Heterotic Complex Structure Moduli Stabilization via Spectral Data

Mohsen Karkheiran

IBS-CTPU
IBS-IFT MultiDark Workshop

Based on a paper with Wei Cui to appear soon

October 13, 2020

Introduction

- In compactification models, besides the gauge/matter fields in the effective theory, one gets (thousands) of neutral mass-less scalars called Moduli Particles.
- These moduli particles correspond to the moduli space of the compactification model.
- We need a way to get rid of these moduli particles, and this can be done by "moduli stabilization" which is mechanism that produces a potential for these scalar particles.

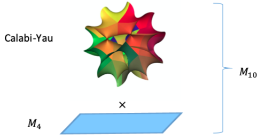
Introduction

- Complex structure moduli stabilization in the heterotic string (at least partially) can be achieved by a holomorphicity condition of the background gauge fields. Anderson, Gray, Lukas, Ovrut 2010
- Mathematically, this is parameterized by Atiyah class. Atiyah 1957
- In practice the models that are used to stabilize the complex structure moduli are limited. Our goal was to find more general models and consider their "image" in a dual F-theory model.

Heterotic String Theory

- In the low energy limit of Heterotic string theory we have a 10d $\mathcal{N} = 1$ supergravity coupled to the $Spin(32)/\mathbb{Z}_2$ or $E_8 \times E_8$ gauge fields.
- We consider solutions that spacetime factors as,

$$M_{10} = M_{int} \times M_4$$



- The field content is,

$$\begin{aligned} &\text{Gravity Multiplet:} && g_{\mu\nu}, B_{\mu\nu}, \phi, \psi_\mu, \lambda \\ &\text{Gauge Multiplet:} && A_\mu, \chi \end{aligned}$$

- We are looking for a solution that, after compactification ($M_{10} = M_4 \times M_{int}$), creates $\mathcal{N} = 1$ SUSY in 4d. For phenomenological reasons, it is preferred to work with the $E_8 \times E_8$ heterotic string theory. (M_4 is Minkowski)

Heterotic String Theory

- In particular if $H = 0$, $d\phi = 0$, $(H = dB + \omega_L - \omega_y)$, we need to have a covariantly constant spinor on the 6d (or 4d) internal manifold,

$$D\eta = 0$$

This condition requires that M_{int} to be a complex Kaehler manifold with SU(3) holonomy. That is the Calabi-Yau manifold (Ricci Flat) which has $c_1(M_{int}) = 0$ (according to Yau's theorem). Candelas, Horowitz, Strominger,Witten, 1985

- In addition, there is a gauge field (Fiber Bundle) over M_{int} with structure group G. The gauge group which will be visible in 4d is the commutant of G in E_8. Again by the $\mathcal{N} = 1$ SUSY in 4d :

$$F_{ab} = F_{\bar{a}\bar{b}} = 0, \quad g^{a\bar{b}} F_{a\bar{b}} = 0.$$

It's a holomorphic, Hermitian Yang-Mills vector bundle.

- The first two equation is satisfied by holomorphicity of the corresponding bundle, the third one is guaranteed by the stability condition,

$$\mathcal{F} \subset V \to \mu(\mathcal{F}) < \mu(V),$$

$$\mu(V) := \frac{\int c_1(V) \wedge \omega^{d-1}}{Rank(V)}.$$

Donaldson-Uhlenbeck,Yau

Spectral Covers

Let $\pi : X \longrightarrow B$ be a Weierstrass elliptic fibration, and V be a holomorphic, stable, degree zero, locally free sheaf (vector bundle) on X,

- Formally, one defines a functor,

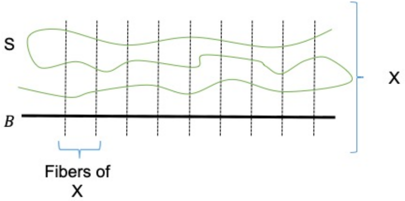
$$\Phi : D^b(X) \longrightarrow D^b(X),$$
$$\Phi(V) = R\pi_{2*}(\pi_1^* V \otimes \mathcal{P}),$$
$$\pi_{1,2} : X \times_B X \longrightarrow X, \qquad \mathcal{P} = \mathcal{I}_\Delta \otimes \pi_1^* \mathcal{O}_X(\sigma) \otimes \pi_2^* \mathcal{O}_X(\sigma) \otimes K_B^*.$$

- "Elements" of $D^b(X)$ are bounded complexes,

$$0 \to \mathcal{C}^i \to \mathcal{C}^{i+1} \to \cdots \to \mathcal{C}^{i+k} \to 0.$$

- Since V is relatively (semi)stable, degree zero,

$$\Phi(V) = i_{S*}\mathcal{N}[-1],$$
$$V \leftrightarrow (S, \mathcal{N}).$$



Anderson, Gao, MK, 2019

Friedman, Morgan, Witten 1997

Andreas, Curio, Ruiperez, Yau, 2000

Complex Structure Moduli Stabilization

$$\text{Moduli} \begin{cases} J_a^b \to J_a^b + \delta J_{\bar{a}}^b, & H^1(TX), \\ A_\mu^a \to A_\mu^a + \delta A_\mu^a, & H^1(V \otimes V^*), \\ J_{a\bar{b}} \to J_{a\bar{b}} + \delta J_{a\bar{b}} & H^{1,1}(X) \end{cases}$$

The complex structure moduli must satisfy the following equation:

$$F_{ab} = F_{\bar{a}\bar{b}} = 0 \Rightarrow \delta J_{[\bar{a}}^c F_{|c|\bar{b}]}^{(0)} = D_{[\bar{a}}^{(0)} \delta A_{\bar{b}]}$$

Atiyah 1957

Anderson, Gray, Lukas, Ovrut 2010

This can be derived by simultaneous deformation of the F-term $F_{ab} = \mathcal{P}_a^\mu \mathcal{P}_b^\nu F_{\mu\nu} = 0$ or the GVW superpotential $W = \int \Omega \wedge \omega_3(A)$.

- Mathematically such deformations are elements of the Atiyah class $H^1(X, Q)$,

$$0 \to V \otimes V^* \to Q \to TX \to 0.$$

Rank 2 Example

Consider a bundle defined by the extension:

$$0 \to \mathcal{L} \to V \to \mathcal{L}^* \to 0.$$

- Choose Kahler structure s.t. split bundle is impossible by the D-term ($g^{a\bar{b}}F_{a\bar{b}} = 0$).
- Then deforming the complex structure s.t. forces $Ext^1(\mathcal{L}^*, \mathcal{L}) = 0$, V becomes non-holomorphic.
- Since $Ext^1(\mathcal{L}^*, \mathcal{L}) = H^1(X, \mathcal{L}^2)$ and $H^1(X, \mathcal{L}^{*2})$ are subspaces of $H^1(V \otimes V^*)$, their elements appear in the GVW,

$$\begin{aligned} W = \int_X \Omega \wedge \omega(A) \sim \lambda_{i,j}(J) C_+^i C_-^j, \\ C_+ \in H^1(X, \mathcal{L}^{*2}), \qquad C_- \in H^1(X, \mathcal{L}^2), \\ V_F \sim |\lambda_{i,j}(J) \left\langle C_-^j \right\rangle|^2. \end{aligned} \tag{1}$$

Anderson, Gray, Lukas, Ovrut 2011

Rank 2 Example from Spectral Data and Generalization

Consider a Weierstrass $\pi : X \to B_2$, and a vector bundle defined as

$$0 \to \mathcal{O}_X(-\sigma + D_b) \to V \to \mathcal{O}_X(\sigma - D_b) \to 0.$$

$$\begin{array}{ccccc} Ext^1(\mathcal{L}^*, \mathcal{L}) & = & H^0(B, \mathcal{O}(2D_b + K_B)) & \oplus & H^0(B, \mathcal{O}(2D_b - K_B)) \\ & & \downarrow & & \downarrow \\ S & = & a_0 X & + & a_2 Z^2 \end{array}$$

Anderson, Gao, MK, 2019

The coefficients of $\mathcal{S}$ maps to the Casimir's of the Higgs field inside the 7-brane,

$$W \sim \int_{7-brane} F^{(2,0)} \wedge \Phi^{(0,2)}.$$

Beasley, Heckman, Vafa 2008

Rank 2 Example from Spectral Data and Generalization

- Example:

$$\begin{aligned} B_2 &= \left[\begin{array}{c|c} \mathbb{P}^1_x & 2 \\ \mathbb{P}^2_y & 2 \end{array} \right], \quad c_1(B_2) = H_2, \\ P &= x_1^2 P_1(y) + x_2^2 P_2(y) + x_1 x_2 P_3(y). \end{aligned}$$

Let us choose $D_b = -H_1 + 2H_2$,

$$S = \begin{cases} a_0 Z^2, & P \quad \text{generic} \\ a_0 Z^2 + a_2 X, & P = x_1^2 P_1(y) + x_2^2 P_2(y). \end{cases}$$

- Generally one can consider a spectral cover of degree n,

$$S_n = a_n X^{n/2} + \cdots + a_0 Z^n,$$

and try to find sub-loci in the complex structure moduli that the coefficients a_i can have new terms. The question is how to find such sub-loci.

Cui, MK, 2020

Jump in the Picard Group of the Base

- Noether-Lefshetz locus: The locus in the complex structure moduli space such that there are divisors that are not induced by ambient space divisors.
- The idea is to take a non-favorable Fano surface as the base of the elliptic fibration, and find the loci where the Picard number jumps to a higher value.

$$B_2^0 = \begin{array}{cccccc|c} x_1 & x_2 & z_1 & z_2 & u_1 & u_2 & P \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 & 1 & 1 & 3 \end{array}$$

$$P = x_1(z_1^3 f_3(u) + z_1^2 z_2 f_2(u) + z_1 z_2^2 f_1 + z_2^3 f_0) + x_2(z_1^2 g_2(u) + z_1 z_2 g_1(u) + z_2^2 g_0) = 0.$$

$$h^{1,1}(B_2) = 8, \qquad \rho = 3.$$

- This surface contains an exceptional divisor, e, consist of six (-1)-curves that can be blown down at the same time.

Jump in the Picard Group of the Base

- In the following locus Picard Group jumps to $\rho = 4$,

$$P = x_1(z_1^3 f_3(u) + z_1^2 z_2 f_2(u) + z_1 z_2^2 f_1 + z_2^3 f_0) + x_2(z_1 h_1(u) + z_2 h_0)(z_1 l_1(u) + z_2 l_0) = 0,$$

- On this locus e reduces to $e_1 + e_2$.
- If the spectral cover depends only on either e_1 or e_2,

$$[S] = [S_0] + e_1,$$

then the spectral cover will be reducible but reduced (hence the vector bundle remains stable) with one component on $\pi^* e_1$.

Cui, MK, 2020

Jump in the Picard Group of the Spectral Cover

- Another way to use the Noether-Lefshetz theorem is to deform the complex structure of the Calabi-Yau and the spectral cover at thre same time such that Picard number of the Spectral cover jumps.

$$
\begin{aligned}
&y^2 + x^3 + fx + g, \\
&g \to \alpha \cdot g', \\
&\Longrightarrow [C] := \{x = y = \alpha = 0\} \in H^{2,2}(X).
\end{aligned}
$$

Braun, Collinucci, Valando 2011

- This curve intersects a generic spectral cover at finite number of points.

$$
\begin{aligned}
S(V_n) &= a_0 z^n + a_2 z^{n-2} x + \cdots + a_n x^{n/2}, \\
a_0 &\to \alpha \cdot a_0'. \\
&\Longrightarrow [C] \in H^{1,1}(S).
\end{aligned}
$$

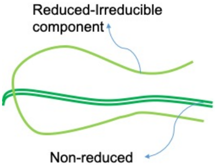
Donagi, Wijnholt, 2009

- It is the spectral sheaf in this case that (partially) stabilizes both complex structure and vector bundle moduli. Cui, MK, 2020

$$
\begin{aligned}
Ch(V_n) &= n - (\sigma \cdot \eta + \omega[f] + [C]) + \frac{1}{2} c_3(V_n), \\
[S(V_n)] &= n\sigma + \eta + \pi_*[C], \\
c_1(\mathcal{N}) &= \frac{1}{2}\left([S(V_n)] - c_1(B_2)\right) + \lambda\left(n\sigma - \eta + nc_1(B_2)\right) + \left(\frac{1}{n}\pi_*[C] - [C]\right).
\end{aligned}
$$

Future direction/F-theory Dual

- So far we restricted to the cases that the spectral cover was reduced $\Rightarrow$ Stabilize Complex structure/Bundle. However one add consider non-reduced components.
- Possibility of (partially) stabilizing the Complex/Kahler/Vector bundle moduli at the same time.



- Since everything is already described in terns of spectral data $\Longrightarrow$ F-theory image.