On U(1)'s

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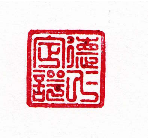
Jihn E. Kim Fest



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Preface

This morning session also seems to be about the CERN Theory Division in 1983–1984 (with the subsidiary support of Geneva Univ.).

We (Hans-Peter Nilles, Antonio Masiero, I) were around, with some switches between CERN and the University (both ways).

And then Jihn E. Kim visited CERN and wrote papers with us (and some others) in various combinations...

And some of these papers did attract a fair amount of attention over the next years ...

A prehistoric era without personal computers (but with typing machines and the CERN typing pool), no internet, with papers published on paper, distributed by real mail ("fast" airmail for well-connected persons, very slow surface mail for all others), new papers displayed once a week (Tuesday), in the CERN library (and many stolen the same day).
With lots of creative, joyful, fruitful corridor science.

At this time: [not an exhaustive list of course. . .]

- $\sin^2\theta_W(M_W) \sim .215$ (now $\sim .233$), in good agreement with $SU(5)$ prediction (the non-supersymmetric version)
- Proton decay experiments were on their way to disprove the $SU(5)$ prediction, and push the limit too far for minimal unified models (now $\tau_p > (6$ to $8)10^{33}$ years in positron modes)
- The interface particle physics — cosmology was developing into a central subject (now “astroparticle physics")
- Supersymmetric or supergravity models where building up, largely motivated by curiosity, hierarchy and naturalness
- String theories were (almost) asleep, until a noisy wake-up call at the end of 1984

ANTI-SU(5)

J.-P. DERENDINGER, Jihn E. KIM and D.V. NANOPOULOS
CERN, Geneva, Switzerland

Received 25 January 1984

We discuss ordinary as well as supersymmetric SU(5) × $\tilde{\mathrm{U}}$(1) models in the hope of accommodating acceptable τ_p and $\sin^2\theta_W$. The ordinary SU(5) × $\tilde{\mathrm{U}}$(1) model does not have the monopole. The supersymmetric SU(5) × $\tilde{\mathrm{U}}$(1) model can be unified in SO(10).

…accomodating acceptable τ_p and $\sin^2\theta_W$, at the time of writing of course.

The paper is in large part concerned with the "Flipped $SU(5)$" model. For some obscure reasons, it was called "Anti $SU(5)$" …

And this model has gauge group $SU(5) \times U(1)$, hence today's talk title.

(May 2015, Ioannina)

In this talk, I should:

- Review briefly the $U(1)$'s relevant to the Standard Model and their role in baryon number violating processes in Grand Unified Theories (GUT), and in nucleon decay, as a motivation to ...
- Describe the development of the flipped SU(5) model
- Briefly mention some recent uses of flipped models.
- Move to some more recent, completely unrelated, (personal) research on $U(1)$'s in supersymmetric models.
- And ultimately, as usual, run out of time ...

On $U(1)$'s in the Standard Model

The Standard Model has two perfect $U(1)$'s:

- $U(1)_Q$, the electromagnetic $U(1)$, perfectly vector-like, parity-conserving, partner of colour $SU(3)$ in exact gauge symmetries.
- $U(1)_{B-L}$, perfectly vector-like, perfectly symmetric when supplemented with $SU(2)_L \times SU(2)_R$.

  A global symmetry leaving all four-fermion operators invariant (exception: the Majorana character of the gauge-singlet right-handed neutrino N_L^c)

The Standard Model has a fully asymmetric $U(1)$:

The weak hypercharge Y, gauged, asymmetric for complete parity violation of weak interactions.

And also less relevant global $U(1)$'s (approximate) symmetries (B, L)

On $U(1)$'s in the Standard Model

Each quark–lepton generation:

	$SU(3) \times SU(2)_L \times U(1)_Y$	$Q = T_L^3 + Y$	$B - L$
Q_L	$(3, 2, 1/6)$	$2/3, -1/3$	$1/3, 1/3$
U_L^c	$(\overline{3}, 1, -2/3)$	$-2/3$	$-1/3$
D_L^c	$(\overline{3}, 1, 1/3)$	$1/3$	$-1/3$
L_L	$(1, 2, -1/2)$	$0, -1$	$-1, -1$
E_L^c	$(1, 1, 1)$	1	1
N_L^c	$(1, 1, 0)$	0	1

B-violating four-fermion processes:

$$\psi_L^c = \mathcal{C}\,\overline{\psi}_R{}^{\tau}$$

Relevant four-fermion operators:
$$\begin{cases} QQQL & B = L = 1 \\ QQDN & B = L = 1 \\ QQUE & B = L = 1 \\ UUDE & B = L = 1 \\ UDDN & B = L = 1 \end{cases}$$

And $B - L$ violations due to (Majorana) ambiguity $N_L^c \sim N_L$ ($\Delta(B-L) = \pm 2$)

On $U(1)$'s in the Standard Model

$B - L$ selects processes quark + quark $\longrightarrow$ antiquark + antilepton

Quark level: $\begin{cases} u + u \longrightarrow \overline{d} + \overline{\ell} \\ u + d \longrightarrow \overline{u} + \overline{\ell}, \quad \overline{d} + \overline{\nu} \\ d + d \longrightarrow \overline{u} + \overline{\nu} \end{cases}$

Nucleon level: $\begin{cases} p \longrightarrow \pi^0 + e^+, \quad \pi^+ + \overline{\nu} \\ n \longrightarrow \pi^0 + \overline{\nu}, \quad \pi^- + e^+ \end{cases}$ (Or heavier mesons/leptons)

Mediated by two classes of leptoquarks (massive gauge fields of a GUT)

X : $(3, 2, -5/6)$ [$Q = -1/3, -4/3$]

Y : $(3, 2, 1/6)$ [$Q = 2/3, -1/3$] Same as quark doublet Q_L

X is in $SU(5)$, Y is in flipped $SU(5) \times U(1)$, both are in $SO(10)$ or extensions.

X favours $p \longrightarrow \pi^0 + \overline{e}^+$ Y favours $p \longrightarrow \pi^+ + \overline{\nu}$

Branching ratios provide indications on M_X/M_Y and then on the type of GUT inducing the decay ... *ideally of course*

On proton decay and flipped $SU(5)$

At the origin of the first appearance of the “flipped $SU(5)$", in the context of an $SO(10)$ grand unified theory ($SO(10)$ is actually used to model the needed flavour mixings)

Flavor Goniometry by Proton Decay

A. De Rújula[(a)]

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

H. Georgi and S. L. Glashow

The Physics Laboratories, Harvard University, Cambridge, Massachusetts 02138

(Received 3 June 1980)

Unification of strong and electroweak forces implies that protons (and bound neutrons) decay. Modes like $\bar{\nu}\pi$, $e^+\pi$, and μ^+K^0 are expected, while modes like $\mu^+\pi$ and e^+K^0 are “Cabibbo” suppressed. Branching ratios can reveal much about the nature of the unifying group and the origin of fermion masses. Plausible models of unification and flavor mixing give surprisingly different predictions for two-body branching ratios.

Phys. Rev. Lett. 45, August 11, 1980

Flavor Goniometry by Proton Decay

A. De Rújula[a]

Center for Theoretical Physics, Laboratory for Nuc

Massachusetts Institute of Technology, Ca

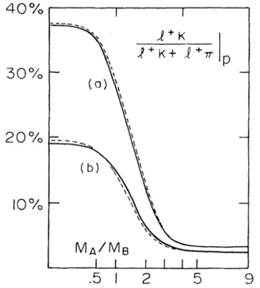

FIG. 2. Branching ratios of strange relative to nonstrange decay modes, with the same conventions as in Fig. 1. The curves labeled *a* correspond to the nonrelativistic model of Ref. 12, the curves labeled *b* are extracted from Ref. 13. Here $l^+K = e^+K + \mu^+K$ and $l^+\pi = e^+\pi + \mu^+\pi$.

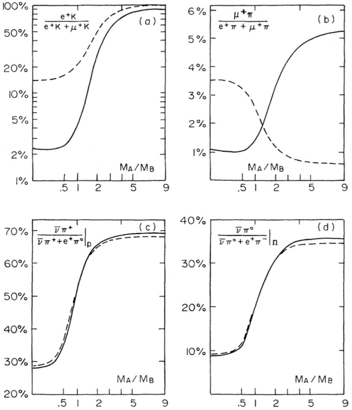

FIG. 1. Relative branching ratios of two-body modes with the same strangeness. The horizontal scales are linear in $(M_A/M_B)^{1/2}$. Continuous (dashed) lines correspond to F (J) masses. Figure 1(a) refers to proton decay; the e^+K^- and μ^+K^- decays of neutrons are forbidden. Figure 1(b) applies to either proton or bound-neutron decay. Figures 1(c) and 1(d) refer to protons and neutrons, respectively.

Flipped $SU(5)$

Next: analysis of SO(10) symmetry breaking with intermediate flipped $SU(5) \times U(1)_X$ symmetry

A NEW SYMMETRY BREAKING PATTERN FOR SO(10) AND PROTON DECAY

S M BARR

Department of Physics, University of Washington, Seattle, WA 98195, USA

Received 22 January 1982

We discuss a "new" pattern of symmetry breaking for SO(10) grand unified models in which SO(10) breaks to $SU(5)' \times U(1)$ and then further breaks to $SU(3)_C \times SU(2)_L \times U(1)$ at a lower scale In this pattern of breaking electric charge is *not* a generator of the $SU(5)'$ group The predictions for proton decay branching ratios are markedly different from those of the Georgi–Glashow SU(5) model

Phys. Lett. B112, January 22, 1982

Next: our "Anti $SU(5)$" paper of January 1984, which actually analyses also other models with $SU(N) \times U(1)$ gauge group.

Flipped $SU(5)$, construction

- In $SU(5)$, a unique embedding of $SU(3) \times SU(2) \times U(1)_Y$:

$\overline{5} = (\overline{3}, 1, 1/3) + (1, 2, -1/2)$	D_L^c, L_L
$10 = (\overline{3}, 1, -2/3) + (3, 2, 1/6) + (1, 1, 1)$	U_L^c, Q_L, E_L^c
$1 = (1, 1, 0)$	N_L^c
$24 = (8, 1, 0) + (1, 3, 0) + (1, 1, 0)$	
$+(3, 2, -5/6) + (\overline{3}, 2, 5/6)$	(adjoint)

In 24: $(1, 1, 0)$ breaks into $SU(3) \times SU(2) \times U(1)$

- A unique anomaly-free $U(1)_X$ can be added:

$$\overline{5}_{-3} = (\overline{3}, 1, 1/3, -3) + (1, 2, -1/2, -3)$$
$$10_1 = (\overline{3}, 1, -2/3, 1) + (3, 2, 1/6, 1) + (1, 1, 1, 1)$$
$$1_5 = (1, 1, 0, 5) \qquad N_L^c \text{ needed}$$

- Two definitions of the weak hypercharge:

Either Y in $SU(5)$ or $\widetilde{Y} = \frac{1}{5}(X - Y)$ not in $SU(5)$

Flipped $SU(5)$, construction

- Under $SU(3) \times SU(2) \times U(1)_{\tilde{Y}}$:

$$\begin{array}{ll} \bar{5}_{-3} = (\bar{3}, 1, -2/3) + (1, 2, -1/2) & U_L^c,\, L_L \\ 10_1 = (\bar{3}, 1, 1/3) + (3, 2, 1/6) + (1, 1, 0) & D_L^c,\, Q_L,\, N_L^c \\ 1_5 = (1, 1, 1) & E_L^c \end{array}$$

- $SU(5) \leftrightarrow SU(5) \times U(1)_X$: $\quad D_L^c \leftrightarrow U_L^c \quad E_L^c \leftrightarrow N_L^c$
- The direction $(1, 1, 0)$ in 10_1 breaks

$$SU(5) \times U(1)_X \longrightarrow SU(3) \times SU(2) \times U(1)_{\tilde{Y}}$$

i.e. to the Standard Model.

- Minimal Higgs system:

$\underline{10_1} + \underline{\overline{10}_{-1}}$ to break into $SU(3) \times SU(2) \times U(1)_{\tilde{Y}}$

$\underline{5_{-2}} + \underline{\bar{5}_2}$ to break into $SU(3) \times U(1)_Q$

Simple antisymmetric tensors, easy to obtain in string theories.

Susy $SU(5) \times U(1)_X$: doublet-triplet splitting

Supersymmetric version, cubic holomorphic superpotential for GUT breaking:

- $SU(5)$ breaking: $W \supset M_1[\overline{5} \times 5] + \lambda_2[\overline{5} \times 24 \times 5]$ with $\langle 24 \rangle \sim M_{GUT}$.

  $5 = (3, 1, -1/3) + (1, 2, 1/2)$:

  triplet, induced mass: $M_1 - \frac{1}{3}\lambda_2 M_{GUT} \sim M_{GUT}$
  doublet, induced mass: $M_1 + \frac{1}{2}\lambda_2 M_{GUT} \sim M_W$: strong fine tuning

- $SU(5) \times U(1)$ breaking:

  $10 + \overline{10}$ for $SU(5) \times U(1) \to SU(3) \times SU(2) \times U(1)$
  $5 + \overline{5}$ for $SU(3) \times SU(2) \times U(1) \to SU(3) \times U(1)_Q$

  $W \supset M_1[10 \times 10 \times 5] + M_2[\overline{10} \times \overline{10} \times \overline{5}]$ with $\langle 10 \rangle \sim M_{GUT}$
  $5 = (3, 1, -1/3) + (1, 2, -1/2)$:

  triplet, induced mass: $M_1 \langle 10 \rangle \sim M_{GUT}$
  doublet, induced mass: zero, doublet partner missing in 10

[Antoniadis, Ellis, Hagelin, Nanopoulos, 1987]

Flipped $SU(5) \times U(1)_X$: string realisations

- Provides a relation between the two gauge couplings at $\sim M_P$.

Flipped $SU(5)$ string solutions have been found using various approaches:

- Direct constructions of $SU(5) \times U(1)_X \times \ldots$ and appropriate massless spectra.
- Models with larger symmetries at the $M_P/M_{Str.}$ scale and intermediate scale breaking(s). Simple to construct but poor control on the intermediate scales.
- Flipped scheme favoured over many others by the simplicity of the required massless representations (simple antisymmetric tensors, no adjoint or higher), ...
- $SU(5)$ is not the unique source of flipped models. Simple Calabi-Yau string compactifications may admit a flipped $SU(6)$ structure with

$$E_6 \subset SU(6) \times U(1)_{T_R^3}$$

[Cecotti, J.-P.D., Ferrara, Girardello, Roncadelli, 1985]

Second part

On $U(1)$ symmetries, supersymmetry and non-dynamical fields.

Supersymmetry, two $U(1)$ curiosities

Formulated on $N = 1$ superspace, with coordinates $(x^\mu, \theta, \overline{\theta})$, gauge fields and gauge symmetries are extended:

- $A_\mu \quad \longrightarrow \quad \mathcal{A}(x, \theta, \overline{\theta}) = \ldots + \theta\sigma^\mu\overline{\theta}\, A_\mu(x) + \ldots$
- Abelian: $\delta A_\mu = \partial_\mu \Lambda(x) \quad \longrightarrow \quad \delta\mathcal{A} = \Lambda + \overline{\Lambda}$
  with a chiral superfield parameter $\overline{D}_{\dot{\alpha}}\Lambda = 0$
- Nonabelian: $\delta A_\mu = \partial_\mu \Lambda(x) + i[\Lambda, A_\mu] \quad \longrightarrow \quad e^{\mathcal{A}'} = e^{-\overline{\Lambda}} e^{\mathcal{A}} e^{-\Lambda}$
- $\mathcal{A}$ includes $8_B + 8_F$ component fields
  $\Lambda(x, \theta, \overline{\theta})$ generates $4_B + 4_F$ gauge symmetries (instead of 1_B)
- There is a gauge where $\mathcal{A}$ has $5_B + 4_F$ fields and one (1_B) residual ordinary gauge variation, but this (Wess-Zumino) gauge is not invariant under supersymmetry.
- In this gauge, $\mathcal{A}$ includes the gauge field with its ordinary gauge variation, a gaugino, and a real auxiliary field D (which transforms with a derivative).

The Fayet-Iliopoulos term for susy $U(1)$ theories

- $U(1)$ $N = 1$ gauge theory: gaugino and auxiliary D are gauge invariant. Then

$$\xi\, D = 2\,\xi \int d^2\theta d^2\overline{\theta}\, \mathcal{A} \qquad\qquad \xi : (\text{energy})^2$$

is a lagrangian contribution (Fayet-Iliopoulos term, FI) which potentially spontaneously breaks supersymmetry.

- Now: for coupling to supergravity, a gauge-invariant off-shell multiplet is needed: neither $\mathcal{A}$ nor D convenient.

The trick is: introduce a non-dynamical superfield:

$$\widehat{\mathcal{A}} = \mathcal{A} + S + \overline{S} \qquad\qquad \delta S = -\Lambda$$

Since $$\int d^2\theta d^2\overline{\theta}\, \widehat{\mathcal{A}} = \int d^2\theta d^2\overline{\theta}\, \mathcal{A} + \text{derivative}$$

S does not contribute to the dynamical equations, but it does contribute to the Noether currents.

The Fayet-Iliopoulos term for susy $U(1)$ theories

Ferrara-Zumino supercurrent structure (for a theory with Kähler potential K and chiral superfields Φ): $\overline{D}^{\dot{\alpha}}\widehat{J}_{\alpha\dot{\alpha}} = D_\alpha\widehat{X}$

$$\begin{aligned}
\widehat{J}_{\alpha\dot{\alpha}} &= 2\,K_{\phi_i\overline{\phi}_j}(\mathcal{D}_\alpha\phi_i)(\overline{\mathcal{D}}_{\dot{\alpha}}\overline{\phi}_j) - 2(g+\overline{g})\mathcal{W}_\alpha\overline{\mathcal{W}}_{\dot{\alpha}} \\
&\quad -\frac{2}{3}[D_\alpha,\overline{D}_{\dot{\alpha}}]\Big[K+\xi(A+S+\overline{S})\Big] \\
\widehat{X} &= 4W - \frac{1}{3}\overline{DD}[K+\xi(A+S+\overline{S})]
\end{aligned}$$

This structure is equivalent (via a supersymmetric improvement) with

$$\begin{aligned}
\overline{D}^{\dot{\alpha}}J_{\alpha\dot{\alpha}} &= D_\alpha X + \chi_\alpha \qquad\qquad X = 4W \\
J_{\alpha\dot{\alpha}} &= 2\,K_{\phi_i\overline{\phi}_j}(\mathcal{D}_\alpha\phi_i)(\overline{\mathcal{D}}_{\dot{\alpha}}\overline{\phi}_j) - 2(g+\overline{g})\mathcal{W}_\alpha\overline{\mathcal{W}}_{\dot{\alpha}} \\
\chi_\alpha &= \overline{DD}D_\alpha K - 4\,\xi\,\mathcal{W}_\alpha \;=\; \overline{DD}D_\alpha(K+\xi A).
\end{aligned}$$

The Fayet-Iliopoulos terms for susy $U(1)$ theories

Hence: [Arnold, J.-P. D., Hartong, 2012]

- Since the second structure does not depend on S and all quantities are gauge-invariant, all current conservation equations are S–independent: S does not affect the physics of the model.
- Since the first structure is a supersymmetric improvement of the second, all dependence on S is trivial. Like:

  $$\partial^\mu J_\mu = \Delta \qquad \text{and} \qquad J_\mu \quad \rightarrow \quad J_\mu + \mathcal{V}_\mu \qquad \Delta \quad \rightarrow \quad \Delta + \partial^\mu \mathcal{V}_\mu$$

  For instance, the $U(1)_R$ current acquires a gauge-invariant piece

  $$\xi(A_\mu + 2\,\mathrm{Im}\,\partial_\mu s)$$

- S is non-dynamical, the theory has a well-defined Ferrara-Zumino structure with the FI term, it couples well to old/new minimal supergravity if the superpotential is R–symmetric (this condition disappears in the $\kappa \rightarrow 0$ gravity-decoupling limit). Nothing to worry about ...

A non-dynamical field in super-Yang-Mills, $U(1)_R$

Super-Yang-Mills in $\mathcal{N} = 1$ superspace can be written as

$$\mathcal{L}_{SYM} = \frac{1}{4}\int d^2\theta\, \mathcal{W}\mathcal{W} + \text{h.c.} = \int d^2\theta d^2\overline{\theta}\, \hat{L}$$

Where: Ω: Chern-Simons superfield, L: real linear superfield

$$\hat{L} = L - 2\Omega \qquad \overline{DD}\Omega = \mathcal{W}\mathcal{W} \qquad \overline{DD}L = 0, \quad L = L^\dagger$$

L is non-dynamical, its role is to preserve gauge invariance of the theory with

$$\delta_{\text{gauge}} L = 2\,\delta_{\text{gauge}}\Omega$$

Now we have two gauge invariant field/operators

$\mathcal{W}\mathcal{W}$ (chiral) $\hat{L} = L - 2\,\Omega$ (real)

to construct effective lagrangian of super-Yang-Mills, and L does provide the gauge coupling background field. [*Effective couplings should be fields*]

A non-dynamical field in super-Yang-Mills, $U(1)_R$, ...

Explicitly: $$\mathcal{L}_{eff.} = \int d^2\theta d^2\overline{\theta}\, \mathcal{H}(\hat{L}) = \frac{\partial}{\partial C}\mathcal{H}(C)\, \mathcal{L}_{SYM}$$

$$\frac{1}{g^2} = \frac{\partial}{\partial C}\mathcal{H}(C) \qquad C = L|_{\theta=0}$$ A real function

To determine the function $\mathcal{H}(\hat{L})$, use anomaly matching / cancellation of the two "U(1)'s" of the $N = 1$ superconformal algebra:

- $U(1)_R$: R–symmetry rotating Grassmann coordinates and gauginos: one-loop (Adler-Bardeen) anomaly
- $O(1,1)$ dilatation / scale transformation, which also has a (formally one-loop) anomaly
- These two anomalies deliver two coefficients depending on $C(G)$
- Anomaly-matching / cancellation delivers the all-order NSVZ β function, which is then derived from a simple algebraic argument.

[J.-P. D., Ferrara, Kounnas, Zwirner, 1991; J.-P. D., Hartong et al., to appear]

The interplay of supersymmetry and gauged or global $U(1)$ (or nonabelilan) symmetries hides some surprises and mysterious concepts.

Their resolution may be a useful tool to uncover dynamical aspects of super-Yang-Mills theories.