Quantum Corrections for 4d N=1 Emergent Strings and the Weak Gravity Conjecture

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based on:
D. Kläwer, S.-J. Lee, T. Weigand, MW [2010.xxxxx]

UNIVERSIDAD AUTONOMA DE MADRID

5th IBS-IFT-MultiDark Workshop
October 13, 2020

"la Caixa" Foundation

Funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 713673

Introduction

Swampland Program: [Vafa '05]

Landscape	**Swampland**
EFT + quantum gravity = ✔	EFT + quantum gravity = ✗

Consistency conditions imposed by Quantum Gravity

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Swampland Conjectures:

- No Global Symmtery
  [Banks, Dixon '88]
- Completeness Hypothesis
  [Polchinski '03]
- Weak Gravity Conjecture
  [Arkani-Hamed, Motl, Nicolis, Vafa '06]
- Swampland Distance Conjecture
  [Ooguri, Vafa '06]
- dS/AdS conjecture
  [Palti, Shiu, Ooguri, Vafa '18, Lüst, Palti, Vafa'19]
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[review: Palti '19]

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Focus of this talk

- Swampland Distance Conjecture
  [Ooguri, Vafa '06]
- dS/AdS conjecture
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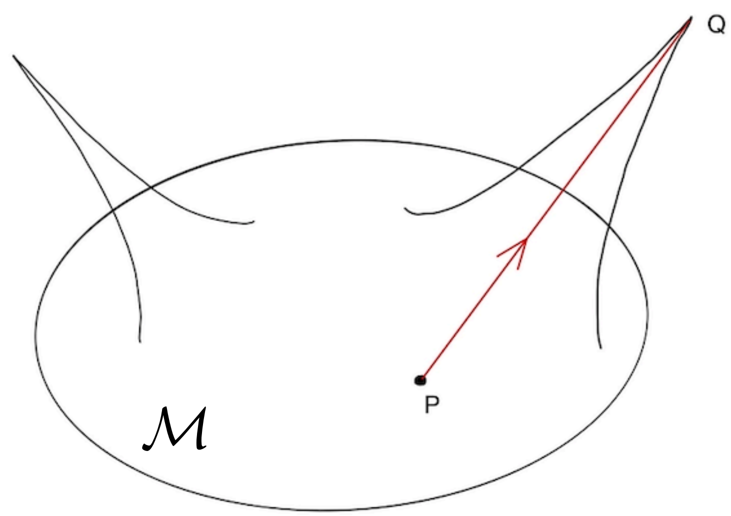
Swampland Distance Conjecture [Ooguri, Vafa '06]

At infinite distance in moduli space an infinite tower of states becomes exponentially massless in Planck units.



$$\frac{M(Q)}{M_{pl}} \sim e^{-Ad(P,Q)}$$

Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa '06]

Decay of extremal black holes $\Rightarrow$ Self-repulsive state:

$$|F_{\text{grav.}}| \sim \frac{m^2}{M_P^2} \lesssim (gq)^2 \sim |F_{\text{gauge}}| \,.$$

Introduction

TEST SDC AND WGC IN 4D $\mathcal{N}=1$.

- Setup: F-theory on CY 4-folds.

- Look at **infinite distance** limits in *classical* Kähler moduli space: "$t \sim \lambda \to \infty$" [cf. Lee, Lerche, Weigand '19]

- if $\exists$ gauge theory with $g \sim e^{-a\lambda}$: [Heidenreich, Reece, Rudelius '15-'18; Kläwer, Palti '16]
  $\Rightarrow$ WGC requires states with $m_n \lesssim q_n e^{-b\lambda} M_{pl}$

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$\Rightarrow$ WGC requires states with $m_n \lesssim q_n e^{-b\lambda} M_{pl}$
- What are these states?

KK-tower

$m_n^2 \sim n^2 M_{KK}^2$

(partial) **decompactification!**

String states

$m_n^2 \sim n M_S^2$

not necessarily decompactification:

[Lee, Lerche, Weigand '19]

Emergent String Proposal:

Theory reduces to weakly coupled ST and massless states are excitations of weakly-coupled critical *string.*

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Three Core Messages

[Kläwer, Lee, Weigand, MW (to appear)]

1. **Uniqueness:** For equi-dimensional infinite distance points the emergent tensionless and weakly-coupled string is *unique*.

2. **Quantum Consistency:** Leading perturbative corrections at $\mathcal{O}(\alpha'^2)$ ensure *consistency* of emergent string limits.

3. **WGC:** To leading order the WGC is *fulfilled* and *constraints* on the form of α' corrections can be deduced from heterotic/F-theory duality.

Setup

- F-theory on elliptic CY 4-fold: $E_\tau \to X_4 \to B_3$.
- Kähler moduli space spanned by $h^{1,1}(B_3)$ moduli v^α.

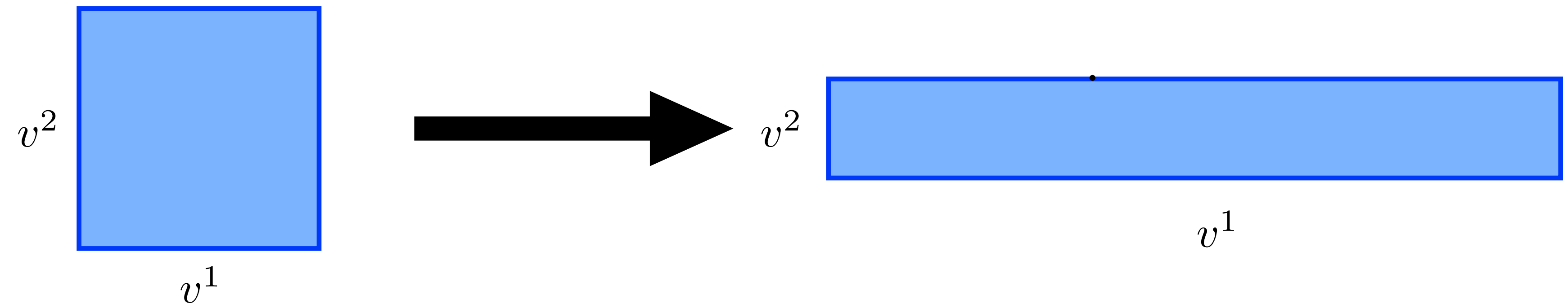
$$J = v^\alpha J_\alpha \qquad \Rightarrow \qquad \mathcal{V}_{B_3} = \frac{1}{6}\int_{B_3} J^3 = \frac{1}{6} k_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma \,.$$

- To avoid decompactification limit, cannot just take

$$v^\alpha \sim \mu \to \infty \qquad \forall \alpha \,.$$

- Have to take a non-trivial limit:

Intuition:



[Lee, Lerche, Weigand '19]

- possible if $J_0^3 = 0$ for some generator J_0.

Setup

[Lee, Lerche, Weigand '19]

Two possibilities

$J_0^3 = 0$ → $J_0^2 = 0$ J-class B

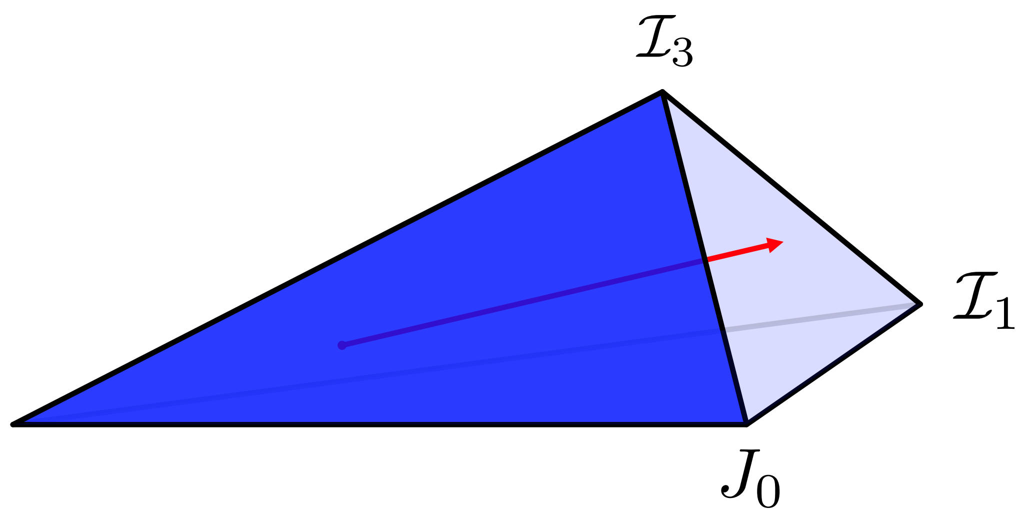
$J_0^3 = 0$ → $J_0^2 \neq 0$ J-class A

Focus here

$$J = \lambda J_0 + \sum_{\mu \in \mathcal{I}_1} \frac{1}{\lambda^2} J_\mu + \sum_{r \in \mathcal{I}_3} v^r J_r \,,$$

$$J_0^2 . J_\mu \neq 0 \qquad \forall \mu \in \mathcal{I}_1 \,,$$
$$J_0^2 . J_r = J_0 . J_s . J_r = 0 \qquad \forall r, s \in \mathcal{I}_3$$

→ $\mathcal{V}_{B_3}$ =const. for $\lambda \to \infty$



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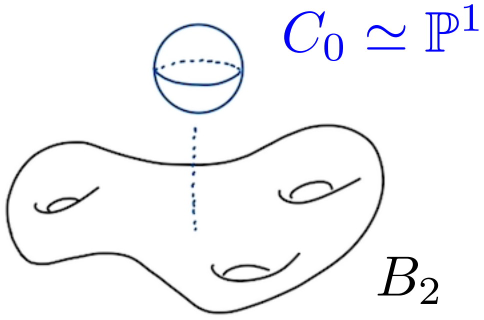
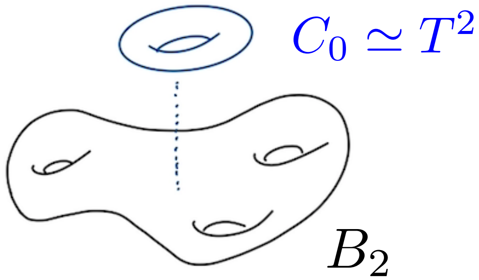
- The curve $C_0 = J_0.J_0$ shrinks in the limit: $\mathcal{V}_{C_0} \sim \frac{1}{\lambda^2}$
- B_3 is fibration $C_0 \to B_2$:

1



$\bar{K}_{B_3}.C_0 = 2$

2



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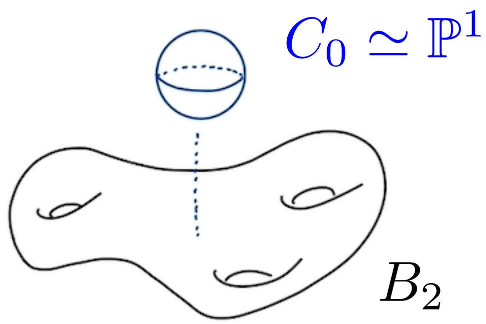
Emergent Strings

[Lee, Lerche, Weigand '19]



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- D3-brane wrapped on C_0 gives rise to *emergent heterotic string*:

$$\frac{M_{\rm het}^2}{M_S^2} \sim \lambda^{-2}$$

- Gauge theory on divisor $\mathcal{S}$ with $\mathcal{S}.C_0 \neq 0$ becomes weakly coupled:

$$\frac{1}{g_{YM}^2} = \mathcal{V}_{\mathcal{S}} \sim \lambda^2$$

- String excitation modes satisfy WGC!

$$M_k^2 \sim M_{\rm het}^2(n_k - 1)$$
$$\exists q_k\colon\; q_k^2 \geq 4mn_k$$

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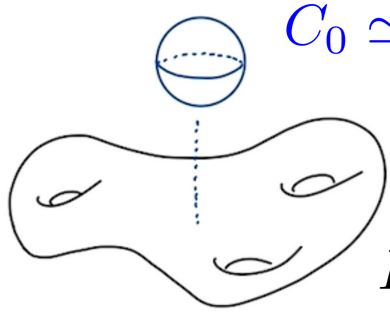

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So everythings fine?

Open Problems

1 Can there be multiple strings that become tensionless at the same rate?

$$B_3 = \underbrace{\mathbb{P}^1}_{\sim\lambda^{-1/2}} \times \underbrace{\mathbb{P}^1}_{\sim\lambda^{-1/2}} \times \underbrace{\mathbb{P}^1}_{\sim\lambda^{1}}$$

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2 Analysis so far was purely classical!

- $\mathcal{N} = 1$ moduli space is **not** classically exact.

- Quantum Corrections can obstruct classical infinite distance limits!

Already observed in $\mathcal{N} = 2$ setups:

1. Vector multiplet moduli space of IIA on CY_3.
   [Lee, Lerche, Weigand '19]
2. Hypermultiplet moduli space of IIB on CY_3.
   [Marchesano, MW '19]

- Shrinking of fibral curve can be problematic:

$$\mathcal{V}_{C_0} = \frac{M_{\text{het}}^2}{M_S^2} \precsim \frac{M_{KK}^2}{M_S^2} \,.$$

$\Rightarrow$ Critical 4D string theory?!

In type IIB hypermultiplet moduli space the quantum corrections precisely obstruct limits with classically intrinsically 4D strings! [Baume, Marchesano, MW '19]

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3 Does the classical WGC relation also receive quantum corrections?

Summary of Results

[Kläwer, Lee, Weigand, MW (to appear)]

1 **Uniqueness:** For any J-class A or J-class B limit one of the following holds:

1. B_3 has a *unique* $\mathbb{P}^1$-fibration and the fiber shrinks at the fastest rate in the limit (unique emergent heterotic string).

2. B_3 has a *unique* T^2-fibration and the fiber shrinks at the fastest rate in the limit. (unique emergent type II string)

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2 For T^2-fibrations:

- α'-corrections do **not** ostruct infinite distance limits with *finite* volume: $\to$ Classical analysis goes through!

- Reason: Winding modes along the torus fiber give the leading KK tower $\Rightarrow M^2_{IIB}/M^2_{KK} \sim \mathcal{O}(1)$: Emergent string limit!

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2 For $\mathbb{P}^1$-fibrations:

- α'-corrections obstruct infinite distance limits with *finite* volume:
  $\rightarrow$ Perturbative control lost at finite distance in Kähler moduli space
  $\Rightarrow$ No emergent string limit!

- Limit: $J = \tilde{\lambda} J_0 + \frac{1}{\tilde{\lambda}} J_{\mathcal{I}_1} + b J_{\mathcal{I}_3}$ *marginally* allowed with $M_{\text{het}}/M_{\text{KK}} \sim \mathcal{O}(1)$.
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3 The WGC relation (only applies to $\mathbb{P}^1$-fibrations):

- Including α'-corrections to F-theory geometry, the WGC-relation is still fulfilled at leading order in the weak coupling limit.

- $\exists$ sublattice of super-extremal states in all cases.

Quantum Corrections

[Grimm, Keitel, Mayer, Pugh, Savelli, Weissenbacher '13-'19]

- The classical 4D effective theory has $h^{(1,1)}(B_3)$ chiral multiplets with saxionic component

$$\mathrm{Re}\, T^0_\alpha = \frac{1}{2}\int_{D_\alpha} J\wedge J = \frac{1}{2}k_{\alpha\beta\gamma}v^\beta v^\gamma\,,$$

with Kähler potential and dual linear multiplets

$$K = -2\log \mathcal{V}^0_{B_3}\,,\qquad L_0^\alpha = -\frac{\partial K}{\partial \mathrm{Re}\, T_\alpha} = \frac{v^\alpha}{\mathcal{V}^0_{B_3}}\,.$$

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- Higer derivative contributions to the 11D M-theory supergravity effective action give α' corrections to these quantities:

$$\mathrm{Re}\, T_\alpha = \mathrm{Re}\, T^0_\alpha + \alpha^2\left((\kappa_3+\kappa_5)\frac{\mathrm{Re}\, T^0_\alpha \mathcal{Z}}{\mathcal{V}^0_{B_3}} + \kappa_5\frac{\mathrm{Re}\, T^0_\alpha\mathcal{T}}{\mathcal{V}^0_{B_3}} + \kappa_4\mathcal{Z}_\alpha\log\mathcal{V}^0_{B_3} + \kappa_6\mathcal{T}_\alpha + \kappa_7\mathcal{Z}_\alpha\right)\,,$$

$$e^{K/2} = \mathcal{V}^0_{B_3} + \alpha^2\left((\tilde\kappa_1+\tilde\kappa_2)\mathcal{Z} + \tilde\kappa_2\mathcal{T}\right)\,.$$

$$\mathcal{Z}_\alpha = \int_{Y_4} c_3(Y_4)\wedge\pi^*(J_\alpha)\,,$$

$$\mathcal{T}_\alpha = -18(1+\alpha_2)\frac{1}{\mathrm{Re}\, T^{\mathrm{cl.}}_\alpha}\int_{D_\alpha} c_1(B_3)\wedge J\int_{D_\alpha} J_\alpha\wedge J\,,$$

$$\mathcal{T} = \mathcal{T}_\alpha v^\alpha\,,\qquad \mathcal{Z} = \mathcal{Z}_\alpha v^\alpha\,,$$

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[Kläwer, Lee, Weigand, MW (to appear)]

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$B_3 : \mathbb{P}^1 \to B_2$:

- In generic models: $\mathcal{Z}_0 \neq 0$ but precise value model dependent.
- $\bar{K}.C_0 = 2 \Rightarrow \mathcal{T}_0 = -36(1 + \alpha_2)$ independently of chosen model.

$\Rightarrow$ Classical limit leads out of perturbative F-theory regime!

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[Kläwer, Lee, Weigand, MW (to appear)]

$\bar{K}.C_0 \neq 0$ vs. $\bar{K}.C_0 = 0$ a significantly different $\leftrightarrow$ different emergent strings

- D3-brane on T^2 gives fundamental Type II string:
  - emergent string limit is protected from quantum corrections $\leftrightarrow$ has enhanced $\mathcal{N} = (2,2)$ worldsheet supersymmetry.
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- D3-brane on $\mathbb{P}^1$ gives fundamental Heterotc string:
  - classical emergent string limit obstructed.
  - quantum corrections in particular affect the gauge coupling for gauge theory along divisor $\mathcal{S}$:

$$\frac{1}{g_{YM}^2} = \underbrace{\mathrm{Re}\, T_{\mathcal{S}}^0}_{=g_{YM,0}^{-2}} \left(1 + \alpha^2 \underbrace{\frac{\mathcal{Z} + \mathcal{T}}{\mathcal{V}_{B_3}^0}}_{\gg 1} \right)$$

    $\rightarrow$ Enter a non-perturbative phase also for the gauge theory: no weak-coupling limit anymore!
  - Following this trajectory stability of heterotic string cannot be ensured!

Quantum Obstructions — Consequences

[Kläwer, Lee, Weigand, MW (to appear)]

To keep weakly-coupled, tensionless heterotic string limit have to rescale:

$$J \to J' = \mu J$$

- For $\mu = \lambda^{1/2}$ have $M_{\text{het}}/M_{KK} \sim \mathcal{O}(1)$ which is *marginally* allowed by the quantum corrections (only numerical suppression).

- No further obstruction due to non-perturbative superpotential.

Question: Is the WGC relation affected by the quantum corrections?

WGC: Quantum Corrections

[Kläwer, Lee, Weigand, MW (to appear)]

- The geometric part $\mathcal{V}_{C_0} \cdot \mathcal{V}_{\mathcal{S}} = \mathcal{V}_{B_3}$ of the WGC relation receives α'-corrections:

$$\mathcal{V}_{C_0} = \mathcal{V}_{C_0}^{(0)} \left[1 + \alpha^2 \left((\tilde{\kappa}_1 + \tilde{\kappa}_2 - \kappa_3 - \kappa_5) \frac{\mathcal{Z}}{\mathcal{V}_{B_3}^{(0)}} + (\tilde{\kappa}_2 - \kappa_5) \frac{\mathcal{T}}{\mathcal{V}_{B_3}^{(0)}} - \delta_1 \right)\right] ,$$

$$\mathcal{V}_{\mathcal{S}} = \mathcal{V}_{\mathcal{S}}^{(0)} \left[1 + \alpha^2 \left((\kappa_3 + \kappa_5) \frac{\mathcal{Z}}{\mathcal{V}_{B_3}^{(0)}} + \kappa_5 \frac{\mathcal{T}}{\mathcal{V}_{B_3}^{(0)}} + \delta_2 \right)\right] ,$$

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- **Expectation:** To leading order in λ the WGC relation should still hold.

- **Problem:** δ_2 is suppressed but δ_1 is not surpressed for $\lambda \to \infty$,

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- **Expectation:** To leading order in λ the WGC relation should still hold.

- **Problem:** δ_2 is suppressed but δ_1 is not surpressed for $\lambda \to \infty$.
  - **Solution:** classical F-theory relation is equivalent to the expression for heterotic gauge kinetic function at tree-level:

$$\mathcal{V}_{\mathcal{S}} = \frac{\mathcal{V}_{B_3}}{\mathcal{V}_{C_0}} \qquad \leftrightarrow \qquad f_{\text{het.}} = S \qquad \text{(the heterotic 4d dilaton)}$$

  - The latter relation only receives corrections at 1-loop!
  - δ_1 cannot be understood as 1-loop correction $\Rightarrow$ $\delta_1 = 0$ by het./F-theory duality!

WGC: Quantum Corrections

[Kläwer, Lee, Weigand, MW (to appear)]

Is $\delta_1 = 0$ possible?

- Higher derivative contributions to M-theory on CY 4-fold yield $\delta_1 \neq 0$.

- But term δ_1 term can be seen as a 1-loop correction when comapctifying the 4d F-theory effective action on an additional S^1. [Grimm, Mayer, Weissenbacher '17, '19]

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Heterotic/F-theory duality: $\delta_1 = 0$

→ WGC fulfilled to leading order in λ!

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What about higher orders?

- F-theory α'-corrections induce $\mathcal{O}(g_{YM}^2)$ corrections to WGC relation.

- Also should take corrections to the mass of the string excitations into account.

Summary

- In equi-dimensional infinite distance limits in the Kähler moduli space of F-theory on CY_4, there is always a **unique** emergent string.

- F-theory α'-corrections ensure that $M_{KK}^2 \precsim M_{\text{string}}^2$ is always fulfilled.

- *finite* volume limits for rationally fibered B_3 are out of perturbative control but allowed for elliptically fibered B_3.

- WGC relation still holds in the presence of α'-corrections.

Future Directions:

- Include corrections to mass of string excitations to compare with BH extremality bound!

- Better understand the regime away from the weak-coupling limit!

Thank you!