scenario & GWs

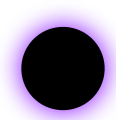
Several GW sources

associated to the PBH-dominating scenario

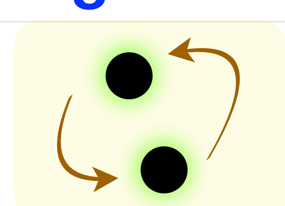
1. SIGWs associated to PBH formation



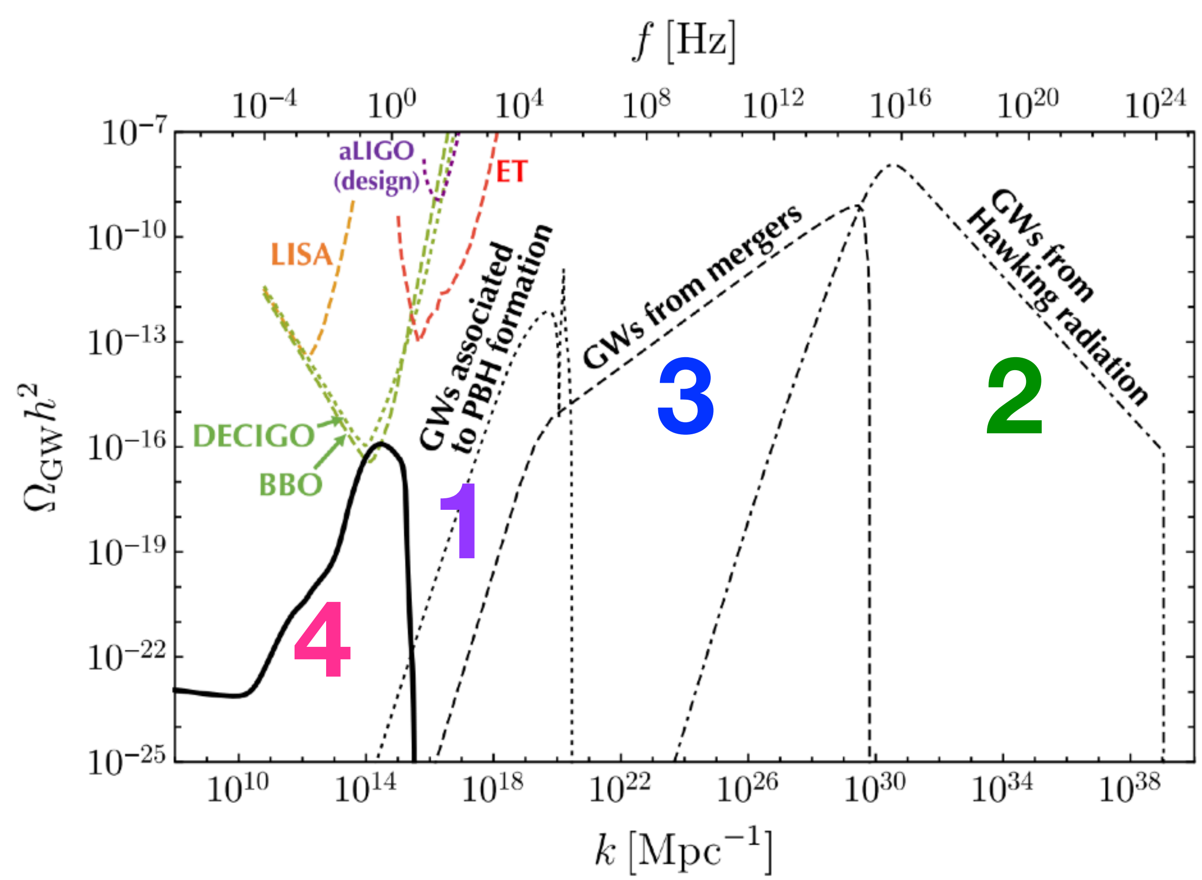
2. GWs from Hawking radiation



3. GWs from mergers of binary PBHs

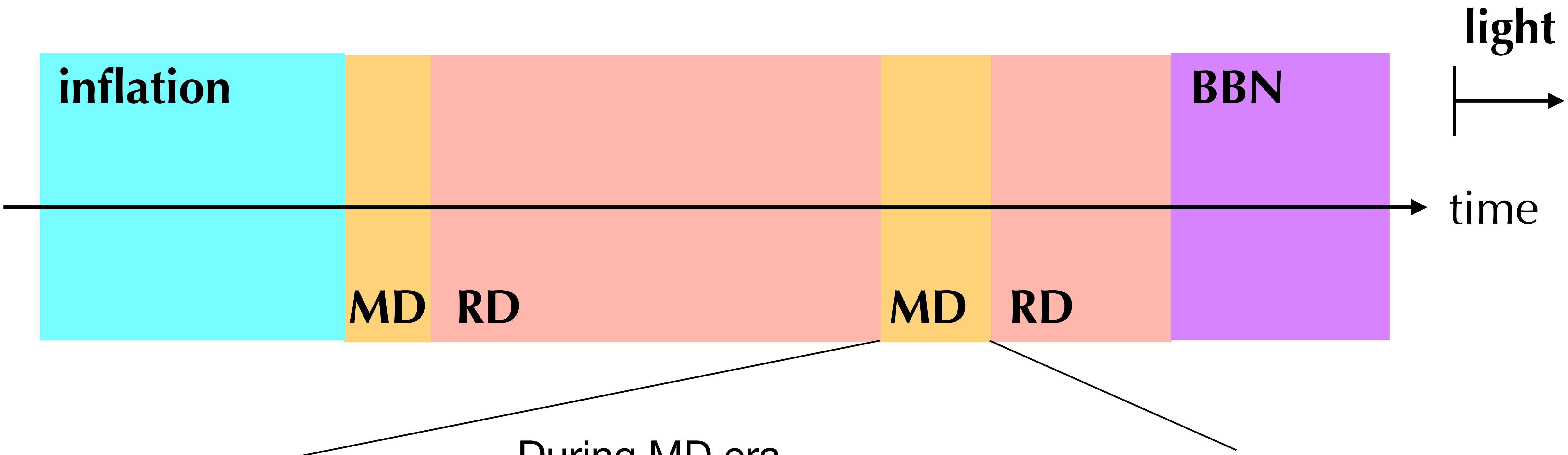


4. SIGWs right after PBH evaporation



$$M_{\rm PBH} = 10^4\,{\rm g}, \beta = 10^{-7}$$

“Charging” during MD era



During MD era,

$$\frac{\delta\rho}{\rho} \propto a(\eta) \quad \Longleftrightarrow \quad \Phi = \text{const.}$$

[Assadullahi, Wands, 0901.0989]

[Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290]

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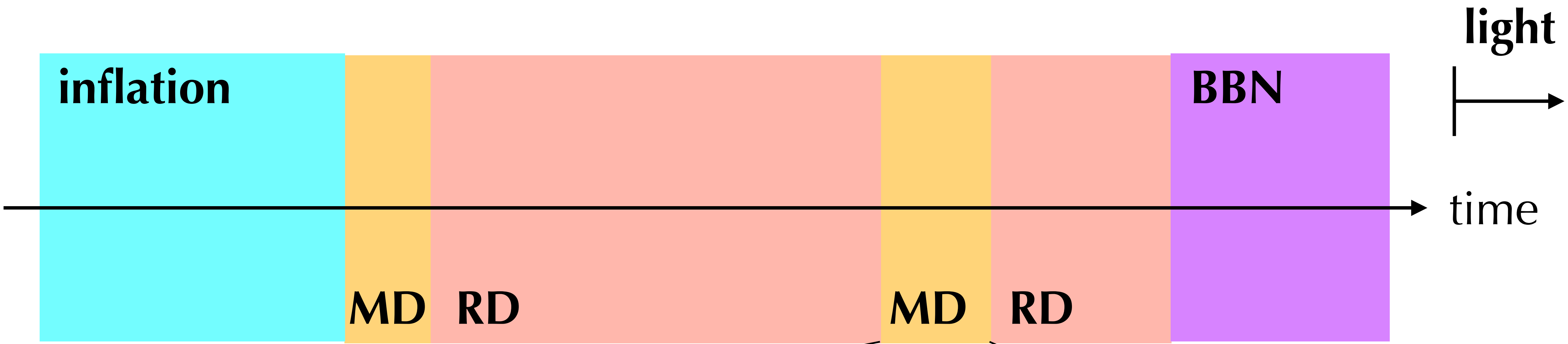


During MD era,

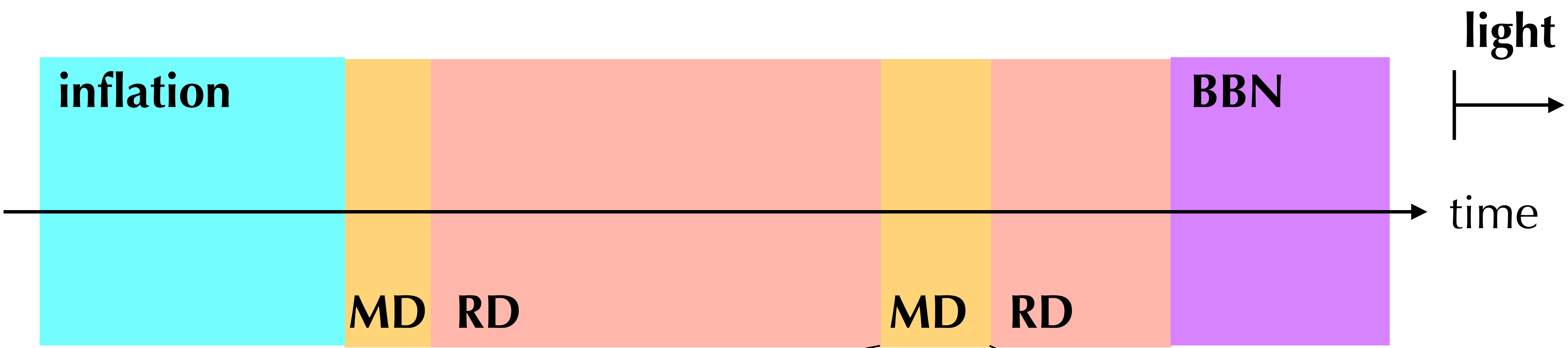
$$\frac{\delta\rho}{\rho} \propto a(\eta) \quad \Longleftrightarrow \quad \Phi = \text{const.}$$

This makes the source term constant.

$$\cancel{h''_{\mathbf{k}}(\eta)} + \cancel{2\mathcal{H}h'_{\mathbf{k}}(\eta)} + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

[Assadullahi, Wands, 0901.0989]
[Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290]

“Charging” during MD era



During MD era,

$$\frac{\delta\rho}{\rho} \propto a(\eta) \quad \Longleftrightarrow \quad \Phi = \text{const.}$$

This makes the source term constant.

$$\cancel{h''_{\mathbf{k}}(\eta)} + \cancel{2\mathcal{H}h'_{\mathbf{k}}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

Constant metric distortion

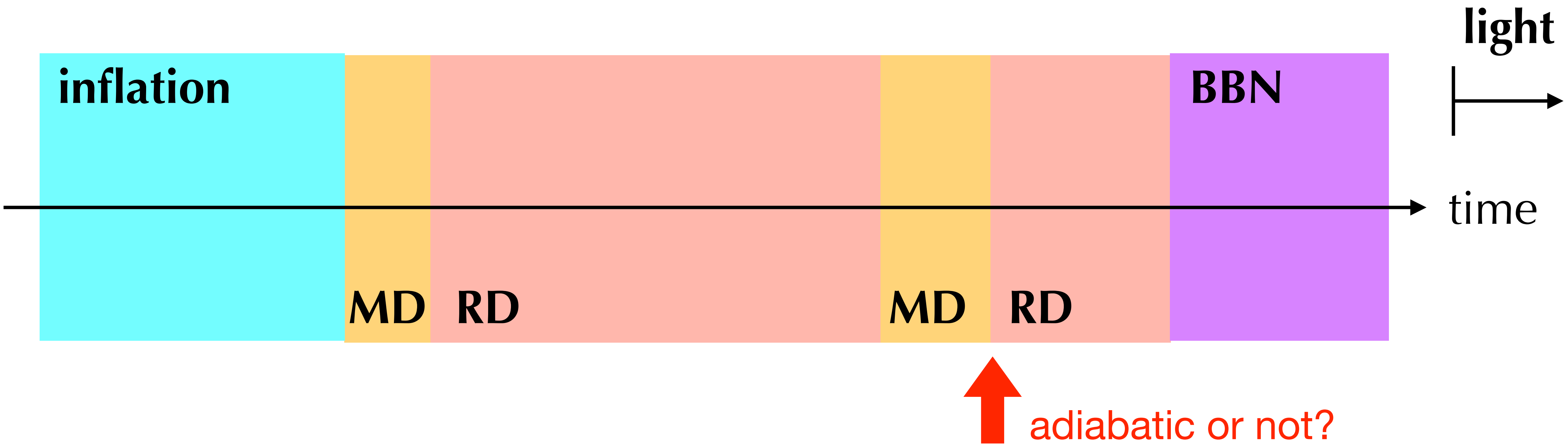
(+ decaying mode)

After MD $\Rightarrow$ Propagating GW

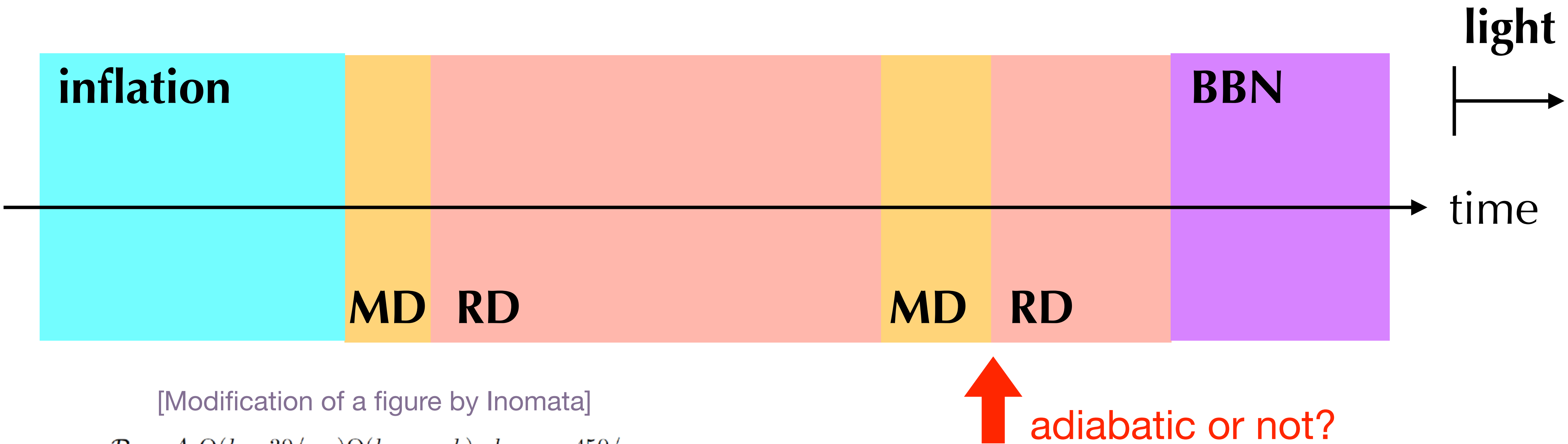
[Assadullahi, Wands, 0901.0989]

[Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290]

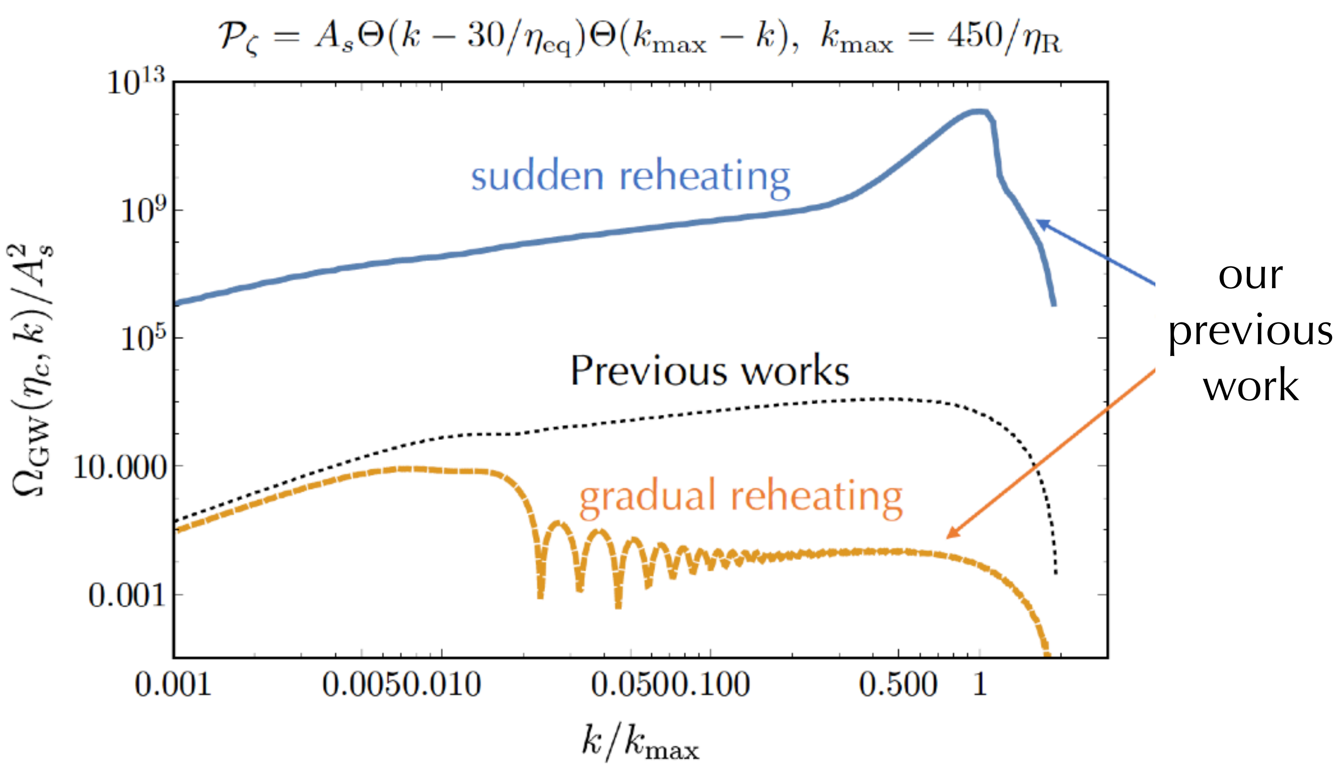
Transition time scale matters



Transition time scale matters

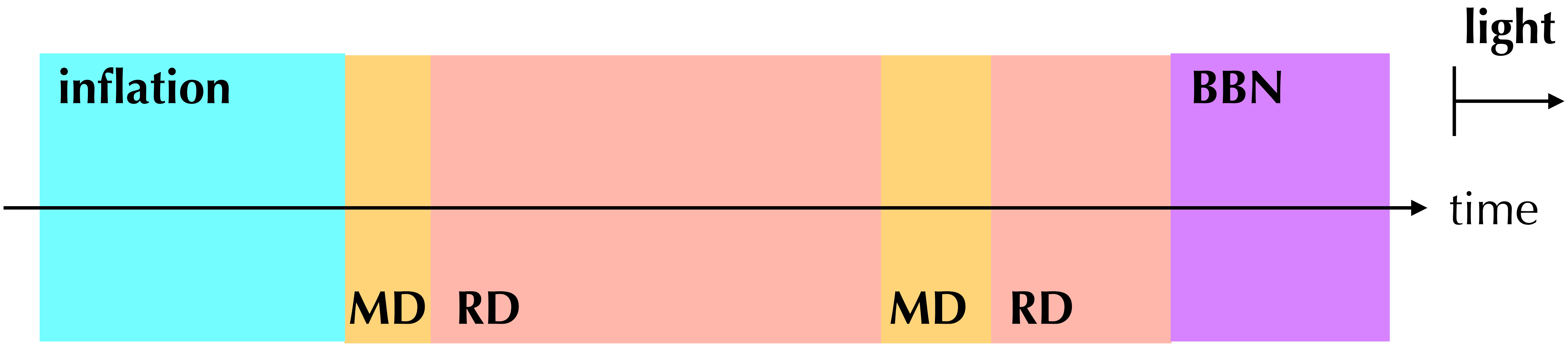


[Modification of a figure by Inomata]



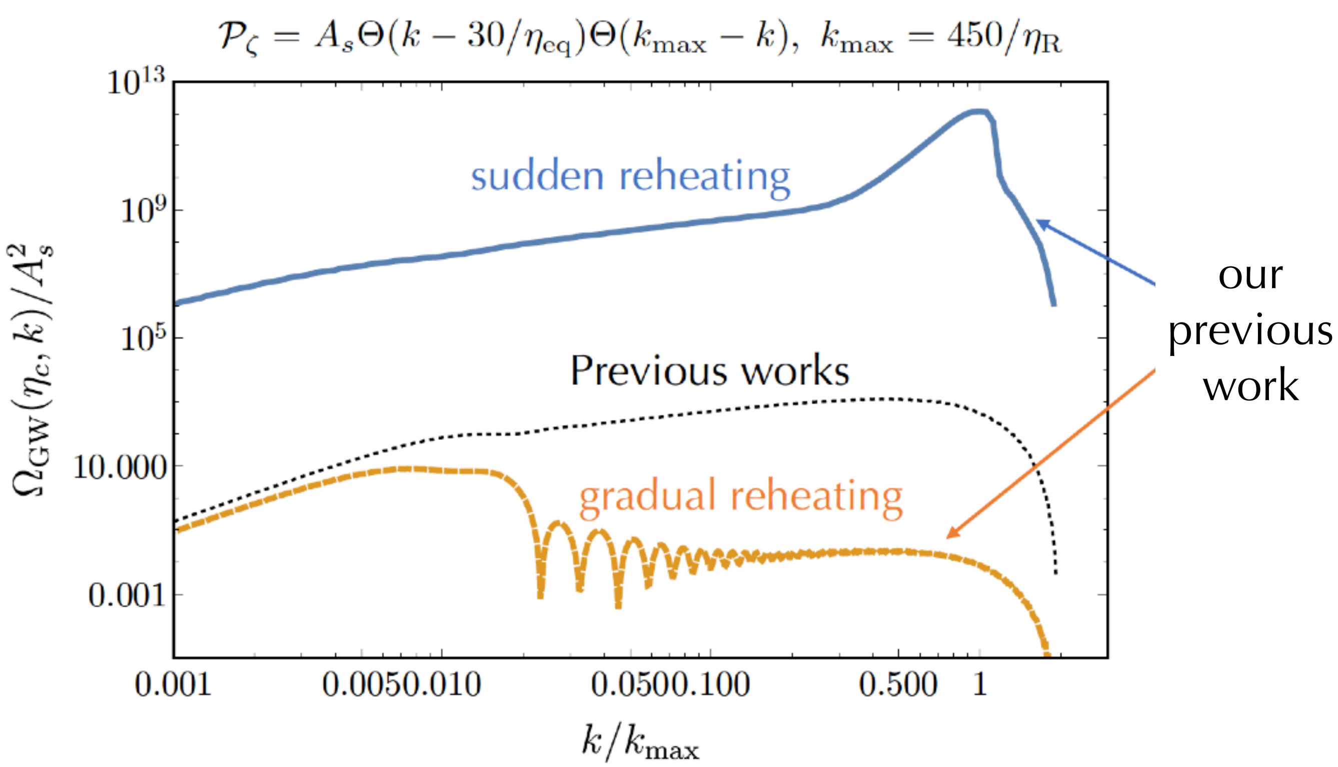
[Inomata, Kohri, Nakama, TT, JCAP 10 (2019) 071, 1904.12878]
[Inomata, Kohri, Nakama, TT, PRD100 (2019) 043532, 1904.12879]

Transition time scale matters



adiabatic or not?

[Modification of a figure by Inomata]

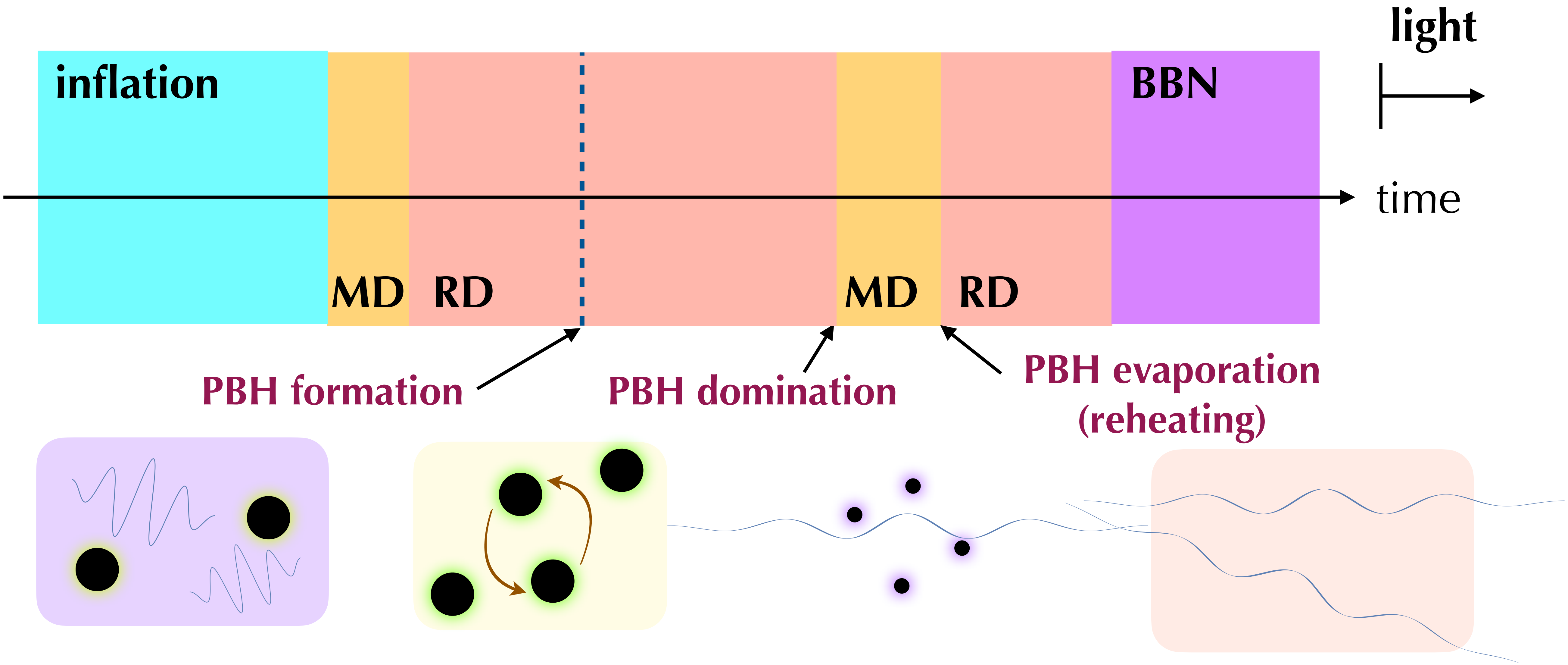


$$\Gamma \equiv -\frac{\mathrm{d}\ln M_{\rm PBH}}{\mathrm{d}t} = \frac{1}{3(t_{\rm evp} - t)}$$

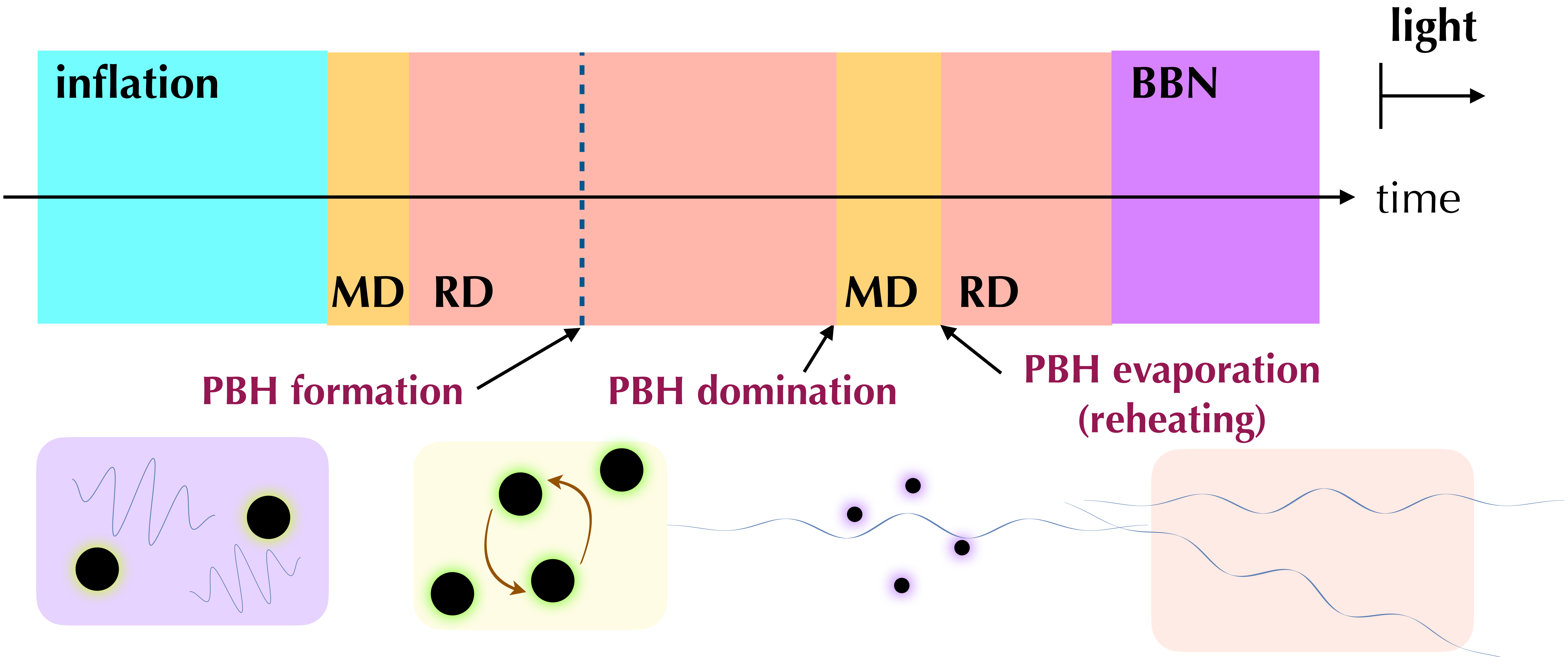
$$M_{\rm PBH} \propto (t_{\rm evp} - t)^{1/3}$$

PBH evaporation is more sudden than the standard exp. decay.

[Inomata, Kohri, Nakama, TT, JCAP 10 (2019) 071, 1904.12878]
[Inomata, Kohri, Nakama, TT, PRD100 (2019) 043532, 1904.12879]



light
inflation
BBN
time
MD
RD
MD
RD
PBH formation
PBH domination
PBH evaporation
(reheating)

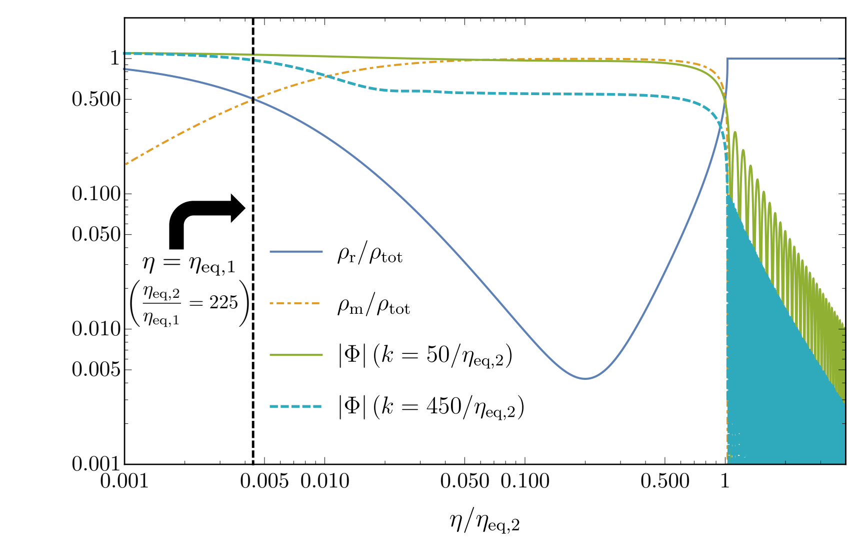


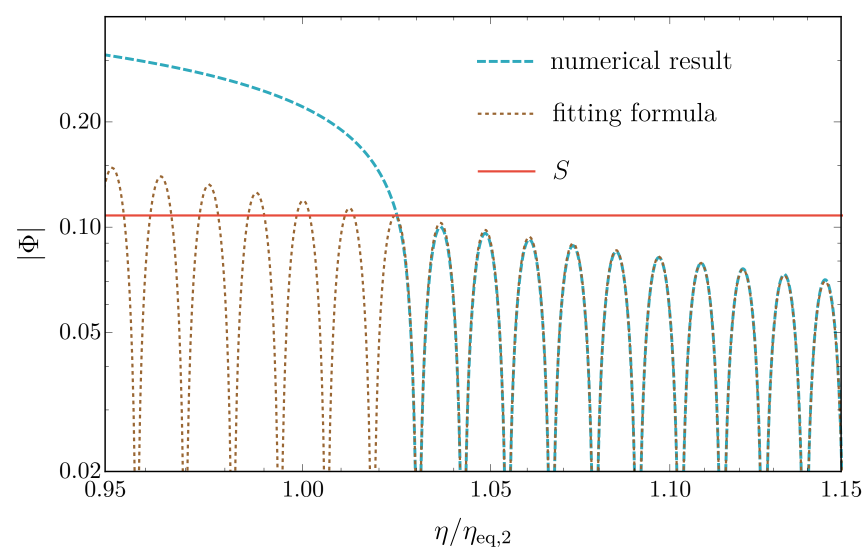
[Figure from http://www.pxfuel.com/en/free-photo-ebwtw]

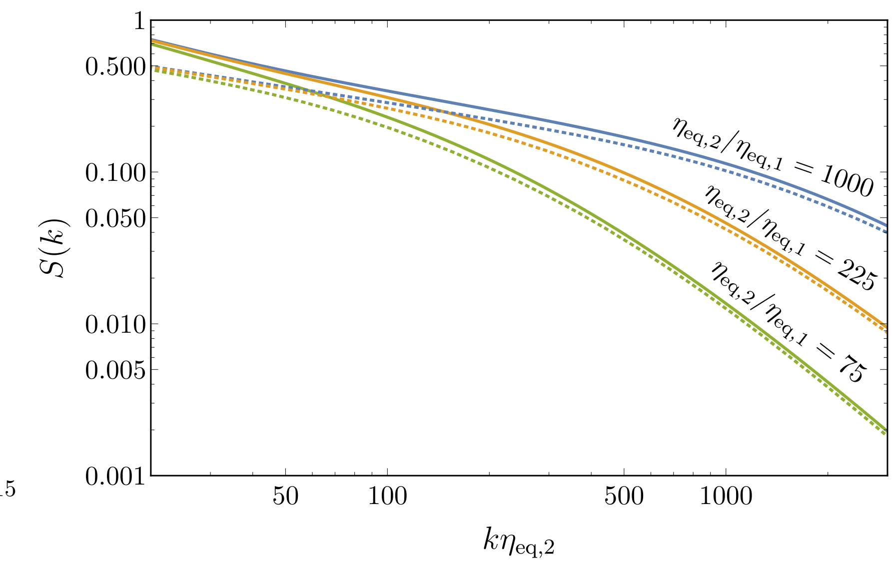
Poltergeist

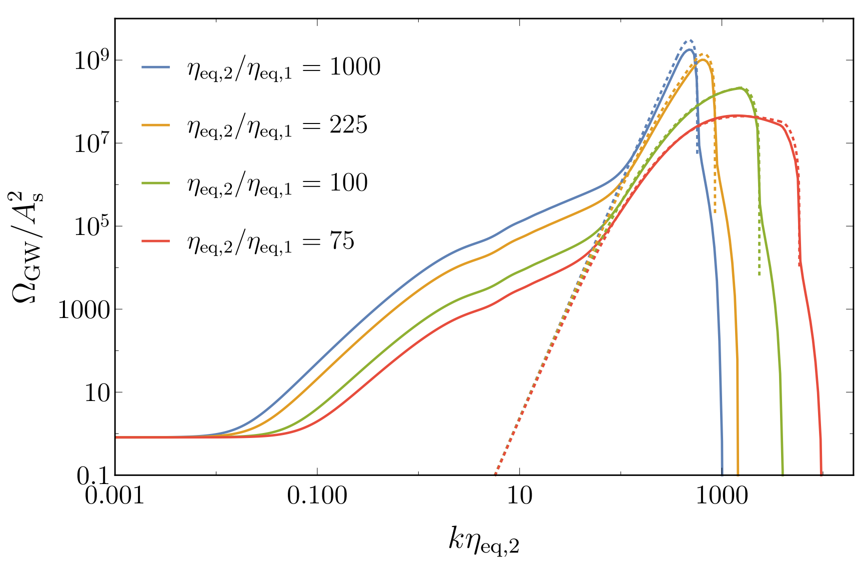
mechanism for gravitational wave production

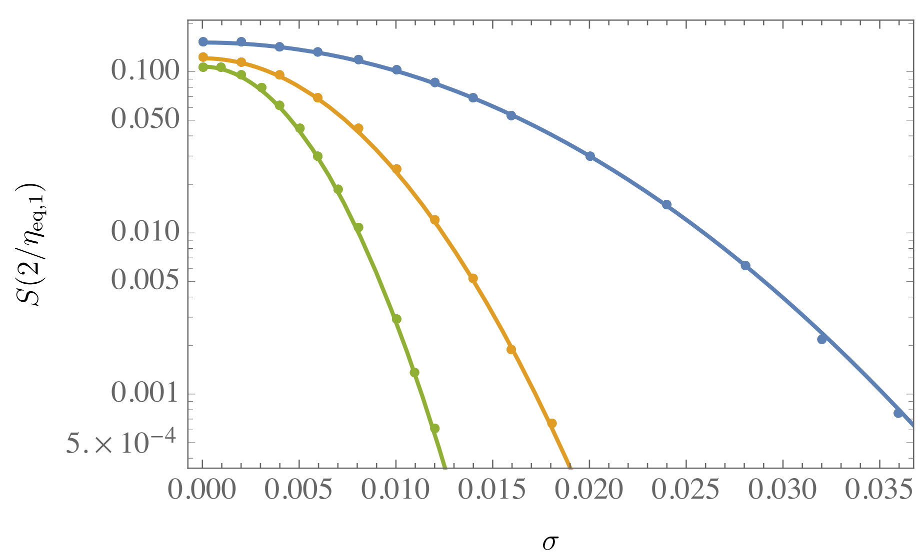
(Skipping all the details...)

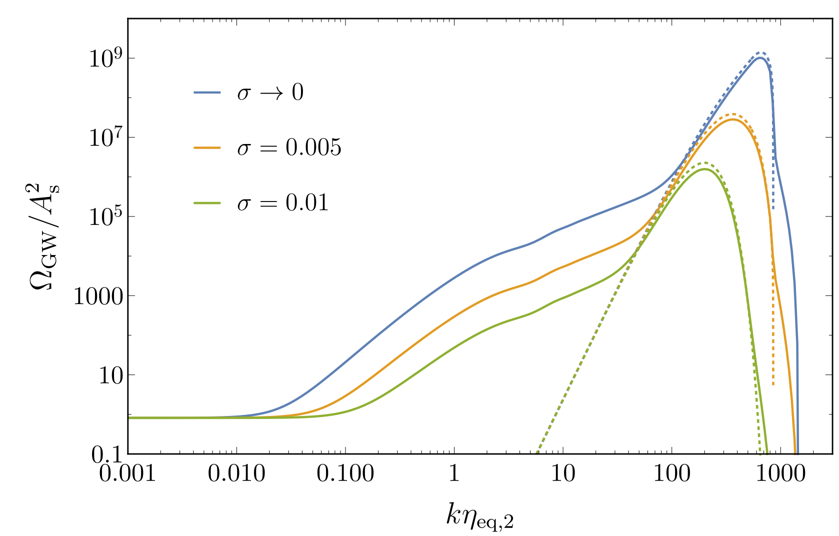




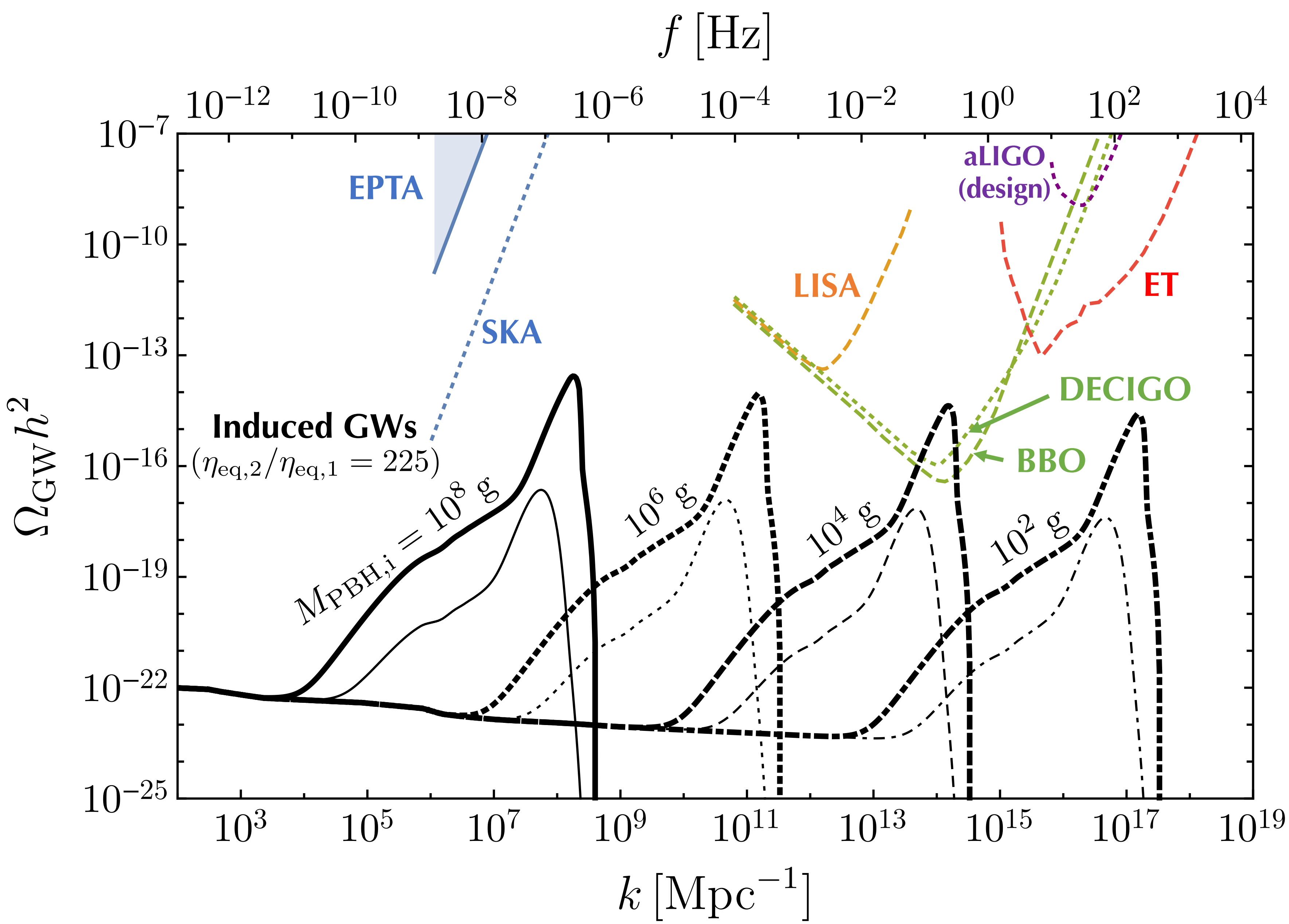


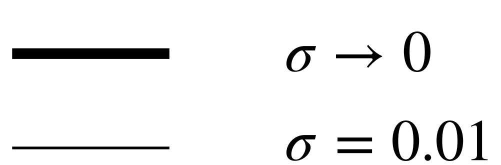






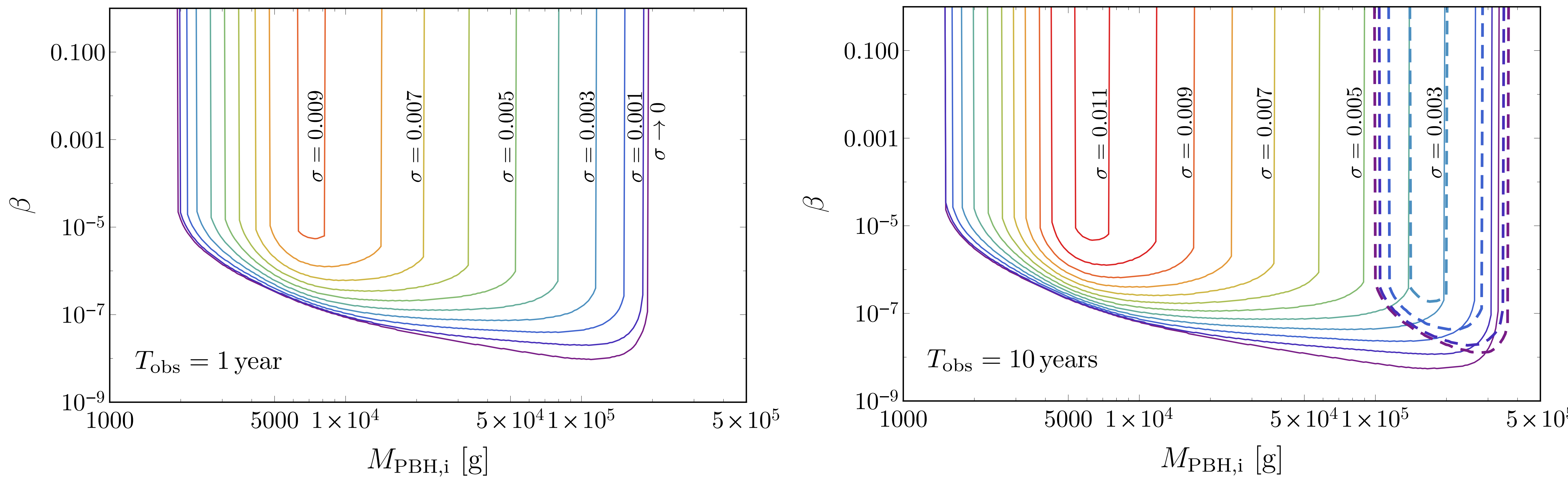
GWs from Poltergeist mechanism





$$\rho_{\rm PBH,form}(M_{\rm PBH,form}) = \frac{\rho_{\rm PBH,form}}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\left(\ln(M_{\rm PBH}/M_{\rm PBH,0})\right)^2}{2\sigma^2}\right)$$

Prospective constraints



DECIGO

LISA

Summary

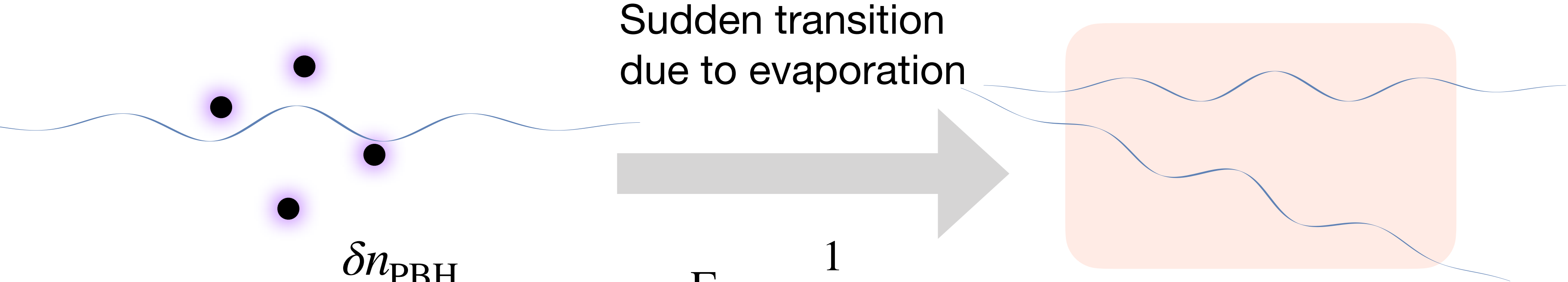
GWs are induced from the curvature perturbations right after the PBH evaporation in the PBH-dominating scenario.

Poltergeist

mechanism for gravitational wave production

Sudden transition due to evaporation

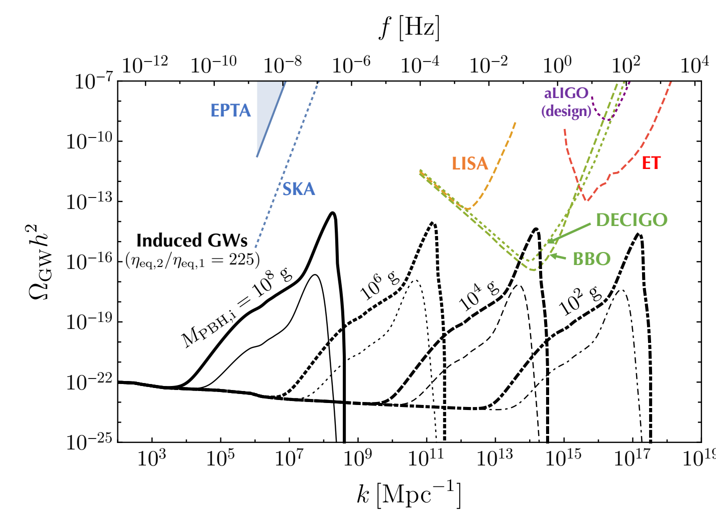
Sound waves in thermal bath



$$\frac{\delta n_{\rm PBH}}{n_{\rm PBH}} \propto a$$

$$\Gamma \sim \frac{1}{t_{\rm evp} - t}$$

Fast oscillation of Φ_k



(Resonant) production of 2nd-order GWs

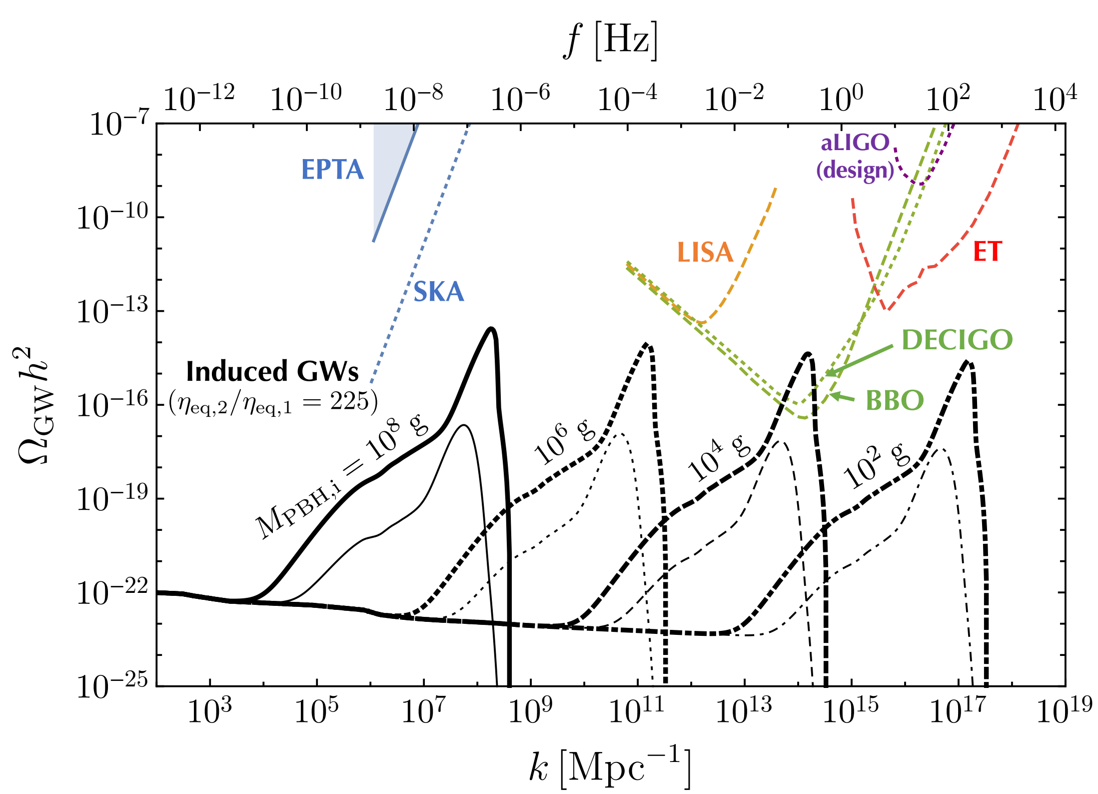
The resultant GWs can be so strong that DECIGO/BBO/LISA may detect them if the PBH mass function is narrow (~1%).

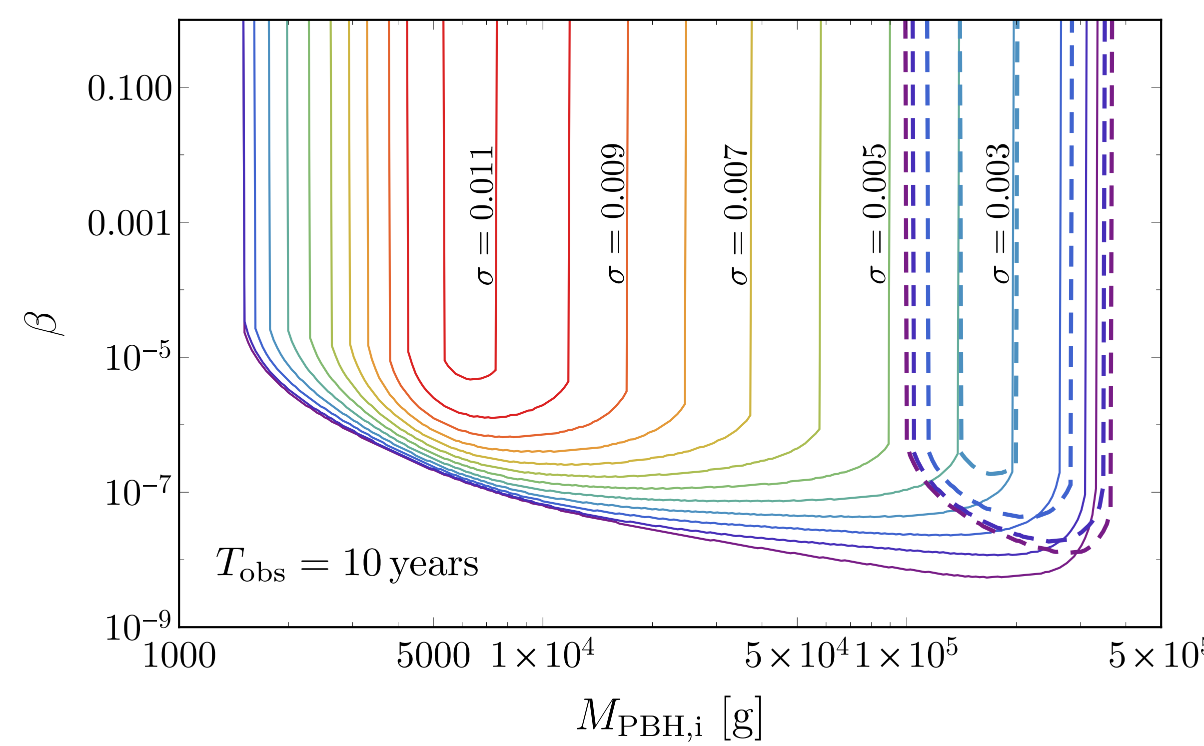
Conclusion

Poltergeist
mechanism for gravitational wave production

is useful to constrain New Physics that triggers a sudden reheating transition.

We have derived a qualitatively new independent prospective constraint on the formation probability of PBHs.





Discussion

- Narrow width of PBH mass:
  - Narrow parametric resonance?
  - Refined criterion for the PBH formation.
  - Effects of binary PBH merger?
- Effects of spin width?
- Compatibility with particle cosmology:
  - Massive DM is overproduced. Axion(-like) DM?
  - Leptogenesis possible in several ways.

Outline of calculations

We are interested in the strength of the scalar-induced gravitational waves (SIGWs).

$$\Omega_{\mathrm{GW}}(\eta,k) = \frac{\rho_{\mathrm{GW}}(\eta,k)}{\rho_{\mathrm{tot}}(\eta)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)\overline{\mathcal{P}_h(\eta,k)}$$

This is given by an integral of the square of the scalar power spectrum.

$$\overline{\mathcal{P}_h(\eta,k)} = 4\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2 \overline{I^2(u,v,k,\eta,\eta_{\mathrm{evp}})}\mathcal{P}_\zeta(uk)\mathcal{P}_\zeta(vk)$$

This encodes the information of time evolution of the scalar perturbations.

<u>Strategy</u>

1. Calculate scalar perturbations in the synchronous gauge, in which the PBH decay rate is spatially uniform. We utilize the formalism for *decaying dark matter*.
2. Translate the results into the Newtonian gauge, which is conventionally adopted in the SIGW calculation.

Scalar perturbations

Perturbation variables

$$\mathrm{d}s^2 = a^2\left[-\mathrm{d}\eta^2 + \left(\delta_{ij} + H_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$ (synchronous gauge)

$$\mathrm{d}s^2 = a^2\left[-(1+2\Phi)\,\mathrm{d}\eta^2 + \left(\left(1-2\Psi\right)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$ (Newtonian gauge)

where $H_{ij} = \hat{k}_i\hat{k}_j\gamma + (\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij})6\epsilon$

The relation between the gauges: $\Psi = \epsilon - \frac{(6\epsilon+\gamma)'}{2k^2}\mathscr{H}$

Scalar perturbations

Perturbation variables

$$\mathrm{d}s^2 = a^2 \left[-\mathrm{d}\eta^2 + \left(\delta_{ij} + H_{ij} + \frac{1}{2} h_{ij} \right) \mathrm{d}x^i \mathrm{d}x^j \right]$$ (synchronous gauge)

$$\mathrm{d}s^2 = a^2 \left[-(1+2\Phi)\,\mathrm{d}\eta^2 + \left(\left(1-2\Psi\right) \delta_{ij} + \frac{1}{2} h_{ij} \right) \mathrm{d}x^i \mathrm{d}x^j \right]$$ (Newtonian gauge)

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Background equations of motion

$$3M_{\mathrm{P}}^2 \mathscr{H}^2 = a^2 \rho_{\mathrm{tot}}$$

$$\rho_{\mathrm{m}}' = -\left(3\mathscr{H} - \frac{\mathrm{d}\ln M_{\mathrm{PBH}}}{\mathrm{d}\eta} \right) \rho_{\mathrm{m}}$$

$$\rho_{\mathrm{r}}' = -4\mathscr{H}\rho_{\mathrm{r}} - \frac{\mathrm{d}\ln M_{\mathrm{PBH}}}{\mathrm{d}\eta} \rho_{\mathrm{m}}$$

Scalar perturbations

Perturbation variables

$$ds^2 = a^2\left[-d\eta^2 + \left(\delta_{ij} + H_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j\right]$$ (synchronous gauge)

$$ds^2 = a^2\left[-(1+2\Phi)\,d\eta^2 + \left((1-2\Psi)\,\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j\right]$$ (Newtonian gauge)

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$$\rho_{\rm r}' = -4\mathscr{H}\rho_{\rm r} - \frac{d\ln M_{\rm PBH}}{d\eta}\rho_{\rm m}$$

Perturbation equations of motion

$\delta \equiv \delta\rho/\rho$ $\quad\theta \equiv \nabla\cdot v$ $\quad$ (One can take $\theta_{\rm m} = 0$.)

$$\delta_{\rm m}' = -\frac{\gamma'}{2} \qquad \delta_{\rm r}' = -\frac{4}{3}\left(\theta_{\rm r} + \gamma'/2\right) - \frac{d\ln M_{\rm PBH}}{d\eta}\frac{\rho_{\rm m}}{\rho_{\rm r}}\left(\delta_{\rm m} - \delta_{\rm r}\right)$$

$$\theta_{\rm r}' = \frac{k^2}{4}\delta_{\rm r} + \frac{d\ln M_{\rm PBH}}{d\eta}\frac{\rho_{\rm m}}{\rho_{\rm r}}\theta_{\rm r}$$

$$k^2\epsilon - \frac{1}{2}\frac{a'}{a}\gamma' = -\frac{3}{2}\mathscr{H}^2\left(\frac{\rho_{\rm m}}{\rho_{\rm tot}}\delta_{\rm m} + \frac{\rho_{\rm r}}{\rho_{\rm tot}}\delta_{\rm r}\right) \qquad k^2\epsilon' = 2\mathscr{H}^2\frac{\rho_{\rm r}}{\rho_{\rm tot}}\theta_{\rm r}$$

cf. decaying DM: [Poulin, Serpico, Lesgourgues, 1606.02073]

Scalar perturbations

Perturbation variables

$$\mathrm{d}s^2 = a^2\left[-\mathrm{d}\eta^2 + \left(\delta_{ij} + H_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$ (synchronous gauge)

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$$\theta_\mathrm{r}' = \frac{k^2}{4}\delta_\mathrm{r} + \frac{\mathrm{d}\ln M_\mathrm{PBH}}{\mathrm{d}\eta}\frac{\rho_\mathrm{m}}{\rho_\mathrm{r}}\theta_\mathrm{r}$$

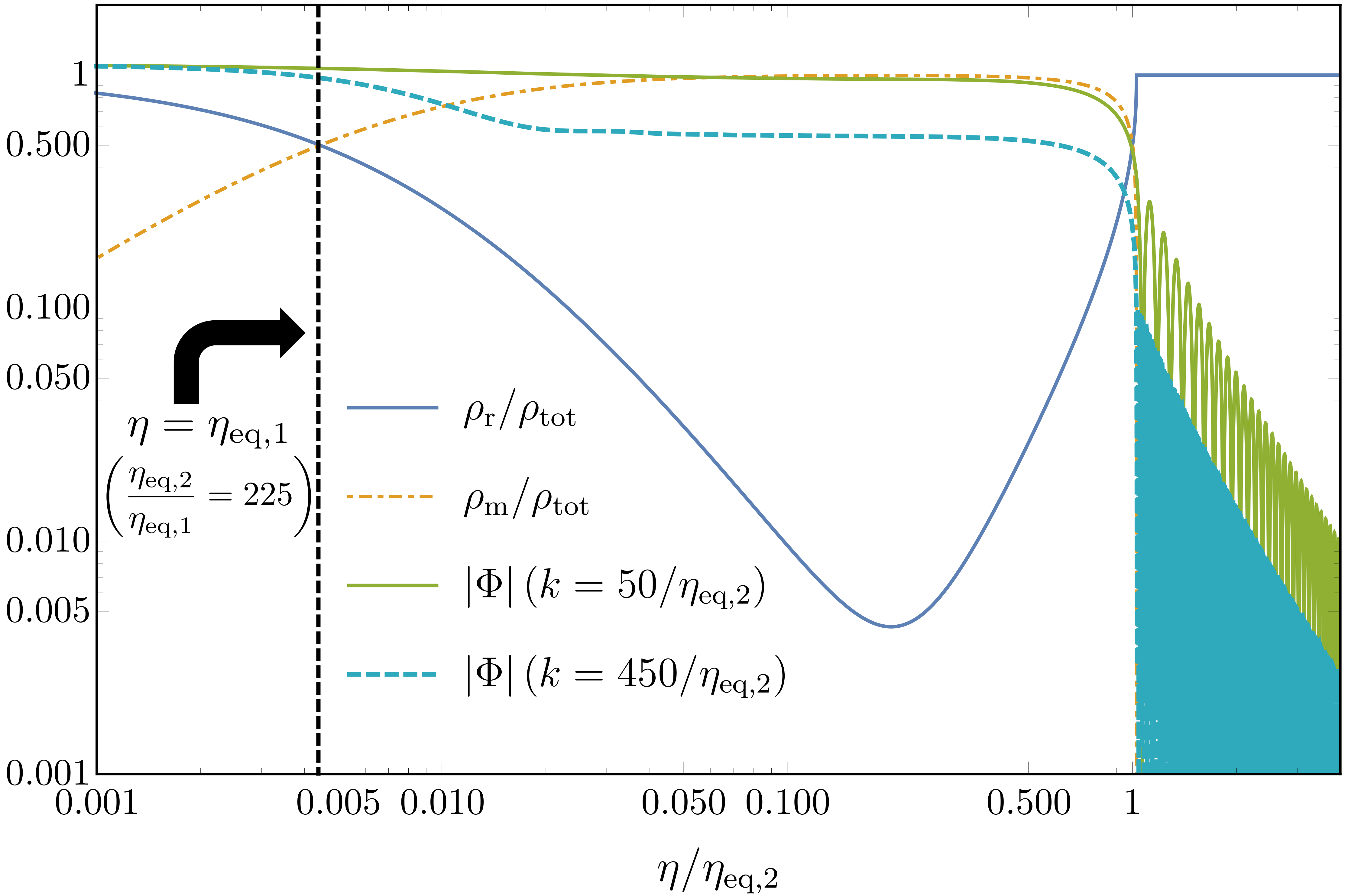
$$k^2\epsilon - \frac{1}{2}\frac{a'}{a}\gamma' = -\frac{3}{2}\mathscr{H}^2\left(\frac{\rho_\mathrm{m}}{\rho_\mathrm{tot}}\delta_\mathrm{m} + \frac{\rho_\mathrm{r}}{\rho_\mathrm{tot}}\delta_\mathrm{r}\right) \qquad k^2\epsilon' = 2\mathscr{H}^2\frac{\rho_\mathrm{r}}{\rho_\mathrm{tot}}\theta_\mathrm{r}$$

Initial conditions

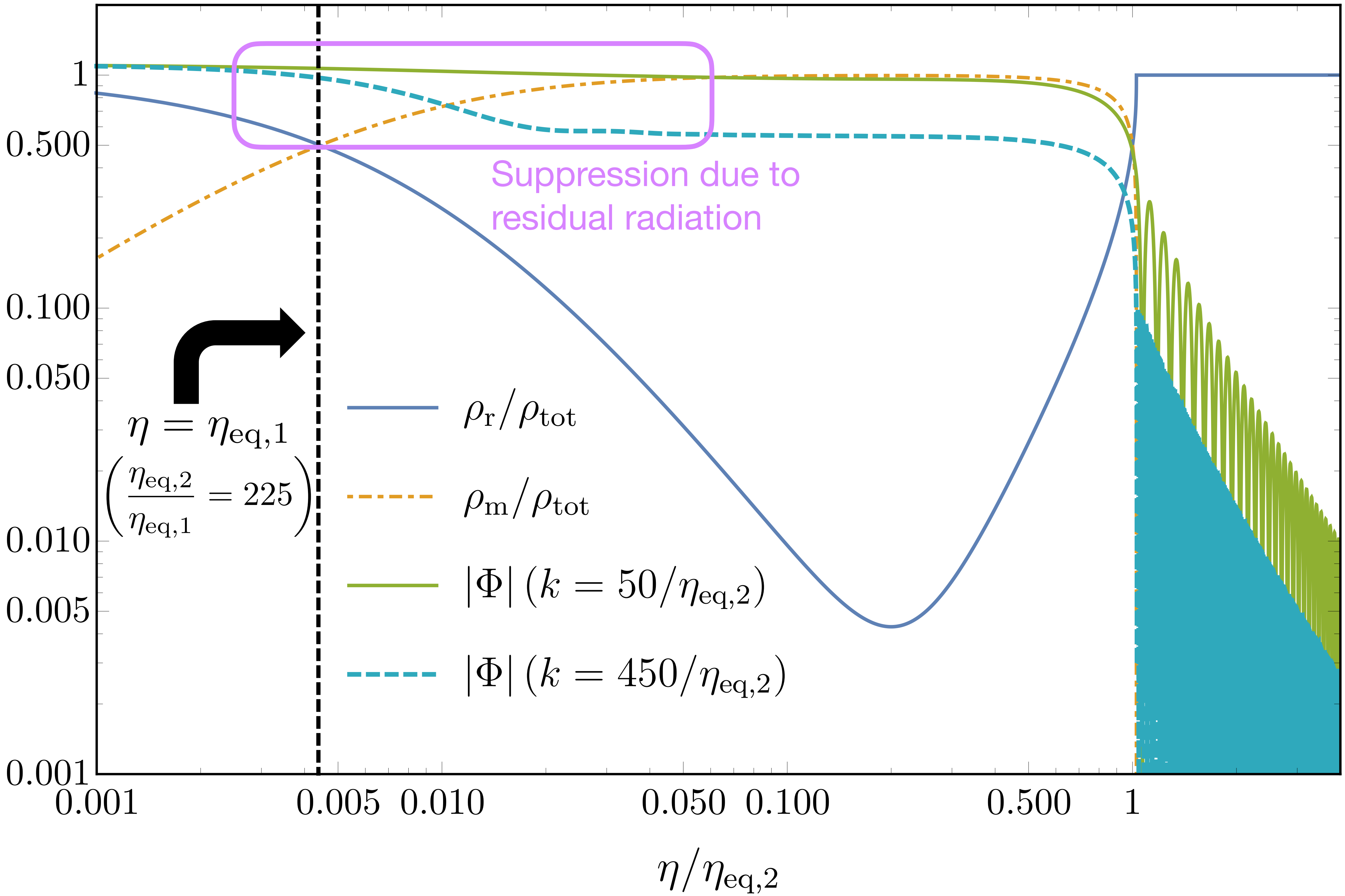
$$\delta_\mathrm{r} = -\frac{2}{3}C\left(k\eta\right)^2,\ \delta_\mathrm{m} = \frac{3}{4}\delta_\mathrm{r},\ \theta_\mathrm{r} = -\frac{1}{18}C\left(k^4\eta^3\right),\ \gamma = C\left(k\eta\right)^2,\ \epsilon = 2C - \frac{1}{18}C\left(k\eta\right)^2$$

cf. decaying DM: [Poulin, Serpico, Lesgourgues, 1606.02073]

Scalar perturbations



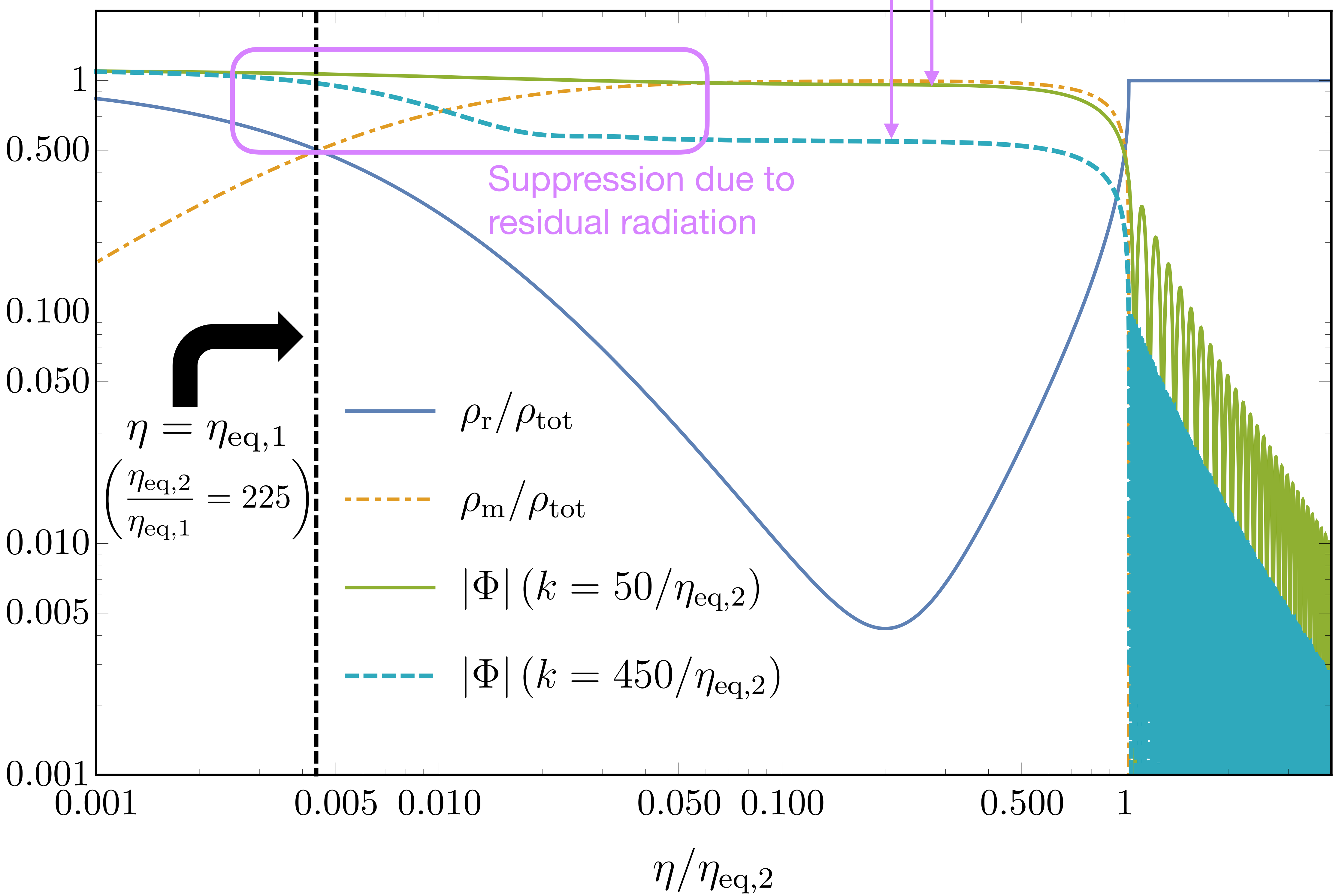
Scalar perturbations



$x \equiv k\eta$

[Bardeen, Bond, Kaiser, Szalay, APJ304, 15 (1986)] [Dodelson, “Modern Cosmology” (2003)]

$$\Phi_{\rm plateau}(x_{\rm eq,1}) \equiv \Phi(x)\big|_{\eta_{\rm eq,1} \ll \eta \lesssim \eta_{\rm eq,2}} \simeq \frac{\ln[1+0.146\,x_{\rm eq,1}]}{\left(0.146\,x_{\rm eq,1}\right)} \left[1 + 0.242\,x_{\rm eq,1} + \left(1.01\,x_{\rm eq,1}\right)^2 + \left(0.341\,x_{\rm eq,1}\right)^3 + \left(0.418\,x_{\rm eq,1}\right)^4\right]^{-0.25}$$



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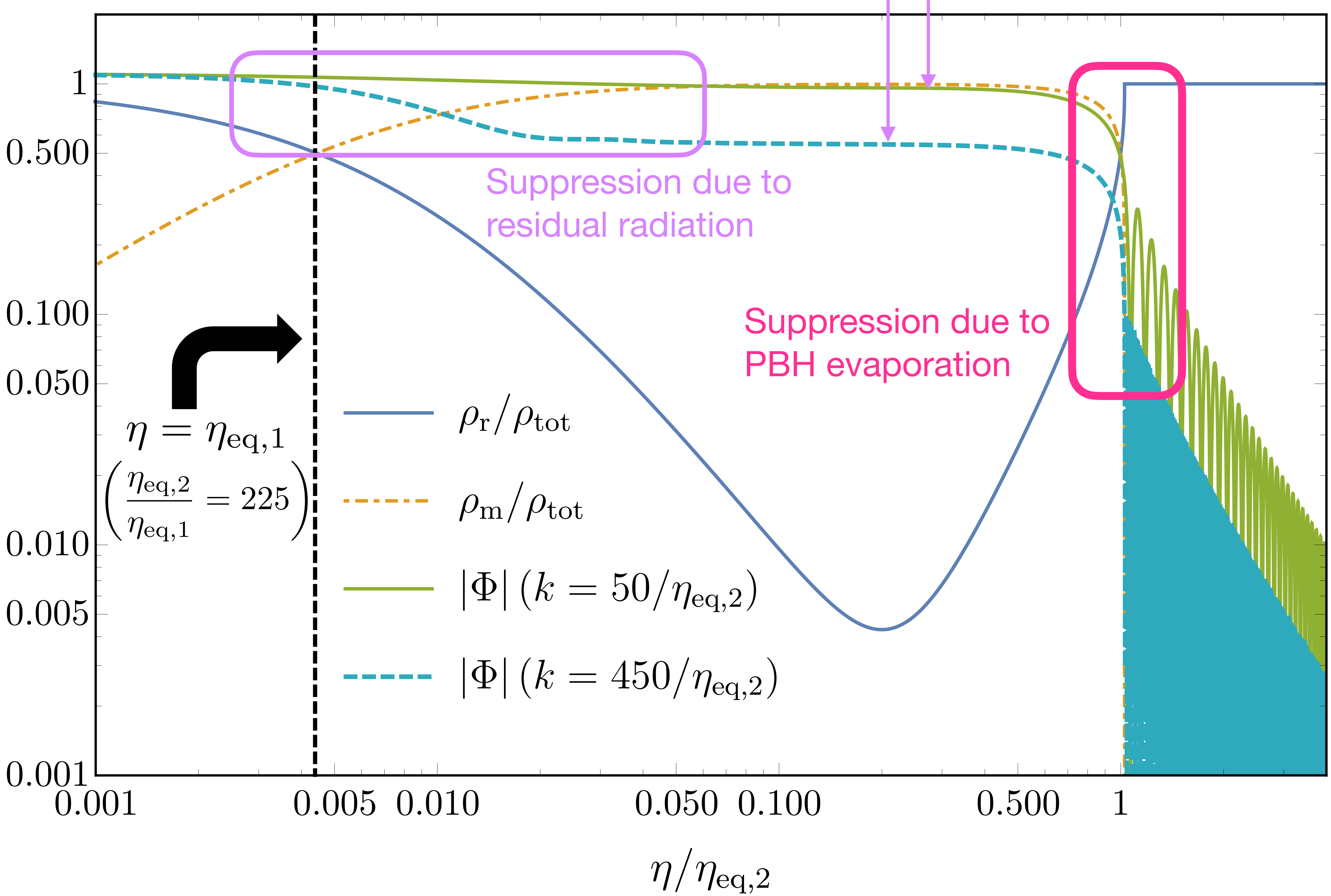


Scalar perturbations

During the decay

$$\frac{\Phi(t)}{\Phi_{\rm plateau}} \simeq \exp\left(-\int_{t_{\rm form}}^{t} d\bar{t}\, \Gamma(\bar{t})\right) \simeq \left[1-\left(\frac{t}{t_{\rm evp}}\right)\right]^{1/3}$$

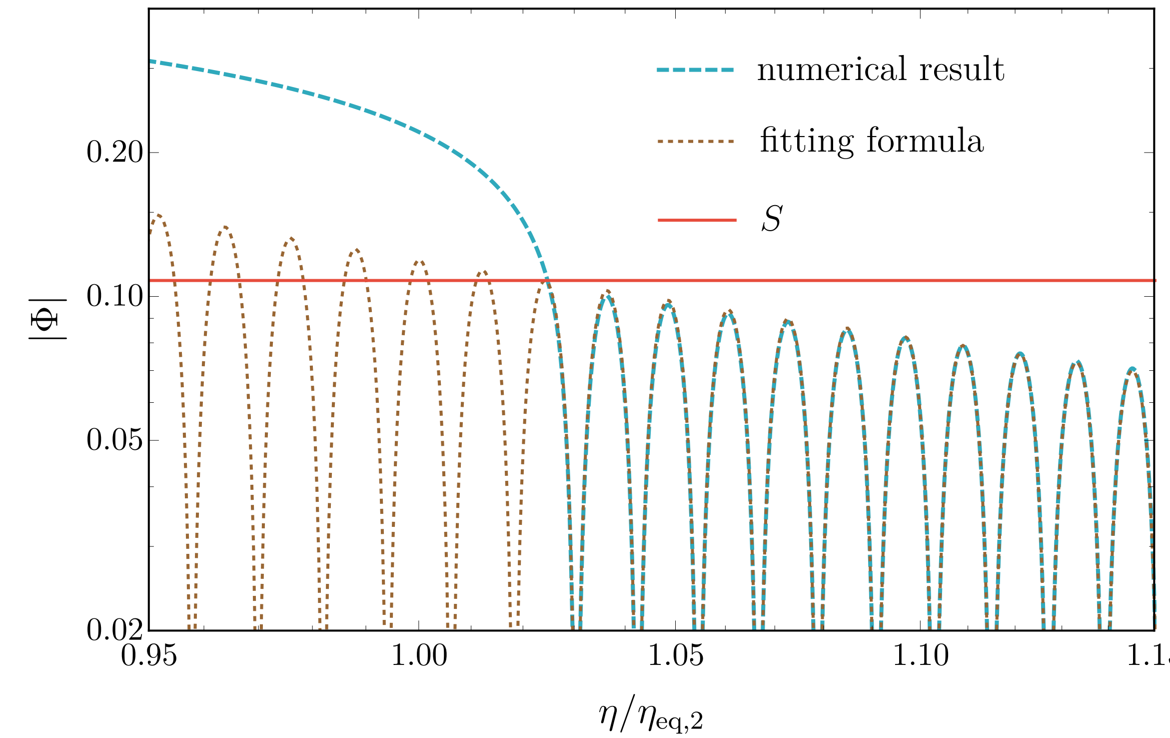
When do the oscillations begin?

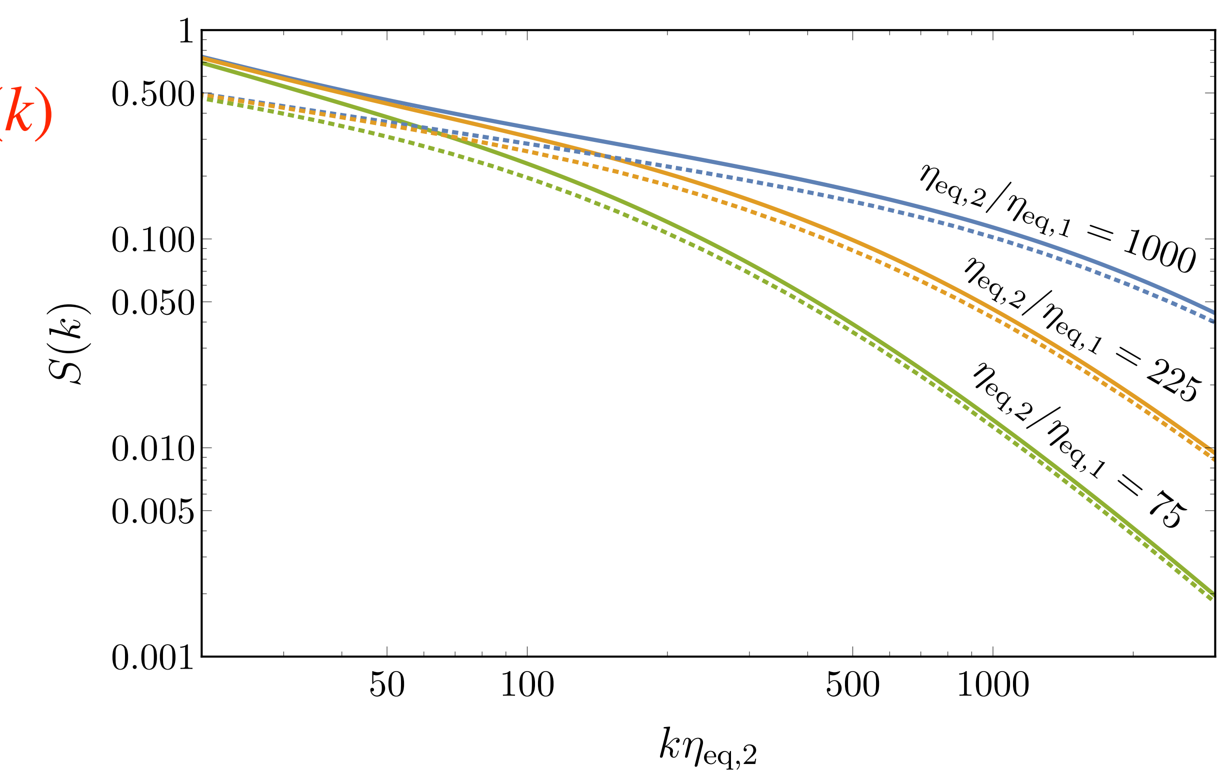
$$\Phi'' + 4\mathscr{H}\Phi' + (k^2/3)\Phi = 0$$

The lower bound on the normalization factor $S(k)$

$$S_{\rm low}(k) \simeq \left(\frac{\sqrt{6}}{k\eta_{\rm evp}}\right)^{1/3} \Phi_{\rm plateau}(x_{\rm eq,1})$$

This is consistent with the numerical fit of $S(k)$.





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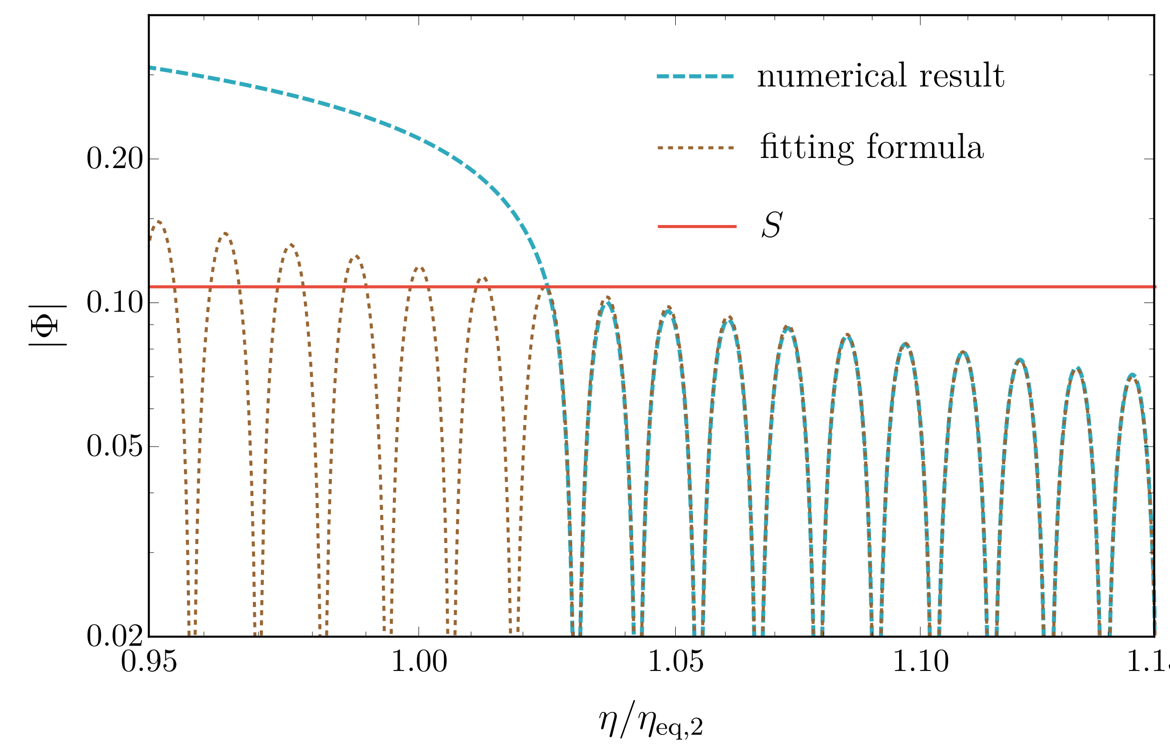

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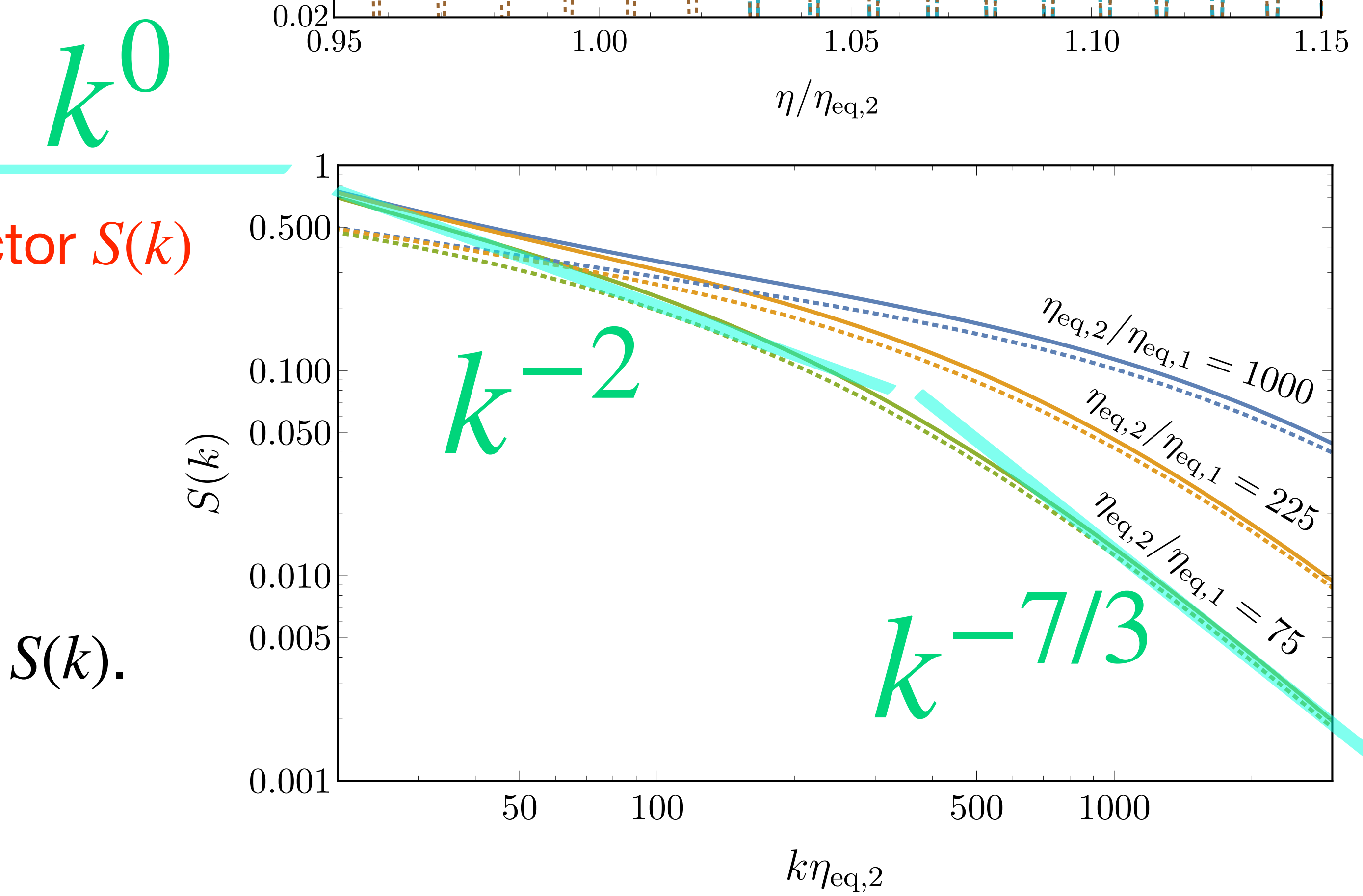

SIGW: Calculation Strategy

$$\Omega_{\mathrm{GW}}(\eta,k)=\frac{1}{6}\left(\frac{k}{\mathscr{H}(\eta)}\right)\int_0^\infty \mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2\overline{I^2(u,v,k,\eta,\eta_{\mathrm{evp}})}\mathscr{P}_\zeta(uk)\mathscr{P}_\zeta(vk)$$

SIGW: Calculation Strategy

$$\Omega_{\mathrm{GW}}(\eta, k) = \frac{1}{6}\left(\frac{k}{\mathcal{H}(\eta)}\right)\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2 \overline{I^2(u,v,k,\eta,\eta_{\mathrm{evp}})}\underbrace{\boxed{\mathcal{P}_\zeta(uk)\mathcal{P}_\zeta(vk)}}_{\text{primordial}}$$

SIGW: Calculation Strategy

$$\Omega_{\mathrm{GW}}(\eta, k) = \frac{1}{6}\left(\frac{k}{\mathscr{H}(\eta)}\right)\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2 \underbrace{\overline{I^2(u,v,k,\eta,\eta_{\mathrm{evp}})}}_{\text{time evolution}} \underbrace{\mathscr{P}_\zeta(uk)\mathscr{P}_\zeta(vk)}_{\text{primordial}}$$

SIGW: Calculation Strategy

$$\Omega_{\mathrm{GW}}(\eta,k)=\frac{1}{6}\left(\frac{k}{\mathcal{H}(\eta)}\right)\int_0^\infty \mathrm{d}v\int_{|1-v|}^{1+v}\mathrm{d}u\left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2\overline{I^2(u,v,k,\eta,\eta_{\mathrm{evp}})}\mathcal{P}_\zeta(uk)\mathcal{P}_\zeta(vk)$$

“kinematic” time evolution primordial

SIGW: Calculation Strategy

$$\Omega_{\rm GW}(\eta,k) = \frac{1}{6}\left(\frac{k}{\mathscr{H}(\eta)}\right)\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \underbrace{\left(\frac{(1+v^2-u^2)^2-4v^2}{4uv}\right)^2}_{\text{“kinematic”}} \underbrace{\overline{I^2(u,v,k,\eta,\eta_{\rm evp})}}_{\text{time evolution}} \underbrace{\mathscr{P}_\zeta(uk)\mathscr{P}_\zeta(vk)}_{\text{primordial}}$$

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"kinematic" time evolution primordial

Green fn.

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redshift scalar source

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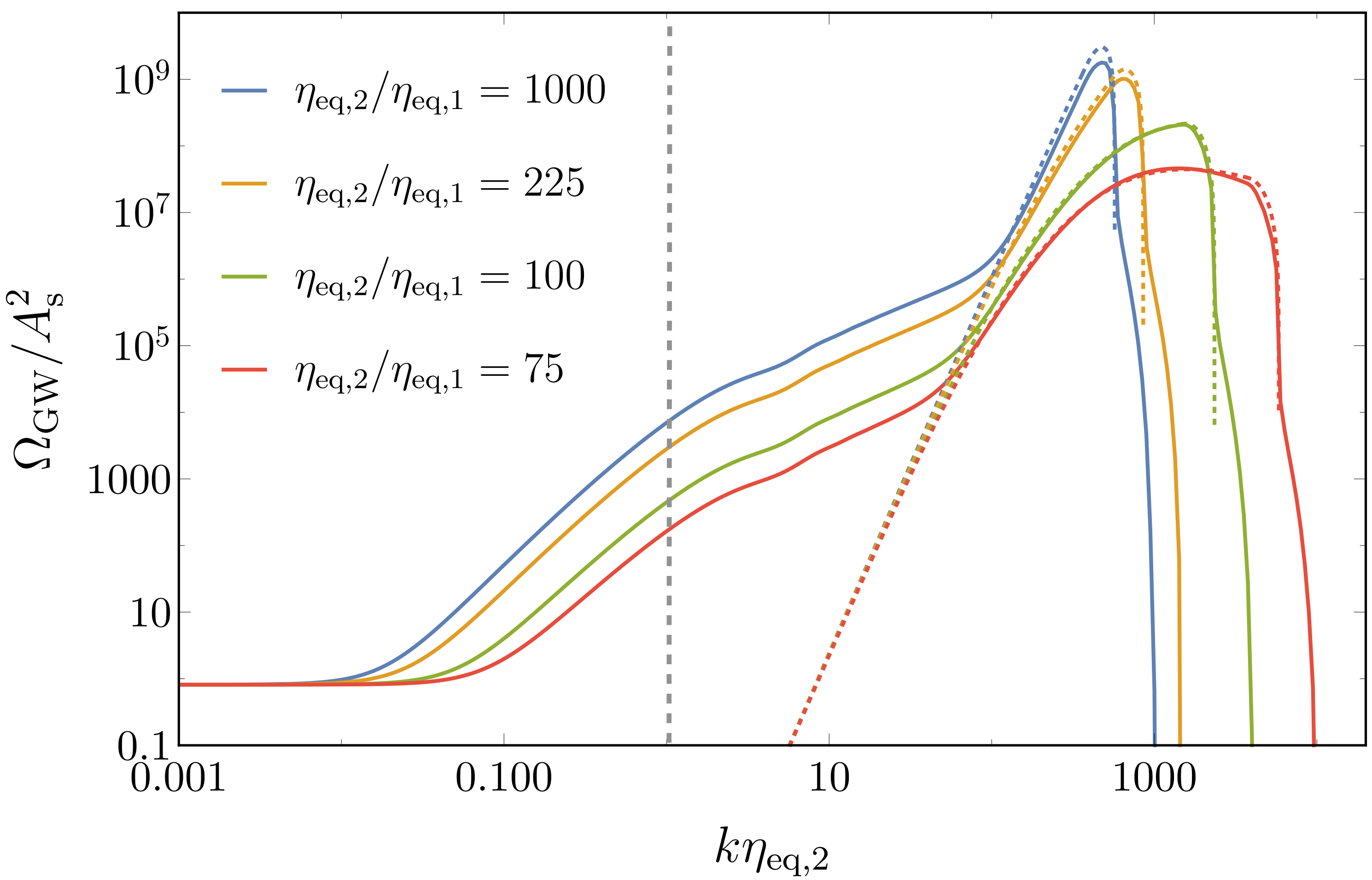
SIGW: Results

Considering the following primordial curvature perturbations $\mathscr{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \Theta(k_{\rm cut} - k)$

(for $n_s = 1$)



We can analytically calculate the resonance contribution (dotted lines).

$$\Omega_{\rm GW}^{(\rm res)}(\eta_c, k) \simeq 1.6 \times 10^{-6} \left(k\eta_{\rm evp}\right)^7 A_s^2 \left(\frac{k}{k_*}\right)^{2(n_s-1)} S_{\rm low}^4(k)\, E\left(\frac{k}{k_{\rm NL}}\right)$$

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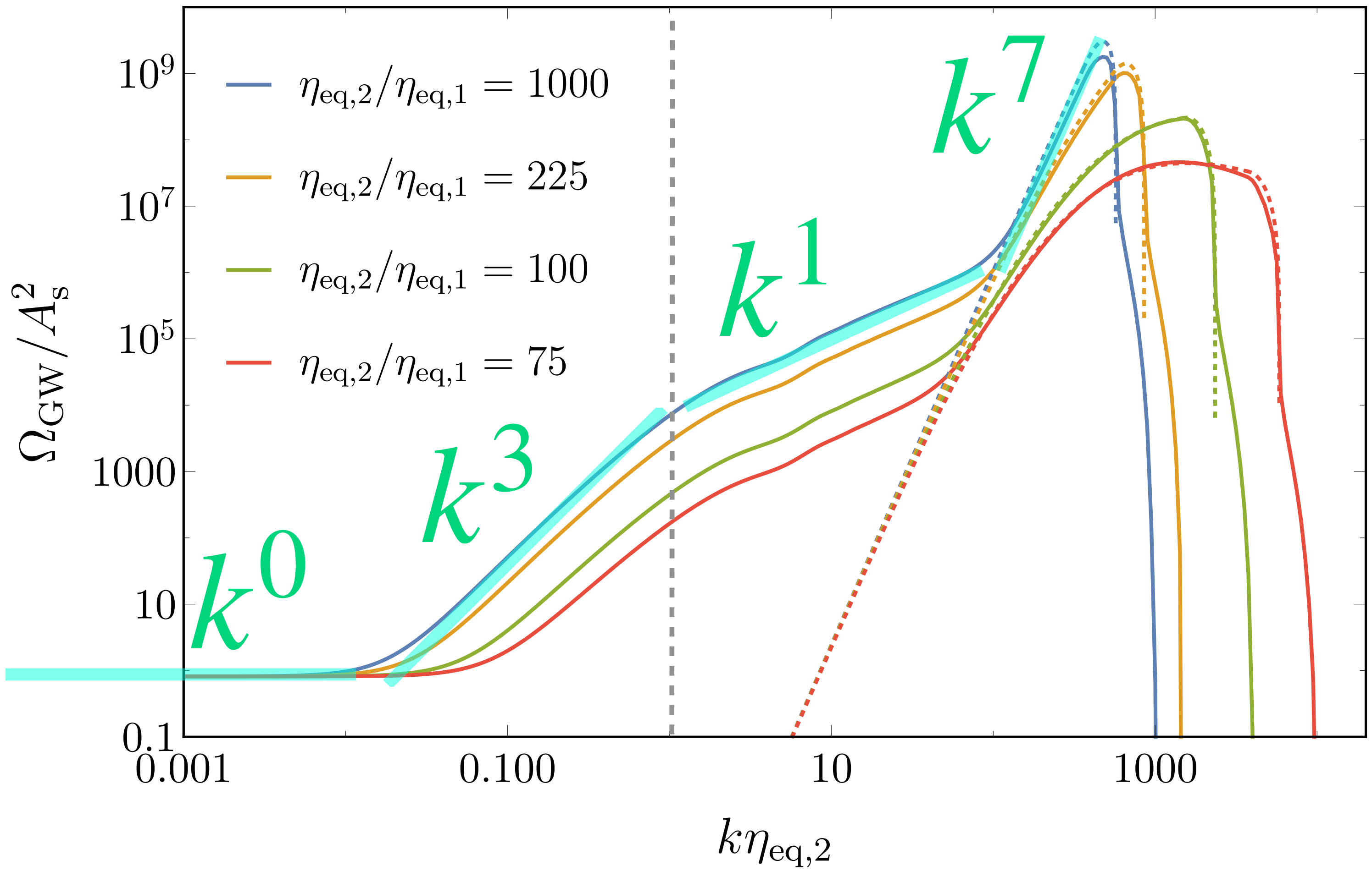

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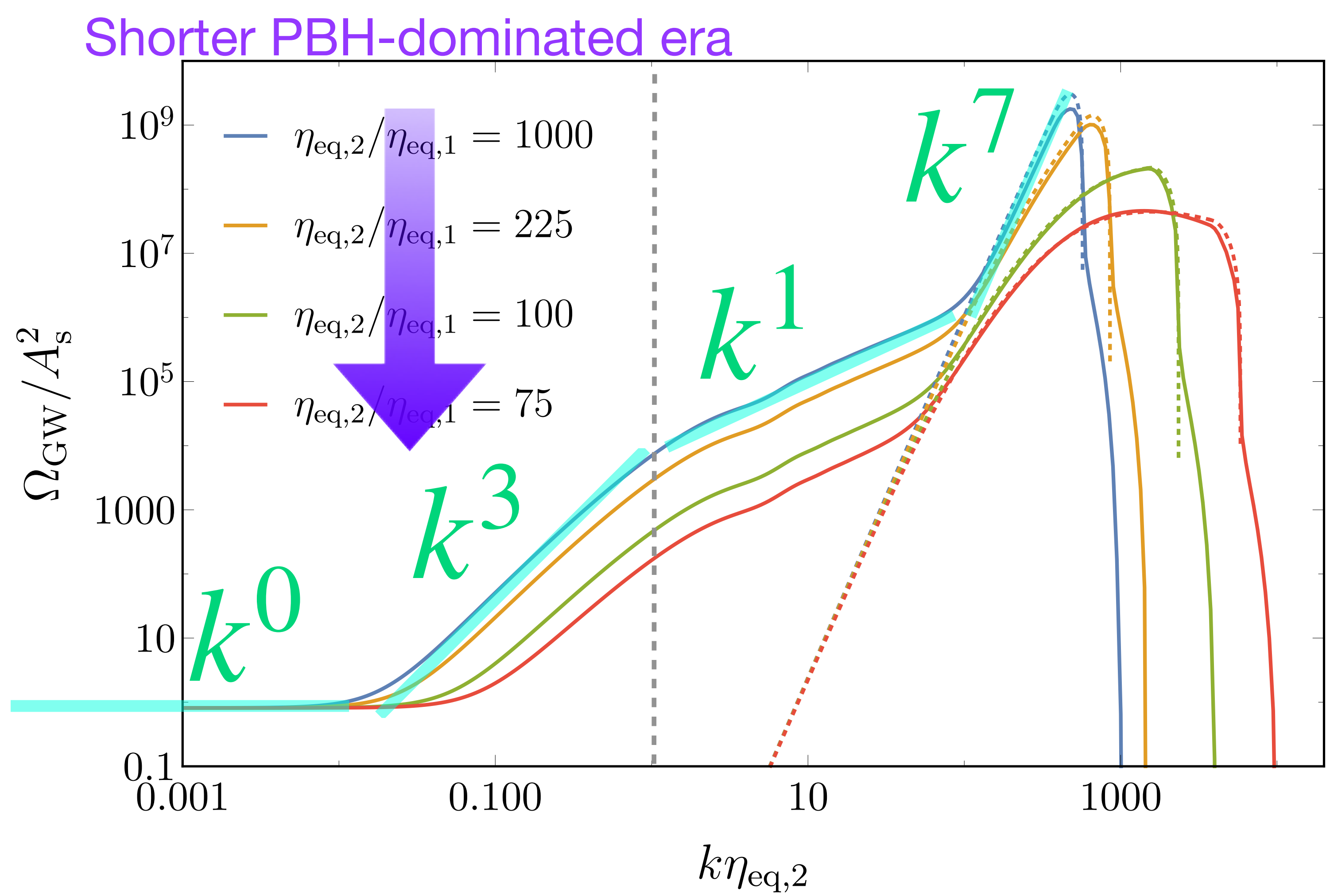

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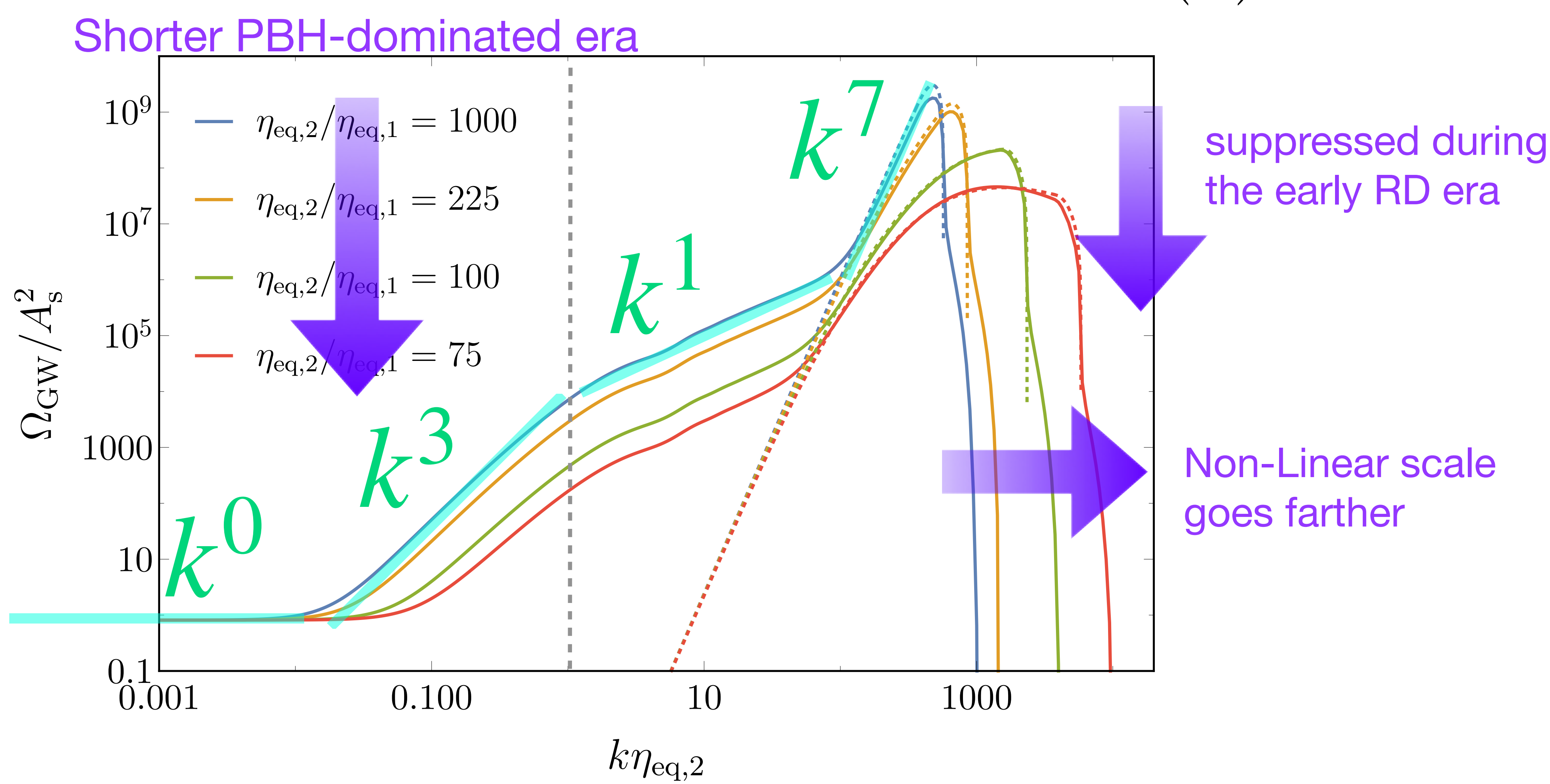

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Finite mass width

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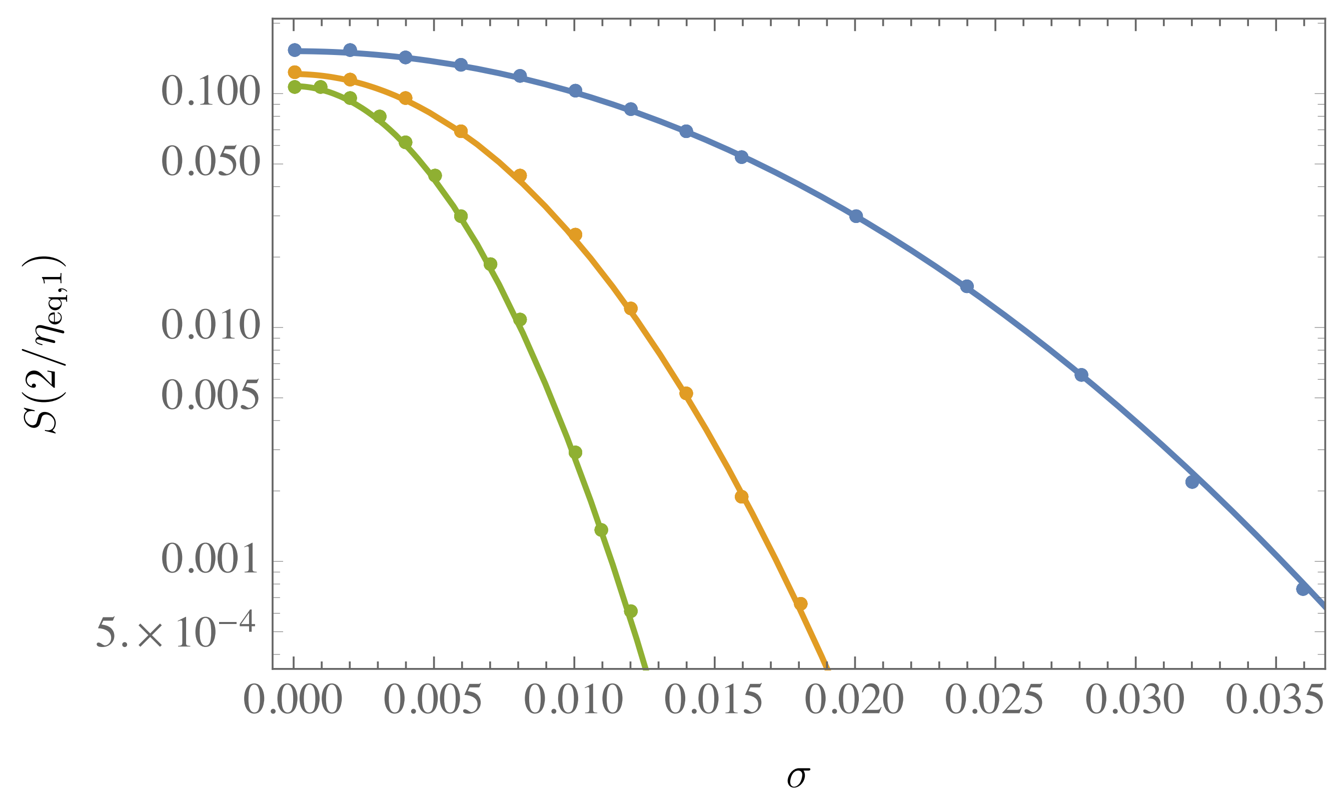

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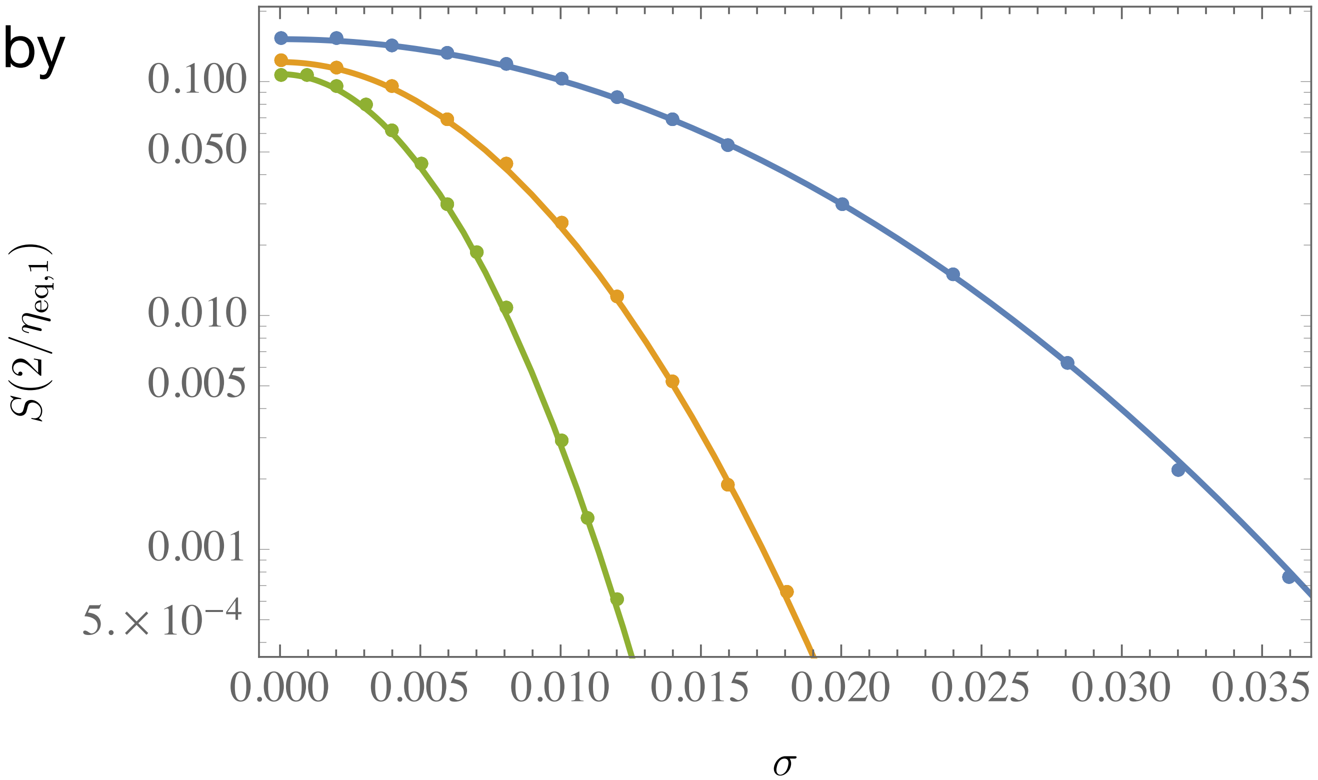
The normalization factor $S(k)$ is well fitted by

$$S(k,\sigma) = S(k)\, \exp\left(-\left(c\sigma k\eta_{\mathrm{eq},2}\right)^2\right)$$

with $c^2 \simeq 0.18$ (solid lines).

Time scales: k^{-1} vs $\sigma\eta_{\mathrm{eq},2}$



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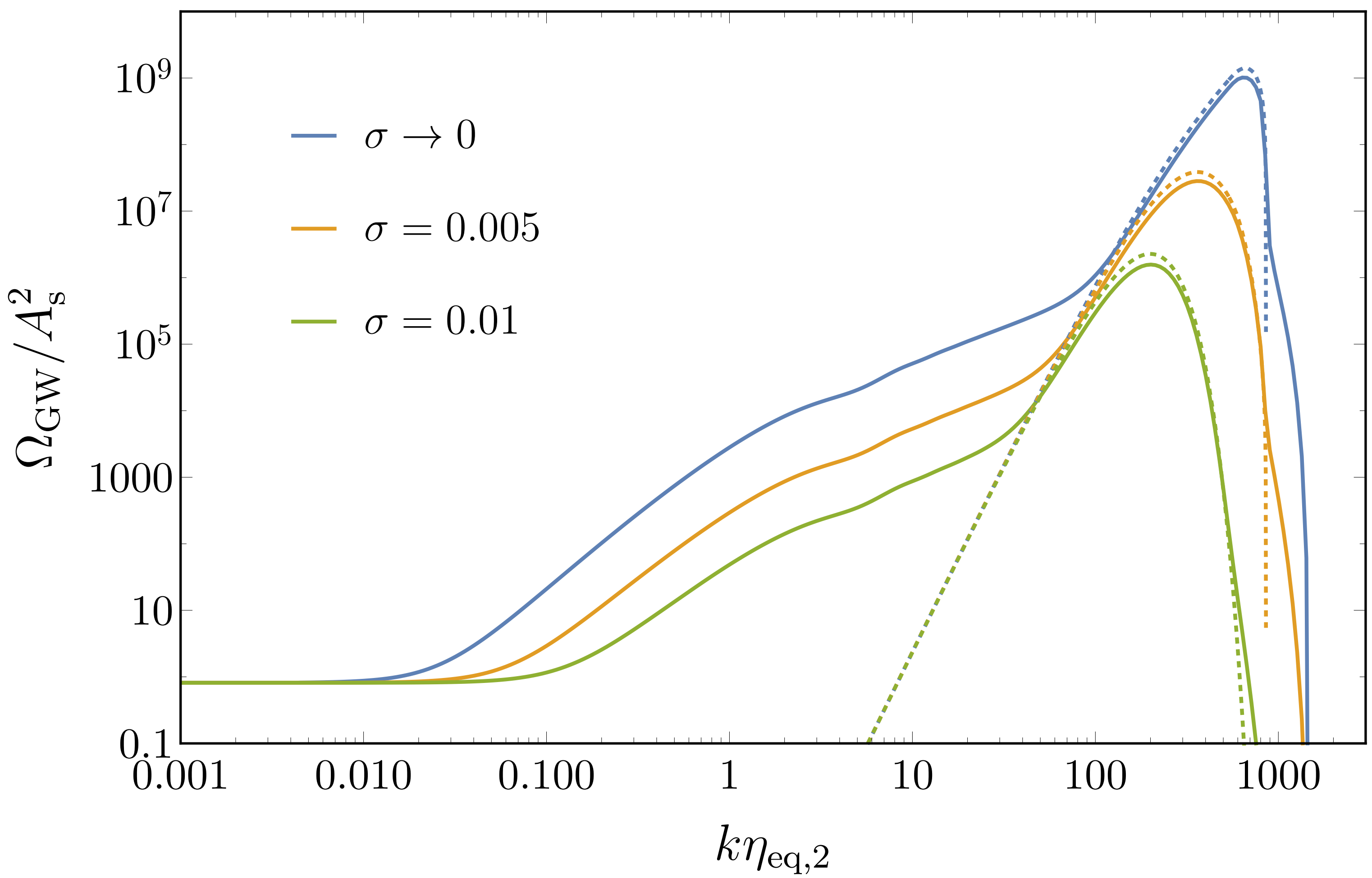

$$\Omega_{\rm GW}^{(\rm res)}(\eta_{\rm c}, k, \sigma) = \Omega_{\rm GW}^{(\rm res)}(\eta_{\rm c}, k, 0)\, G\left((c\sigma k \eta_{\rm eq,2})^2\right)$$

$$G(z) = \frac{15 \mathrm{e}^{-4z}}{64 z^{5/2}} \left(2\sqrt{z}\,(2z-3) + \left(4z^2 - 4z + 3\right) \mathrm{e}^{z} \sqrt{\pi}\,\mathrm{Erf}\left(\sqrt{z}\right)\right)$$