

Yukawa Couplings from T-Brane

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F-theory in 8D

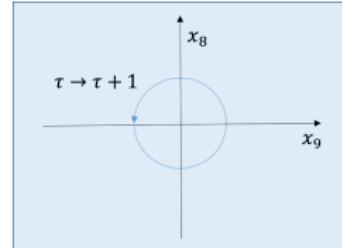
- ▶ Low energy limit of type IIB string theory, is a $\mathcal{N} = 2$ supergravity, where the

Gravity Multiplet: $g_{\mu\nu}, \phi, C_0, B_{\mu\nu}, C_{\mu\nu}, C_4,$
 $\psi_\mu^1, \psi_\mu^2, \lambda_\phi, \lambda_c$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad \left(\begin{array}{c} C'_2 \\ B'_2 \end{array} \right) = \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \left(\begin{array}{c} C_2 \\ B_2 \end{array} \right), \quad ad - bc = 1, (a, b, c, d) \in \mathbb{Z}$$

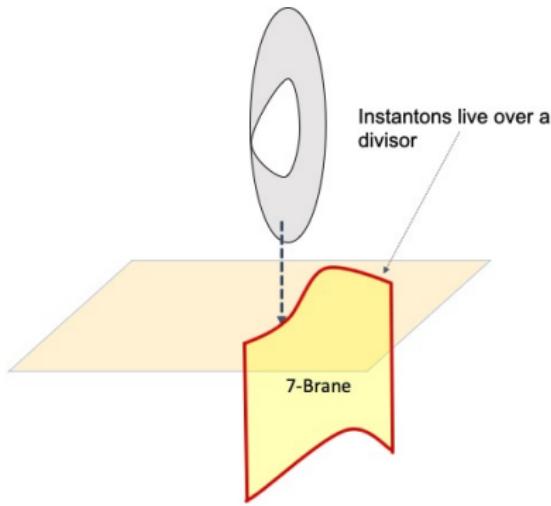
- ▶ Consider a single D7-brane source in type IIB string theory.
It's shown that the

$$\tau = C_0 + \frac{i}{g_s}, \quad \tau = \frac{1}{2\pi i} \ln\left(\frac{z}{\lambda}\right)$$



- ▶ Singularities of K3 (Kodaira) \longleftrightarrow Brane configuration in 8D (ADE gauge group)

F-theory in Lower Dimensions



$\mathcal{N} = 2$ worldvolume theory:

$$A_\mu^a, \quad ad(G)$$

$$\phi = \phi_1 + i\phi_2,$$

$$\psi_\pm, \quad Q_\pm$$

$$U(1) \text{ R-Symmetry}$$

After topological twist $\Rightarrow \mathcal{N} = 1$ effective theory

[Beasley, Heckman, Vafa, 2008](#)

$$\Phi \in H^0(S, K_S \otimes ad(G))$$

$$\text{F-term} \Rightarrow F_{ab} = F_{\bar{a}\bar{b}} = 0, \quad \bar{D}_A \Phi = D_A \bar{\Phi} = 0$$

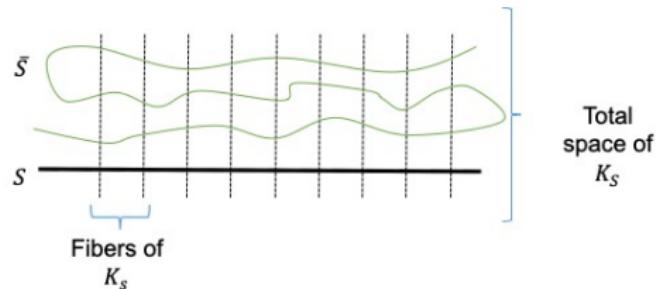
$$\text{D-term} \Rightarrow F \wedge \omega + \frac{i}{2} [\Phi, \bar{\Phi}] = 0$$

Hitchin's Abelianization/Spectral Data

- ▶ Abelianization:

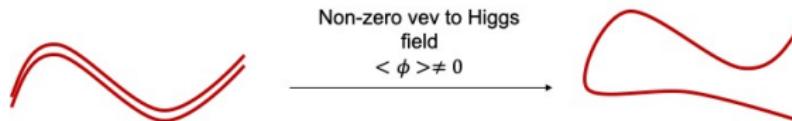
Hitchin, 86

$$\begin{aligned}\bar{S} &:= \{\det(\Phi - \lambda 1) = 0\} \\ (E, \Phi) &\Leftrightarrow (\bar{S}, \mathcal{L})\end{aligned}$$



- ▶ T-brane: $SU(2)$ Example:

$$\Phi = \begin{pmatrix} 0 & 1 \\ \phi = 0 & 0 \end{pmatrix} \Rightarrow \bar{S} = \lambda^2$$

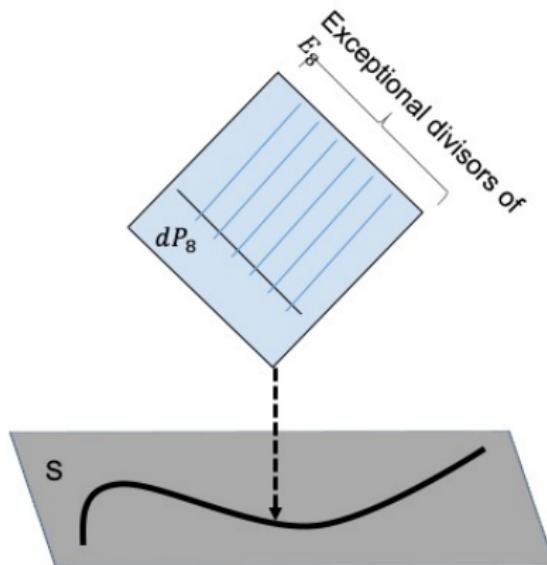


- ▶ Unfolding:

$$\begin{array}{ccc} E_8 \text{ singularity} & y^2 = x^3 - z^5 & \xrightarrow{\text{Unfolding}} \\ & \mathcal{E}_2 & \in \end{array} \quad \begin{array}{c} y^2 = x^3 - z^5 + \mathcal{E}_2 xz^3 + \dots \\ H^0(\mathrm{tr}(\Phi^2)) \dots \end{array}$$

Duality to Heterotic

- ▶ To find more non-trivial spectral covers we can use the Heterotic picture.



- ▶ The local picture determines a flat E_8 bundle over an elliptically fibered Calabi-Yau \implies Heterotic E_8 .

Fourier(-Mukai) Transform/Spectral Covers

- ▶ A flat gauge field over $T^2 \Leftrightarrow$ Wilson lines (direct sum of line bundles) \Leftrightarrow Points in $\text{Jac}(T^2) \simeq T^2$
- ▶ Formally, one defines a functor,

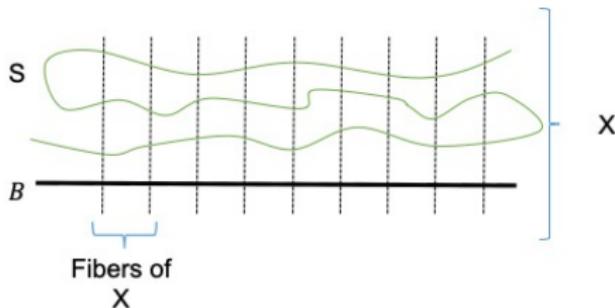
$$\Phi : D^b(X) \longrightarrow D^b(X),$$

$$\Phi(V) = R\pi_{2*}(\pi_1^* V \otimes \mathcal{P}),$$

$$\pi_{1,2} : X \times_B X \longrightarrow X, \quad \mathcal{P} = \mathcal{I}_\Delta \otimes \pi_1^* \mathcal{O}_X(\sigma) \otimes \pi_2^* \mathcal{O}_X(\sigma) \otimes K_B^*.$$

- ▶ Since V is (semi)stable, degree zero,

$$\Phi(V) = i_{S*} \mathcal{L}[-1].$$



Anderson, Gao, MK, 2019
Friedman, Morgan, Witten 1997
Andreas, Curio, Ruiperez, Yau, 2000

Yukawa Couplings from T-brane

- ▶ The bulk Yukawa couplings,

$$\int_S \text{Tr}(F \wedge \Phi) \longrightarrow - \int_S h_{ijk} A^i \wedge A^j \wedge \Phi^k.$$

- ▶ Yukawa coupling from matter curve Σ ,

$$\int_{\Sigma} \text{Tr}(\Lambda^c \partial_{A+A'} \Lambda) \longrightarrow \int_{\Sigma} c_{ijk} \Lambda^{c,i} (A^j + A'^j) \Lambda^k.$$

- ▶ Both contributions coming from the hypercohomology $(\partial_A + \Phi)$,

$$\mathcal{E}^\bullet := E \xrightarrow{\Phi} E \otimes K,$$

$$\mathbb{H}^1(\mathcal{E}^\bullet) \wedge \mathbb{H}^1(\mathcal{E}^\bullet) \wedge \mathbb{H}^2(ad(\mathcal{E}^\bullet)) \longrightarrow \mathbb{C},$$

$$\mathbb{H}^i(\mathcal{E}^\bullet) \Leftrightarrow \text{Ext}_{\text{Tot}(K_S)}^i(\mathcal{O}_{S_0}, \mathcal{L}).$$

- ▶ Heterotic Yukawa couplings,

$$\begin{aligned} \int_X \omega_3 \wedge \Omega_3 &\longrightarrow \int_X c_{ijk} A^i \wedge A^j \wedge A^k \wedge \Omega_3, \\ H^1(V) \wedge H^1(V) \wedge H^1(\Lambda^2 V) &\longrightarrow \mathbb{C}. \end{aligned}$$

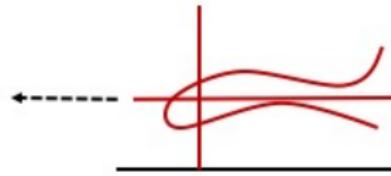
Stable V defined via GLSMs or Extensions $\xrightarrow{\Phi(V)} (S_{H\acute{e}t}, \mathcal{L}_{H\acute{e}t})$

$$\downarrow$$
$$\mathbb{H}^1(\mathcal{E}^\bullet) \longleftarrow (S_{Higgs}, \mathcal{L}_{Higgs})$$

Vanishing Yukawa
couplings.



Non-Vanishing Yukawa
couplings from matter
curve.



Expected non-vanishing
Yukawa couplings.
Working on examples.

