Strong CP problem and axion on the lattice

Ryuichiro Kitano (KEK)

based on 1506.00370 with Nori Yamada (KEK)

and works in progress with Nori Yamada, Julien Frison Shingo Mori, and Hideo Matsufuru

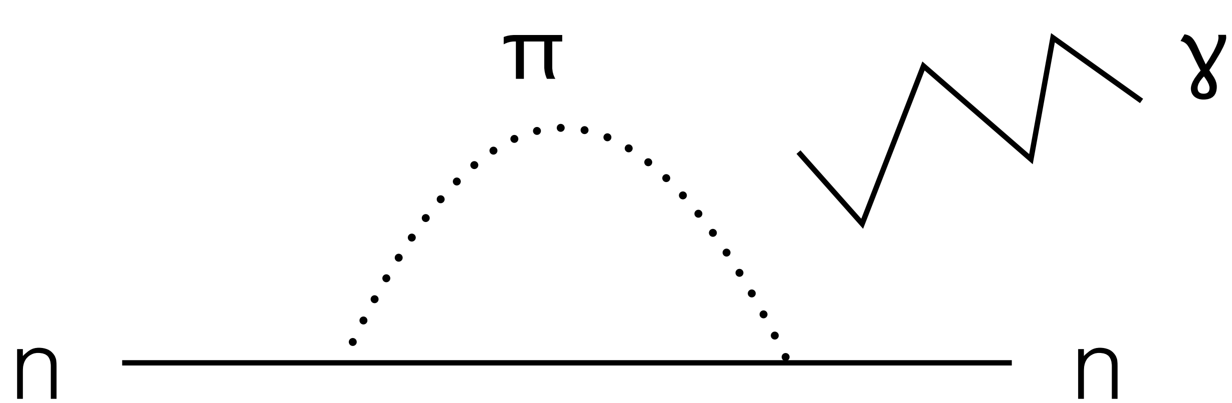
talk@IBS-SNU joint workshop on particle physics, Seoul, May 3, 2015

Strong CP problem

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar{\psi}]e^{-S_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2}F^2 + \frac{i\theta}{32\pi^2}F\tilde{F} + \bar{\psi}(D+m)\psi \right)$$

θ term breaks CP ['t Hooft '76]



$$d_n \sim 10^{-15}\theta e \cdot {\rm cm}$$

$$\theta \lesssim 10^{-10}$$????

[Crewther, Di Vecchia, Veneziano, Witten '79]

Is θ-term really physical?

—> Does the partition function Z depend on θ?

$$\frac{1}{iZ}\frac{dZ}{d\theta}\bigg|_{\theta=0} = \langle \underbrace{\int d^4x \frac{1}{32\pi^2} F\tilde{F}}_{=Q} \rangle\bigg|_{\theta=0} = 0$$

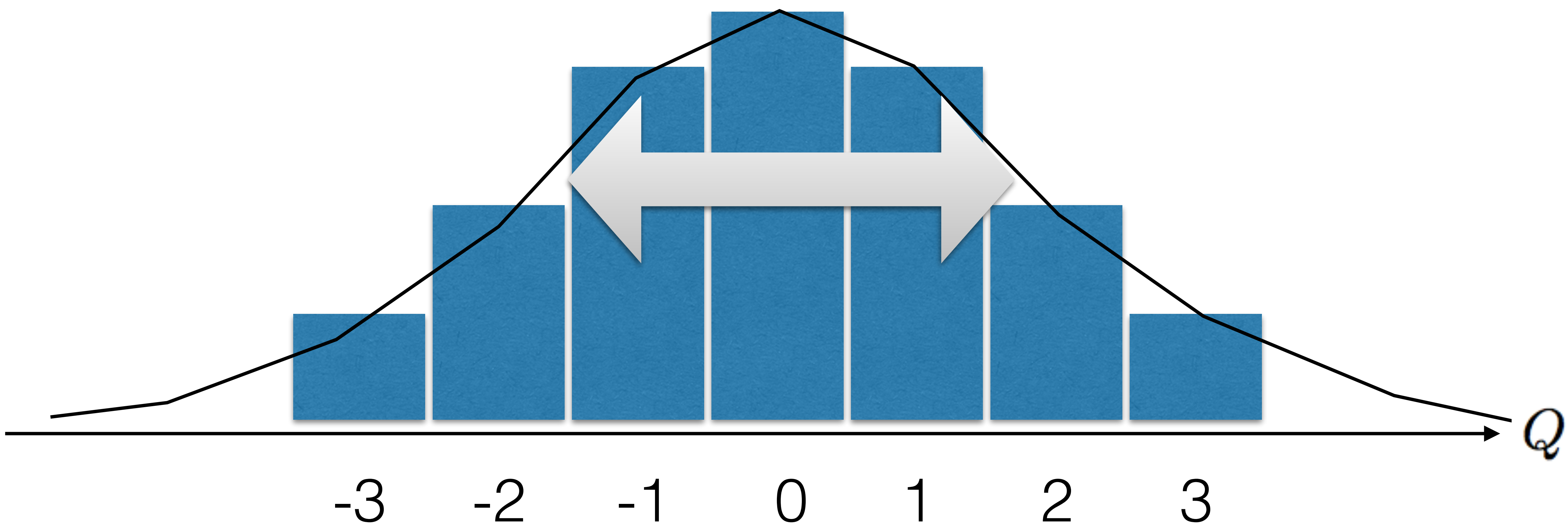
(CP)

(topological charge = integers!)

$$\boxed{\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = \frac{\langle Q^2\rangle}{V}}$$

(topological susceptibility)

χ_t



$$\langle Q^2 \rangle = \chi_t V$$

χ_t measures how often instantons appear in the path integral.

If χ_t is nonzero, θ is physical.

χ_t and m_u

$$Z_{\rm QCD} = \int [dA][d\psi][d\bar\psi] e^{-S_{\rm QCD}} = \int [dA][d\psi][d\bar\psi] e^{-S'_{\rm QCD}}$$

$$S_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \frac{i\theta}{32\pi^2} F\tilde{F} + \bar\psi (D + m)\psi \right)$$

$$S'_{\rm QCD} = \int d^4x \left(\frac{1}{4g^2} F^2 + \bar\psi (D + m e^{-i\gamma_5\theta})\psi \right)$$

$$\chi_t = -\frac{1}{V}\frac{1}{Z}\frac{d^2 Z}{d\theta^2}\bigg|_{\theta=0} = -m_u \langle \bar u u \rangle + O(m_u^2/m_\pi^2)$$

If m_u is non zero, θ is physical.

If m_u=0, physics does **not** depend on θ.
—> no strong CP problem

$m_u=0$?

[PDG]

LIGHT QUARKS (u, d, s)

OMITTED FROM SUMMARY TABLE

u-QUARK MASS

The u-, d-, and s-quark masses are estimates of so-called "current-quark masses," in a mass- independent subtraction scheme such as $\overline{\mathrm{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\mathrm{MS}}$ masses at a renormalization scale of $\mu = 2$ GeV. Results quoted in the literature at $\mu = 1$ GeV have been rescaled by dividing by 1.35. The values of "Our Evaluation" were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
$2.3\ ^{+0.7}_{-0.5}$ OUR EVALUATION	See the ideogram below.			
$2.15 \pm 0.03 \pm 0.10$	[1] DURR	11	LATT	$\overline{\mathrm{MS}}$ scheme
$2.24 \pm 0.10 \pm 0.34$	[2] BLUM	10	LATT	$\overline{\mathrm{MS}}$ scheme

[Peccei and Quinn '77]

Axion

OK, maybe m_u is non zero and θ is physical.

Then, why is θ so small?

The axion provides a nice solution.

$$\theta \rightarrow \theta + \frac{a(x)}{f_a} \qquad \left(\Delta\mathcal{L} = \frac{ia(x)}{32\pi^2 f_a} F\tilde{F}\right)$$

$$\boxed{\frac{\chi_t}{f_a^2} = m_a^2}$$

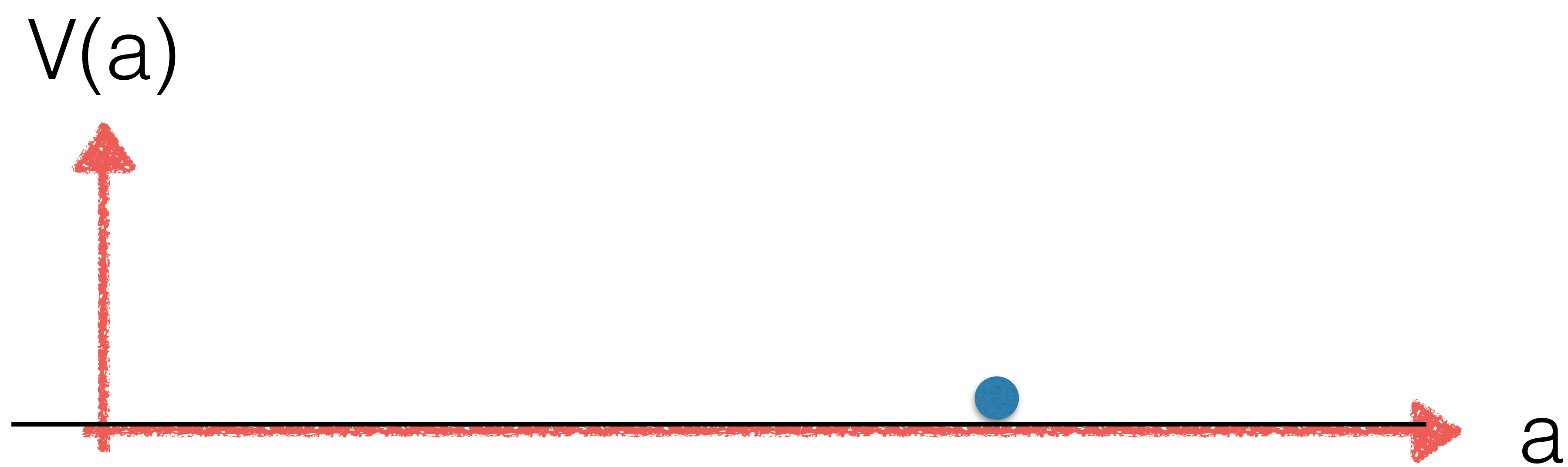
$$\mathcal{L}_{\text{eff}} = \frac{\chi_t}{2}\theta^2 + \cdots \quad \Rightarrow \quad \mathcal{L}_{\text{eff}} = (\partial_\mu a)^2 + \frac{\chi_t}{2}(\theta + \frac{a}{f_a})^2 + \cdots$$

$$\chi_t > 0 \quad \Rightarrow \quad \theta + \frac{a}{f_a} = 0$$

(dynamically selected)

[Preskill, Wise, Wilczek ’83][Abbott, Sikivie ’83]
[Dine, Fischler ’83]

Axion Dark Matter



$$\ddot{a} + 3H\dot{a} = -V'(a) \sim -m_a^2 a$$

$$\left.\frac{n_a}{T^3}\right|_{\rm now} \sim \frac{m_a(T_*) f_a^2 \theta_{\rm ini}^2}{T_*^3}$$

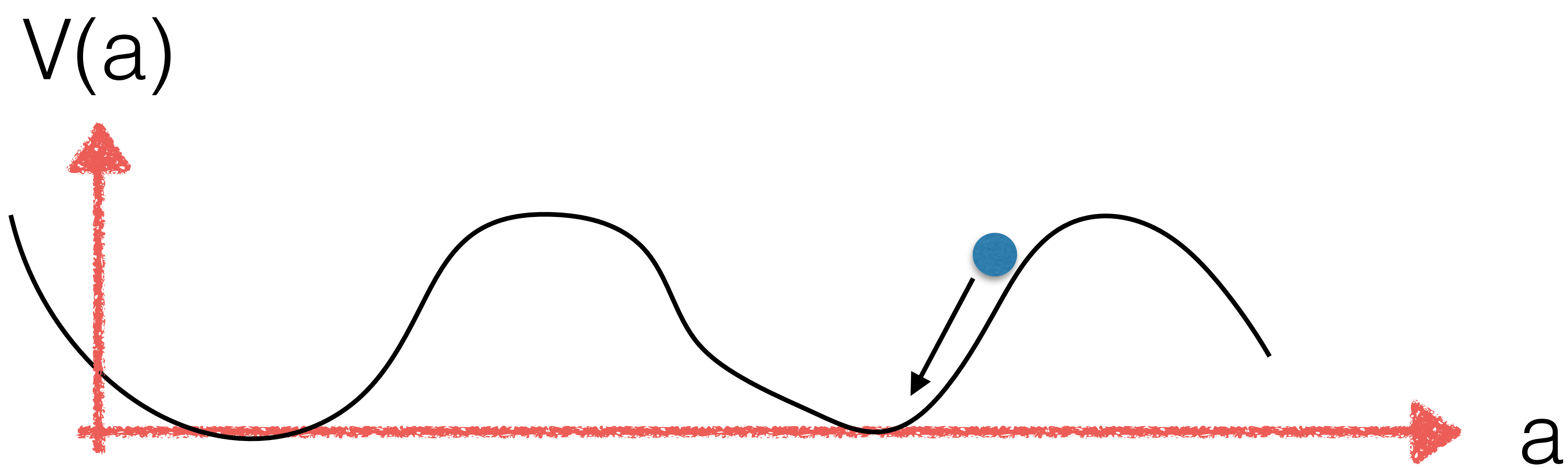
where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

[Preskill, Wise, Wilczek '83][Abbott, Sikivie '83]
[Dine, Fischler '83]

Axion Dark Matter



$$\ddot{a} + 3H\dot{a} = -V'(a) \sim -m_a^2 a$$

$$\left.\frac{n_a}{T^3}\right|_{\rm now} \sim \frac{m_a(T_*) f_a^2 \theta_{\rm ini}^2}{T_*^3}$$

where

$$m_a(T_*) \sim 3H(T_*)$$

temperature dependence of the axion mass
is the essential information to estimate the abundance.

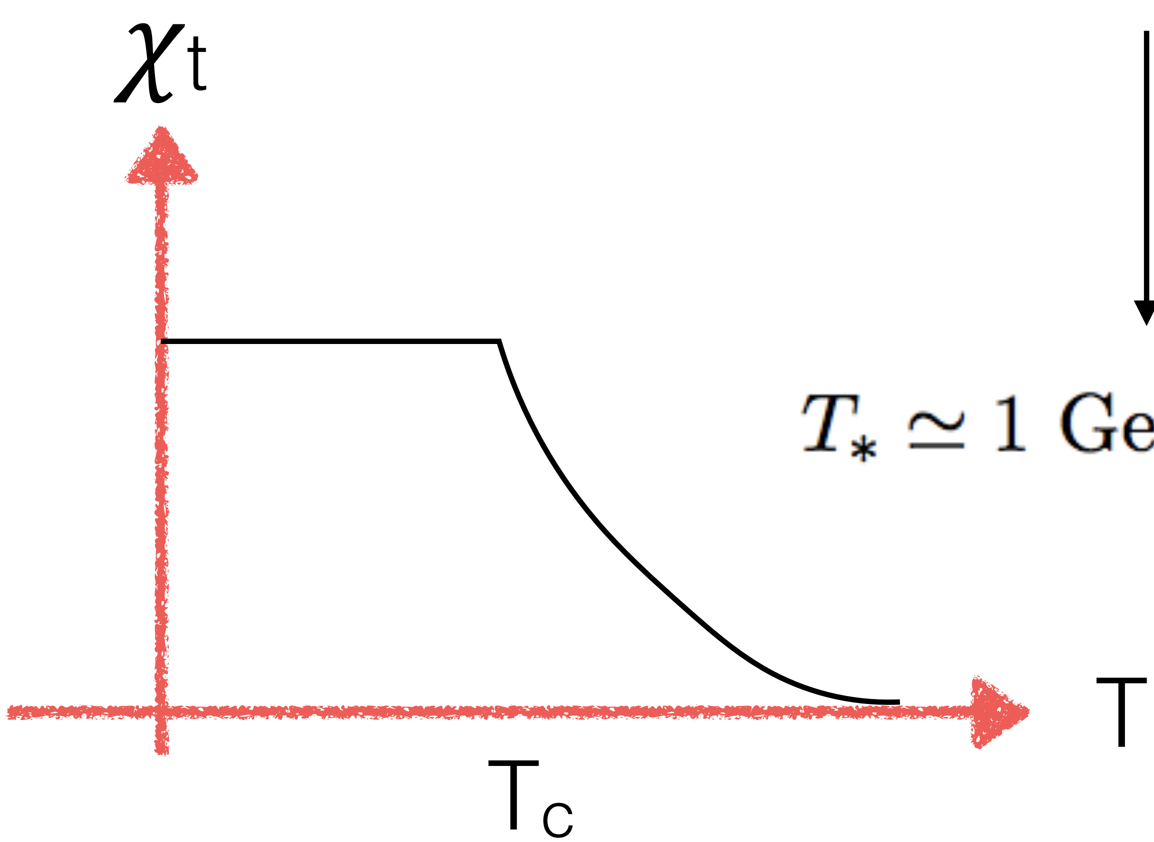
instanton paradigm

The standard way to calculate the temperature dependence of m_a is based on the dilute instanton gas approximation.

[Pisarsky, Yaffe '80]

$$\chi_t(T) = m_a^2(T) f_a^2 \propto m_u m_d m_s \Lambda_{\rm QCD}^b T^{-8} \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f = 9$$

instanton action $e^{-8\pi^2/g^2}$



$$T_* \simeq 1\ \mathrm{GeV} \cdot \left(\frac{m_a}{10^{-5}\ \mathrm{eV}}\right)^{1/6}$$

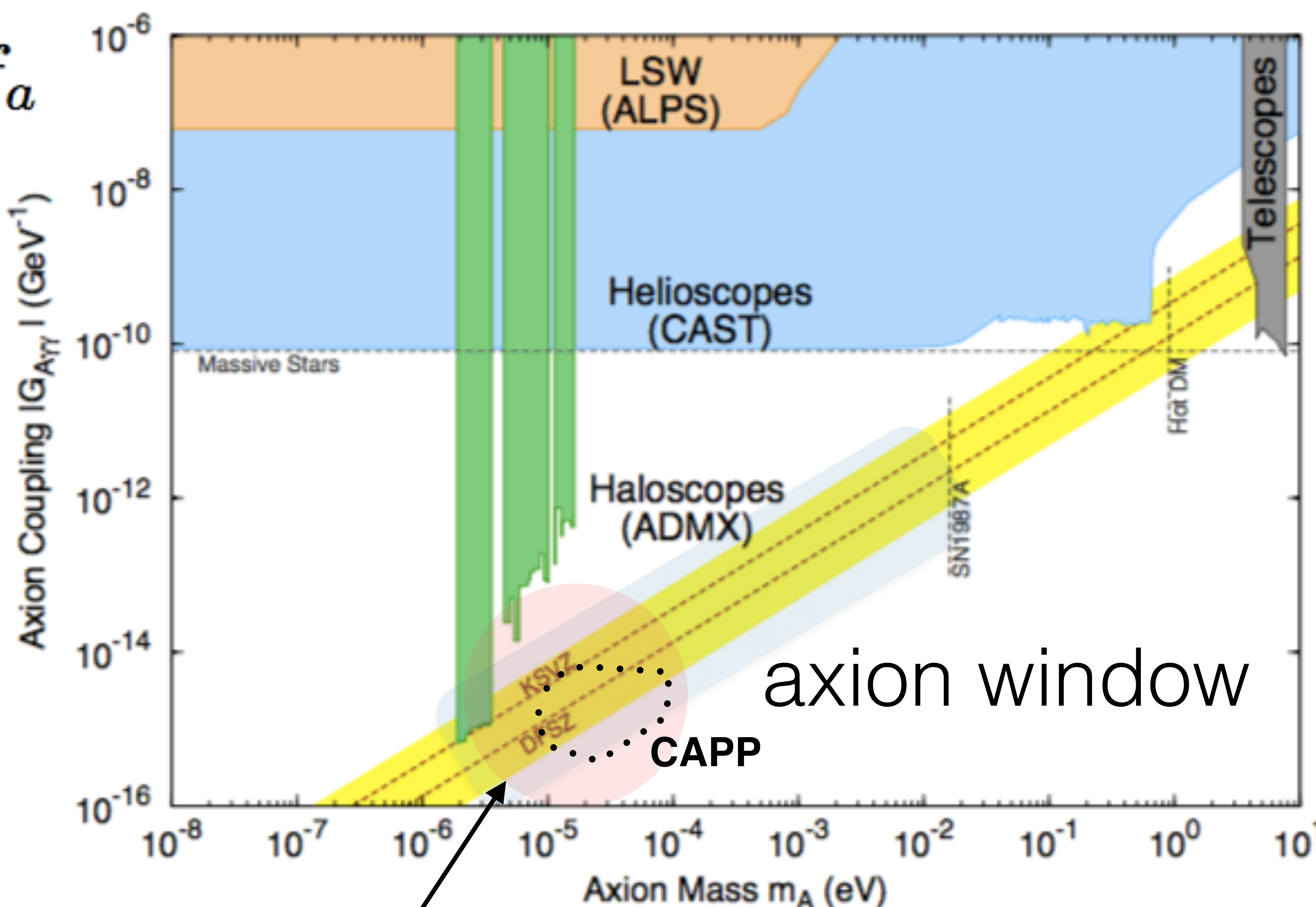
$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ \mathrm{eV}}\right)^{-7/6}$$

Axion Dark Matter

$$\Omega_a \simeq 0.2 \cdot \theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-7/6}$$

[PDG]

$\sim 1/f_a$



good DM abundance

[Cohen '96][Aoki, Fukaya, Taniguchi '12]

Is instanton correct?

Based on $\langle\bar{q}q\rangle = O(m_q)$ at high temperatures and the Ward identities, Cohen has argued

$$\chi_t(T) = O(m_q^4) \qquad \text{for } N_f=2$$

whereas the instanton says

$$\chi_t(T) = O(m_q^2) \qquad \text{for } N_f=2$$

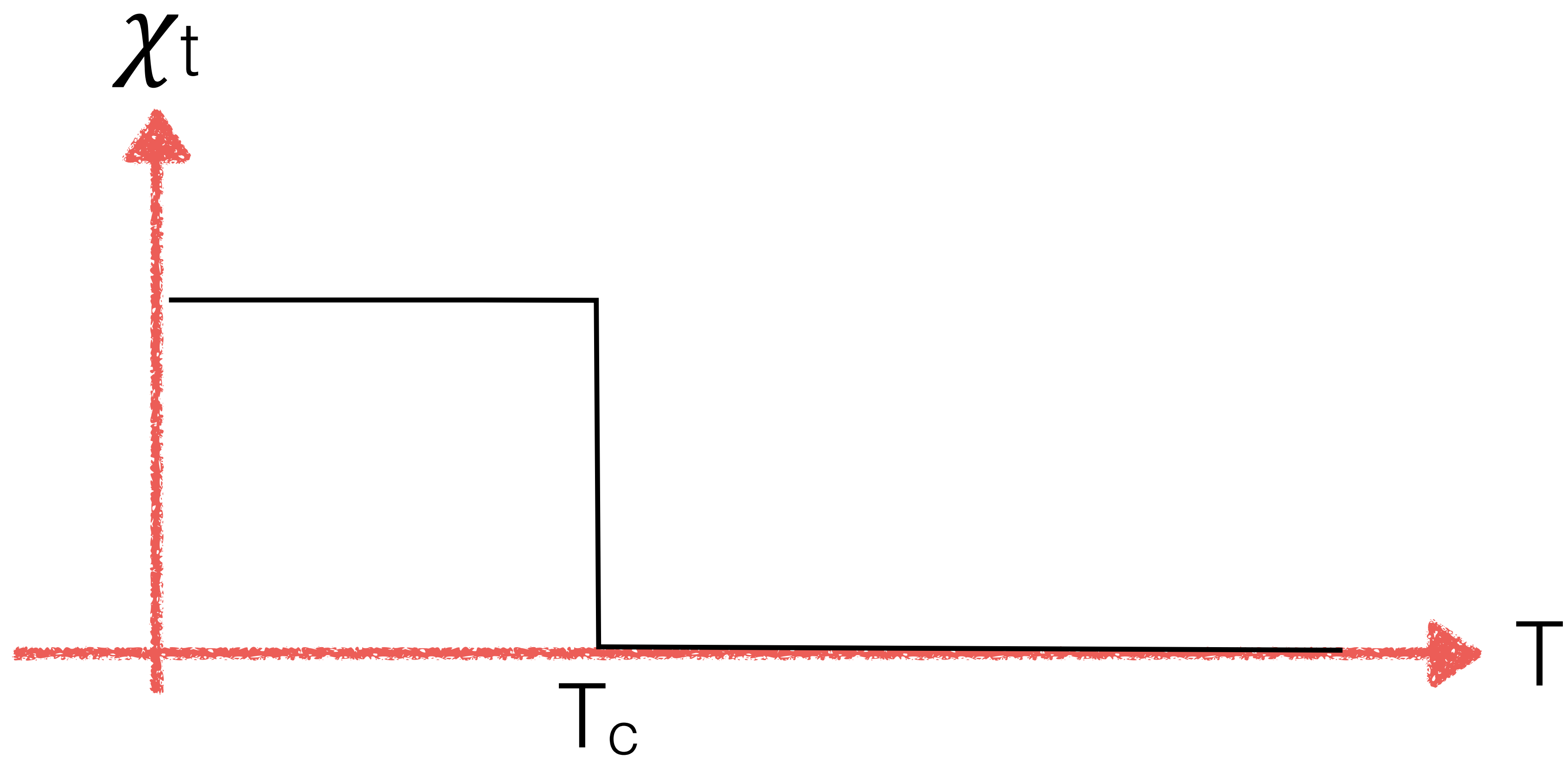
Aoki et al refined the Cohen's analysis and argued

$$\chi_t(T) = 0 \qquad \text{for small but finite } m_q$$

in any case, it is clearly inconsistent with instantons.

if χ_t=0 above T_c~150MeV,

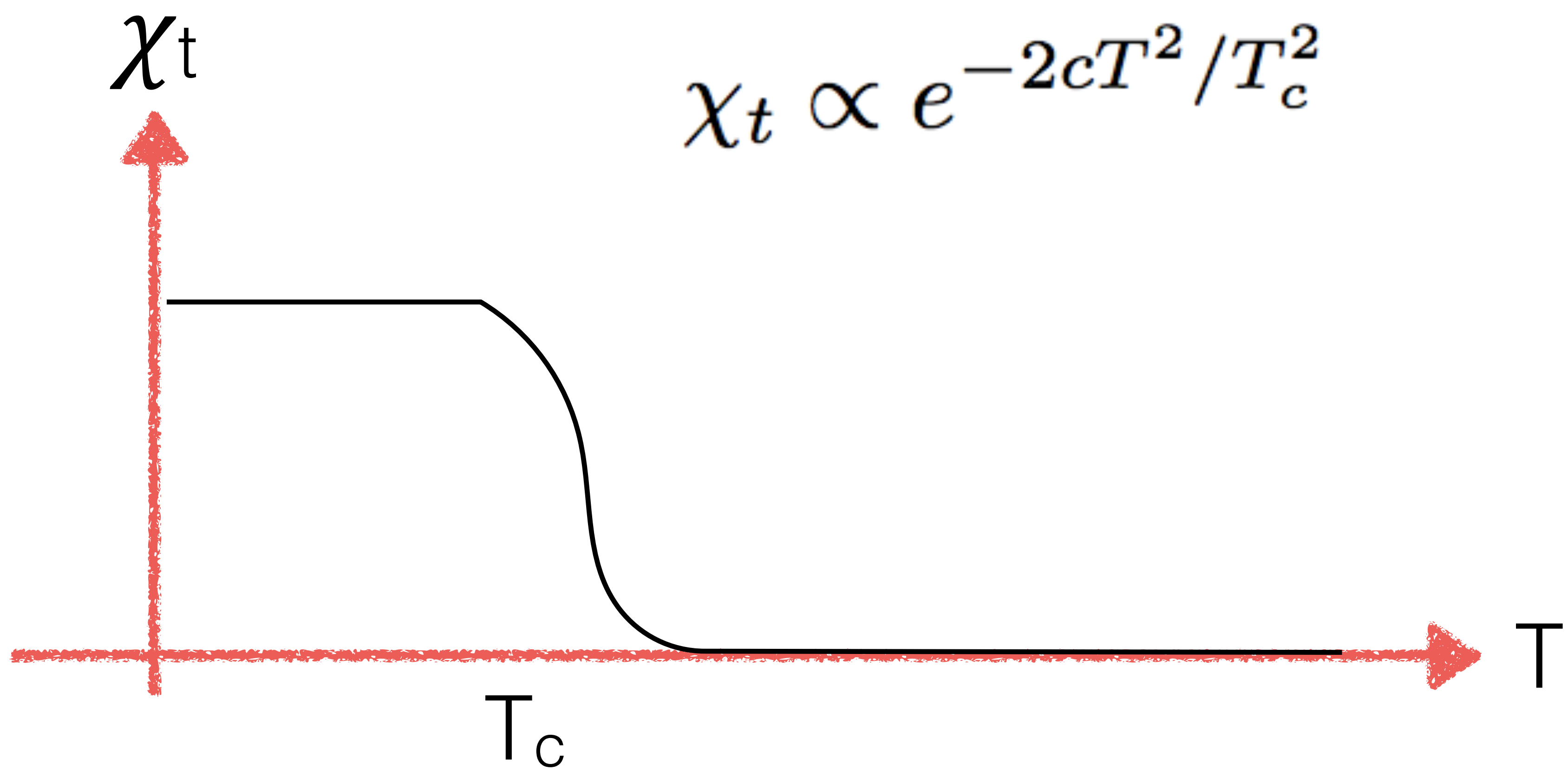
the axion suddenly starts to oscillate at $T=T_c$



$$\Omega_a \sim 2 \times 10^5 \theta_{\rm ini}^2$$ independent of m_a

axion window is gone.

a bit milder case



$$\Omega_a \sim 0.2\theta_{\rm ini}^2 \left(\frac{m_a}{10^{-5}\ {\rm eV}}\right)^{-1} \boxed{\times\, 2.5c} \quad (c \gg 1)$$

enhancement due to the non-adiabatic evolution of the potential.

It seems that
the lattice determination of
χ_{t} is important

χ_t on the lattice

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$

we just need to measure Q in each configuration.

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$

$$= \mathrm{Tr}\gamma_5 = n_+ - n_- \qquad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

There are two ways to measure Q.

Bosonic definition

$$Q = \int d^4x \frac{1}{32\pi^2} F\tilde{F}$$

on the lattice, one would not get integers
due to the ambiguities in the definition of F.

—> The techniques called Cooling or Wilson flow
can make it possible to identify Q.

Fermionic definition

$$Q = \mathrm{Tr}\gamma_5 = n_+ - n_-$$

With a properly defined γ_5, one can get integers.

This method gives unambiguous Q, but the cost of the calculation is high.

Somehow,

in 2015, three independent calculations appeared.

(in the SU(3) Yang-Milles theory, **no quarks yet**)

E. Berkowiz, M. Buchoff, E. Rinaldi (LLNL)

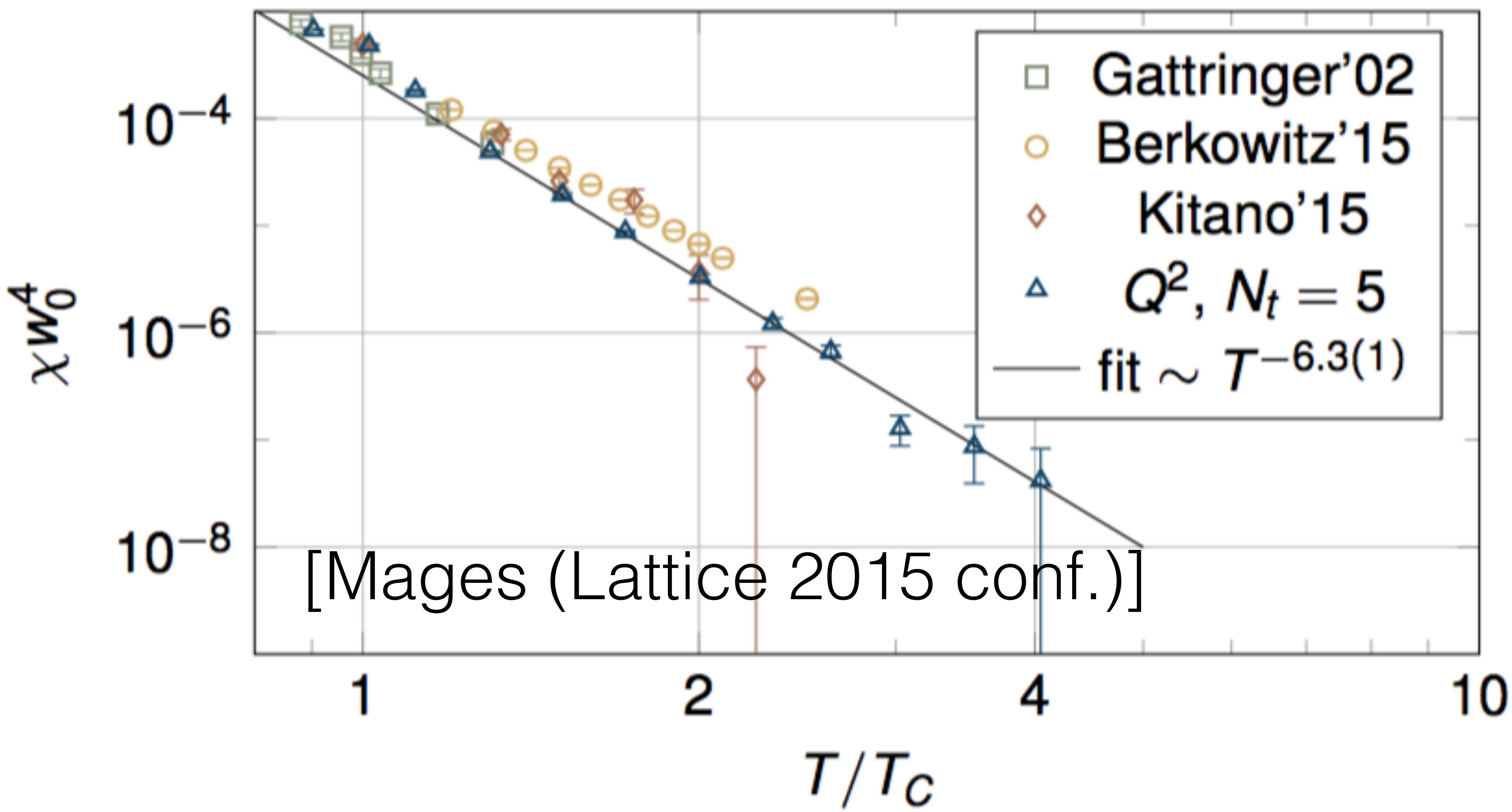
Bosonic (cooling)

RK and N. Yamada (KEK) Fermionic (overlap)

S. Mages et al (BMW) Bosonic (Wilson Flow)

All look consistent

(at least qualitatively)



[Mages (Lattice 2015 conf.)]

We see a clear **power law** even at a very low temperature.

instanton?

The instanton predicts $\chi_t \propto T^{-7}$ for $T \gg T_c$

in SU(3) YM theory

at one-loop level

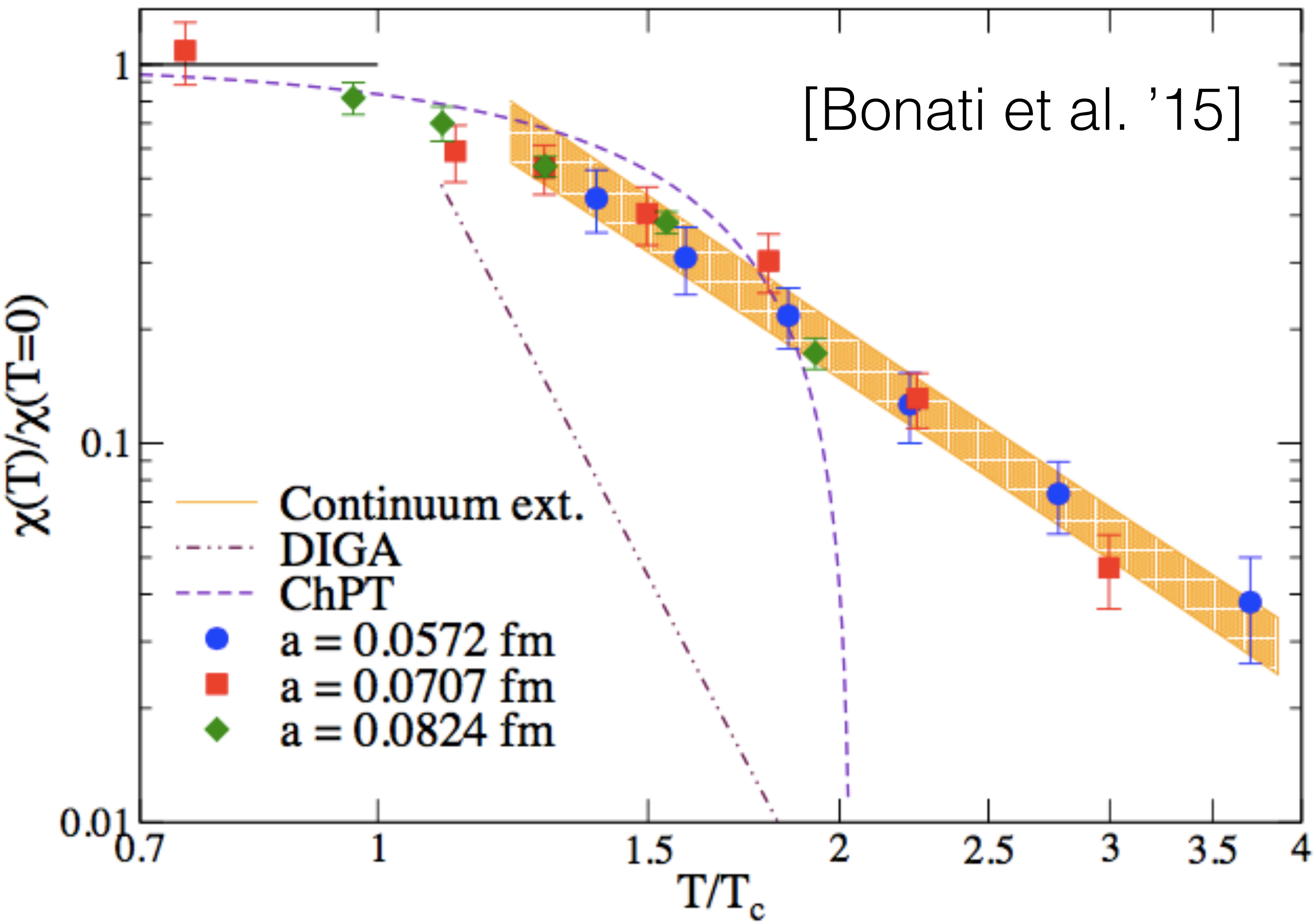
The lattice says

$$\chi_t \propto T^{-6\pm 0.?}$$ T ~ 2-4Tc

It seems that the semiclassical instanton picture
is qualitatively good in YM theories.

But for the axion study, we need to include quarks.

recent progress



very large deviation from instantons!?

(Fukaya seems to get completely different results by using domain wall+overlap reweighting method.)

Summary

χ_t is a fundamental quantity in QCD which measures the effects of topology.

very much related to Strong CP problem

The calculation in YM seems to support the instanton picture, **but anything can happen when we include dynamical quarks**.

more lattice simulations
are necessary to make things clear.