MODULI STABILISATION AND THE HOLOGRAPHIC SWAMPLAND

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(based on JC, Quevedo 1811.06276,
JC, Revello 2006.01021)

MODULI STABILISATION

- String theory

EFT

- 10-dimensional supergravity with alpha' corrections

EFT

- 4-dimensional supergravity of moduli and matter

EFT

- Integrate out heavy modes to get potential for lightest moduli

EFT

- Find vacuum as minimum of effective potential

LARGE VOLUME SCENARIO

Balasubramanian, Berglund, JC, Quevedo

- Perturbative corrections to K and non-perturbative corrections to W

$$W = \int G_3 \wedge \Omega + \sum_i A_i e^{-2\pi a_i T_i}$$

$$K = -2\ln\left(\mathcal{V} + \xi'\right) + \ln\left(\int \Omega \wedge \bar{\Omega}\right) - \ln(S + \bar{S})$$

- Resulting scalar potential has minimum at *exponentially large* values of the volume

$$V = \frac{A\sqrt{\tau_s}\, e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{B\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{C}{\mathcal{V}^3}$$

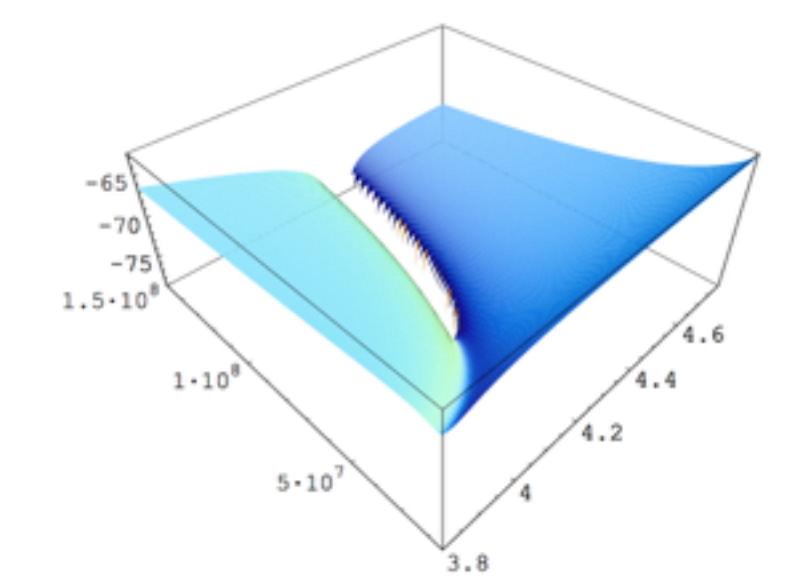


Figure 1: $\ln(V)$ for $P^4_{[1,1,1,6,9]}$ in the large volume limit, as a function of the divisors τ_4 and τ_5. The void channel corresponds to the region where V becomes negative and $\ln(V)$ undefined. As $V \to 0$ at infinite volume, this immediately

WHY LVS?

- In LVS, volume is exponentially large - can easily be

$$\mathcal{V} \sim 10^{50}(2\pi\sqrt{\alpha'})^6$$

- This *generates interesting hierarchies* and ensures *superb parametric decoupling* of heavy modes (KK modes, heavy moduli)

- Decoupling also has a clear geometric origin - large volume

$$\langle \mathcal{V} \rangle \sim e^{\xi/g_s}$$

- $\mathcal{V} \to \infty$ limit of LVS also leads to a unique effective theory

LVS HOLOGRAPHY

- LVS effective theory for volume modulus $\mathbf{\Phi}$ and axion a

$$\left(\lambda = \sqrt{27/2}\right)$$

$$V_{potential} = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right)$$

$$\mathcal{L}_{kinetic} = \frac{1}{2}\partial_\mu\Phi\partial_\mu\Phi + \frac{3}{4}e^{-\sqrt{\frac{8}{3}}\Phi}\partial_\mu a\partial^\mu a$$

- Other terms are subleading in infinite volume limit by $\mathcal{O}\left(\frac{1}{\ln\mathcal{V}}\right)$

LVS HOLOGRAPHY

- LVS effective theory for volume modulus Φ and axion a

$$V_{potential} = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \left(\lambda = \sqrt{27/2}\right)$$

$$\mathcal{L}_{kinetic} = \frac{1}{2}\partial_\mu \Phi \partial_\mu \Phi + \frac{3}{4} e^{-\sqrt{\frac{8}{3}}\Phi} \partial_\mu a \partial^\mu a$$

- Solve for minimum and expand about it to determine masses and couplings

HOLOGRAPHY

- CFT dimensions of dual operators:

$$\Delta(\Delta - 3) = m_\Phi{}^2 R_{AdS}^2$$

- In infinite volume limit can classify modes as

heavy $m_\Phi^2 \gg R_{AdS}^{-2}, \Delta \to \infty \quad \text{as} \quad \mathcal{V} \to \infty$

light $m_\Phi^2 \ll R_{AdS}^{-2}, \Delta \to 3 \quad \text{as} \quad \mathcal{V} \to \infty$

interesting $m_\Phi^2 \sim R_{AdS}^{-2}, \Delta \to \mathcal{O}(1-10) \quad \text{as} \, \mathcal{V} \to \infty$

LVS MASS SPECTRUM

- In LVS we have
- **Heavy:** KK modes, complex structure moduli, all Kahler moduli except overall volume
- **Light:** Graviton, overall volume axion
- **Interesting:** overall volume modulus

LVS HOLOGRAPHY

Mode	Spin	Parity	Conformal dimension
$T_{\mu\nu}$	2	+	3
a	0	-	3
Φ	0	+	$8.038 = \frac{3}{2}\left(1+\sqrt{19}\right)$

Table 1. The low-lying single-trace operator dimensions for CFT duals of the Large Volume Scenario in the limit $\mathcal{V} \to \infty$.

In minimal LVS, AdS effective theory has small number of fields which correspond to specific predictions for dual conformal dimensions

No Landscape! (not true of KKLT)

LVS HOLOGRAPHY

- LVS is attractive as it offers a well-motivated Generalised Free Field Theory
- Large volume limit $\mathcal{V} \to \infty$ gives a unique theory
- Two scalars with fixed and radiatively stable anomalous dimensions
- All AdS interactions are also fixed and radiatively stable

LVS AND THE SWAMPLAND

- LVS effective Lagrangian is

$$V_{potential} = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \left(\lambda = \sqrt{27/2}\right)$$

$$\mathcal{L}_{kinetic} = \frac{1}{2}\partial_\mu \Phi \partial_\mu \Phi + \frac{3}{4} e^{-\sqrt{\frac{8}{3}}\Phi/M_P} \partial_\mu a \partial^\mu a$$

- This has the expected behaviour that

$$f_a / M_P \to 0 \quad \text{as} \quad \mathcal{V} \to \infty$$

LVS AND THE SWAMPLAND

- Now consider this small modification:

$$V_{potential} = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \left(\lambda = \sqrt{27/2}\right)$$

$$\mathcal{L}_{kinetic} = \frac{1}{2}\partial_\mu\Phi\partial_\mu\Phi + \frac{3}{4}e^{+\sqrt{\frac{8}{3}}\Phi/M_P}\partial_\mu a\partial^\mu a$$

- This coupling is equivalent to axion decay constants that *diverge* in the decompactification limit - must be in the swampland!

$$f_a / M_P \to \infty \quad \text{as} \quad \mathcal{V} \to \infty$$

HOLOGRAPHIC SWAMPLAND

- n-point self interactions of volume modulus

$$\mathcal{L}_{n-pt} = (-1)^{n-1}\lambda^n (n-1)\left(-3\frac{M_P^2}{R_{AdS}^2}\right)\frac{1}{n!}\left(\frac{\delta\Phi}{M_P}\right)^n\left(1+\mathcal{O}\left(\frac{1}{\ln\mathcal{V}}\right)\right) \quad \left(\lambda=\sqrt{27/2}\right)$$

- Mixed interactions of volume modulus and axion

$$\mathcal{L}_{\Phi^n aa} = \left(-\sqrt{\frac{8}{3}}\right)^n \frac{1}{2n!}\left(\frac{\delta\Phi}{M_P}\right)^n \partial_\mu a \partial^\mu a$$

- The higher-point interaction define 3- and higher point-correlators within a dual CFT

HOLOGRAPHIC SWAMPLAND

- n-point self interactions of volume modulus

$$\mathcal{L}_{n-pt} = (-1)^{n-1} \lambda^n (n-1)\left(-3\frac{M_P^2}{R_{AdS}^2}\right)\frac{1}{n!}\left(\frac{\delta\Phi}{M_P}\right)^n \left(1+\mathcal{O}\left(\frac{1}{\ln\mathcal{V}}\right)\right)$$

- Now modify interactions of volume modulus and axion $\left(\lambda = \sqrt{27/2}\right)$

$$\mathcal{L}_{\Phi^n aa} = \left(+\sqrt{\frac{8}{3}}\right)^n \frac{1}{2n!}\left(\frac{\delta\Phi}{M_P}\right)^n \partial_\mu a \partial^\mu a$$

- This defines a perturbation to the Generalised Free Field CFT with axion decay constants that *diverge* in the decompactification limit - must be in the swampland!

$$f_a / M_P \to \infty \quad \text{as} \quad \mathcal{V} \to \infty$$

HOLOGRAPHIC SWAMPLAND

- The problem:

1. Generalised Free Field + (some corrections)

   - **consistent theory**

2. Generalised Free Field + (other corrections)

   - **swampland!**

Where does the difference lie? Can one correlate any properties of the CFT with this change from the consistent theory to the swampland theory?

HOLOGRAPHIC SWAMPLAND

- 3-pt interactions in AdS theory relate to 3-pt structure functions in CFT
- Signs of 3-pt functions are not determinate - can change by field redefinitions
- Focus on 3-pt interactions only

HOLOGRAPHIC SWAMPLAND

$$\mathcal{L} \supset \frac{g}{M_P R_{AdS}^2}\varphi^3 - \frac{\mu}{M_P}\varphi\,\partial_\mu a \partial^\mu a$$

- gives rise to CFT structure constants

$$f_{\varphi\varphi}^{\varphi} = \frac{3g\pi^{d/2}\Gamma\left(\frac{3\Delta_\varphi - d}{2}\right)}{\Gamma(\Delta_\varphi)^3}.$$

$$f_{aa}^{\varphi} = \frac{\mu\pi^{d/2}\Gamma\left(\frac{2\Delta_a + \Delta_\varphi - d}{2}\right)}{\Gamma(\Delta_\varphi)\Gamma(\Delta_a)^2}(\Delta_\varphi - 6),$$

HOLOGRAPHIC SWAMPLAND

- Anomalous dimensions are well-defined; can be related to Mellin amplitude for 2 -> 2 scattering

$$\gamma(0,\ell) = -\int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} M(s,0)\ {}_3F_2(-\ell, \Delta_1+\Delta_3+\ell-1, \frac{s}{2}; \Delta_1, \Delta_3; 1)$$
$$\times \Gamma\Big(\Delta_1 - \frac{s}{2}\Big)\Gamma\Big(\Delta_3 - \frac{s}{2}\Big)\Gamma\Big(\frac{s}{2}\Big)^2,$$

where $M(s,t)$ is the Mellin amplitude corresponding to the correlator

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\mathcal{O}_3(x_3)\mathcal{O}_3(x_4)\rangle.$$

HOLOGRAPHIC SWAMPLAND

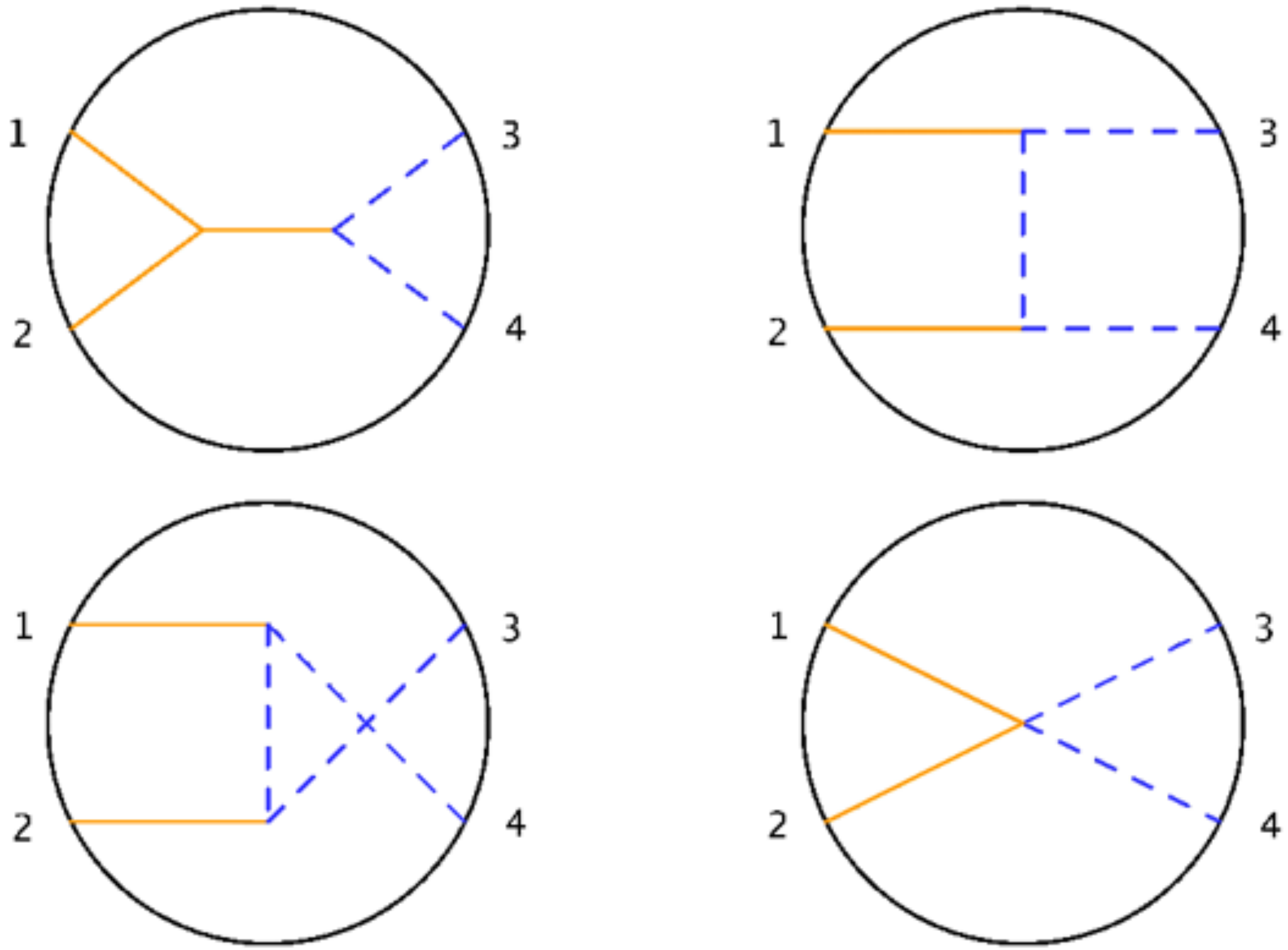


Figure 1: Witten diagrams contributing to the anomalous dimensions of double trace operators of the form $[\mathcal{O}_{\varphi_1}\mathcal{O}_{\varphi_2}]_{(n,\ell)}$. The volume modulus corresponds to the continuous orange lines, and the axion to the blue dashed ones. The last diagram, without internal propagators, only contributes to anomalous dimension for small ℓ.

HOLOGRAPHIC SWAMPLAND

$$\gamma^{\varphi a}(0,\ell) = -2 f_{\varphi\varphi\varphi} f_{\varphi aa} \frac{\Gamma(\Delta_a)\Gamma(\Delta_\varphi)^2}{\Gamma\left(\frac{2\Delta_a-\Delta_\varphi}{2}\right)\Gamma\left(\frac{\Delta_\varphi}{2}\right)^3} \frac{1}{\ell^{\Delta_\varphi}} + \mathcal{O}\left(\frac{1}{\ell}\right)$$

Anomalous dimensions are equivalent to binding energies of 2-particle states in AdS

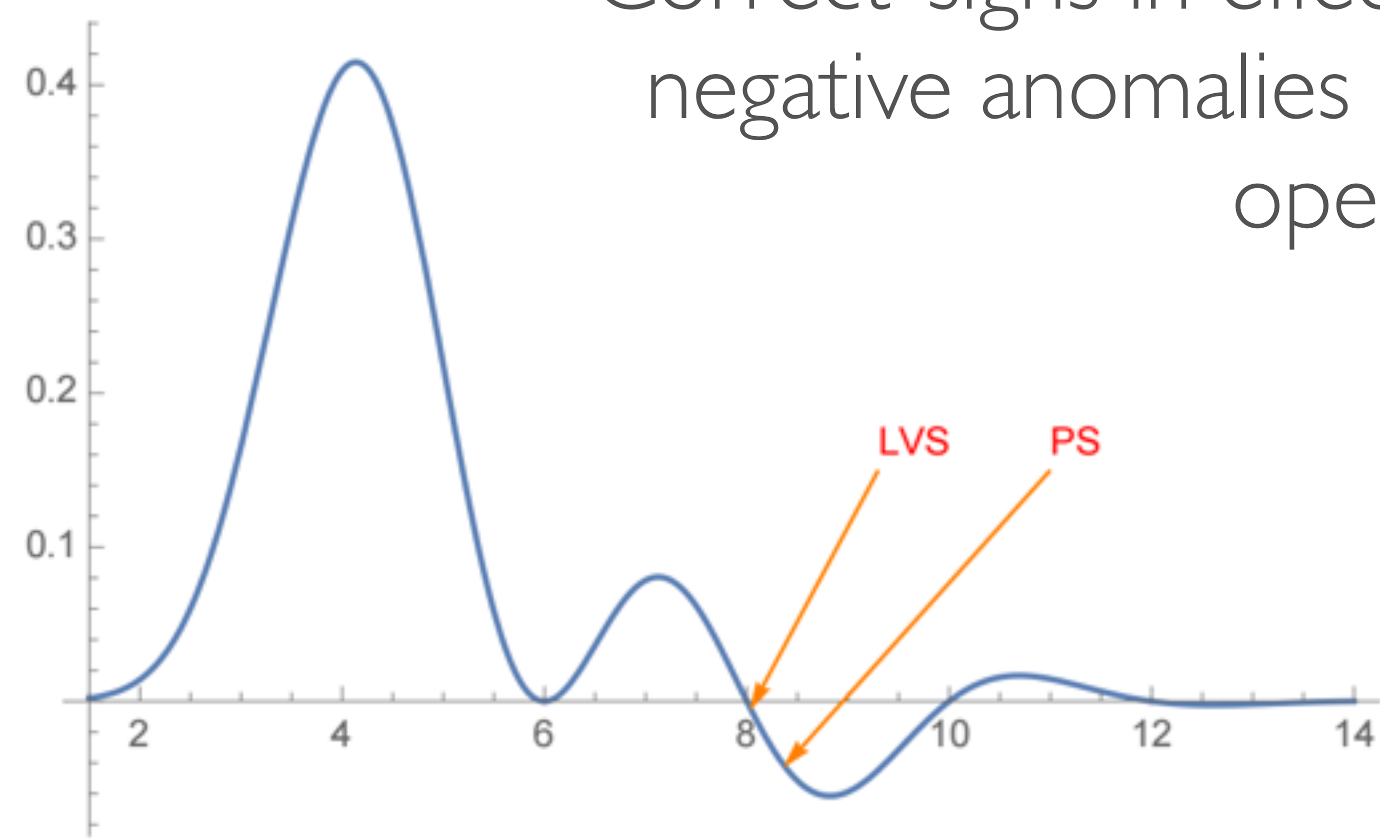
In Mellin amplitude, 'exchange' t-channel diagrams provide dominant contribution at large l

HOLOGRAPHIC SWAMPLAND

LVS 'just' gives a negative anomalous dimension for the mixed volume-axion state

$$\gamma^{\varphi a}(0,\ell) \sim -g\mu \frac{(\Delta_\varphi - 6)}{\Gamma(\frac{6-\Delta_\varphi}{2})}.$$

'Correct' signs in effective AdS equivalent to negative anomalies dimension for mixed operator



HOLOGRAPHIC SWAMPLAND

- In LVS context, right signs of 3-pt AdS couplings are equivalent to negative anomalous dimensions for the mixed double-trace operator.

- A similar result holds for perturbative or KKLT stabilisation (qualitatively different as involves a massive axion)

CONNECTION TO REFINED DISTANCE CONJECTURE

$$\frac{1}{2}e^{-\sqrt{\frac{1}{6}}(\varphi-\varphi_0)/M_P}\partial_\mu\psi\partial^\mu\psi - m_\psi^2 e^{-\frac{5}{\sqrt{6}}(\varphi-\varphi_0)/M_P}\frac{\psi^2}{2}.$$

KK modes have to couple to light volume modulus in a way that their mass decreases with increasing volume

This fixes the sign of the 3-pt function, again in a way that results in a negative anomalous dimension for the mixed double trace operator

CONCLUSIONS

- For many examples, negative CFT anomalous dimensions appear to correspond to the correct signs in the AdS Lagrangian
- However:

  1. Negativity of anomalous dimensions does not seem to hold for fibred LVS with extra light fibre moduli

  2. Axions couple with different signs to the different fibre moduli, resulting in a mixture of signs

- Are earlier results just a feature of the volume modulus? In progress…..