Fine and course structures in the SCFT landscape

Dark World to Swampland: 5th IBS-IFT MultiDark Workshop

Institute for Basic Science

Center for Theoretical Physics of the Universe

Peter R. Merkx

Based on PRM '19, *Pairing 6D SCFTs*, arXiv:1903.00079 and PRM '18 *Classifying Global Symmetries of 6D SCFTs*, JHEP03(2018)163

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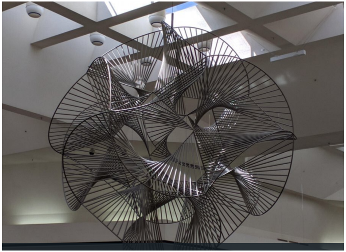
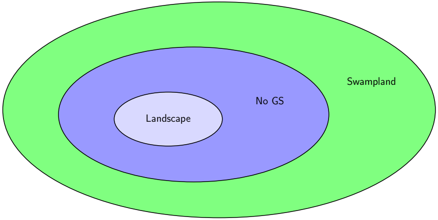


An abundance of EFTs

- There are many EFTs; can we couple them to gravity? i.e. does a given EFT arise as a low energy limit of a theory consistent with string theory / quantum gravity?

(Yes: "Landscape" versus No: "Swampland")



- A minimal, widely accepted, conjectural necessity condition on whether an EFT falls in the Landscape is that the **EFT has no global symmetries** (e.g. Banks, Seiberg '11)

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- There are stronger conjectural criteria: weak gravity conjecture(s), gauge algebra rank restrictions, etc.

4D insights from 6D

- An abundance of EFTs in 4D. Many can be obtained by further compactifying from 6D (e.g. 6D to 4D by torus compactification: Morrison, Vafa '16; many EFTs: $\geq 10^{15}$ Standard Models from F-theory: Cvetic, Halverson, Lin, Liu, Tian '19).

- Classifying 4D SCFTs remains difficult, as does explicitly relating properties of a single 6D SCFT to the **many** 4D SCFTs we may obtain by further compactification.

- Classifying 6D SCFTs is better understood. Explicit classifications $(1,0)$ and $(2,0)$ theories from F-theory are robust, even conjecturally (nearly?) complete (Heckman, Morrison, Rudelius, Vafa '15; PRM '18; Bhardwaj '19).

- One approach for hints about 4D Landscape vs. Swampland structure: classify the (UV) global symmetries of (UV F-theory models for) 6D SCFTs (Bertolini, PRM, Morrison '16; Morrison, Rudelius '16; PRM '18); allows explicit inspection of "no GS" SCFTs in context.

Outline

- Intro to F-theory 6D SCFT models
- Classifying global symmetries of these theories
- Birds eye view of 6D SCFT classification
- Course structure in SCFT classification: on families of bases

- We consider an elliptically fibered Calabi-Yau (CY) threefold X, possibly singular, having fibration $\pi : X \to B$ over a base B with a section defined by the Weierstrass equation

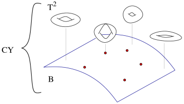
$$y^2 = x^3 + fx + g \ ,$$

where f, g are sections of $\mathcal{O}(-4K_B)$ and $\mathcal{O}(-6K_B)$), respectively, locally $B \cong \mathbb{C}^2/\Gamma$, and Γ is a discrete subgroup of $U(2)$. Locally, this can be thought of as

$$y^2 = x^3 + \tilde{f}(z,w)x + \tilde{g}(z,w) \ ,$$

with $\widetilde{f}, \widetilde{g}$ polynomials in two variables obtained by dropping higher order terms (giving irrelevant deformations of an associated 6D SCFT).

- The generic fiber $\pi^{-1}(p)$ is a smooth elliptic curve (i.e. a torus). The specific types of degenerations respecting the CY condition were classified long-ago by Kodaira.



- Kodaira's classification describes the types of degenerate fibers that can occur over a generic point P in a curve $C \subset B$ via orders of vanishing of two variable polynomials (f, g, Δ); here $\Delta := 4f^3 + 27g^2$ is the discriminant with zero locus over which the singular fibers of π arise.

ord(f)	ord(g)	ord(Δ)	type	singularity	non-abelian algebra
≥ 0	≥ 0	0	I_0	none	none
0	0	1	I_1	none	none
0	0	$n \geq 2$	I_n	A_{n-1}	$\mathfrak{su}(n)$ or $\mathfrak{sp}([n/2])$
≥ 1	1	2	II	none	none
1	≥ 2	3	III	A_1	$\mathfrak{su}(2)$
≥ 2	2	4	IV	A_2	$\mathfrak{su}(3)$ or $\mathfrak{su}(2)$
≥ 2	≥ 3	6	I_0^*	D_4	$\mathfrak{so}(8)$ or $\mathfrak{so}(7)$ or $\mathfrak{g}_2$
2	3	$n \geq 7$	I_{n-6}^*	D_{n-2}	$\mathfrak{so}(2n-4)$ or $\mathfrak{so}(2n-5)$
≥ 3	4	8	IV^*	$\mathfrak{e}_6$	$\mathfrak{e}_6$ or $\mathfrak{f}_4$
3	≥ 5	9	III^*	$\mathfrak{e}_7$	$\mathfrak{e}_7$
≥ 4	5	10	II^*	$\mathfrak{e}_8$	$\mathfrak{e}_8$
≥ 4	≥ 6	≥ 12	non-minimal	-	-

Singularity types with associated non-abelian algebras.

- Tate's extended analysis of the Kodaira classification allows us to remove the ambiguity in above non-abelian algebras.
- The above correspondence of classical singularity types to gauge summands of a physical theory constructed in F-theory appeared decades later (Bershadsky et. al. '96)

- The precise algebra which occurs in the cases of ambiguity is determined by inspection of the auxiliary polynomials below following what is known as Tate's Algorithm. Here Σ is a curve along a component of the discriminant locus at $\{z = 0\}$. Larger algebras result with more complete factorizations (notation: split/semi-split/non-split$\leftrightarrow$irreducible).

type	equation of monodromy cover
$I_n^{s/ns}$, $n \geq 3$	$\psi^2 + (9g/2f)\vert_{z=0}$
$IV^{s/ns}$	$\psi^2 - (g/z^2)\vert_{z=0}$
$I_0^{*s/ss/ns}$	$\psi^3 + (f/z^2)\vert_{z=0} \cdot \psi + (g/z^3)\vert_{z=0}$
$I_{2n-5}^{*s/ns}$, $n \geq 3$	$\psi^2 + \frac{1}{4}(\Delta/z^{2n+1})(2zf/9g)^3\vert_{z=0}$
$I_{2n-4}^{*s/ns}$, $n \geq 3$	$\psi^2 + (\Delta/z^{2n+2})(2zf/9g)^2\vert_{z=0}$
$IV^{*s/ns}$	$\psi^2 - (g/z^4)\vert_{z=0}$

Monodromy cover polynomials determining non-abelian algebras.

- The same algebra dictionary for non-compact components of $S := \{\Delta = 0\}$ give rise to global symmetry summands (upon taking appropriate limits to shrink S and decouple gravity from the SCFT). [Subtle issues: need to run RG flow towards a CFT, multiple GS maxima,...]

SCFT global symmetries using 6D F-theory: 1D Culoumb branch

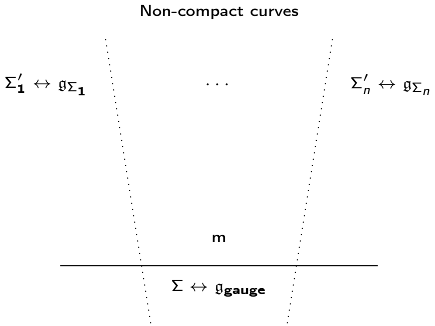
The base case: 6D SCFT models with a **simple non-abelian gauge algebra** (i.e. those from an F-theory model with a single compact curve component of $\{\Delta = 0\} \subset B$.

- Let $\Sigma \cong \mathbb{P}^1$ be a component of Δ with $-\Sigma \cdot \Sigma = m$ and some compatible choice of gauge algebra. (Our choice of m confines the Kodaira types and in turn the algebras that can pair with Σ.)
- We consider a collection of non-compact curves $\Sigma'_1, \ldots \Sigma'_n$ which are also components of $\{\Delta = 0\}$ and are transverse to Σ. In taking an appropriate limit in which Σ is contracted, the direct sum of the non-abelian algebras carried on the curves Σ'_i arises as the global symmetry algebra of an SCFT associated to our F-theory model having gauge algebra matching that from Σ.

  [Caveats: the corresponding field theory may not be conformal, but flows to a conformal theory under renormalization; tracking the algebra content during this flow remains difficult to treat in full generality.]

SCFT global symmetries using 6D F-theory: 1D Culoumb branch

- The challenge is to determine the (relatively) maximal algebras $\mathfrak{g}_{\Sigma'_{\mathbf{1}}} \oplus \cdots \oplus \mathfrak{g}_{\Sigma'_n}$ which can be supported simultaneously the Σ'_i.



Goal: identify collections with $[\Sigma'_{\mathbf{1}}, \cdots, \Sigma'_n] \cap_{trans} \Sigma$ allowable

such that $\mathfrak{g}_{\Sigma'_{\mathbf{1}}} \oplus \cdots \oplus \mathfrak{g}_{\Sigma'_n}$ is (relatively) maximal.

Constraints provided by factoring 2 variable polynomials

We can provide general local models for nearly all intersections we encounter in 6D F-theory models to obtain adequate bounds. For example:

- Consider a base B with a compact curve Σ of Kodaira type IV at $\{z = 0\}$ carrying $\mathfrak{su}(3)$ gauge algebra, i.e. the monodromy cover polynomial is split, meaning $\widetilde{g}|_{z=0} = (g/z^2)|_{z=0}$ is a square.
- Suppose additionally that we have an intersection with a Kodaira type II curve Σ_1' at $\{\sigma = 0\}$. We can expand f, g, Δ to get naïve contributions before considering monodromy:

$$
\begin{aligned}
f &= z^{\mathbf{2}}\sigma(f_{\mathbf{2}} + f_{\mathbf{3}}z + O(z^{\mathbf{2}})) \\
g &= z^{\mathbf{2}}\sigma(g_{\mathbf{2}} + g_{\mathbf{3}}z + O(z^{\mathbf{2}})) \\
\Delta &= z^{\mathbf{4}}\sigma^{\mathbf{2}}(\Delta_{\mathbf{4}} + \Delta_{\mathbf{5}}z + O(z^{\mathbf{2}})) \qquad \implies \textbf{contributions to } (\widetilde{a}, \widetilde{b}, \widetilde{d}) \textbf{ are } \geq (\mathbf{1}, \mathbf{1}, \mathbf{2}).
\end{aligned}
$$

From the monodromy condition, we find raised contributions:

$$
\begin{aligned}
f &= z^{\mathbf{2}}\sigma(f_{\mathbf{2}} + f_{\mathbf{3}}z + O(z^{\mathbf{2}})) \\
g &= z^{\mathbf{2}}\sigma(\sigma g_{\mathbf{2}} + g_{\mathbf{3}}z + O(z^{\mathbf{2}})) \\
\Delta &= z^{\mathbf{4}}\sigma^{\mathbf{2}}(\sigma^{\mathbf{2}}\Delta_{\mathbf{4}} + \Delta_{\mathbf{5}}z + O(z^{\mathbf{2}})) \implies \textbf{contributions to } (\widetilde{a}, \widetilde{b}, \widetilde{d}) \textbf{ are } \geq (\mathbf{1}, \mathbf{2}, \mathbf{4}).
\end{aligned}
$$

Following this type of analysis gives the desired GS classification. (Can bypass hypermultiplet counting based mixed anomaly cancellation checks; the converse is not true.)

Some cases are delicate; e.g. Σ_1' having Kodaira type I_6 in $\{\sigma = 0\}$. We can find the general form of f, g, Δ:

$$\begin{aligned}
f &= -\tfrac{1}{48}\mu^2\alpha^4\beta^4 - \tfrac{1}{6}\mu\alpha^2\beta^3\nu\sigma + \tfrac{1}{3}(-\tfrac{1}{2}\mu\alpha^2\beta\phi_2 - \beta^2\nu^2)\sigma^2 \\
&\quad + (-\tfrac{1}{3}\nu\phi_2 - 3\beta\lambda)\sigma^3 + f_4\sigma^4 + f_5\sigma^5 + f_6\sigma^6 + O(\sigma^7)\ , \\
g &= \tfrac{1}{864}\mu^3\alpha^6\beta^6 + \tfrac{1}{72}\mu^2\alpha^4\beta^5\nu\sigma + \tfrac{1}{18}\left(\mu\alpha^2\beta^4\nu^2 + \tfrac{1}{4}\mu^2\alpha^4\beta^3\phi_2\right)\sigma^2 \\
&\quad + \left(\tfrac{1}{12}\mu\alpha^2\beta^2\nu\phi_2 + \tfrac{2}{27}\beta^3\nu^3 + \tfrac{1}{4}\mu\alpha^2\beta^3\lambda\right)\sigma^3 \\
&\quad + \left(\tfrac{1}{36}\mu\alpha^2\phi_2^2 + \tfrac{1}{9}\beta\nu^2\phi_2 + \beta^2\nu\lambda - \tfrac{1}{12}\mu\alpha^2\beta^2 f_4\right)\sigma^4 \\
&\quad + \left(\phi_2\lambda - \tfrac{1}{3}\beta\nu f_4 - \tfrac{1}{12}\mu\alpha^2\beta^2 f_5\right)\sigma^5 + \left(\widehat{g}_6 - \tfrac{1}{3}\beta\nu f_5 - \tfrac{1}{12}\mu\alpha^2\beta^2 f_6\right)\sigma^6 + O(\sigma^7)\ , \\
\Delta &= \tfrac{1}{432}\mu^2\alpha^4\beta^3\left(27\mu\alpha^2\beta^3\widehat{g}_6 + 9\mu\alpha^2\beta^2\phi_2 f_4 + \mu\alpha^2\phi_2^3\right. \\
&\qquad \left. -243\lambda^2\beta^3 + 54\phi_2\nu\lambda\beta^2 - 3\beta\nu^2\phi_2^2\right)\sigma^6 + O(\sigma^7)\ .
\end{aligned}$$

We can now impose that $z^{1+A}|f, z|g$ to study a transverse intersection with Σ of type II. Results depend on A, monodromy along Σ_1', and case analysis to avoid terminal singularities over $\Sigma \cap \Sigma_1'$ (preventing Calabi-Yau resolution). In treating a single curve, we can take all $A = B = 0$, curves transverse to Σ are non-compact, and they carry non-abelian gauge algebra. (Here A, B are additional orders of vanishing of f, g along Σ resp.)

Constraints provided by factoring 2 variable polynomials (1D Culoumb branch)

Together with a few cases requiring considering pairs of transverse curves, we obtain constraints that are strictly stronger than hypermultiplet counting on GS maxes for all curves with non-abelian gauge algebra (Bertolini, PRM, Morrison, '16; PRM '18).

type along Σ	algebra on Σ	$-\Sigma^2$	max. global symmetry algebra(s)
I_2	$\mathfrak{su}(2)$	2	$\mathfrak{su}(4)$
		1	$\mathfrak{so}(20)$
$I_{n\geq 3}$, n odd	$\mathfrak{sp}([n/2])$	1	$\mathfrak{so}(13+2n)$ $\mathfrak{so}(7+2p) \oplus \mathfrak{so}(7+2n-2p)$, $0 \leq p \leq \frac{n+1}{2}$
	$\mathfrak{su}(n)$	2	$\mathfrak{su}(2n)$
		1	$\mathfrak{su}(8+n)$
$I_{n\geq 4}$, n even	$\mathfrak{sp}(n/2)$	1	$\mathfrak{so}(16+2n)$
	$\mathfrak{su}(n)$	2	$\mathfrak{su}(2n)$
		1	$\mathfrak{su}(8+n)$
I_6	$\mathfrak{su}(6)^*$	1	$\mathfrak{su}(15)$
III	$\mathfrak{su}(2)$	2	$\mathfrak{so}(7)$
		1	$\mathfrak{so}(7) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$ $\mathfrak{so}(7) \oplus \mathfrak{sp}(3)$ (†) $\mathfrak{sp}(5)$ (†)

Global symmetries of gauged F-theory models (I).

- Results respect the weaker constraints provided by field theory (below).
- The presence of multiple GS maxima remains mysterious. These may disappear in RG flow or hint at a finer SCFT classification.

Constraints provided by factoring 2 variable polynomials (1D Culoumb branch)

type along Σ	algebra on Σ	$-\Sigma^2$	max. global symmetry algebra
IV	$\mathfrak{su}(2)$	2	$\mathfrak{g}_2$ (†)
		1	$\mathfrak{g}_2 \oplus \mathfrak{g}_2 \oplus \mathfrak{su}(3)$ (†) $\mathfrak{g}_2 \oplus \mathfrak{sp}(2)$ (†) $\mathfrak{sp}(3)$ (†)
	$\mathfrak{su}(3)$	3	-
		2	$\mathfrak{su}(3) \oplus \mathfrak{su}(3)$ $\mathfrak{sp}(2)$
		1	$\mathfrak{su}(3)^{\oplus 4}$ $\mathfrak{su}(3)^{\oplus 2} \oplus \mathfrak{sp}(2)$ $\mathfrak{su}(3) \oplus \mathfrak{sp}(3)$ $\mathfrak{sp}(4)$
I_0^*	$\mathfrak{g}_2$	3	$\mathfrak{sp}(1)$
		2	$\mathfrak{sp}(4)$
		1	$\mathfrak{sp}(7)$
	$\mathfrak{so}(7)$	3	$\mathfrak{sp}(2)$
		2	$\mathfrak{sp}(4) \oplus \mathfrak{sp}(1)$
		1	$\mathfrak{sp}(6) \oplus \mathfrak{sp}(2)$
	$\mathfrak{so}(8)$	4	-
		3	$\mathfrak{sp}(1) \oplus \mathfrak{sp}(1) \oplus \mathfrak{sp}(1)$
		2	$\mathfrak{sp}(2) \oplus \mathfrak{sp}(2) \oplus \mathfrak{sp}(1)^{\oplus 2}$
		1	$\mathfrak{sp}(3) \oplus \mathfrak{sp}(3) \oplus \mathfrak{sp}(1)^{\oplus 3}$

Global symmetries of gauged F-theory models (II).

- The same approach turns out to work for cases where the single compact curve in the base does not carry a non-abelian gauge algebra (Morrison, Rudelius, '16).

Constraints from field theory (1D Culoumb branch)

$\mathfrak{g}$	representation	global symmetry
$\mathfrak{su}(2)$	$(32+12\Sigma^2)\frac{1}{2}F$	$\mathfrak{so}(32+12\Sigma^2)$
$\mathfrak{su}(3)$	$(18+6\Sigma^2)F$	$\mathfrak{su}(18+6\Sigma^2)$
$\mathfrak{su}(4)$	$(16+4\Sigma^2)F+(2+\Sigma^2)\Lambda^2$	$\mathfrak{su}(16+4\Sigma^2)\oplus\mathfrak{sp}(2+\Sigma^2)$
$\mathfrak{su}(5)$	$(16+3\Sigma^2)F+(2+\Sigma^2)\Lambda^2$	$\mathfrak{su}(16+3\Sigma^2)\oplus\mathfrak{su}(2+\Sigma^2)$
$\mathfrak{su}(6)$	$(16+2\Sigma^2)F+(2+\Sigma^2)\Lambda^2$	$\mathfrak{su}(16+2\Sigma^2)\oplus\mathfrak{su}(2+\Sigma^2)$
$\mathfrak{su}(6)^*$	$(16+\Sigma^2)F+\frac{1}{2}(2+\Sigma^2)\Lambda^3$	$\mathfrak{su}(16+\Sigma^2)\oplus\mathfrak{so}(2+\Sigma^2)$
$\mathfrak{su}(n),\ n\geq 7$	$(16+(8-n)\Sigma^2)F+(2+\Sigma^2)\Lambda^2$	$\mathfrak{su}(16+(8-n)\Sigma^2)\oplus\mathfrak{su}(2+\Sigma^2)$
$\mathfrak{sp}(n),\ n\geq 2$	$(16+4n)\frac{1}{2}F$	$\mathfrak{so}(16+4n)$
$\mathfrak{so}(7)$	$(3+\Sigma^2)V+2(4+\Sigma^2)S_*$	$\mathfrak{sp}(3+\Sigma^2)\oplus\mathfrak{sp}(8+2\Sigma^2)$
$\mathfrak{so}(8)$	$(4+\Sigma^2)V+(4+\Sigma^2)(S_++S_-)$	$\mathfrak{sp}(4+\Sigma^2)\oplus\mathfrak{sp}(4+\Sigma^2)\oplus\mathfrak{sp}(4+\Sigma^2)$
$\mathfrak{so}(9)$	$(5+\Sigma^2)V+(4+\Sigma^2)S_*$	$\mathfrak{sp}(5+\Sigma^2)\oplus\mathfrak{sp}(4+\Sigma^2)$
$\mathfrak{so}(10)$	$(6+\Sigma^2)V+(4+\Sigma^2)S_*$	$\mathfrak{sp}(6+\Sigma^2)\oplus\mathfrak{su}(4+\Sigma^2)$
$\mathfrak{so}(11)$	$(7+\Sigma^2)V+(4+\Sigma^2)\frac{1}{2}S_*$	$\mathfrak{sp}(7+\Sigma^2)\oplus\mathfrak{so}(4+\Sigma^2)$
$\mathfrak{so}(12)$	$(8+\Sigma^2)V+(4+\Sigma^2)\frac{1}{2}S_*$	$\mathfrak{sp}(8+\Sigma^2)\oplus\mathfrak{so}(4+\Sigma^2)$
$\mathfrak{so}(13)$	$(9+\Sigma^2)V+(2+\frac{1}{2}\Sigma^2)\frac{1}{2}S_*$	$\mathfrak{sp}(9+\Sigma^2)\oplus\mathfrak{so}(2+\frac{1}{2}\Sigma^2)$
$\mathfrak{so}(n),\ n\geq 14$	$(n-8)V$	$\mathfrak{sp}(n-8)$
$\mathfrak{e}_6$	$(6+\Sigma^2)27$	$\mathfrak{su}(6+\Sigma^2)$
$\mathfrak{e}_7$	$(8+\Sigma^2)\frac{1}{2}56$	$\mathfrak{so}(8+\Sigma^2)$
$\mathfrak{e}_8$	none	none
$\mathfrak{f}_4$	$(5+\Sigma^2)26$	$\mathfrak{sp}(5+\Sigma^2)$
$\mathfrak{g}_2$	$(10+3\Sigma^2)7$	$\mathfrak{sp}(10+3\Sigma^2)$

Notes: $\Sigma^2=-1$ for $\mathfrak{su}(6)^*$; $\Sigma^2=-1$ for $\mathfrak{sp}(n),\ n\geq 2$; $\Sigma^2=-4$ for $\mathfrak{so}(n),\ n\geq 14$; and $\Sigma^2=-12$ for $\mathfrak{e}_8$.

Global symmetry constraints from field theory.

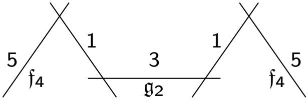
Towards 6D SCFT global symmetries in the general case

(For simplicity, let us for now exclude frozen divisors, i.e. $O7^+$ planes, from our discussion.)

The data to specify a 6D SCFT model is a tree of pairwise intersecting $\mathbb{P}^1$'s (i.e. spheres) $\{\Sigma_i, | i \in I\}$ obeying the following.

- $\Sigma_i \cdot \Sigma_j \leq 1$ for $i \neq j$ (Distinct spheres intersect in at most a point).
- $1 \leq m_i := -\Sigma_i \cdot \Sigma_i \leq 12$ (All spheres have bounded negative self-intersection; we drop minus signs for notational ease).
- Restrictive branching rules (most bases are linear chains with only nearest neighbor intersections).

Finally, a decoration on each curve with a simple gauge algebra summand obeying a few restrictions including constraints for m_i compatibility. An example of a valid configuration giving an SCFT with $\mathfrak{g}_2 \oplus \mathfrak{f}_4^{\oplus 2}$ gauge symmetry:



(Preliminary classification: Heckman, Morrison, Rudelius, Vafa '15 [H-M-R-V15]

Refinements: PRM '18.

Including frozen cases: Bhardwaj, Morrison, Tachikawa, Tomasiello '18; Bhardwaj '19.)

Structure of 6D SCFT classification

The only m_i occurring together can be described concisely via rules for attaching "nodes" $m_i \in \{4, 6, 7, 8, 9, (10), (11), (12)\}$ to the outer -1 curves of finitely many "links", namely trees having only $m_{j,i} \in \{1, 2, 3, 5\}$, e.g.

$$\underset{\text{end-link}}{(2321)}(8)\ \underset{\text{interior-link}}{(12321)}\ (8)\underset{\text{end-link}}{(12315)} \sim 2321812321812315$$

- Links can be explicitly listed (H-M-R-V15; mild trimming for branched links PRM'18):

Interior links	End links		0-links	
131	132	13	232	2315123
1231	1322	123	2312	22315123
12231	1232	1223	23132	23151313
12321	13132	1313	21322	223151313
151321	123132	12313	21512	215131513
151231	131322	122313	223132	2315131513
1513221	131512	131513	215132	215
1512321	1223132	1231513	2315132	2315
1315131	1315132	1315123	2231322	2215
12315131	1231322	1321513	2151322	22315
122315131	1231512	12231513	2215132	23215
123151321	12315132	12321513	2151232	22215
151315131	12231322	12315123	22315132	231315
1223151321	13151322	122315123	23215132	2151315
1513151321	13151232	1315131513	22151322	23151315
12231513221	13215132	12315131513	223151322	21513215
15131513221	12231512	122315131513	232151322	231513215
	122315132	1315	23151315132	2315132215
	123215132	12315	21513151322	2231513215
	123151322	13215	2231513151322	22315132215
	123151232	122315	23	313
	132151322	123215	223	3213
	1223151322	132215	213	31513
	1223151232	13151315	2313	315123
	1232151322	123151315	22313	3131513
	13151315132	131513215	23213	315131513
	123151315132	1223151315	21513	315
	131513151322	1231513215	231513	3215
	1223151315132	12231513215	221513	32215
	1231513151322		215123	31315
	12231513151322		2231513	313215
			2321513	3151315

All linear links with length ≥ 2 (excluding "instanton links", those of the form $(1)2222\cdots$)

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Structure of 6D SCFT classification

- Explicit listing of gauge assignments on links is possible (H-M-R-V15, PRM‘18):

(Again omitting instanton links)

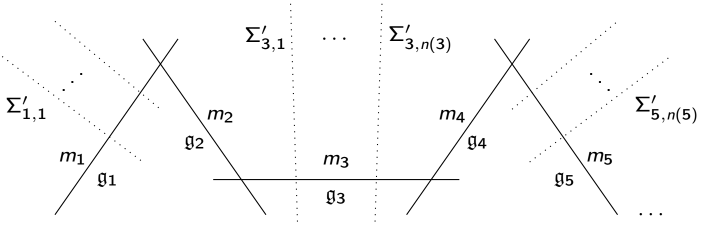
- Similar explicit listings for essentially **any base** (i.e. not only for links) are available **including with GS** (PRM ‘18).

(Caveats: only *sans* frozen divisors, for “geometrically realizable” GS)

SCFT global symmetries in the general case

- Our earlier methods extend to instead treat collections of non-compact curves on each Σ_i, say $\Sigma'_{i,1}, \ldots \Sigma'_{i,n(i)}$, also components of $\{\Delta = 0\}$ transversally intersecting Σ_i.

Which $\Sigma'_{i,j}$ collections give (relative) maxima for $\oplus_{i,j}[\mathfrak{g}_{i,j}]$?



- Taking an appropriate limit in which we contract all Σ_i, the direct sum of the non-abelian algebras carried on the curves $\Sigma'_{i,j}$ arises as a global symmetry algebra of an SCFT associated to our F-theory model.
- Our implementation calls on computer book-keeping since the number of cases is large, even for any single SCFT. Relevant CY3 constraints (perhaps nearly all) are derived and encoded.
- Results include: (1) that continuous degrees of freedom are lost in gauging geometrically realizable global symmetries; (2) concerns about constraint propagation through the curve tree can be more safely ignored; (3) searches for SCFTs without GS can be compiled.

6D SCFT classification from a birds eye view

In place of the above fine-scale ingredients of 6D SCFT classification structure, bases offer course-grained groupings of these theories.

- Each base B is an orbifold that we can treat as $\mathbb{C}^2/\Gamma$, where Γ is a discrete subgroup of $U(2)$.

- Not all discrete $U(2)$ subgroups appear

- Each discrete $U(2)$ subgroup falling in $SU(2)$ does; these were classified in the mathematics literature long ago.

- To determine which trees fall in isomorphic orbifolds, we can simply iteratively blow-down all -1 curves (e.g. $13151 \to 2151 \to 141 \to 31 \to 2$). Call these **endpoints**.

- Bases themselves fall into countable families (indexed by $n \in \mathbb{Z}_{\geq 0}$) determined by a rational function with integer coefficients A, B, C, D in the form $\frac{An+B}{Cn+D}$. Endpoints have been classified and an appropriate collection of rational functions is known.
  (Morrison, Taylor '12; Heckman, Morrison, Vafa '13; with outliers in families: Morrison, Vafa '16).

6D SCFT classification from a birds eye view

As in the classical mathematics literature on resolution of cyclic surface quotient singularities, we have the following.

- The action of Γ on $\mathbb{C}^2$ is given by

$$(s,t) \mapsto (e^{2\pi i/p} s, e^{2\pi i q/p} t)$$

  for p/q given by the Hirzebruch-Jung continued fraction from $\alpha \sim m_1 \cdots m_k$ as

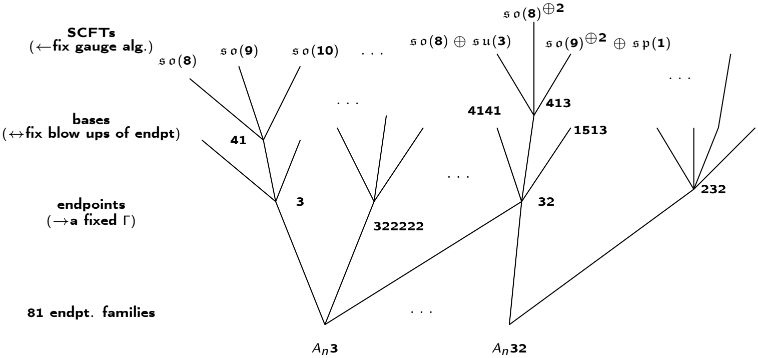
$$p/q = m_1 - \cfrac{1}{m_2 - \cfrac{1}{\ddots \, m_{k-2} - \cfrac{1}{m_{k-1} - \cfrac{1}{m_k}}}} \, .$$

  (A similar statement can be made for the few branching endpoint families; the action is generally not diagonal for these cases. The determining rationals nonetheless fall in the above families.)

- All (linear) bases appear (uniquely in cases with large enough n) from one of 78 rational functions with the above form (Morrison, Vafa '16) via a choice of integer $n \geq 0$.
- Note 1: gauge algebra ranks (strictly) grow with n. (I.e. large n and the Swampland...).
- Note 2: family structure descends in some form to 4D counterparts.

¿More structure among the base families?

- Cartoon of the picture so far:



- The 78 linear endpoint families are structured nicely. They are uniquely represented by the truncations of a certain base family

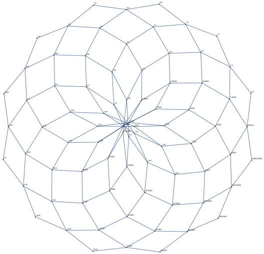
$$12231513221(12)12231513221\cdots(12)12231513221$$

(Heckman, Rudelius '19, PRM '19)

¿More structure among the base families?

How does this family structure "look"?

- This unique endpoint representative truncation structure yields a graph with automorphisms. (Define vertices as adjacent if endpoints have nearest unique base representative truncations.)
- A distinguished order 2 automorphism corresponds to a nontrivial $n \mapsto -n$ map on endpoint families (PRM '19). Moreover, automorphisms respect appropriate E_8 nilpotent orbit structure (in the sense of Heckman, Rudelius '19); explicit checks confirm each automorphism non-trivially respects gauge algebra SCFT structure within endpoint families in a "level" n preserving manner.



- Remaining puzzles include resolving the following. (1) Breakdowns in the typically one-to-one (structure preserving) gauge algebra correspondences paired with these automorphisms. (2) orbifold geometry underpinnings of the distinguished SCFT family pairing. (3) novel paired elementary combinatorics. (4) Small n endpoint family degeneracies.

¡El fin!

Thank you! Questions?

α \ β	32222	3222	4	33	322	3	5	42	32	6	7	$\emptyset$
22223	$\frac{36n+96}{30n+79}$	$\frac{30n+79}{25n+65}$	$\frac{18n+45}{15n+37}$	$\frac{30n+67}{25n+55}$	$\frac{24n+50}{20n+41}$	$\frac{12n+28}{10n+23}$	$\frac{24n+62}{20n+51}$	$\frac{30n+73}{25n+60}$	$\frac{18n+39}{15n+32}$	$\frac{30n+61}{25n+50}$	$\frac{36n+72}{30n+59}$	$\frac{6n+11}{5n+9}$
2223	$\frac{30n+79}{24n+62}$	$\frac{25n+65}{20n+51}$	$\frac{15n+37}{12n+29}$	$\frac{25n+55}{20n+43}$	$\frac{20n+41}{16n+32}$	$\frac{10n+23}{8n+18}$	$\frac{20n+51}{16n+40}$	$\frac{25n+60}{20n+47}$	$\frac{15n+32}{12n+25}$	$\frac{25n+50}{20n+39}$	$\frac{30n+59}{24n+46}$	$\frac{5n+9}{4n+7}$
4	$\frac{18n+45}{12n+28}$	$\frac{15n+37}{10n+23}$	$\frac{9n+21}{6n+13}$	$\frac{15n+31}{10n+19}$	$\frac{12n+23}{8n+14}$	$\frac{6n+13}{4n+8}$	$\frac{12n+29}{8n+18}$	$\frac{15n+34}{10n+21}$	$\frac{9n+18}{6n+11}$	$\frac{15n+28}{10n+17}$	$\frac{18n+33}{12n+20}$	$\frac{3n+5}{2n+3}$
33	$\frac{30n+67}{18n+39}$	$\frac{25n+55}{15n+32}$	$\frac{15n+31}{9n+18}$	$\frac{25n+45}{15n+26}$	$\frac{20n+33}{12n+19}$	$\frac{10n+19}{6n+11}$	$\frac{20n+43}{12n+25}$	$\frac{25n+50}{15n+29}$	$\frac{15n+26}{9n+15}$	$\frac{25n+40}{15n+23}$	$\frac{30n+47}{18n+27}$	$\frac{5n+7}{3n+4}$
223	$\frac{24n+50}{6n+11}$	$\frac{20n+41}{5n+9}$	$\frac{12n+23}{3n+5}$	$\frac{20n+33}{5n+7}$	$\frac{16n+24}{4n+5}$	$\frac{8n+14}{2n+3}$	$\frac{16n+32}{4n+7}$	$\frac{20n+37}{5n+8}$	$\frac{12n+19}{3n+4}$	$\frac{20n+29}{5n+6}$	$\frac{24n+34}{6n+7}$	$\frac{4n+5}{n+1}$
3	$\frac{12n+28}{6n+11}$	$\frac{10n+23}{5n+9}$	$\frac{6n+13}{3n+5}$	$\frac{10n+19}{5n+7}$	$\frac{8n+14}{4n+5}$	$\frac{4n+8}{2n+3}$	$\frac{8n+18}{4n+7}$	$\frac{10n+21}{5n+8}$	$\frac{6n+11}{3n+4}$	$\frac{10n+17}{5n+6}$	$\frac{12n+20}{6n+7}$	$\frac{2n+3}{n+1}$
5	$\frac{24n+62}{18n+45}$	$\frac{20n+51}{15n+37}$	$\frac{12n+29}{9n+21}$	$\frac{20n+43}{15n+31}$	$\frac{16n+32}{12n+23}$	$\frac{8n+18}{6n+13}$	$\frac{16n+40}{12n+29}$	$\frac{20n+47}{15n+34}$	$\frac{12n+25}{9n+18}$	$\frac{20n+39}{15n+28}$	$\frac{24n+46}{18n+33}$	$\frac{4n+7}{3n+5}$
24	$\frac{30n+73}{12n+28}$	$\frac{25n+60}{10n+23}$	$\frac{15n+34}{6n+13}$	$\frac{25n+50}{10n+19}$	$\frac{20n+37}{8n+14}$	$\frac{10n+21}{4n+8}$	$\frac{20n+47}{8n+18}$	$\frac{25n+55}{10n+21}$	$\frac{15n+29}{6n+11}$	$\frac{25n+45}{10n+17}$	$\frac{30n+53}{12n+20}$	$\frac{5n+8}{2n+3}$
23	$\frac{18n+39}{6n+11}$	$\frac{15n+32}{5n+9}$	$\frac{9n+18}{3n+5}$	$\frac{15n+26}{5n+7}$	$\frac{12n+19}{4n+5}$	$\frac{6n+11}{2n+3}$	$\frac{12n+25}{4n+7}$	$\frac{15n+29}{5n+8}$	$\frac{9n+15}{3n+4}$	$\frac{15n+23}{5n+6}$	$\frac{18n+27}{6n+7}$	$\frac{3n+4}{n+1}$
6	$\frac{30n+61}{6n+11}$	$\frac{25n+50}{5n+9}$	$\frac{15n+28}{3n+5}$	$\frac{25n+40}{5n+7}$	$\frac{20n+29}{4n+5}$	$\frac{10n+17}{2n+3}$	$\frac{20n+39}{4n+7}$	$\frac{25n+45}{5n+8}$	$\frac{15n+23}{3n+4}$	$\frac{25n+35}{5n+6}$	$\frac{30n+41}{6n+7}$	$\frac{5n+6}{n+1}$
7	$\frac{36n+72}{6n+11}$	$\frac{30n+59}{5n+9}$	$\frac{18n+33}{3n+5}$	$\frac{30n+47}{5n+7}$	$\frac{24n+34}{4n+5}$	$\frac{12n+20}{2n+3}$	$\frac{24n+46}{4n+7}$	$\frac{30n+53}{5n+8}$	$\frac{18n+27}{3n+4}$	$\frac{30n+41}{5n+6}$	$\frac{36n+48}{6n+7}$	$\frac{6n+7}{n+1}$
$\emptyset$	$\frac{6n+11}{6n+5}$	$\frac{5n+9}{5n+4}$	$\frac{3n+5}{3n+2}$	$\frac{5n+7}{5n+2}$	$\frac{4n+5}{4n+1}$	$\frac{2n+3}{2n+1}$	$\frac{4n+7}{4n+3}$	$\frac{5n+8}{5n+3}$	$\frac{3n+4}{3n+1}$	$\frac{5n+6}{5n+1}$	$\frac{6n+7}{6n+1}$	$\frac{n+1}{n}$

Continued fraction values $\frac{p}{q}$ for linear endpoints of the form $\alpha A_n \beta$.

Appendices II

α \ β	32222	7	6	42	33	3222	5	322	4	32	3	$\emptyset$
7	0	$-\frac{1}{3}$	$-\frac{11}{30}$	$\frac{13}{30}$	$\frac{7}{30}$	$\frac{1}{30}$	$-\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{6}$
22223	$\frac{1}{3}$	0	$-\frac{1}{30}$	$-\frac{7}{30}$	$-\frac{13}{30}$	$\frac{11}{30}$	$-\frac{1}{12}$	$\frac{5}{12}$	$-\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$
2223	$\frac{11}{30}$	$\frac{1}{30}$	0	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{1}{20}$	$\frac{9}{20}$	$-\frac{2}{15}$	$-\frac{7}{15}$	$-\frac{3}{10}$	$\frac{1}{5}$
33	$-\frac{13}{30}$	$\frac{7}{30}$	$\frac{1}{5}$	0	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{3}{20}$	$-\frac{7}{20}$	$\frac{1}{15}$	$-\frac{4}{15}$	$-\frac{1}{10}$	$\frac{2}{5}$
24	$-\frac{7}{30}$	$\frac{13}{30}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$-\frac{1}{5}$	$\frac{7}{20}$	$-\frac{3}{20}$	$\frac{4}{15}$	$-\frac{1}{15}$	$\frac{1}{10}$	$-\frac{2}{5}$
6	$-\frac{1}{30}$	$-\frac{11}{30}$	$-\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	0	$-\frac{9}{20}$	$\frac{1}{20}$	$\frac{7}{15}$	$\frac{2}{15}$	$\frac{3}{10}$	$-\frac{1}{5}$
223	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{20}$	$-\frac{3}{20}$	$-\frac{7}{20}$	$\frac{9}{20}$	0	$\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{5}{12}$	$-\frac{1}{4}$	$\frac{1}{4}$
5	$-\frac{1}{12}$	$-\frac{5}{12}$	$-\frac{9}{20}$	$\frac{7}{20}$	$\frac{3}{20}$	$-\frac{1}{20}$	$\frac{1}{2}$	0	$\frac{5}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$-\frac{1}{4}$
23	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{2}{15}$	$-\frac{1}{15}$	$-\frac{4}{15}$	$-\frac{7}{15}$	$\frac{1}{12}$	$-\frac{5}{12}$	0	$-\frac{1}{3}$	$-\frac{1}{6}$	$\frac{1}{3}$
4	$-\frac{1}{6}$	$\frac{1}{2}$	$\frac{7}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	$-\frac{2}{15}$	$\frac{5}{12}$	$-\frac{1}{12}$	$\frac{1}{3}$	0	$\frac{1}{6}$	$-\frac{1}{3}$
3	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{1}{10}$	$-\frac{1}{10}$	$-\frac{3}{10}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{6}$	$-\frac{1}{6}$	0	$\frac{1}{2}$
$\emptyset$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{5}$	$-\frac{2}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0

Singular values of p in terms of n reduced modulo 1. Lead ordering respecting T^* dual (as transposition) and $\det : \Gamma \to U(1)$ block diagonal groupings.

Appendices III

End-pair		# Enhancements
$\{22223\ A_{10}\ 3222,$	$6\ A_{10}\ 7\}$	$\{1,1\}$
$\{22223\ A_{10}\ 33,$	$24\ A_{10}\ 7\}$	$\{1,1\}$
$\{22223\ A_{10}\ 42,$	$33\ A_{10}\ 7\}$	$\{1,1\}$
$\{22223\ A_{10}\ 6,$	$2223\ A_{10}\ 7\}$	$\{1,1\}$
$\{22223\ A_{10}\ 322,$	$5\ A_{10}\ 7\}$	$\{2,1\}$
$\{22223\ A_{10}\ 5,$	$223\ A_{10}\ 7\}$	$\{1,2\}$
$\{22223\ A_{10}\ 4,$	$23\ A_{10}\ 7\}$	$\{3,3\}$
$\{22223\ A_{10}\ 32222,$	$7\ A_{10}\ 7\}$	$\{1,1\}$
$\{22223\ A_{10}\ 3,$	$3\ A_{10}\ 7\}$	$\{8,8\}$
$\{22223\ A_{10}\ 32,$	$4\ A_{10}\ 7\}$	$\{3,3\}$
$\{22223\ A_{10}\text{-}\ ,$	$\text{-}\ A_{10}\ 7\}$	$\{38,38\}$
$\{6\ A_{10}\ 33,$	$24\ A_{10}\ 3222\}$	$\{1,1\}$
$\{6\ A_{10}\ 42,$	$33\ A_{10}\ 3222\}$	$\{1,1\}$
$\{6\ A_{10}\ 6,$	$2223\ A_{10}\ 3222\}$	$\{1,1\}$
$\{6\ A_{10}\ 322,$	$5\ A_{10}\ 3222\}$	$\{2,1\}$
$\{6\ A_{10}\ 5,$	$223\ A_{10}\ 3222\}$	$\{1,2\}$
$\{6\ A_{10}\ 4,$	$23\ A_{10}\ 3222\}$	$\{3,3\}$
$\{6\ A_{10}\ 3,$	$3\ A_{10}\ 3222\}$	$\{8,8\}$
$\{6\ A_{10}\ 32,$	$4\ A_{10}\ 3222\}$	$\{3,3\}$
$\{6\ A_{10}\text{-}\ ,$	$\text{-}\ A_{10}\ 3222\}$	$\{38,38\}$
$\{24\ A_{10}\ 42,$	$33\ A_{10}\ 33\}$	$\{1,1\}$
$\{24\ A_{10}\ 322,$	$5\ A_{10}\ 33\}$	$\{2,1\}$
$\{24\ A_{10}\ 5,$	$223\ A_{10}\ 33\}$	$\{1,2\}$
$\{24\ A_{10}\ 4,$	$23\ A_{10}\ 33\}$	$\{3,3\}$
$\{24\ A_{10}\ 3,$	$3\ A_{10}\ 33\}$	$\{8,8\}$
$\{24\ A_{10}\ 32,$	$4\ A_{10}\ 33\}$	$\{3,3\}$
$\{24\ A_{10}\text{-}\ ,$	$\text{-}\ A_{10}\ 33\}$	$\{38,38\}$
⋮	⋮	⋮

Number of gauge algebras arising on linear quivers in T^* endpoint pairs at $n=10$..

Gauge Alg. $\mathfrak{g}_a$ on $\alpha \sim A_4 3 A_{10}$	Gauge Alg. $\mathfrak{g}_b$ on $T^*\alpha \sim A_{10}7$	Rank($\mathfrak{g}_a$)	Rank($\mathfrak{g}_b$)
$A_1^{16} \oplus e_8^7 \oplus f_4^8 \oplus g_2^{16}$	$A_1^{14} \oplus e_8^7 \oplus f_4^7 \oplus g_2^{14}$	136	126
$A_1^{18} \oplus e_8^8 \oplus f_4^9 \oplus g_2^{18}$	$A_1^{16} \oplus e_8^8 \oplus f_4^8 \oplus g_2^{16}$	154	144
$A_1^{20} \oplus B_3 \oplus e_7 \oplus e_8^8 \oplus f_4^9 \oplus g_2^{18}$	$A_1^{18} \oplus B_3 \oplus e_7 \oplus e_8^8 \oplus f_4^8 \oplus g_2^{16}$	166	156
$A_1^{19} \oplus e_8^9 \oplus f_4^{10} \oplus g_2^{20}$	$A_1^{17} \oplus e_8^9 \oplus f_4^9 \oplus g_2^{18}$	171	161
$A_1^{19} \oplus A_2 \oplus e_8^9 \oplus f_4^{10} \oplus g_2^{19}$	$A_1^{17} \oplus A_2 \oplus e_8^9 \oplus f_4^9 \oplus g_2^{17}$	171	161
$A_1^{19} \oplus A_2^2 \oplus e_6 \oplus e_8^9 \oplus f_4^{10} \oplus g_2^{19}$	$A_1^{17} \oplus A_2^2 \oplus e_6 \oplus e_8^9 \oplus f_4^9 \oplus g_2^{17}$	179	169
$A_1^{21} \oplus e_7 \oplus e_8^9 \oplus f_4^{10} \oplus g_2^{21}$	$A_1^{19} \oplus e_7 \oplus e_8^9 \oplus f_4^9 \oplus g_2^{19}$	182	172
$A_1^{21} \oplus B_3 \oplus e_7 \oplus e_8^9 \oplus f_4^{10} \oplus g_2^{20}$	$A_1^{19} \oplus B_3 \oplus e_7 \oplus e_8^9 \oplus f_4^9 \oplus g_2^{18}$	183	173
$A_1^{21} \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{21}$	$A_1^{19} \oplus e_8^{10} \oplus f_4^9 \oplus g_2^{19}$	183	173
$A_1^{21} \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{21}$	$A_1^{19} \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{19}$	187	177
$A_1^{21} \oplus D_4 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{21}$	$A_1^{19} \oplus D_4 \oplus e_8^{10} \oplus f_4^9 \oplus g_2^{19}$	187	177
$A_1^{21} \oplus D_4^2 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{21}$	$A_1^{19} \oplus D_4^2 \oplus e_8^{10} \oplus f_4^9 \oplus g_2^{19}$	191	181
$A_1^{21} \oplus e_8^{10} \oplus f_4^{12} \oplus g_2^{22}$	$A_1^{19} \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{20}$	193	183
$A_1^{21} \oplus A_2 \oplus e_8^{10} \oplus f_4^{12} \oplus g_2^{21}$	$A_1^{19} \oplus A_2 \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{19}$	193	183
$A_1^{21} \oplus A_2 \oplus e_6 \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{21}$	$A_1^{19} \oplus A_2 \oplus e_6 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{19}$	195	185
$A_1^{22} \oplus e_6 \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{20} \oplus e_6 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{20}$	196	186
$A_1^{22} \oplus e_7 \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{20} \oplus e_7 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{20}$	197	187

Gauge algebras and ranks for SCFTs in each endpoint of the $n = 10$ pair $A_4 3 A_{10} \leftrightarrow A_{10}7$ (Part I). Difference by an $A_1^2 \oplus \mathfrak{f}_4^2 \oplus \mathfrak{g}_2^2$ summand accounts the uniform rank 10 differences.

Gauge Alg. $\mathfrak{g}_a$ on $\alpha \sim A_4 3 A_{10}$	Gauge Alg. $\mathfrak{g}_b$ on $T^*\alpha \sim A_{10}7$	Rank($\mathfrak{g}_a$)	Rank($\mathfrak{g}_b$)
$A_1^{23} \oplus e_7 \oplus e_8^{10} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{21} \oplus e_7 \oplus e_8^{10} \oplus f_4^{10} \oplus g_2^{20}$	198	188
$A_1^{22} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{20} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	198	188
$A_1^{23} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{21} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	199	189
$A_1^{24} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{22} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	200	190
$A_1^{23} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{21} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	201	191
$A_1^{23} \oplus A_2 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{21} \oplus A_2 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	201	191
$A_1^{25} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{23} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	201	191
$A_1^{24} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{22} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	202	192
$A_1^{24} \oplus A_2 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{22} \oplus A_2 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	202	192
$A_1^{23} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{24}$	$A_1^{21} \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{22}$	203	193
$A_1^{23} \oplus A_2 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{21} \oplus A_2 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	203	193
$A_1^{23} \oplus A_2^2 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{21} \oplus A_2^2 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	203	193
$A_1^{23} \oplus B_3 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{21} \oplus B_3 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	204	194
$A_1^{23} \oplus A_2 \oplus A_3 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	$A_1^{21} \oplus A_2 \oplus A_3 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{20}$	204	194
$A_1^{23} \oplus e_8^{11} \oplus f_4^{12} \oplus g_2^{23}$	$A_1^{21} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{21}$	205	195
$A_1^{23} \oplus D_4 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{21} \oplus D_4 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	205	195
$A_1^{23} \oplus B_4 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{23}$	$A_1^{21} \oplus B_4 \oplus e_8^{11} \oplus f_4^{10} \oplus g_2^{21}$	205	195
$A_1^{24} \oplus e_8^{11} \oplus f_4^{12} \oplus g_2^{23}$	$A_1^{22} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{21}$	206	196
$A_1^{23} \oplus e_8^{11} \oplus f_4^{12} \oplus g_2^{24}$	$A_1^{21} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	207	197
$A_1^{23} \oplus A_2 \oplus e_8^{11} \oplus f_4^{12} \oplus g_2^{23}$	$A_1^{21} \oplus A_2 \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{21}$	207	197
$A_1^{24} \oplus e_8^{11} \oplus f_4^{12} \oplus g_2^{24}$	$A_1^{22} \oplus e_8^{11} \oplus f_4^{11} \oplus g_2^{22}$	208	198

Gauge algebras and ranks for SCFTs in each endpoint of the $n = 10$ pair $A_4 3 A_{10} \leftrightarrow A_{10}7$ (Part II). Difference by an $A_1^2 \oplus \mathfrak{f}_4^2 \oplus \mathfrak{g}_2^2$ summand accounts the uniform rank 10 differences.