Cosmic neutrino background and (invisible) neutrino decay

MASAHIDE YAMAGUCHI

(Tokyo Institute of Technology)

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Kensuke Akita, MY : 2005.07047, JCAP08(2020)012
Kensuke Akita, Gaetano Lambiase, MY : 2109.02900

(Some slides and figures are used by courtesy of Kensuke)

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

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- **Introduction**
  - Cosmic background (radiations)
  - Spectrum distortion
  - Direct detection experiment
- **(Invisible) decay of a neutrino**
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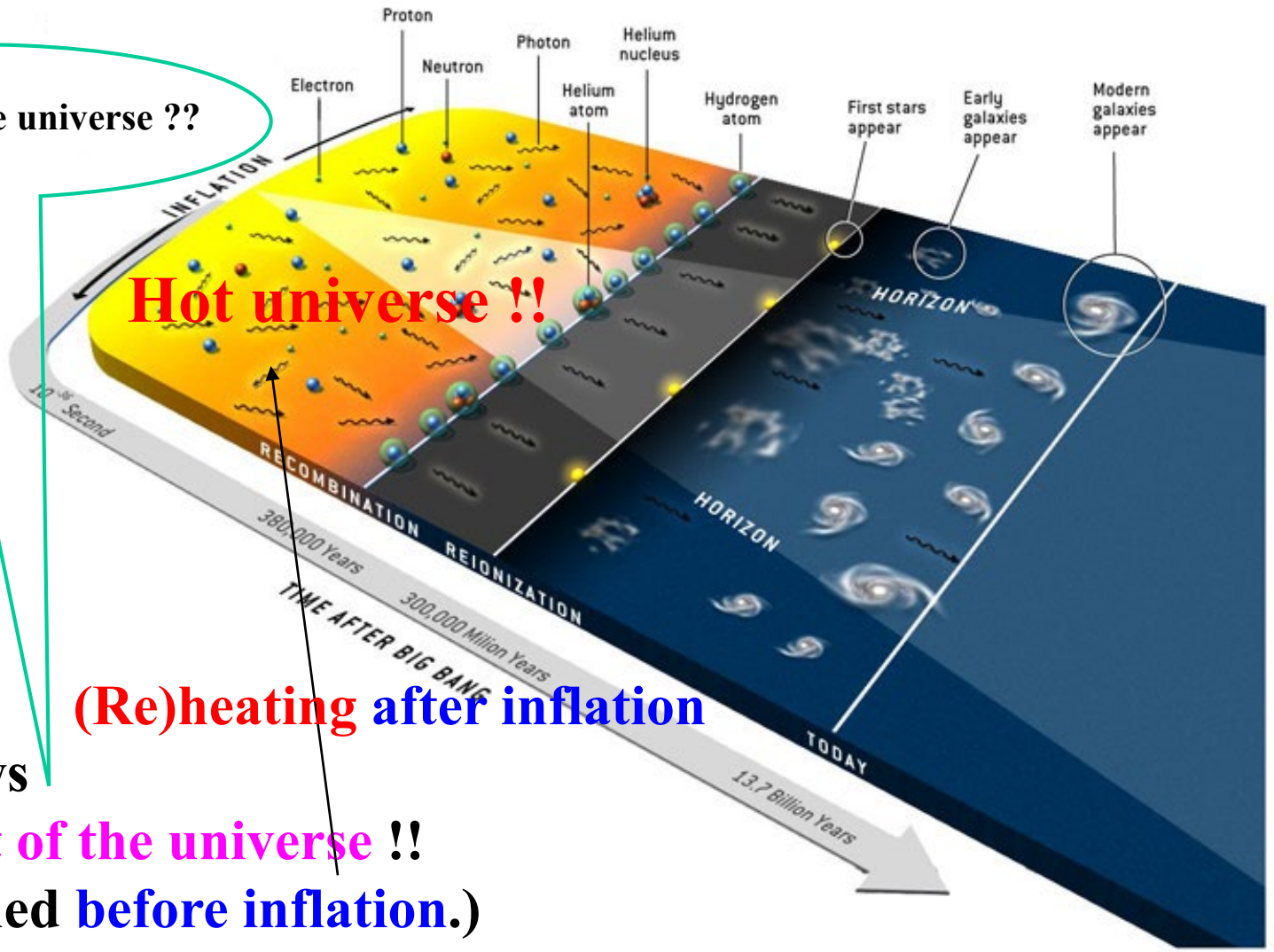
Introduction

Brief history of the universe

http://astrokatie.blogspot.jp/2012/08/the-long-dark-tea-time-of-cosmos.html

The birth of the universe ??



Hot universe !!

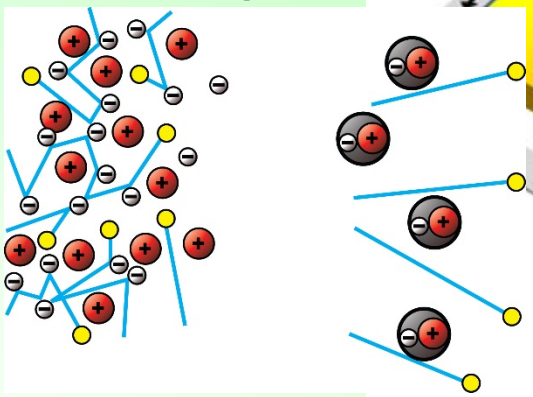
(Re)heating after inflation

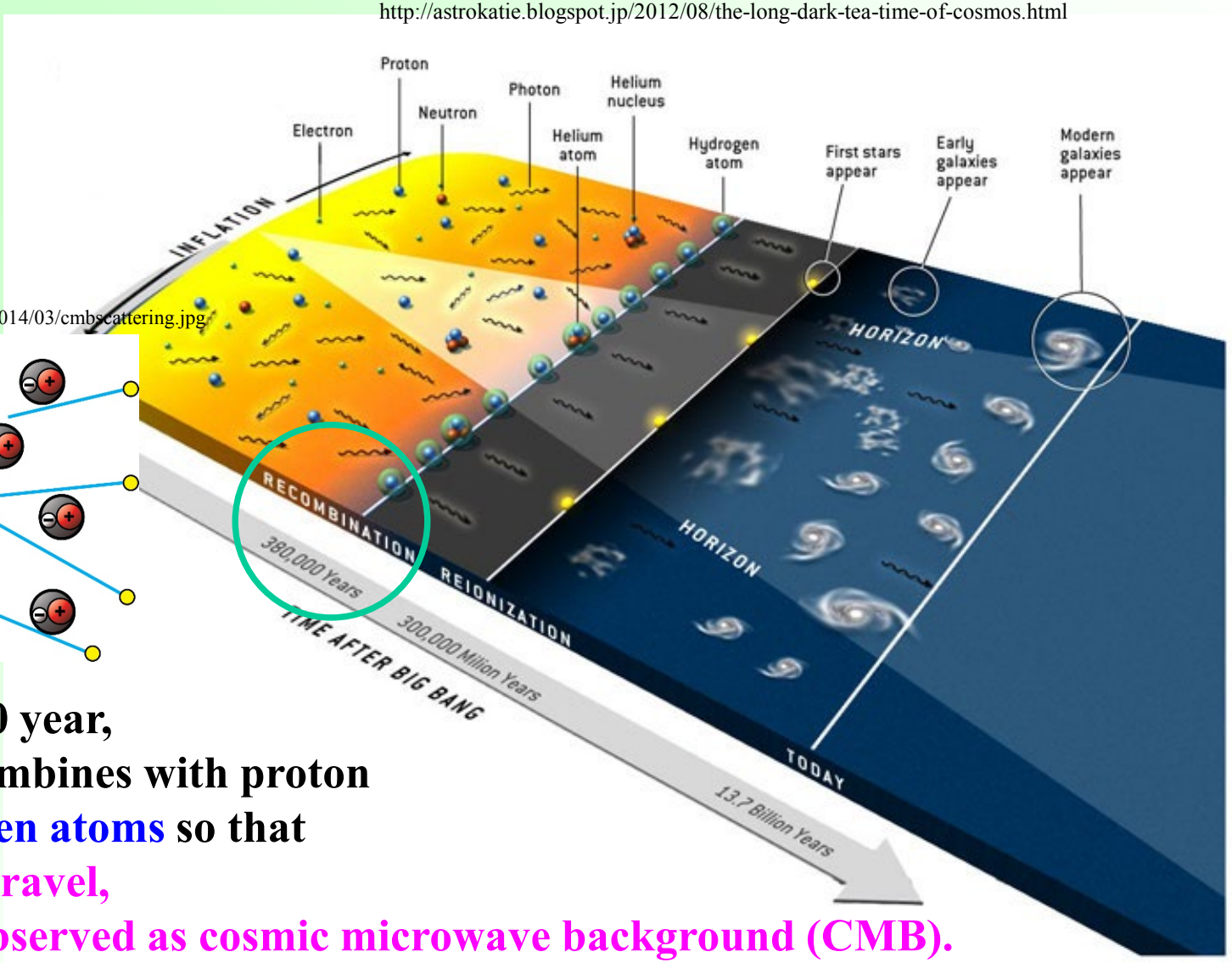
Nobody knows
the true onset of the universe !!
(what happened before inflation.)

Three important primordial background (radiations)

http://astrokatie.blogspot.jp/2012/08/the-long-dark-tea-time-of-cosmos.html

https://writescience.files.wordpress.com/2014/03/cmbscattering.jpg



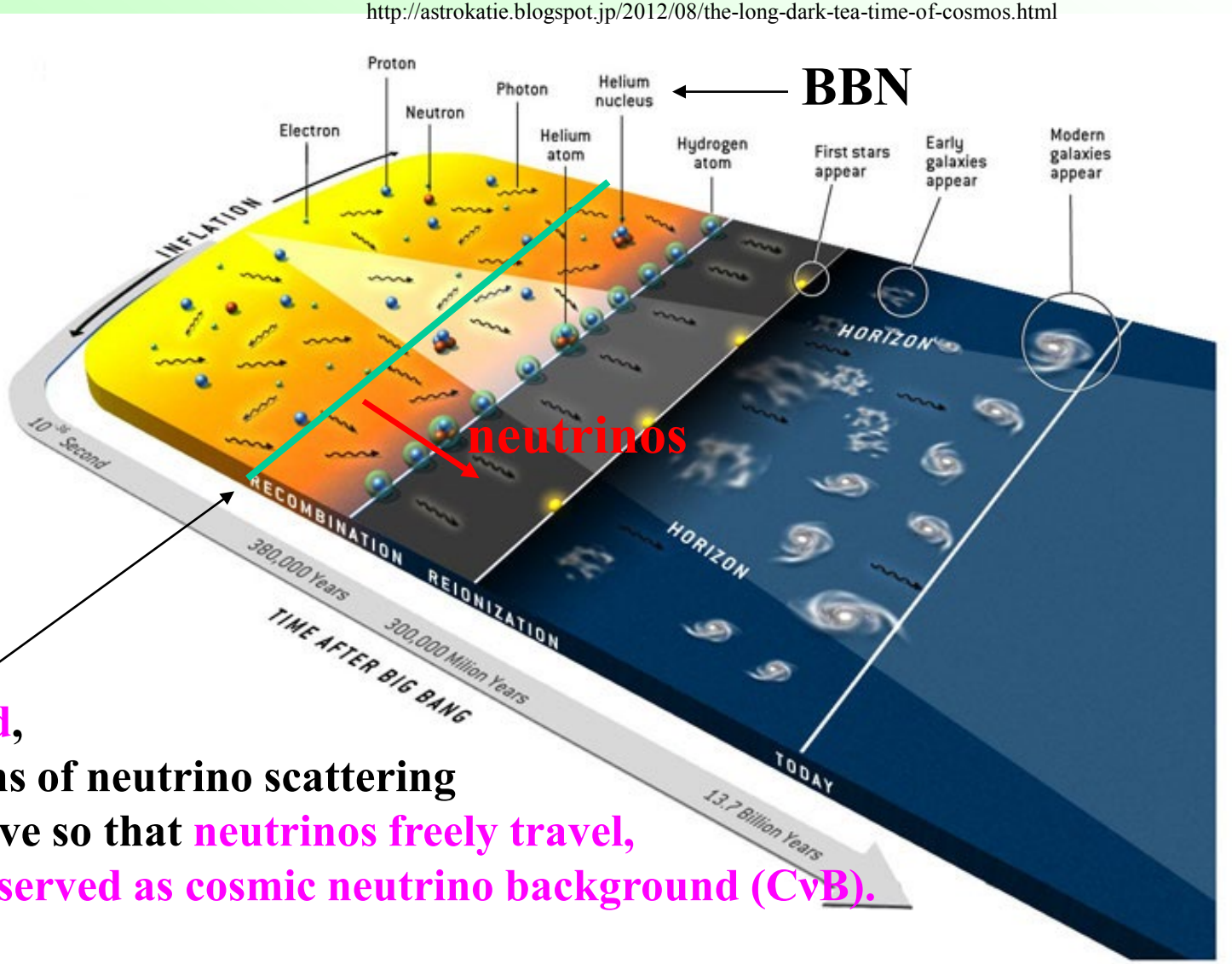


Around 380,000 year,
free electron combines with proton
to form hydrogen atoms so that
photons freely travel,
which can be observed as cosmic microwave background (CMB).

One of them is CMB and another is GW generated during inflation.

Cosmic neutrino background

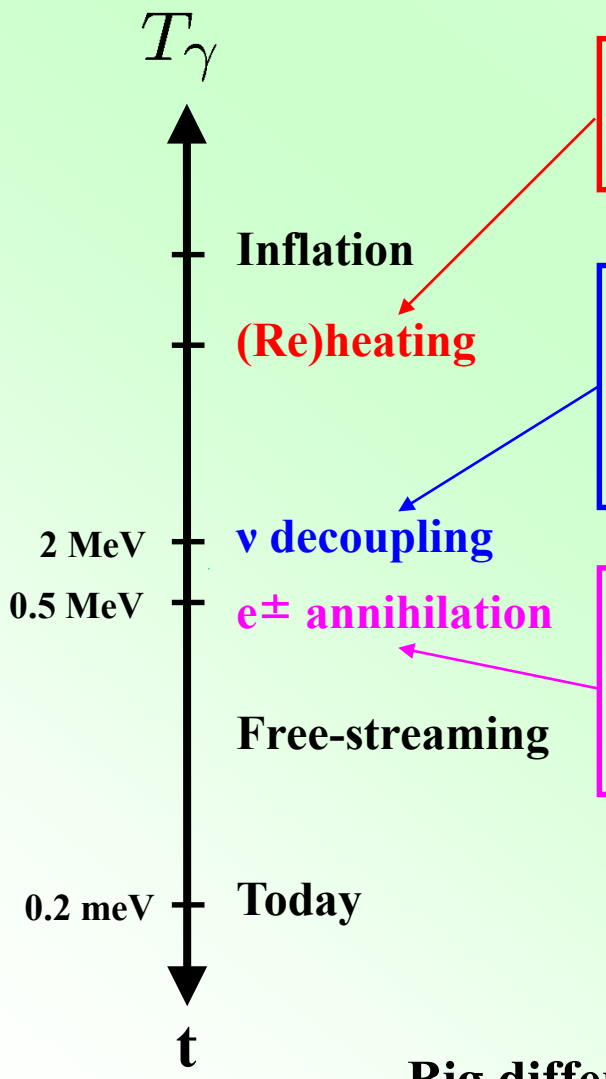
http://astrokatie.blogspot.jp/2012/08/the-long-dark-tea-time-of-cosmos.html



Around 1 second,
weak interactions of neutrino scattering
become ineffective so that neutrinos freely travel,
which can be observed as cosmic neutrino background (CvB).

Yet another is CvB.

History of cosmic neutrino background



The SM particles (including neutrinos) are produced by the decay of an inflaton and are in thermal equilibrium.

Weak interaction rate ~ Hubble expansion rate

$$\Gamma_w \sim G_F^2 T_\nu^2 \quad \sim \quad H \sim T_\nu^2 / M_{\rm pl}$$

$$\Rightarrow \quad T_\nu \sim 2\,{\rm MeV}$$

$$\nu\nu \leftrightarrow \nu\nu$$
$$\nu\bar{\nu} \leftrightarrow \nu\bar{\nu}$$
$$\nu e^\pm \leftrightarrow \nu e^\pm$$
$$\nu\bar{\nu} \leftrightarrow e^+ e^-$$

At $T_\gamma \sim m_e$, $e^+ e^- \to \gamma\gamma$

Photons are heated in comparison to neutrinos :
(Total entropy is conserved)

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

$$\Rightarrow \quad N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\left(\frac{\rho_{\rm rad}}{\rho_\gamma} - 1\right) = 3$$

Instantaneous decoupling approximation

Big difference from CMB (& GW) :

Fermion, massive, weak interaction, three flavors, …

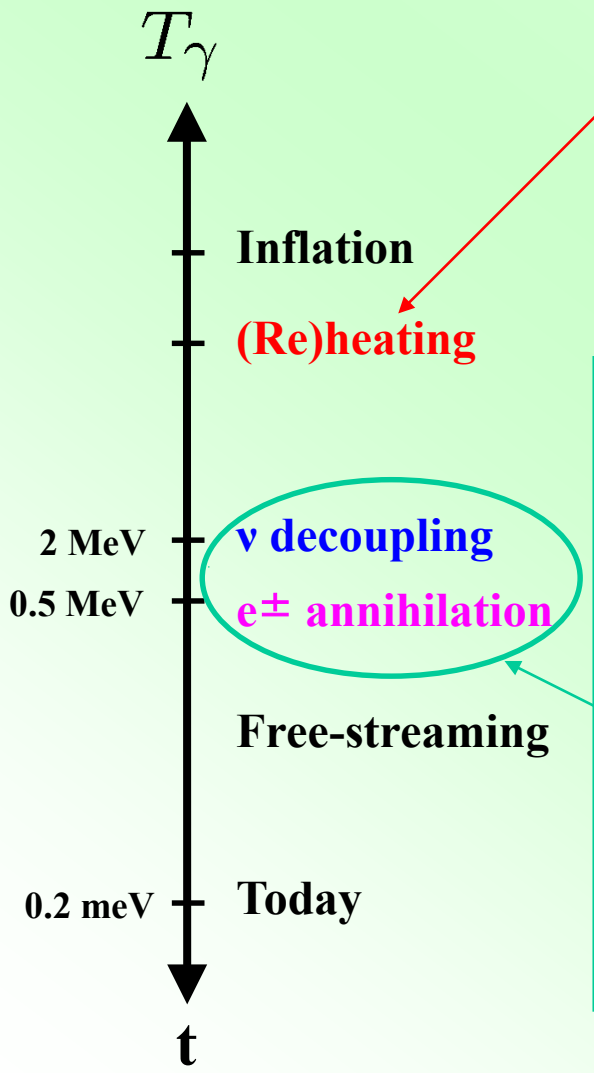
Two important features of background (radiations).

- **Spectrum**
  (deviation of thermal or scale-invariant spectrum)

- **Anisotropy**

- **Spectrum**
  (deviation of thermal or scale-invariant spectrum)

Effective neutrino number (deviation from Fermi-Dirac distribution)



The SM particles (including neutrinos) are produced by the decay of an inflaton and are in thermal equilibrium.

$T_{dec} \sim 2$ MeV and $T_{ann} \sim 0.5$ MeV are very close.

⇒ Neutrinos with high energies still interact with e±.

⇒ e± pairs slightly annihilate into neutrinos.

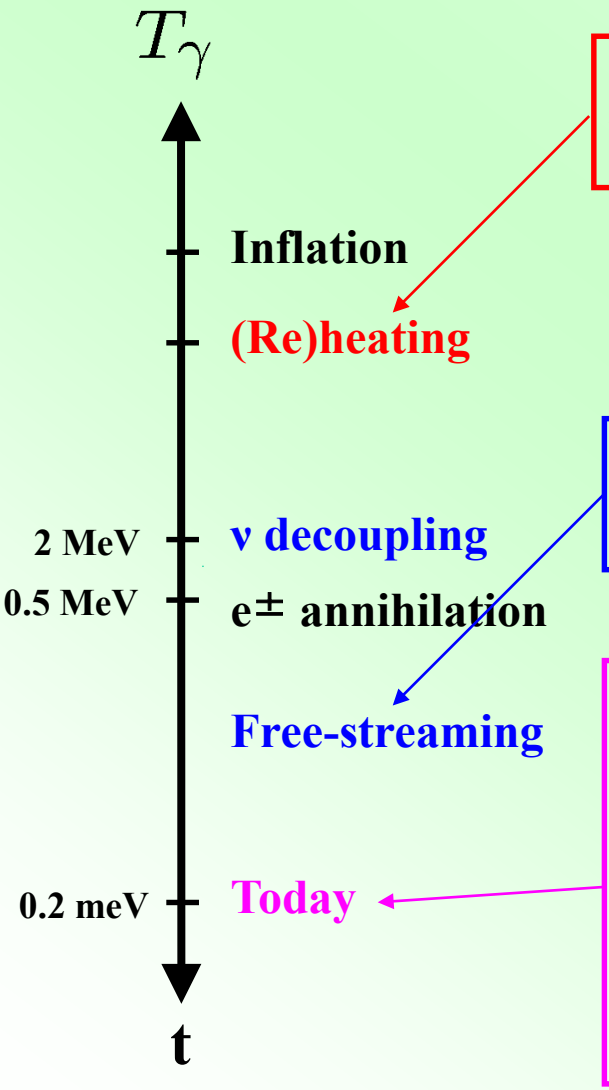
⇒ N_{eff} will be increased.

$N_{eff} = 3.044$ (Akita & MY 2020, Froustey et al. 2020, Bennett et al. 2020)

(Three flavor oscillation & finite temperature correction up to $O(e^3)$)

Any deviation from this value suggests new physics.

Non-relativistic neutrinos (at present)



The SM particles (including neutrinos) are produced by the decay of an inflaton and are in thermal equilibrium.

All of neutrinos were highly relativistic !!

During free streaming, the physical momenta of neutrinos decrease due to cosmic expansion.

$\langle p \rangle \sim 0.53\,\mathrm{meV} \ll \sqrt{\Delta m^2_{21}}, \sqrt{|\Delta m^2_{31}|}$

⇒ *At least two neutrino species are non-relativistic.*

⇒ The formulation in the mass diagonal basis rather than that in the flavor basis is necessary.
(Akita & MY 2020)

(No mode function $\propto e^{i(\sqrt{k^2+m^2}\,t - \boldsymbol{k}\cdot\boldsymbol{x})}$ in the flavor basis)

Background kinetic equations

Background equations ➔ Spectral distortion of CvB

Information of neutrino mass

- **Boltzmann Equation :** Diagonal in mass basis; Matter effects of e±; (Sigl & Raffelt 1993)

$$(\partial_t - Hp\partial_p)\rho_p(t) = -i\left[\left(\frac{M^2}{2p} - \frac{8\sqrt{2}G_F p}{3m_W^2}E\right),\ \rho_p(t)\right] + C[\rho_p(t)]$$

Neutrino oscillation in flavor basis

Collision term (scattering, annihilation)

Density matrix : $\langle a^\dagger_\beta(\boldsymbol{p},t)a_\alpha(\boldsymbol{p}',t)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')\,(\rho_p)_{\alpha\beta}$

(diagonal: spectrum, off-diagonal: transition amplitude)

- **Energy conservation Equation :** $\dfrac{d\rho}{dt} = -3H(\rho + P)$

Akita & MY 2020 made the formulation in the mass diagonal basis.

➔ **Easily extrapolate the spectrum to the current (non-relativistic) epoch.**

(There are still some uncertainties such as the clustering effects.)

Current observational constraints on effective neutrino number

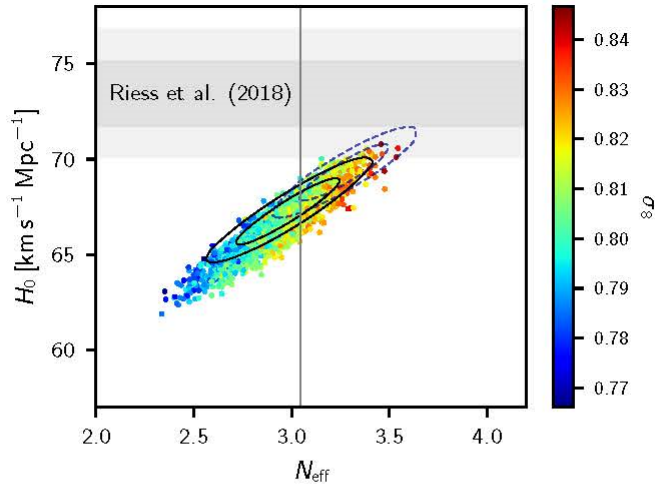


Fig. 35. Samples from *Planck* TT,TE,EE+lowE chains in the $N_{\rm eff}$–H_0 plane, colour-coded by σ_8. The grey bands show the local Hubble parameter measurement $H_0 = (73.45 \pm 1.66)$ km s^{-1} Mpc^{-1} from Riess et al. (2018a). Solid black contours show the constraints from *Planck* TT,TE,EE+lowE+lensing+BAO, while dashed lines the joint constraint also including Riess et al. (2018a). Models with $N_{\rm eff} < 3.046$ (left of the solid vertical line) require photon heating after neutrino decoupling or incomplete thermalization.

Planck 2018 results VI.

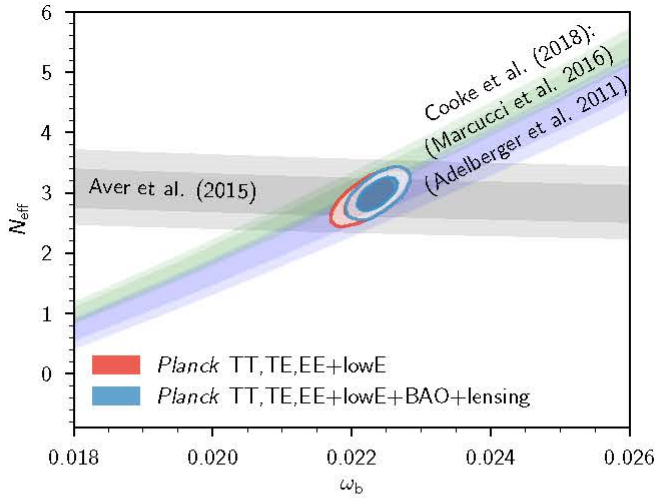


Fig. 39. Constraints in the $\omega_{\rm b}$–$N_{\rm eff}$ plane from *Planck* TT,TE,EE+lowE and *Planck* TT,TE,EE+lowE+BAO+lensing data (68% and 95% contours) compared to the predictions of BBN combined with primordial abundance measurements of helium (Aver et al. 2015, in grey) and deuterium (Cooke et al. 2018, in green and blue, depending on which reaction rates are assumed). In the CMB analysis, $N_{\rm eff}$ is allowed to vary as an additional parameter to the base-ΛCDM model, while $Y_{\rm P}$ is inferred from $\omega_{\rm b}$ and $N_{\rm eff}$ according to BBN predictions. For clarity we only show the deuterium predictions based on the `PArthENoPE` code with two assumptions on the nuclear rate $d(p,\gamma)^3$He (case (a) in blue, case (b) in green). These constraints assume no significant lepton asymmetry.

Consistent with $\mathrm{Neff} = 3.0439 \pm 0.0005$ **(Akita & MY 2020)**

(Future CMB-S4 observation will measure Neff with $\sigma(N_{\rm eff}) \sim 0.03$ at 1σ **)**

Any deviation from this value suggests new physics.

- **Anisotropy**

Perturbed kinetic equations

(Linear) perturbation equations ➔ Anisotropy of CvB

- **Boltzmann Equation :** (e.g. Tully & Zhang 2103.01274)

$$\frac{1}{f_0}L[f] = \underbrace{\frac{\partial \Psi}{\partial \tau} + i\frac{q}{\epsilon}(\boldsymbol{k}\cdot\hat{n})\Psi}_{\text{Free streaming}} + \underbrace{\frac{d\ln f_0}{d\ln q}\left(\dot{\phi} - i\frac{\epsilon}{q}(\boldsymbol{k}\cdot\hat{n})\psi\right)}_{\text{gravity effects}} = \underbrace{\frac{1}{f_0}C[\Psi]}_{\text{Collision term}}$$

$$f(x^i, q, n_j, \tau) = f_0(q)(1+\Psi(x^i, q, n_j, \tau)) = \frac{1}{e^{\frac{q}{T_0}}+1}(1+\Psi(x^i, q, n_j, \tau))$$

comoving momentum q = ap

Line-of-sight direction

Metric perturbations in conformal Newtonian gauge.

$$ds^2 = a(\tau)^2(-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i)$$

- **Poisson Equation & Boltzmann equations for other species**

Nobody has yet solved these equations including the collision term.

How to observe
such low energy (momentum)
neutrinos ?

How to observe CvB directly ?

Weinberg proposed the use of beta-decaying nuclei in 1962.

PTOLEMY type experiment :

Cosmic neutrino capture on tritium

$$\nu_i + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

P on-
T ecorvo
O bservatory for
L ight,
E arly-universe,
M assive-neutrino
Y ield

No threshold energy because $m_{^3\mathrm{H}} > m_{^3\mathrm{He}} + m_e$

Why tritium ? ⟸

- **Long lifetime : $t_{1/2}$ = 12.32 years**
- **High capture cross section**

Difficulty in PTOLEMY experiment

- **Cosmic neutrino capture on tritium :**

$$\nu_i + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

Electron energy from the Cvi B :

$$E_e^i \simeq K_{\mathrm{end}}^0 + m_e + E_{\nu_i}$$

- **Contamination (β- decay) :**

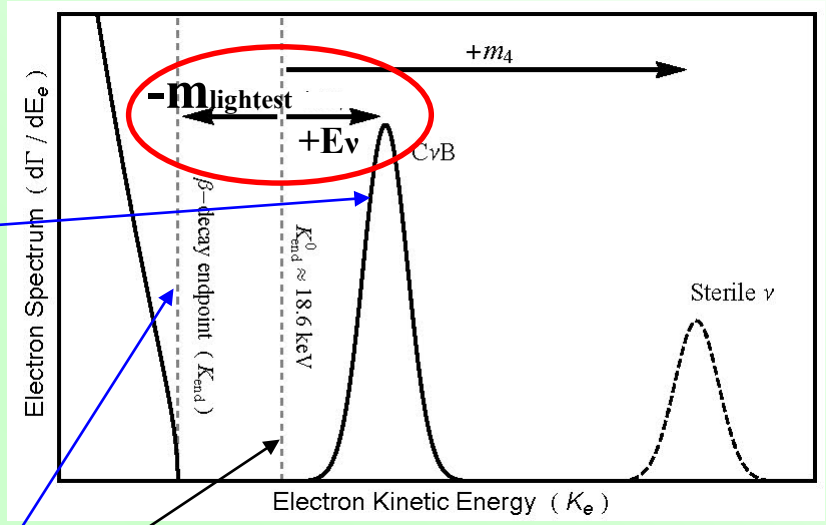
$${}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + \bar{\nu}_i + e^-$$

Maximal electron energy from β-decay:

$$E_{\mathrm{end}} \simeq K_{\mathrm{end}}^0 + m_e - m_{\mathrm{lightest}}$$

(lightest neutrino mass)

(Figure taken from Long et al. 2014 and slightly modified)



$$K_{\mathrm{end}}^0 = \frac{(m_{^3\mathrm{H}} - m_e)^2 - m_{^3\mathrm{He}}^2}{m_{^3\mathrm{H}}}.$$

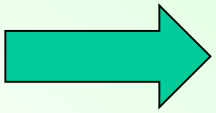
(β- decay endpoint for massless neutrino)

Very precise energy resolution (at least, $m_{\mathrm{lightest}} + E\nu$) is necessary.

Once we would have such experiments which directly capture neutrinos,

it is, of course, fantastic to detect CνB directly !!

Anything else ?? What can we do ??



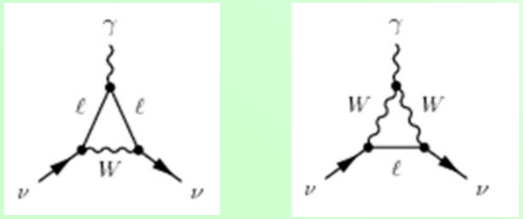
Neutrino decay !!

Increase/decrease of the number of detected neutrinos implies decay into/of neutrinos.

Neutrino "radiative" decay

Even the SM interactions (with right handed neutrinos) induce radiative neutrino decays and then heavier neutrinos can decay into lighter ones :

$\nu_i \rightarrow \nu_j + \gamma$



But, its lifetime is extremely long :

$$\tau_{\nu_i} \sim 10^{36} \left(\frac{m_{\nu_i}}{\mathrm{eV}}\right)^{-5} \ \mathrm{yr} \gg t_0 \simeq 1.4 \times 10^{10} \ \mathrm{yr}$$

(current age of the universe)

If one introduces e.g. a (hypothetical) charged scalar, its lifetime can be shorter.
However, there is stringent constraint from observations :

$\tau_\nu \gtrsim 10^{18}$ yr (electron-neutrino scattering), 10^{20} yr (astrophysical observations), 10^{12} yr (CMB spectral distorsion), etc

Neutrino "invisible" decay

On the other hand, the lifetime (decay rate) of a neutrino invisible decay is far less constrained.

$\nu_i \rightarrow \nu_j + \varphi$

invisible (dark sector) particle like majoron etc

- **Experimental (solar, baseline, etc) constraints :**

$$\tau_{\nu_1}/m_{\nu_1} \gtrsim 4 \times 10^{-3}\ \mathrm{s\ eV^{-1}},\ \tau_{\nu_2}/m_{\nu_2} \gtrsim 1.5 \times 10^{-3}\ \mathrm{s\ eV^{-1}},\ \tau_{\nu_3}/m_{\nu_3} \gtrsim 2.2 \times 10^{-5}\ \mathrm{s\ eV^{-1}}.$$

(Berryman et al. 2015, Aharmim et al. 2019, Funcke et al. 2020)

- **Astrophysical (CMB, BBN, etc) constraints :**

$$\tau_{\nu_i} \sim 1.2 \times 10^{9} \left(\frac{m_\nu}{50\mathrm{meV}}\right)^3 \mathrm{s}$$ **(when non-relativistic neutrinos decay)** (Archidiacono & Hannestad 2014, Escudero & M. Fairbairn 2019)

$$\tau_{\nu_i} \sim 4 \times 10^{5} \left(\frac{m_\nu}{50\mathrm{meV}}\right)^3 \mathrm{s}$$ **(when relativistic neutrinos decay)** (Barenboim et al. 2021)

A direct detection experiment is suitable to constrain such an invisible decay of a neutrino more. **(Akita, Gaetano, MY, arXiv 2109.02900)**

Decay of a heavier neutrino

Assume the lifetime of a heavier neutrino (in the rest frame) : τ_ν

Its number density at present (t0) : $n_\nu(t_0) = e^{-\lambda_\nu} f_c n_\nu^0$

$$\lambda_\nu = \int_{t_d}^{t_0} \frac{dt}{\gamma(t)\tau_\nu} \simeq t_0/\tau_\nu$$

(for currently non-relativistic one, $\gamma \sim 1$)

Lorentz factor

$$n_\nu^0 \equiv n_\nu(t_d)[a(t_d)/a(t_0)]^3 \simeq 56 \text{ cm}^{-3}$$

neutrino decoupling time

(the would-be current number without decays)

fc : the enhancement factor of gravitational clustering by galaxies

m (meV)	f_c
10	1.053
50	1.12
100	1.5
200	3

(Mertsch et al 2019).

Produced lighter neutrinos

- **The number density of a produced lighter neutrino νj at present (t0) :**

$$\tilde{n}^0_{\nu_j} \simeq N_{\nu_j} n^0_{\nu_i} \left(1 - e^{-t_0/\tau_{\nu_i}}\right)$$

$N_{\nu j}$: the number of νj produced by the decay of one νi particle,

$$\nu_i \to \nu_j \phi : \quad N_{\nu_j} = 1$$
$$\nu_i \to \nu_j \nu_j \bar{\nu}_k : \quad N_{\nu_j} = 2$$

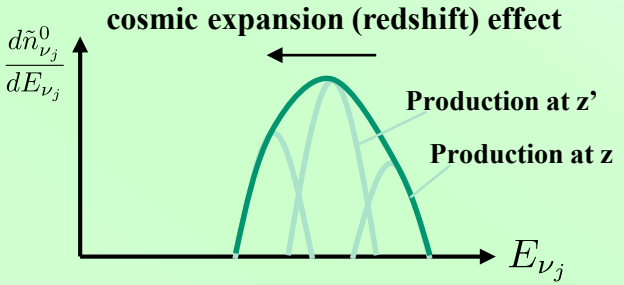
$n^0_{\nu_i} \equiv n_{\nu_i}(t_d)[a(t_d)/a(t_0)]^3 \simeq 56\ \mathrm{cm}^{-3}$ (the would-be current number without decays)

- **The present energy spectrum of the produced lighter neutrinos νj :**

$$\frac{d\tilde{n}^0_{\nu_j}}{dE_{\nu_j}} = \frac{n^0_{\nu_i}}{\tau_{\nu_i}} \int_0^\infty dz\ e^{-t(z)/\tau_{\nu_i}} \frac{1}{H(z)} \frac{E_{\nu_j}}{p_{\nu_j}} \frac{p_{\nu_j}(z)}{E_{\nu_j}(z)} \frac{dN}{dE_{\nu_j}(z)}(E_{\nu_j}(z))$$

Superposition of the decays at z

Spectrum produced by the decay of one νi particle



$$p_{\nu_j}(z) = p_{\nu_j}(1+z),$$
$$E_{\nu_j}(z) = \sqrt{p^2_{\nu_j}(1+z)^2 + m^2_{\nu_j}}.$$

2-body decays

$\nu_i \rightarrow \nu_j + \varphi$: **e.g.** $\mathcal{L}_{\rm int} = i\lambda_{ij}\phi\bar{\nu}_i\gamma^5\nu_j + {\rm h.c.}$

$$\left(\Gamma_{\nu_i\to\nu_j\phi} = \frac{\lambda_{ij}^2}{4\pi m_{\nu_i}}\left[\left(1-\frac{m_{\nu_j}}{m_{\nu_i}}\right)^2 - \frac{m_\phi^2}{m_{\nu_i}^2}\right]\sqrt{\left[m_{\nu_i}^2-(m_{\nu_j}+m_\phi)^2\right]\left[m_{\nu_i}^2-(m_{\nu_j}-m_\phi)^2\right]} \simeq t_0^{-1}\left(\frac{m_{\nu_i}}{50\ {\rm meV}}\right)\left(\frac{\lambda_{ij}}{6.2\times10^{-16}}\right)^2 .\right)$$

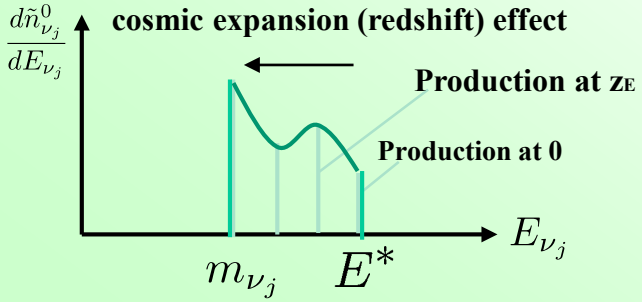
$$(m_i \gg m_j, m_\phi)$$

- **Kinematics :** $\nu_j \longleftarrow \nu_i \times \longrightarrow \phi$

Monochromatic momentum and energy at the decay:

$$p^* = \frac{1}{2m_{\nu_i}}\sqrt{[m_{\nu_i}^2-(m_{\nu_j}+m_\phi)^2][m_{\nu_i}^2-(m_{\nu_j}-m_\phi)^2]},$$

$$E^* = \frac{m_{\nu_i}^2+m_{\nu_j}^2-m_\phi^2}{2m_{\nu_i}}.$$



- **Spectrum produced by the decay of one particle (at its decay with z_E)**

$$\left.\frac{dN}{dE_{\nu_j}}\right|_{\rm 2-body} = \delta\left(E_{\nu_j}-E^*\right), \qquad \textbf{Monochromatic}$$

$$\Rightarrow \quad \frac{d\tilde{n}^0_{\nu_j}}{dE_{\nu_j}} = n^0_{\nu_i}\frac{e^{-t(z_E)/\tau_{\nu_i}}}{H(z_E)\tau_{\nu_i}}\frac{E_{\nu_j}}{E_{\nu_j}^2-m_{\nu_j}^2}.$$

$$1+z_E = \frac{p^*}{\sqrt{E_{\nu_j}^2-m_{\nu_j}^2}}.$$

(present energy spectrum)

Observed "electron" spectrum in PTOREMY-type experiment

$$\nu_i + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

- **Capture rate on tritium of lighter neutrinos ν_j produced by the neutrino decays :**

$$\tilde{\Gamma}^j_{C\nu B} \simeq 2|U_{ej}|^2 \bar{\sigma} \tilde{n}^0_{\nu_j} N_T$$

Total tritium mass M_T = 100g

Number of Tritium : $N_T = M_T / m_{^3\mathrm{H}}$

PMNS matrix

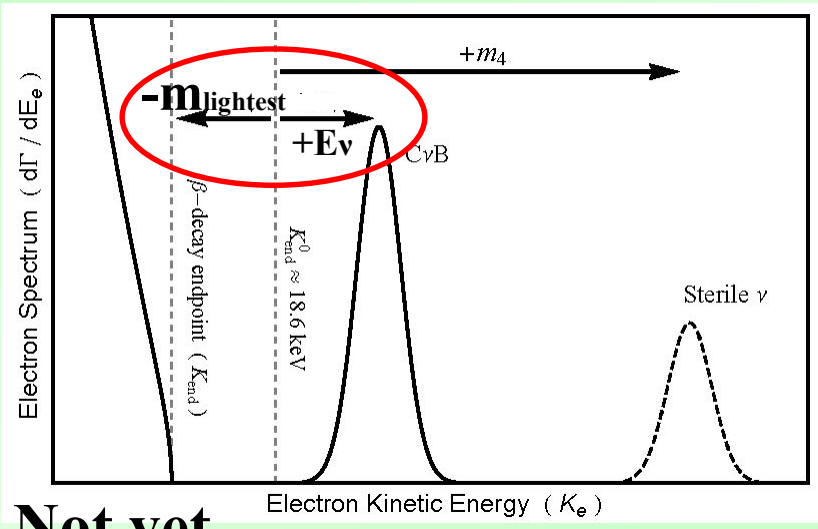
$\bar{\sigma} = 3.76 \times 10^{-45}\ \mathrm{cm}^2$

- **The spectrum of the emitted electrons :**

$$\frac{d\tilde{\Gamma}^j_{C\nu B}}{dE_e} = 2|U_{ej}|^2 \bar{\sigma} N_T \frac{d\tilde{n}^0_{\nu_j}}{dE_e}.$$

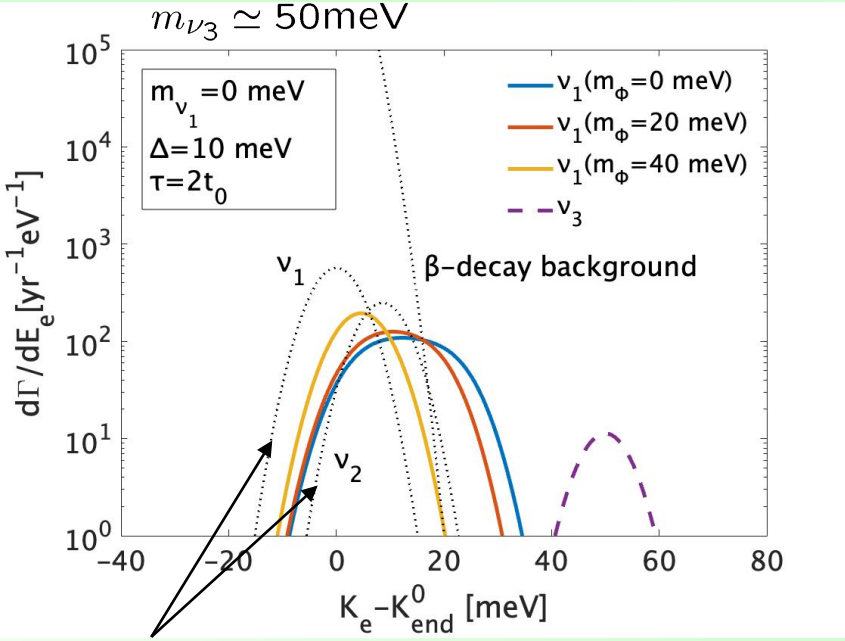
Take the decay into account ⇒

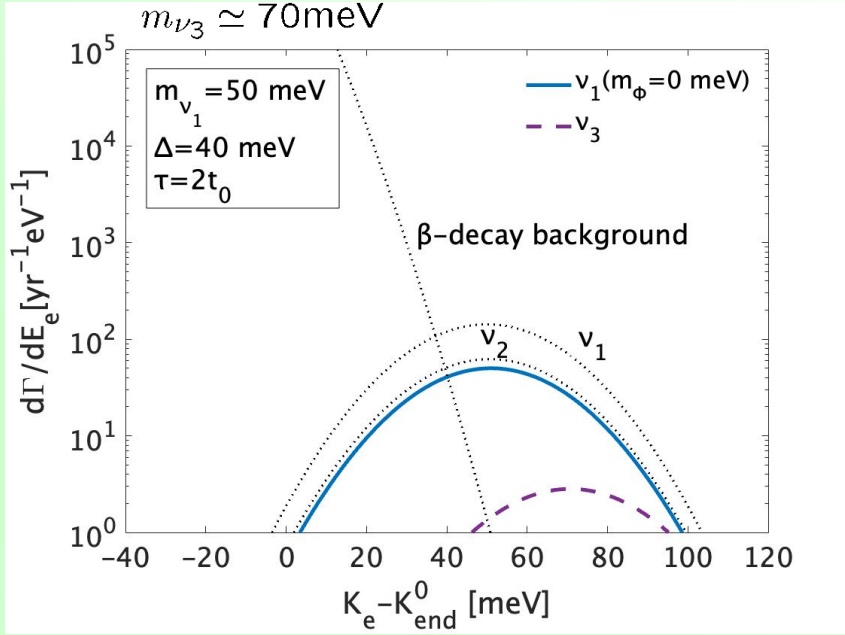
(Figure taken from Long et al. 2014 and slightly modified)



Not yet

Observed "electron" spectrum in normal ordering (NO) : ν3 → ν1 + φ





Original CνBs from reheating

We model the Gaussian-smeared spectrum as an actual spectrum to take the energy resolution of the detector Δ into account. **(νi (=3) → νj(=1) + φ)**

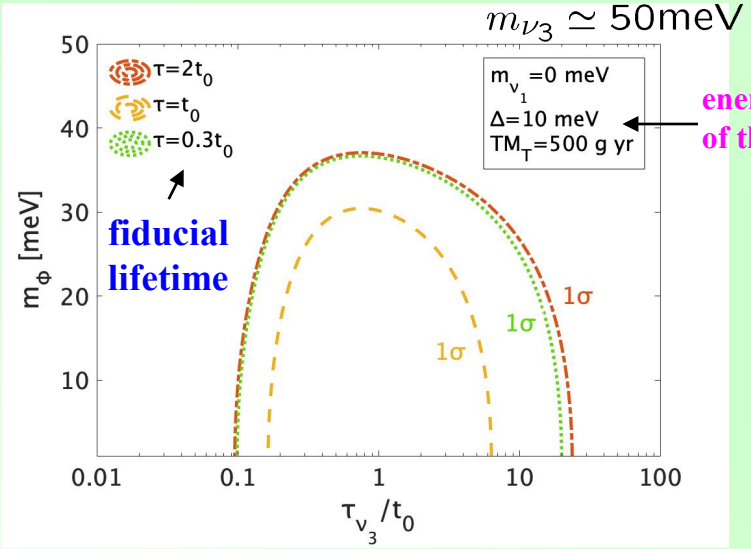
$$\begin{cases} \dfrac{d\Gamma^{i}_{C\nu B}}{dE_e} = \dfrac{1}{\sqrt{2\pi}\sigma}\displaystyle\int_{-\infty}^{\infty} dE'_e\ \Gamma^{i}_{C\nu B}(E'_e)\,\delta[E'_e - (E_{\rm end} + E_{\nu_i} + m_{\rm lightest})]\exp\left[-\dfrac{(E'_e - E_e)^2}{2\sigma^2}\right], \\ \dfrac{d\tilde{\Gamma}^{j}_{C\nu B}}{dE_e} = \dfrac{1}{\sqrt{2\pi}\sigma}\displaystyle\int_{-\infty}^{\infty} dE'_e\ \dfrac{d\tilde{\Gamma}^{j}_{C\nu B}}{dE_e}(E'_e)\exp\left[-\dfrac{(E'_e - E_e)^2}{2\sigma^2}\right]. \end{cases}$$

$$\left(\sigma = \Delta/\sqrt{8\ln 2}\right)$$

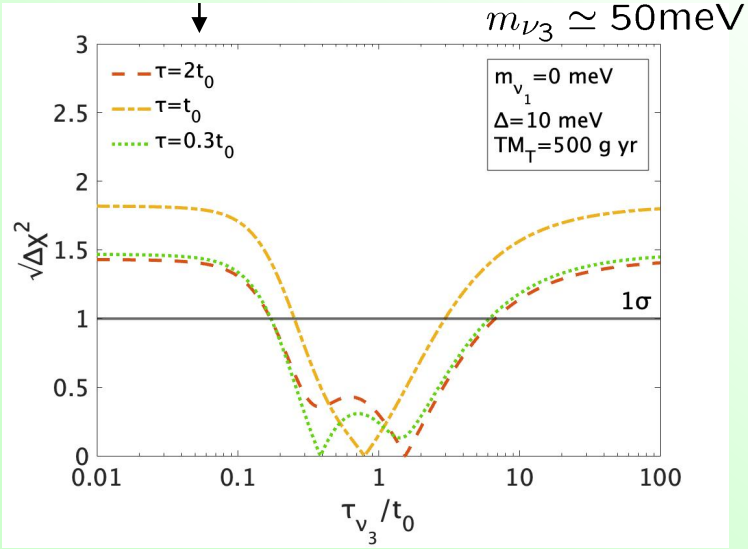
Constraints on a neutrino lifetime

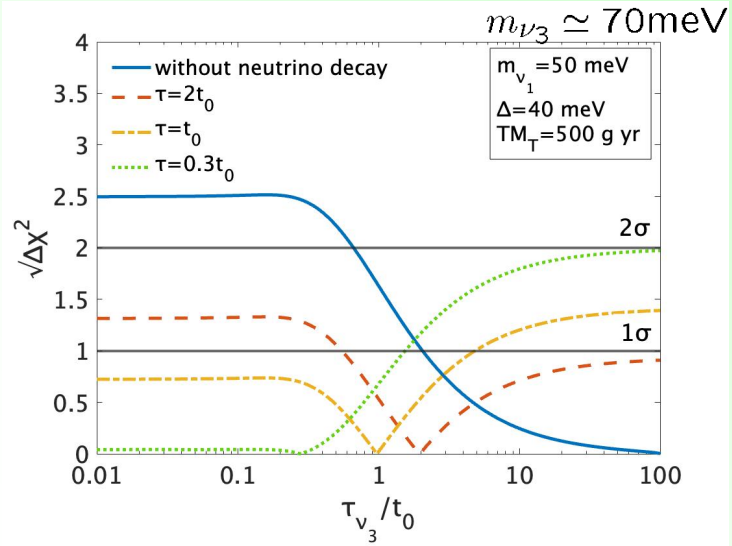
Possible constraints in normal ordering (NO) : ν3 → ν1 + φ

We employ χ2 test.



fiducial lifetime





Ordering	m_{ν_1} (meV)	Δ (meV)	TM_T (g yr)	$\hat{\tau}_{\nu_3}$	$1\sigma(\tau_{\nu_3})$
NO	0	10	500	t_0	$0.2t_0 \lesssim \tau_{\nu_3} \lesssim 3t_0$
	50	40		$\lesssim t_0$	$\tau_{\nu_3} \lesssim 5t_0$
	100	60			$\tau_{\nu_3} \lesssim 3t_0$
	200	120			$\tau_{\nu_3} \lesssim 7t_0$

Table 3: Marginalized constraints on the neutrino lifetime τ_{ν_3} at 1σ confidence level for $\nu_3 \to \nu_1\phi$ with a fiducial value of $\hat{\tau}_{\nu_3} = t_0$ in the NO case and for various values of the lightest neutrino mass, m_{ν_1}, and of the detector resolution, Δ. We assume an exposure of 500 g yr.

Summary

- **Cosmic neutrino background is a powerful tool to probe cosmology as well as particle physics including neutrino physics itself.**
- **For example, any deviation from the theoretical prediction for N_{eff} based on the standard model of particle physics and cosmology will be a signal for new physics.**
- **A direct detection experiment of such cosmic neutrino background is planned.**
- **If it would be detected, it will give us fruitful information.**
- **The lifetime of a neutrino will be constrained more, for example, if one would use such a detection experiment.**