Hubble selection of the weak scale

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Self-organized criticality

~~Why is the SM Higgs mass so small?~~
Why is the Higgs mass so close to a critical point = 0?

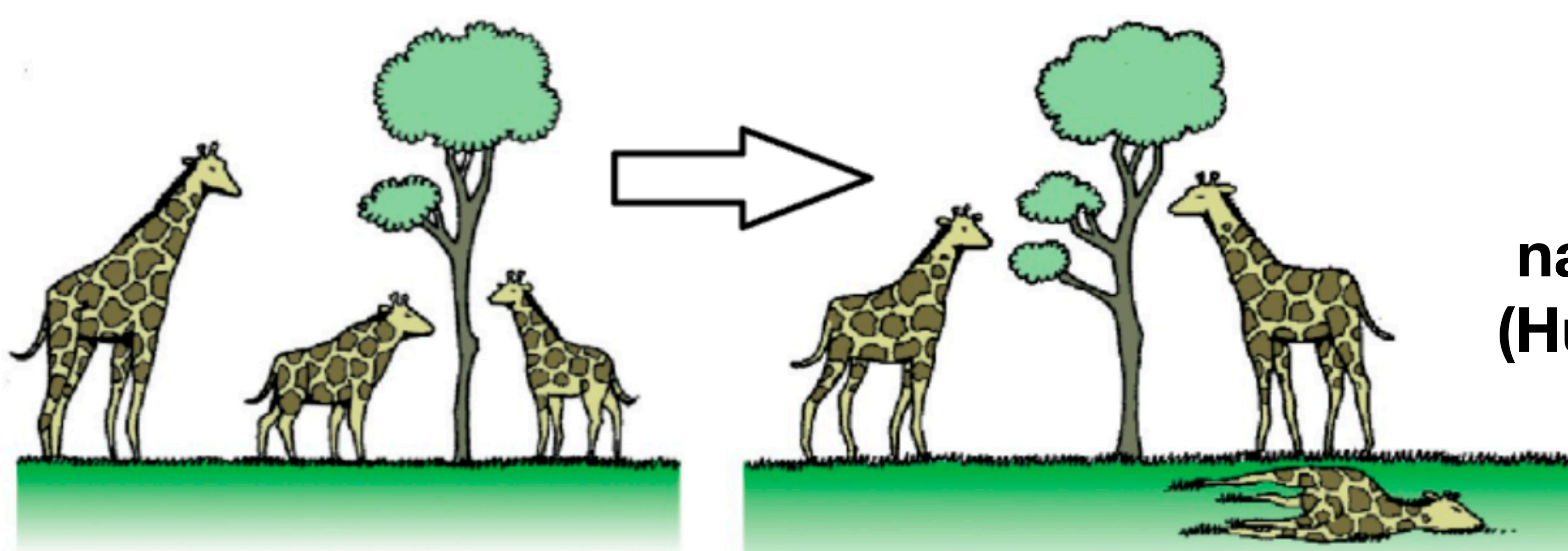
Is there any sharp critical point near the weak scale?
How is the weak scale localized close to it?
And, can it be done naturally?

Quantum driven Hubble selection

In the evolution of the universe,
"quantum fluctuations" during inflation occasionally take us to the regime which is *inaccessible classically*.

Albeit very rare, occasionally it can dominate the evolution afterwards.

mutations (quantum fluctuations)



natural selection (Hubble selection)

Most Hubble patches

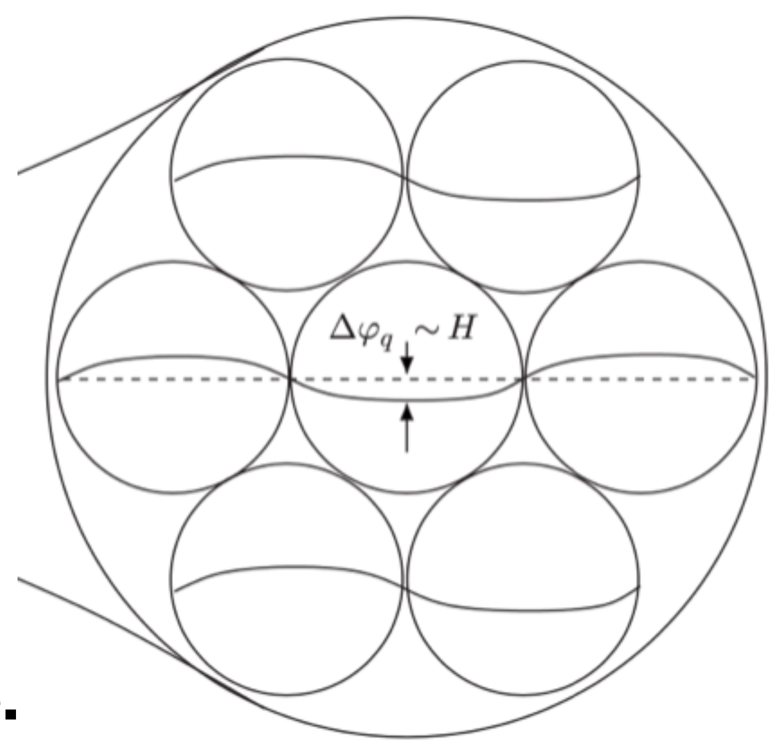
Hubble patches form a statistical ensemble of the universe:

- A large number of patches; Causally disconnected.

- Own theory parameters, varied by scalar field values whose long wave modes are distributed across them.

- Our goal is to realize that the majority of patches have a common weak scale, using that it might be near critical.

“ Hubble selection of criticality ”.



Sum of the long modes determine the field value.

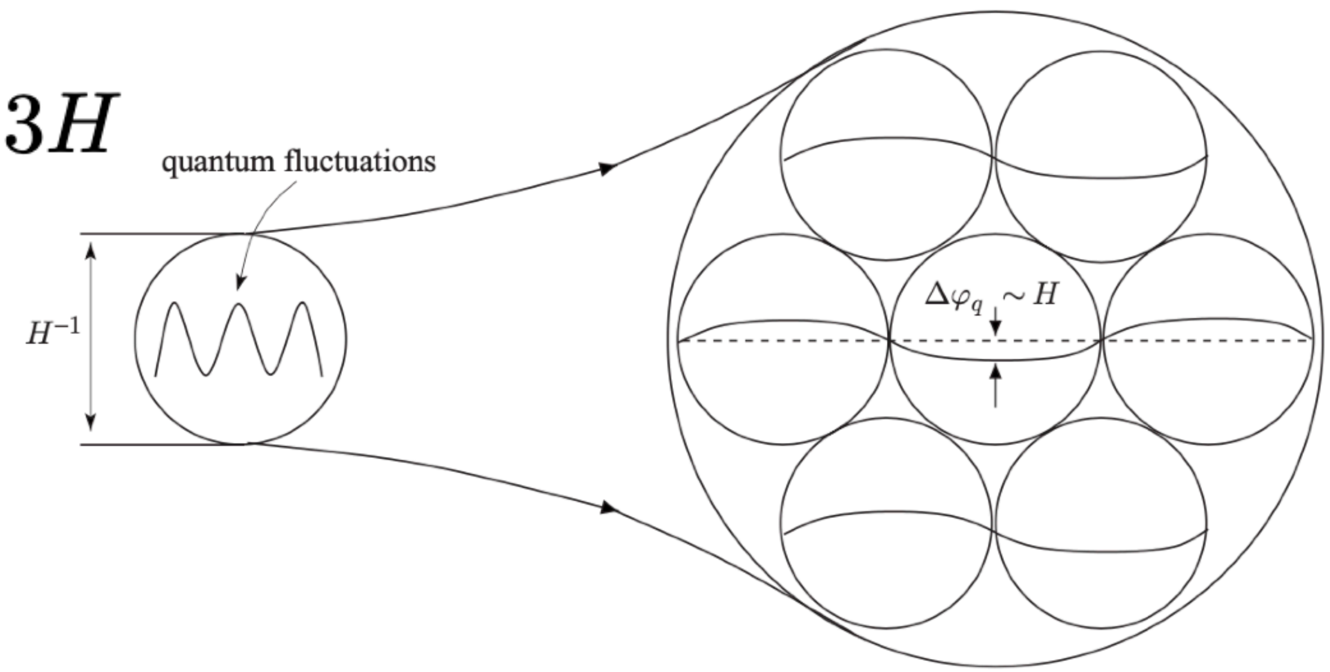
Topics

1. Hubble selection
2. QCD quantum critical points
3. The weak scale criticality
4. Naturalness

1. Scalar field value

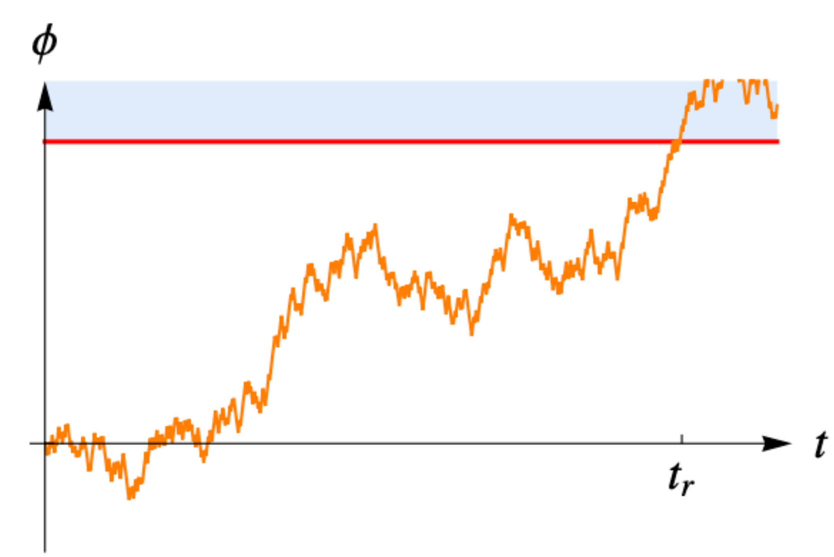
During inflation, scalar field value is subject to

(1) classical rolling $\dot{\phi}_c = -V'/3H$



(2) random walk ~ H
from the long modes that
just exited the horizon.

Since quantum fluctuations are symmetric,
it always rolls down on average
(albeit in a stochastic motion).



Global field value distribution

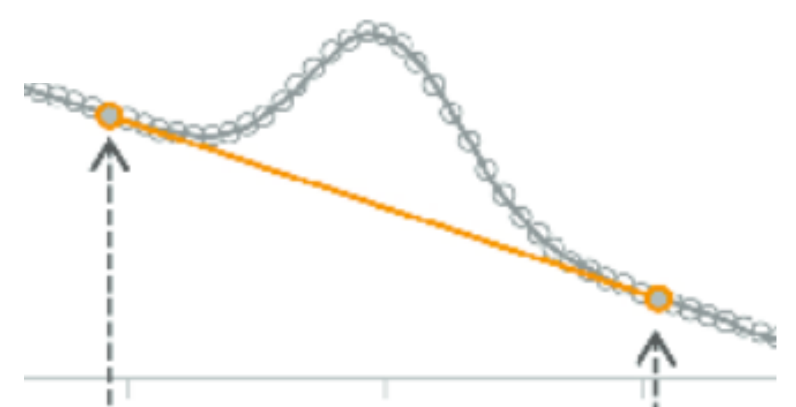
Its probability distribution (among Hubble patches) is diffused due to the random quantum fluctuation of order Hawking T.

$$\sigma_\phi(\bar{t})^2 = \frac{H^2}{4\pi^2} H t$$

Finite Hubble rate difference within the distribution can make a difference in the *global* field value distribution (FPV)

Hubble volume effect

$$\frac{\partial \rho(\phi,t)}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'}{3H}\rho\right) + \frac{1}{8\pi^2}\frac{\partial^2 (H^3 \rho)}{\partial \phi^2} + 3\Delta H \rho$$



Hubble selection

In the linear regime of a potential (no boundaries),
FPV is solved by

$$\rho(\phi,t) \propto \exp\left\{-\frac{1}{2\sigma_\phi}\left[\phi-\left(\phi_0+\dot{\phi}_c t+\tfrac{3}{2}(\Delta H)'\sigma_\phi^2 t\right)\right]^2\right\}$$

$$\sigma_\phi(t)^2 = \frac{H^2}{4\pi^2}Ht \qquad \dot{\phi}_H = 3(\Delta H)'\sigma_\phi^2$$

Hubble selection starts to operate when the peak *climbs* via the overturn between classical and quantum rollings:

$$\sigma_\phi^2 \simeq \tfrac{2}{3}M_{\rm Pl}^2.$$

The width at this moment is always Planckian (quantum nature).

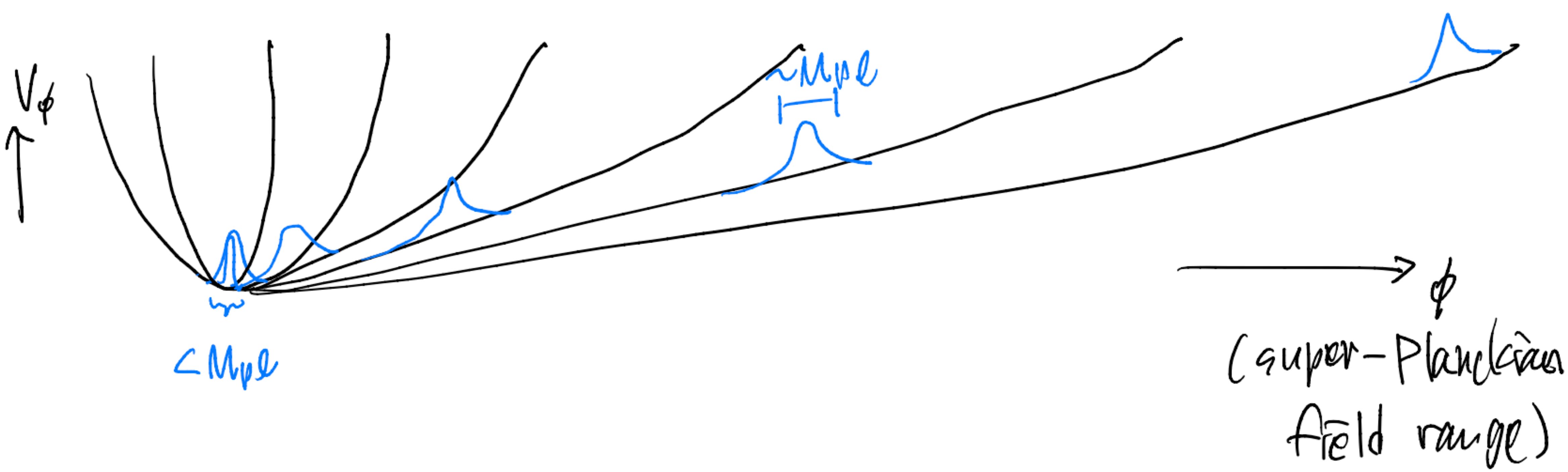
It takes time, field range, Planckian width to be Hubble driven.

Self-organized criticality

G.Giudice et al 21

The global distribution equilibrates close to the top or upper boundary of the potential (most usefully, critical point).

- the flatter the potential, the closer to the crit pt.
- cannot be closer than Mpl (uncertainty principle).
- the eq width is always Mpl (unless too close to the crit).



Eternal inflation

N.Arkani-Hamed et al 07

e-folding until it starts climbing $>$ de-Sitter entropy bound

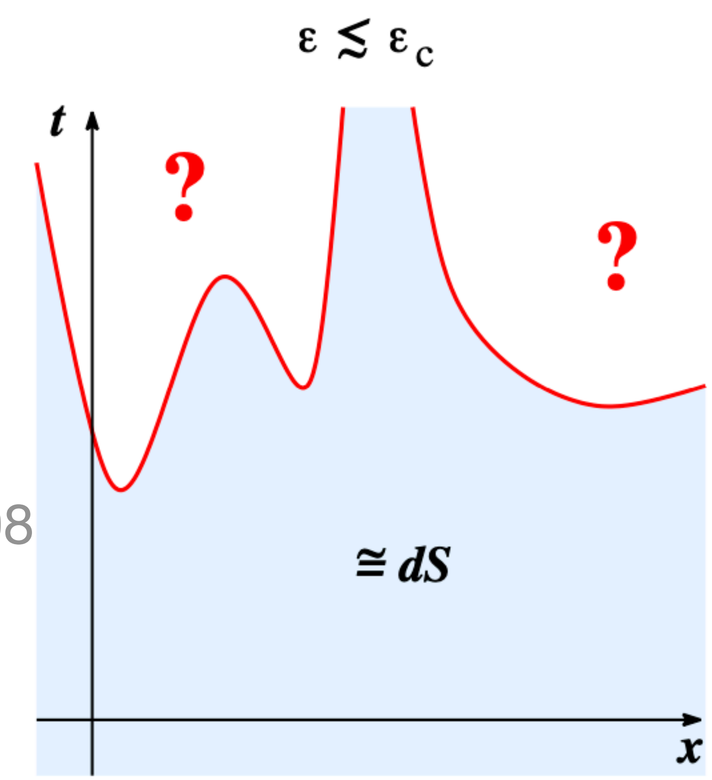
→ Eternal inflation is necessary for Hubble selection.

Then, how do we define probability distribution?

It is defined among Hubble patches that have reached reheating.

→ Stationary states or eq distribution only matters.

A.Vilenkin 95,97
L.Senatore et al 08

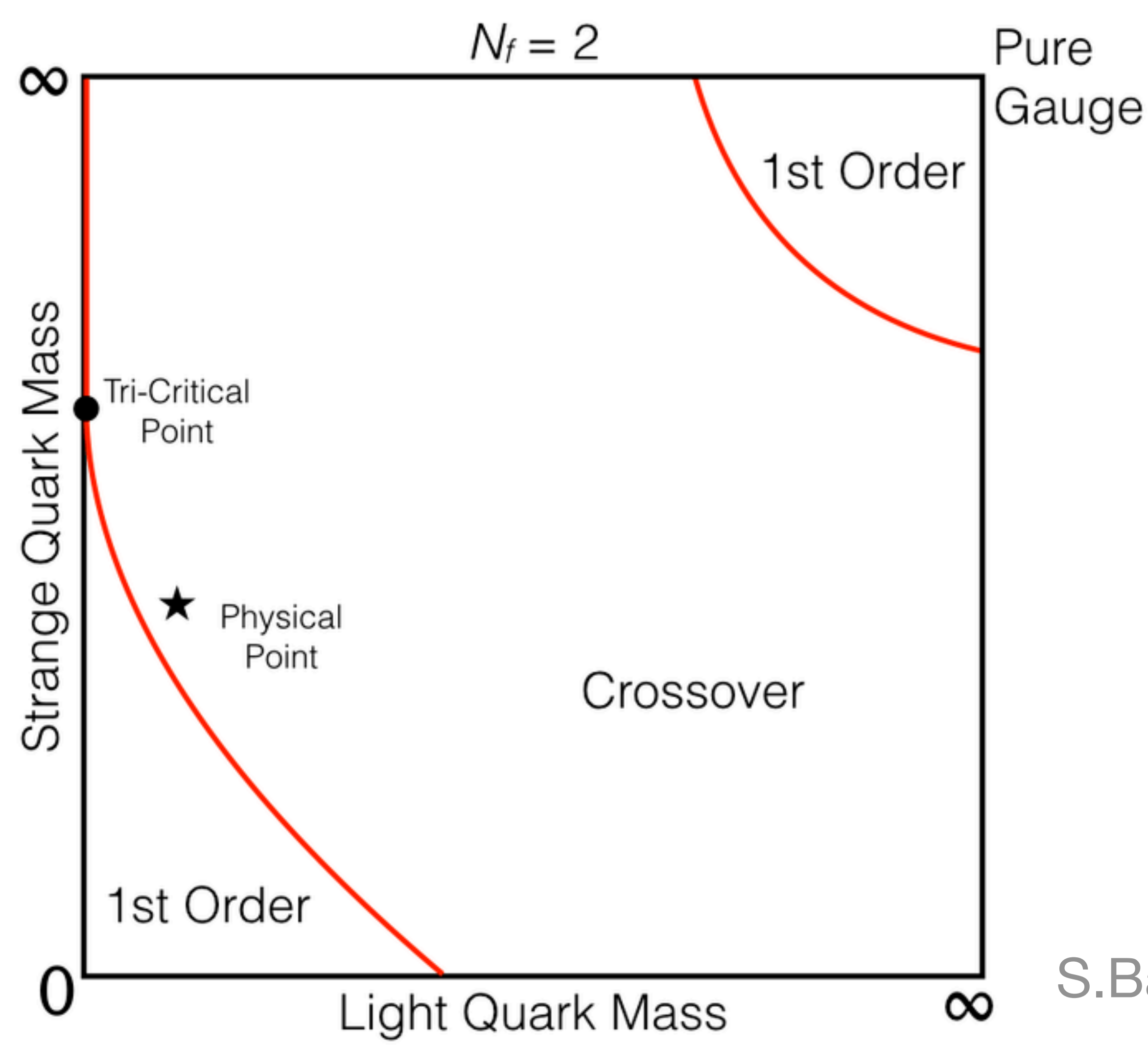


2. QCD quantum critical points

If m_s were slightly smaller, QCD (finite-T) phase transition could have been first order! (but not confirmed yet) F.Wilczek et al 84

But the possibility of QCD quantum phase transition as a fn of quark masses has never been studied.

The QCD scale is close to the observed weak scale and naturally small by dimensional transmutation.



S.Bartz et al 17

QCD quantum critical points

QCD vacuum structure at T=0 can be studied with linear sigma model (LSM) with Nf=3:

$$V_\Sigma = \mu^2 \mathrm{Tr}[\Sigma\Sigma^\dagger] + \lambda_1 (\mathrm{Tr}[\Sigma\Sigma^\dagger])^2 + \lambda_2 \mathrm{Tr}[(\Sigma\Sigma^\dagger)^2]$$
$$-c(\det\Sigma + \det\Sigma^\dagger) - \mathrm{Tr}[\mathcal{H}(\Sigma + \Sigma^\dagger)],$$

Gell-Mann, Levy 60
B.W.Lee 70

- SU(Nf) x SU(Nf) flavor symmetry.
- Meson condensation is an order parameter for QCD chiral symmetry breaking.
- LSM is relatively easy to handle, as a particle physicist.

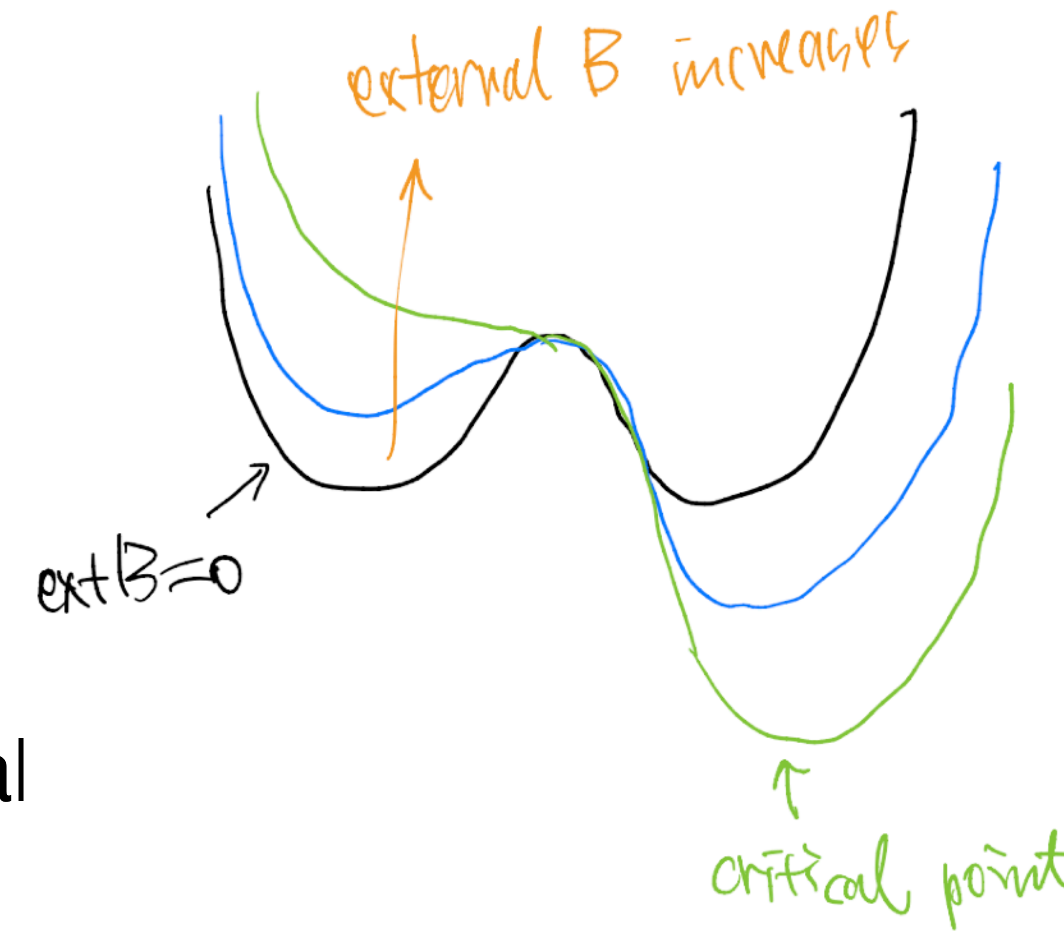
QCD quantum critical points

Most importantly, QCD LSM indeed contains necessary features for *quantum critical point!*

$$V_\Sigma = \mu^2 \mathrm{Tr}[\Sigma\Sigma^\dagger] + \lambda_1(\mathrm{Tr}[\Sigma\Sigma^\dagger])^2 + \lambda_2 \mathrm{Tr}[(\Sigma\Sigma^\dagger)^2]$$
$$-c(\det\Sigma + \det\Sigma^\dagger) - \mathrm{Tr}[\mathcal{H}(\Sigma + \Sigma^\dagger)],$$

- c: cubic instanton interactions,
creating co-existing vacua.

- H: mass (Higgs vev dependence!)
External B-field for ferromagnets,
destabilizing the local vacua at vh*.

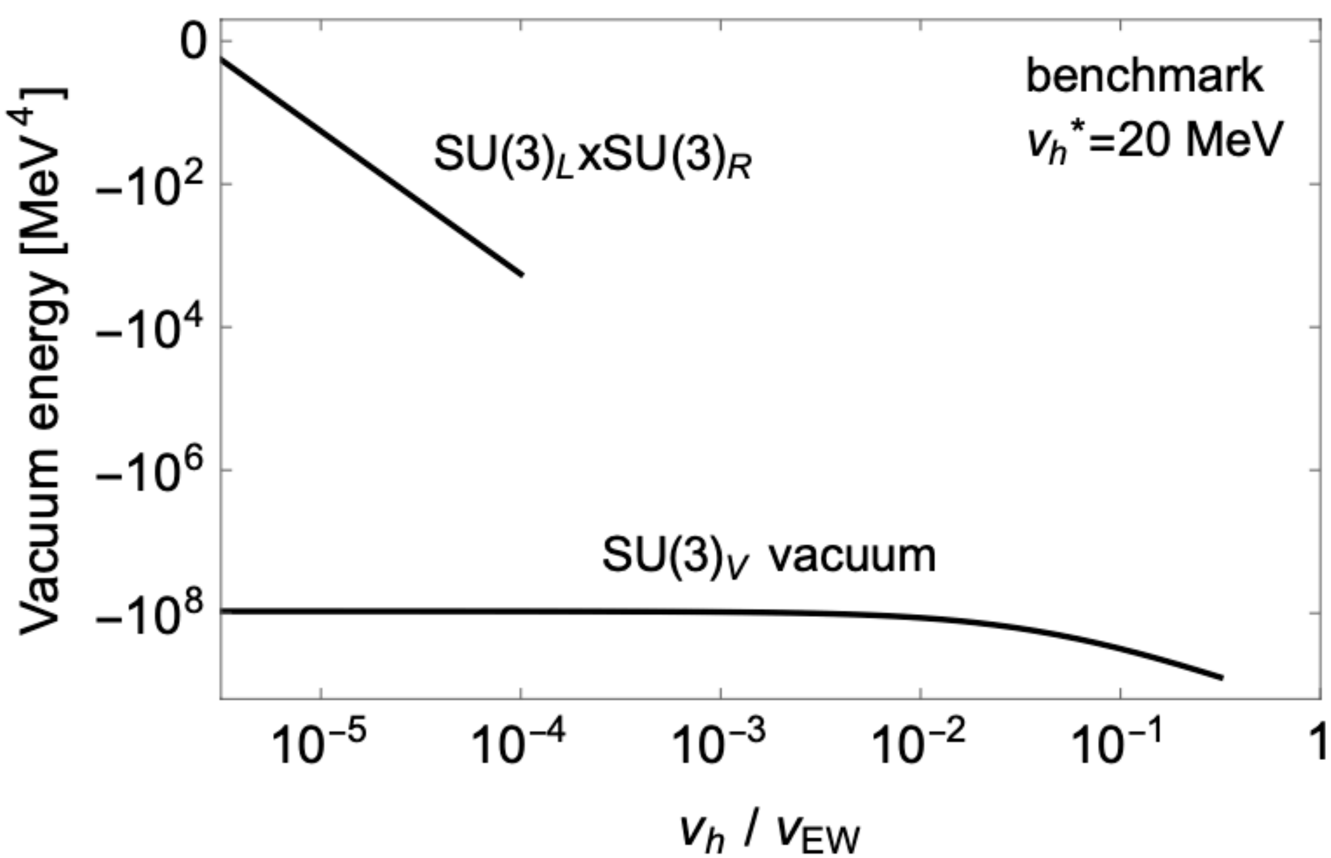


But QCD LSM is at least 18 dimensional

QCD quantum critical points

We have successfully found a range of LSM parameter space:

- containing co-existing vacua at vh=0.
- consistent with meson spectrum, (chi^2/dof ~ 3)



vh* = 20 MeV

(but any value near LQCD is found)

3. Model for weak scale criticality

Quantum-driven relaxion:

- Relaxion scans the Higgs vev as usual, while climbing up.
- If evolves toward vh*, which has criticality in the QCD.
- At the time of reheating, most Hubble patches vh ~ vh*.

$$V_\phi = \Lambda_\phi^4 \cos \frac{\phi}{f_\phi}$$

$$V_h = \frac{1}{2}(M^2 - g\widetilde{\phi})h^2 + \frac{\lambda_h}{4}h^4 \rightarrow -\frac{1}{2}(g\phi)h^2 + \frac{\lambda_h}{4}h^4$$

Near the QCD critical point

- Total V must peak near vh*.
- Total V should drop significantly and never be compensated.

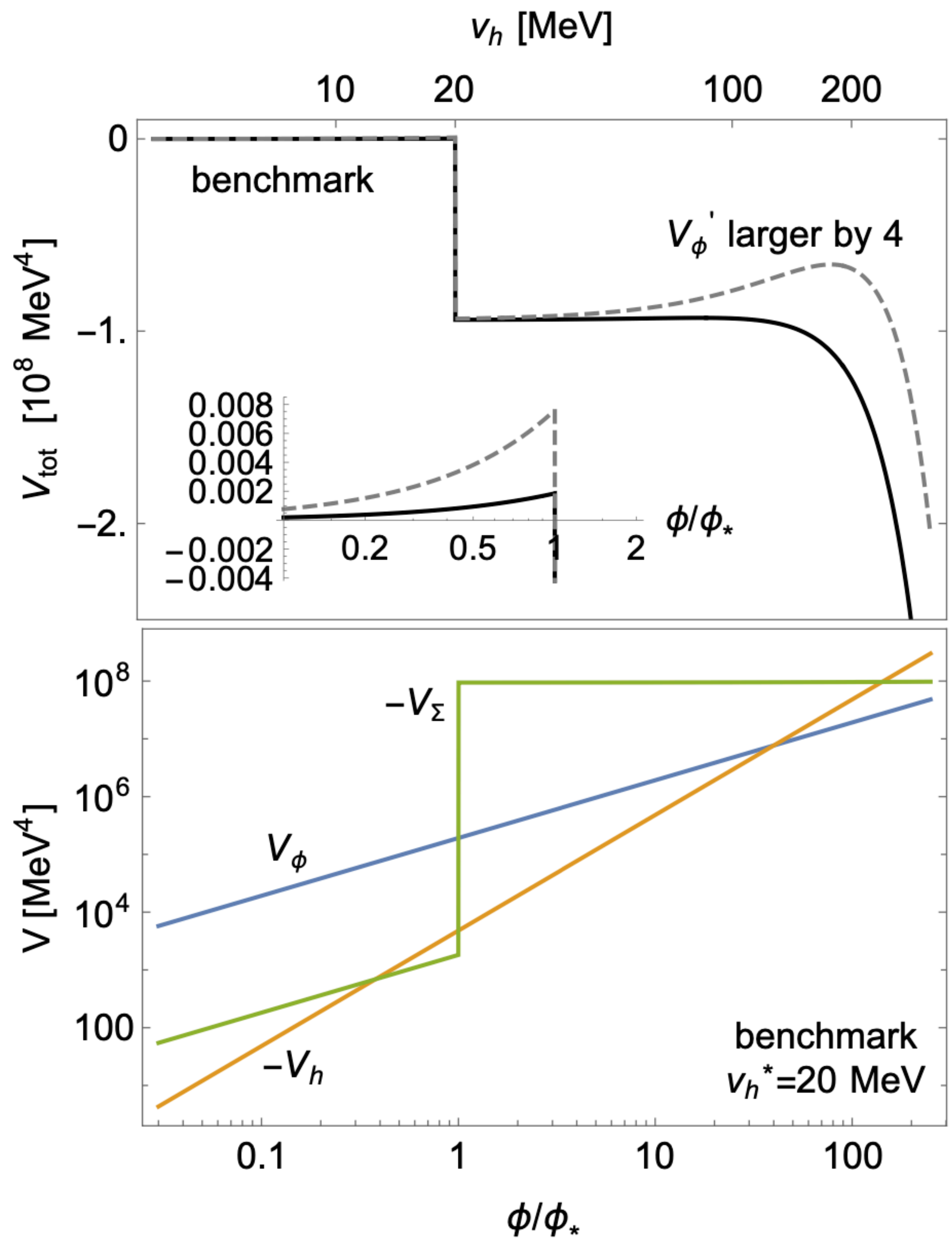
$$v_h^* \lesssim \Lambda_\phi^2/M \lesssim \Lambda_{\rm QCD}.$$

- h and Sigma should be well equilibrated in their local min.

$$H \lesssim v_h^*.$$

- Relaxion should not dominate the inflation dynamics.

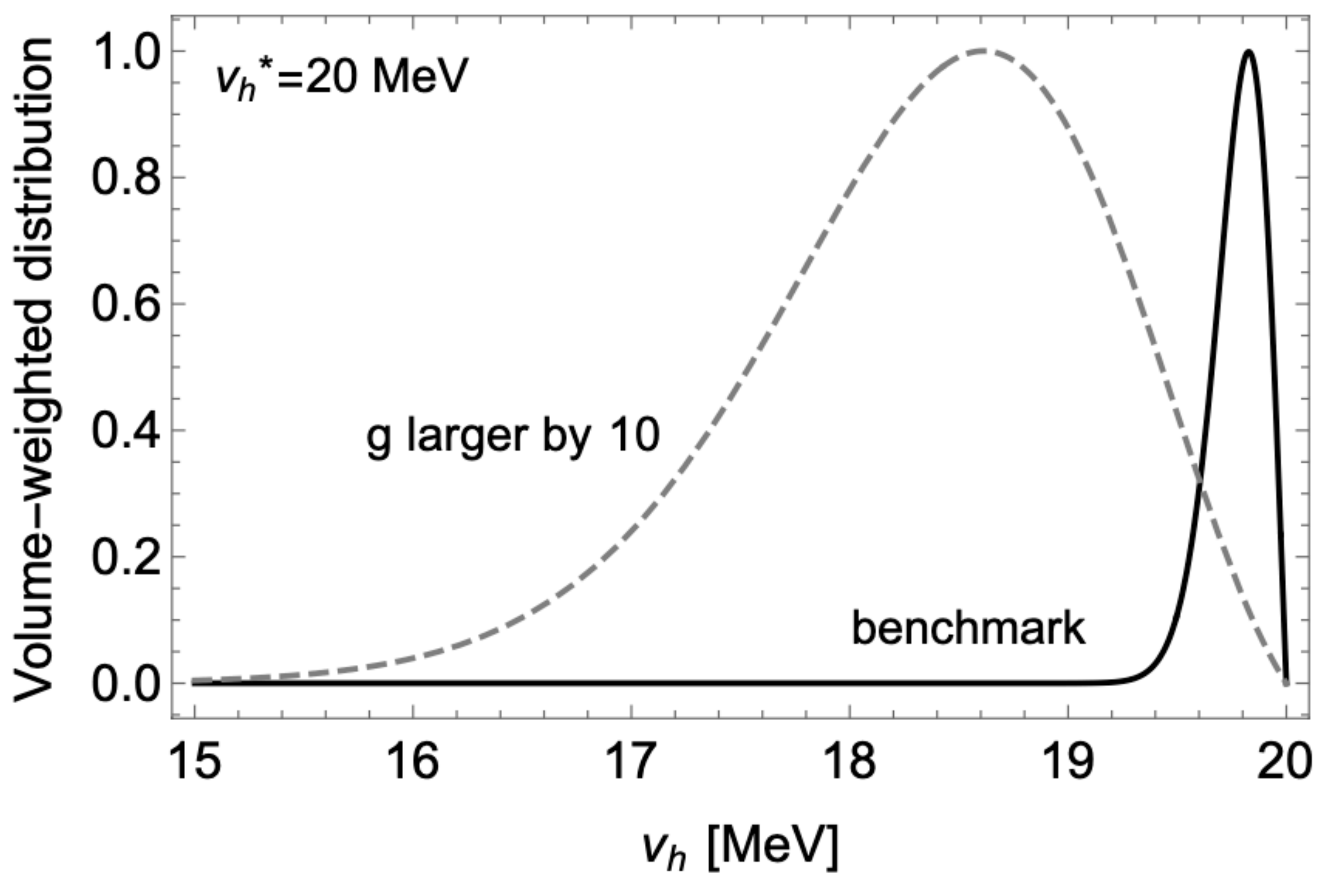
$$\Lambda_\phi^4 \lesssim H^2 M_{\rm Pl}^2.$$



vh probability distribution

The phi width ~ Planckian, independently of potential slope.

Then the vh width decreases with the flatter potential (as phi field range increases, the width fraction decreases.).

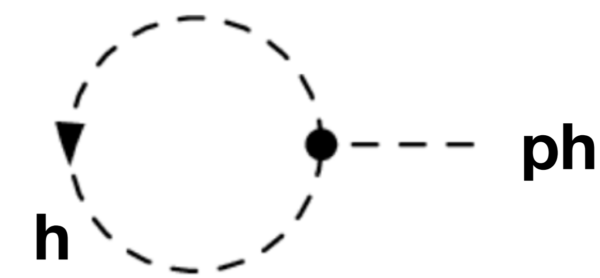


Naturalness

The minimal model itself does not solve hierarchy problem.

$\Lambda_\phi \ll M$ is required for LQCD-scale criticality to determine the dynamics of the order Λ_ϕ^2/M .

But this separation is quantum unstable.



Hubble selection of criticality can work well,
but still need fine-tuning in the model parameters.

(Scale hierarchy can be translated to other fine-tuning.)

Summary

Is our universe a result of the self-organized criticality?

Q: What quantum critical points does the SM have and does the Higgs mass have relevance to?

We have explored possible QCD quantum critical points and calculated their roles for the weak scale criticality. (But more dedicated studies are needed to verify.)

Q: (How) can it be realized (naturally)?

Hubble selection is one way for a theory to be well localized. But natural selection of the weak scale is still challenging.

Thank you