Discrete Symmetries, Weak Coupling Conjecture and Scale Separation in AdS Vacua

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Dark World to Swampland

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Background

1. We have a considerable amount of swampland conjectures constraining effective field theories to be compatible with Quantum Gravity.[1]
2. Most of them focus on the properties of continuous gauge symmetries.
3. Few of them have obtained constraints for discrete symmetries.[2]

[1] T. D. Brennan *et al.*, arXiv: 1711.00864 (hep-th); E. Palti, arXiv: 1903.06239 (hep-th).

[2] T. Banks, N. Seiberg, arXiv: 1011.5120 (hep-th); D. Harlow, H. Ooguri, arXiv: 1810.05337 (hep-th) (2018); D. Harlow, H. Ooguri, arXiv: 1810.05338 (hep-th) (2018); N. Craig *et al.*, *JHEP* **05**, 140, arXiv: 1812.08181 (hep-th) (2019).

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3. $\mathbf{Z}_k$ Weak Coupling Conjecture
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Introduction and Conclusions

1. We uncovered interesting **quantitative** links among both discrete and continuous gauge symmetries.
2. It is an important step in understanding the nature of discrete gauge symmetries in quantum gravity.
3. We propose the $\mathbf{Z}_k$ **Weak Coupling Conjecture** (WCC) that we tested in concrete string theory examples.
4. We argued that discrete symmetries for $3-$**forms** play an important role in the problem of **scale separation**.
5. We propose the $\mathbf{Z}_k$ **Refined Strong AdS Distance Conjecture** that we tested in type IIA AdS_4 vacua obtained in CY orientifold compactification with NSNS and RR fluxes.

Conjectures

$\mathbf{Z}_k$ **Weak Coupling Conjecture**: In a quantum gravity theory with a discrete $\mathbf{Z}_k$ gauge symmetry and a U(1) gauge symmetry with coupling g, the gauge coupling scales as $g \sim k^{-\alpha}$ for large k, with α a positive order 1 coefficient.

$\mathbf{Z}_k$ **Refined Strong AdS Distance Conjecture**: Consider quantum gravity on an AdS vacuum with a $\mathbf{Z}_k$ discrete symmetry for domain walls (with k large). In the flat-space limit $\Lambda \to 0$ (with $\Lambda k \to 0$ as well) there exists an infinite tower of states at a scale M_{cutoff}, with the relation

$$\Lambda \sim \frac{M_{\text{cutoff}}^2}{k}\,.$$

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Higgs model

1. We consider a continuous U(1) broken by a charge-k scalar VEV
$$\Phi = |\Phi| e^{i\phi}$$
2. We focus on the physics of ϕ:
   - Action: $|\partial_\mu \phi - k A_\mu|^2$.
   - Gauge invariance: $A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda$ and $\phi \longrightarrow \phi + k\Lambda$.
3. There is also the dual description in the usual BF coupling theory[3]
   - Defining $H_3 = dB_2 = \star d\phi$ and $F_2 = dA_1$.
   - Action: $|H_3|^2 + kB_2 \wedge F_2 + |F_2|^2$.
   - Dual action: $|H_3|^2 + |d\tilde{A}_1 - kB_2|^2$.
   - Gauge invariance: $B_2 \longrightarrow B_2 + d\Lambda_1$ and $\tilde{A}_1 \longrightarrow \tilde{A}_1 + k\Lambda_1$.

[3] T. Banks, N. Seiberg, arXiv: 1011.5120 (hep-th).

Charged objects

1. There are $\mathbf{Z}_k$ charged particles and $\mathbf{Z}_k$ charged strings
$$\mathcal{O}_p \sim e^{2\pi i n \int_C A_1} \quad \text{and} \quad \mathcal{O}_s \sim e^{2\pi i m \int_\Sigma B_2}$$
2. The gauge invariance means that
   - k particles annihilate into instanton.
   - k strings annihilate into a monopole line.
3. There a no long range fields, but they are detectable because a particle $\mathcal{O}_p$ with charge n will pick up an Aharonov-Bohm phase when moved around a charge m string $\mathcal{O}_s$.
4. **Remark**: It is not always obvious identify the underlying $\mathrm{U}(1)$ from which the $\mathbf{Z}_k$ symmetry derive.

Generalization

1. It is possible to define higher p-forms in n-dimensions[4]
   - (p+1)-form eating up a p-form
   - Dual (n-p-2)-form eating a dual (n-p-3)-form
   - BF coupling given by a (p+1)- and (n-p-2)-forms
2. In particular we can introduce in 4d, the Dvali-Kaloper-Sorbo axion monodromy[5]
   - Action: $|F_4|^2 + k\phi F_4 + |d\phi|^2$
   - Dual action: $|F_4|^2 + |dB_2 - kC_3|^2$
   - Gauge invariance: $C_3 \longrightarrow C_3 + d\Lambda_2$ and $B_2 \longrightarrow B_2 + k\Lambda_2$
   - Charged objects: k domain walls annihilate into string.
   - Axion monodromy:[6] $|d\phi|^2 + |\phi + kN|^2$.

[4] M. Berasaluce-Gonzalez *et al.*, *JHEP* **09**, 059, arXiv: 1206.2383 (hep-th) (2012).

[5] G. Dvali, arXiv: hep-th/0507215 (hep-th) (2005); N. Kaloper, L. Sorbo, *Phys. Rev. Lett.* **102**, 121301, arXiv: 0811.1989 (hep-th) (2009); F. Marchesano *et al.*, *JHEP* **09**, 184, arXiv: 1404.3040 (hep-th) (2014).

[6] E. Silverstein, A. Westphal, *Phys. Rev.* **D78**, 106003, arXiv: 0803.3085 (hep-th) (2008).

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$\mathbf{Z}_k$ Weak Coupling Conjecture

In a quantum gravity theory with a discrete $\mathbf{Z}_k$ gauge symmetry and a U(1) gauge symmetry with coupling g, the gauge coupling scales as $g \sim k^{-\alpha}$ for large k, with α a positive order 1 coefficient.

$\mathrm{AdS}_5 \times \mathbf{S}^5$ orbifolds - generalities

We consider type IIB string theory on $\mathrm{AdS}_5 \times \mathbf{S}^5/\mathbf{Z}_k$ with N units of 5-form flux.

1. There are continuous U(1)'s: "mesonic" U(1)'s and the R-symmetry.
2. Discrete gauge symmetry:[7] Heisenberg group $\mathbf{H}_k$
$$AB = CBA \quad \text{with} \quad A^k = 1\,, \quad B^k = 1 \quad \text{and} \quad C \text{ is central.}$$
3. The charged particles under it are D3-branes wrapped on torsion 3-cycles carrying non-trivial flat gauge bundles. The minimally charged particle is obtained by wrapping the D3-brane on a maximal $\mathbf{S}^3/\mathbf{Z}_k$.[8]

[7] S. Gukov *et al.*, arXiv: hep-th/9811048 (hep-th); B. A. Burrington *et al.*, arXiv: hep-th/0602094 (hep-th); E. Garcia-Valdecasas *et al.*, arXiv: 1907.06938 (hep-th).

[8] A. Mikhailov, *JHEP* **11**, 027, arXiv: hep-th/0010206 (hep-th) (2000).

AdS$_5 \times$ S^5 orbifolds - $\mathbf{Z}_k$-WCC

Given

$$\begin{matrix} R^4 = 4\pi(\alpha')^2 g_s N k \\ M_{P,5}^3 = \frac{M_s^8 R^5}{g_s^2 k} \end{matrix} \implies M_s \sim M_{P,5} g_s^{1/4} N^{-5/12} k^{-1/12} \text{ and } R \sim M_{P,5}^{-1} N^{2/3} k^{1/3}$$

1. the mass m of the D3-brane particle in 5d is
$$m = \frac{M_s^4 R^3}{g_s k} \sim M_{P,5} N^{1/3} k^{-1/3} .$$
2. From KK reduction from 10d to 5d
$$g = \frac{g_s k^{1/2}}{M_s^4 R^{7/2}} \sim R^{-1} M_{P,5}^{-3/2} \implies \boldsymbol{g M_{P,5}^{1/2} = N^{-2/3} k^{-1/3}} , \quad \alpha = \frac{1}{3} .$$
3. **Consistent** with the WGC bound for BPS particles
$$m = (g M_{P,5}^{1/2}) N M_{P,5}$$

Type IIA on $\mathrm{AdS}_4 \times \mathbf{CP}^3$ - generalities

1. M-theory on $\mathrm{AdS}_4 \times \mathbf{S}^7/\mathbf{Z}_k$ with N units of 7-form flux $\Longleftrightarrow$ Type IIA on $\mathrm{AdS}_4 \times \mathbf{CP}^3$ with N units of 6-form flux and k units of 2-form flux.[9]

2. There are continuous U(1)'s: "mesonic" U(1)'s and the R-symmetry.

3. Discrete symmetries:
   - **Stückelberg couplings** of the form $NB_2 \wedge F_2 + kB_2 \wedge F_2'$, with $F_2' = \int_{\mathbf{CP}^2} F_6$, leave a massless and a massive U(1) linear combinations

$$J = kQ_{D0} - NQ_{D4} \text{ and } Q_{\text{broken}} = NQ_{D0} + kQ_{D4}\,.$$

   - The discrete symmetry is $\mathbf{Z}_{N^2+k^2}$.
   - Meaning: N D0-branes (each of charge N) and k D4-branes (each of charge k) annihilate to the vacuum.

[9] O. Aharony *et al.*, arXiv: 0806.1218 (hep-th).

Type IIA on $\mathrm{AdS}_4 \times \mathbf{CP}^3$ - $\mathbf{Z}_k$-WCC

Given

$$M_s \sim M_{P,4} N^{-1/2} k^{-1/2} \text{ and } R_s \sim M_{P,4}^{-1} N^{3/4} k^{1/4}$$

1. the mass m_{D0} and m_{D4} are

$$m_{D0} \sim M_{P,4} N^{-3/4} k^{3/4} \text{ and } m_{D4} \sim M_{P,4} N^{1/4} k^{-1/4} \,.$$

2. From CS terms of the branes and charge normalization

$$g_{D0}^{-2} \sim N^{3/2} k^{-3/2} \text{ and } g_{D4}^{-2} \sim N^{-1/2} k^{1/2}$$
$$g^{-2} = \frac{k^2}{g_{D0}^2} + \frac{N^2}{g_{D4}^2} \sim N^{3/2} k^{1/2} \Longrightarrow \boldsymbol{g \sim N^{-3/4} k^{-1/4}}$$

3. **Consistent** with the WGC bound for BPS particles

$$m_{D0} = M_{P,4} g k \text{ and } m_{D4} = M_{P,4} g N \,.$$

Type IIA on $\mathrm{AdS}_4 \times \mathbf{CP}^3$ - Remark

1. In ABJM we have two different kind of states:

$$m_{D0} \sim M_{P,4} N^{-3/4} k^{3/4} \text{ and } m_{D4} \sim M_{P,4} N^{1/4} k^{-1/4} \,.$$

2. Charged under the $J = kQ_{D0} - NQ_{D4}$ unbroken $\mathrm{U}(1)$ respectively with charge k and charge N.
3. They are consistent with the WGC bound for BPS particles with respect to $g \sim N^{-3/4} k^{-1/4}$, i.e.

$$m_{D0} = M_{P,4} g k \text{ and } m_{D4} = M_{P,4} g N \,.$$

4. The two gauge couplings g_{D4} and g_{D0} must scale in a specific way:

$$g_{D4} \sim N k^{-1} g_{D0} \,.$$

The ratio follows just from **symmetry** and the BPS condition. Without knowing the details of the compactification.

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Scale Separation

AdS Distance Conjecture: Consider a quantum gravity on d-dimensional AdS space with cosmological constant Λ. There exists an infinite tower of states with mass scale m which, as $\Lambda \longrightarrow 0$, behaves as

$$m \sim |\Lambda|^{\alpha} ,$$

where α is a positive order-one number.[10]

1. Strong ADC: $\alpha = 1/2$ for SUSY theories.
2. Many examples satisfy the conjecture, e.g. all the holographic cases satisfy it.
3. In type IIA flux vacua there are examples that seem to violate the Strong ADC, e.g. DGKT-vacua.[11]

Upshot: DGKT-vacua may suggest a refinement of the Strong ADC using $\mathbf{Z}_k$ discrete symmetries.

[10] D. Lüst *et al.*, arXiv: 1906.05225 (hep-th) (2019).

[11] O. DeWolfe *et al.*, arXiv: hep-th/0505160 (2005).

DGKT-vacua

1. Compactification of type IIA on $\mathbf{T}^6/\mathbf{Z}_3$ orientifold with O6-planes and fluxes:
$$F_0 = m\,, \quad \int_{\tilde{\Sigma}} F_4 = e_{\tilde{i}}\,, \quad \int_{\alpha_a} H_3 = p_a\,.$$
2. F_0 and H_3 contribute to the tadpole, bounded and order 1.
3. The F_4 flux is unbounded.
4. In DGKT, $e_{\tilde{i}} \sim \overline{e}_{\tilde{i}} k$ with large k and we have the following scalings:
$$v_i, b_i \sim k^{1/2}\,, \quad e^{-D}, \xi \sim k^{3/2}\,, \quad M_s^2 \sim k^{-3} M_{P,4}^2$$
5. Scale separation:
$$\Lambda \sim k^{-9/2}\,, \quad m_{KK} \sim k^{-7/4} M_{P,4} \Longrightarrow m_{KK}^2 \sim \Lambda^{7/9}$$

How to use discrete symmetries?

1. Let us rewrite the expression as
$$\Lambda \sim k^{-9/2}\,, \quad m_{KK} \sim k^{-7/4} \Longrightarrow \Lambda = \frac{m_{KK}^2}{k}\,.$$
2. From the 10d CS $F_4 \wedge F_4 \wedge B_2$ we obtain the 4d coupling
$$k \left(\sum_i \bar{e}_{\tilde{i}} \phi_i \right) F_4 = k\phi' F_4\,.$$
3. From the 10d CS $mB_2 \wedge F_8$ defining $F_{4,\tilde{i}} = \int_{\tilde{\Sigma}_i} F_8$, we obtain $m \sum_i \phi_i F_{4,\tilde{i}} + k\phi' F_4$
4. Take $\quad Q' = \sum_i \bar{e}_{\tilde{i}} Q_i \quad \Longrightarrow \quad m \left(\sum_i \bar{e}_{\tilde{i}} \phi_i \right) F_4' = m\phi' F_4'$

How to use discrete symmetries?

5. We finally obtain

$$\boldsymbol{\phi' \left(m F_4' + k F_4 \right)} \,.$$

6. As in ABJM we have:
   - Massless 3-form symmetry:

$$Q_{\mathrm{U}(1)} = k \sum_i \bar{e}_{\tilde{i}} Q_{\tilde{i}} - m Q \,.$$

   - Approximate $\mathbf{Z}_k$ symmetry:

$$Q_\perp = m \sum_i \bar{e}_{\tilde{i}} Q_{\tilde{i}} + k Q \,.$$

7. We can look at the gauge coupling of the domain walls charged under the massless $\mathrm{U}(1)$, i.e. D2-branes and D6-branes on combination of 4-cycles.
8. We compute the gauge coupling:

$$\frac{1}{g^2} = k^2 \sum_i \left(\bar{e}_{\tilde{i}}\right)^2 \frac{1}{g^2_{D6,\tilde{i}}} + m^2 \frac{1}{g^2_{D2}}$$

How to use discrete symmetries?

9. If you compute the gauge couplings of the domain walls in general toroidal compactifications[12]. Using **just** the relation derived between g_{D6} and g_{D2}, we can derive:

$$v_i \sim k^{1/2}, \quad \overline{\mathcal{V}} \sim k^{3/2}.$$

10. Using **flux monodromy**:

$$N + k\bar{e}_{\bar{i}}\phi_i + m\kappa_{ijk}\phi_i\phi_j\phi_k + p_a\xi_a$$

you can obtain the scaling of

$$\phi_i \sim k^{1/2} \quad \text{and} \quad e^{-D}, \xi \sim k^{3/2}$$

11. Using this information we can compute

$$m_{KK} \sim k^{-7/4}$$

[12] A. Font *et al.*, *JHEP* **08**, 044, arXiv: 1904.05379 (hep-th) (2019).

How to use discrete symmetries?

12. We use the tensions of the domain walls to compute the vacuum energy.
13. Using D4-branes domain walls, that are responsible for changing the value of k:[12]

$$\frac{1}{g_{4,i}^2} \sim k^{13/2} \Longrightarrow T_{D4} \sim k^{-13/4} \Longrightarrow e^{\mathcal{K}/2} W \sim k^{-9/4} \Longrightarrow \Lambda \sim k^{-9/2} \,.$$

14. We have recovered the scale separation:

$$\Lambda = \frac{m_{KK}^2}{k} \,.$$

[12] L. Randall, R. Sundrum, arXiv: hep-ph/9905221 (hep-ph); L. Randall, R. Sundrum, arXiv: hep-th/9906064 (hep-th).

$\mathbf{Z}_k$ Refined Strong AdS Distance Conjecture

Consider quantum gravity on an AdS vacuum with a $\mathbf{Z}_k$ discrete symmetry for domain walls (with k large). In the flat-space limit $\Lambda \to 0$ (with $\Lambda k \to 0$ as well) there exists an infinite tower of states at a scale M_{cutoff}, with the relation

$$\Lambda \sim \frac{M_{\text{cutoff}}^2}{k} \, .$$

Conjectures

$\mathbf{Z}_k$ Weak Coupling Conjecture: In a quantum gravity theory with a discrete $\mathbf{Z}_k$ gauge symmetry and a $U(1)$ gauge symmetry with coupling g, the gauge coupling scales as $g \sim k^{-\alpha}$ for large k, with α a positive order 1 coefficient.

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Thank you!