Small flux superpotential in F-theory compactifications

Hajime Otsuka (IBS-CTPU)

Reference :

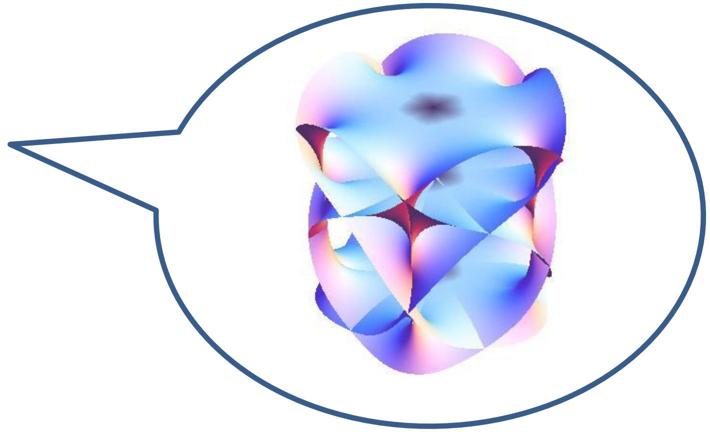
Y. Honma and H.O., Phys. Rev. D **103** (2021) 12, 126022 [arXiv: 2103.03003]

Dark Word to Swampland 2021, The 6th IBS-IFT-MultiDark Workshop, Nov. 15th, 2021

Introduction — Moduli stabilization in the string theory

(Perturbative) superstring theory predicts the extra 6D space

10＝4＋6



・ Metric deformations of extra 6D dimensional space
= 4D scalar fields (called moduli)

・ Unless they are stabilized, it will lead to unobserved fifth forces

・ Stabilization of the extra dimensional space
→ Moduli stabilization (creating a moduli potential)

・ relevant to the construction of de Sitter (dS) spacetime

Introduction — Flux compactifications

- Flux compactification is useful to stabilize the moduli fields.

Let us consider higher-dimensional Maxwell's theory on $R^{1,3} \times M$,

$$\int_{R^{1,3}\times M} F_p \wedge * F_p$$

When there exists a magnetic flux F_p in a cycle Σ_p of M

$$\int_{\Sigma_p} F_p = n \in \mathbb{Z}$$

induces the moduli potential through "$*$", depending on the metric

— Type IIB string theory : Three-forms F_3 and H_3

— F/M- theory : Four-form G_4

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E.g., In type IIB string theory on Calabi-Yau orientifolds
4D N=1 effective superpotential :

$$W_{\text{flux}} \simeq \int_{\text{CY}} (F_3 - SH_3) \wedge \Omega$$

S : Axio-dilaton
Ω : Holomorphic 3-form

KKLT scenario — Prototypical example of dS vacua

3 steps :

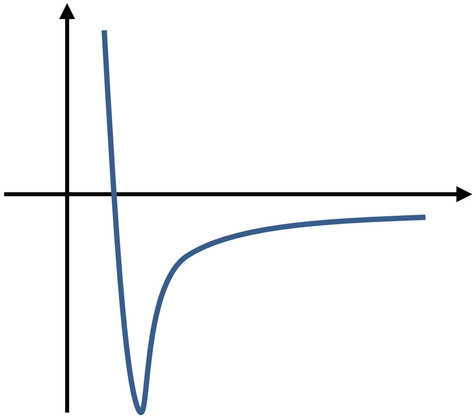
[Kachru-Kallosh-Linde-Trivedi '03]

(i) Flux compactification(Stabilization of complex structure and dilaton)

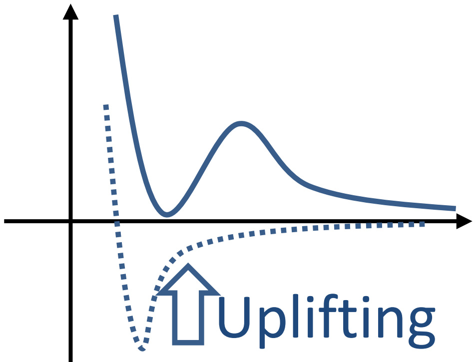
(ii) Stabilization of CY volume by non-perturbative effects

$$W = \langle W_{\rm flux} \rangle + W_{\rm np}$$

$$\langle W_{\rm flux} \rangle \sim W_{\rm np} << 1$$



(iii) Uplifting mechanism (e.g., anti-D3 brane at the conifold)



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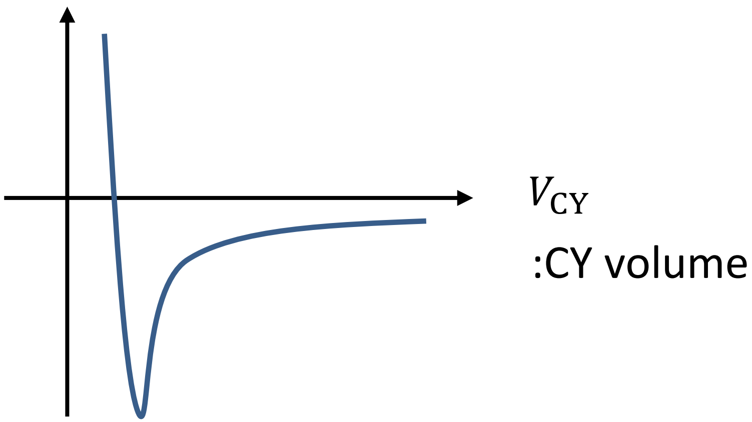


How do we realize $\langle W_{\rm flux} \rangle << 1$?

- Statistical approach [Ashok, Douglas 03, Denef, Douglas 04, …]
- Analytical approach [Demirtas, Kim, Mcallister, Moritz 19, 20, Alvarez-Garcia, Blumenhagen, Brinkmann, Schlechter 20, Honma, Otsuka 21, …]

Short Summary

- We explicitly demonstrate an exponentially small flux superpotential in F-theory flux compactifications on Calabi-Yau fourfolds

$$W_{\text{flux}} = W_{\text{flux}}^{(\text{classical})} + W_{\text{flux}}^{(\text{quantum})}$$

1. SUSY minima with one flat direction at the perturbative level

$$\left\langle W_{\text{flux}}^{(\text{classical})} \right\rangle = 0$$

2. Stabilizing the flat direction by non-perturbative corrections (utilizing the techniques of mirror symmetry)

$$\langle W_{\text{flux}} \rangle = \left\langle W_{\text{flux}}^{(\text{quantum})} \right\rangle << 1$$

E.g., $\langle W_{\text{flux}} \rangle \sim 10^{-9}$

- Our method is broadly applicable to F-theory compactifications on CY4, but Kähler moduli stabilization remains an open problem

Outline

1. Introduction/Short summary

2. F-theory flux compactifications

3. Small flux superpotential

4. Conclusion

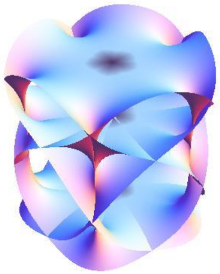
Why F-theory ?

- GUT model building (due to the strong coupling)

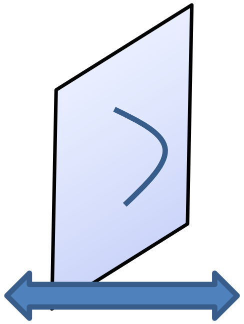
[Beasley-Heckman-Vafa '08, Donagi-Wijnholt'08,….]

- Systematic study of both the open and closed string moduli

Closed string moduli
(Dilaton, Complex structure moduli)



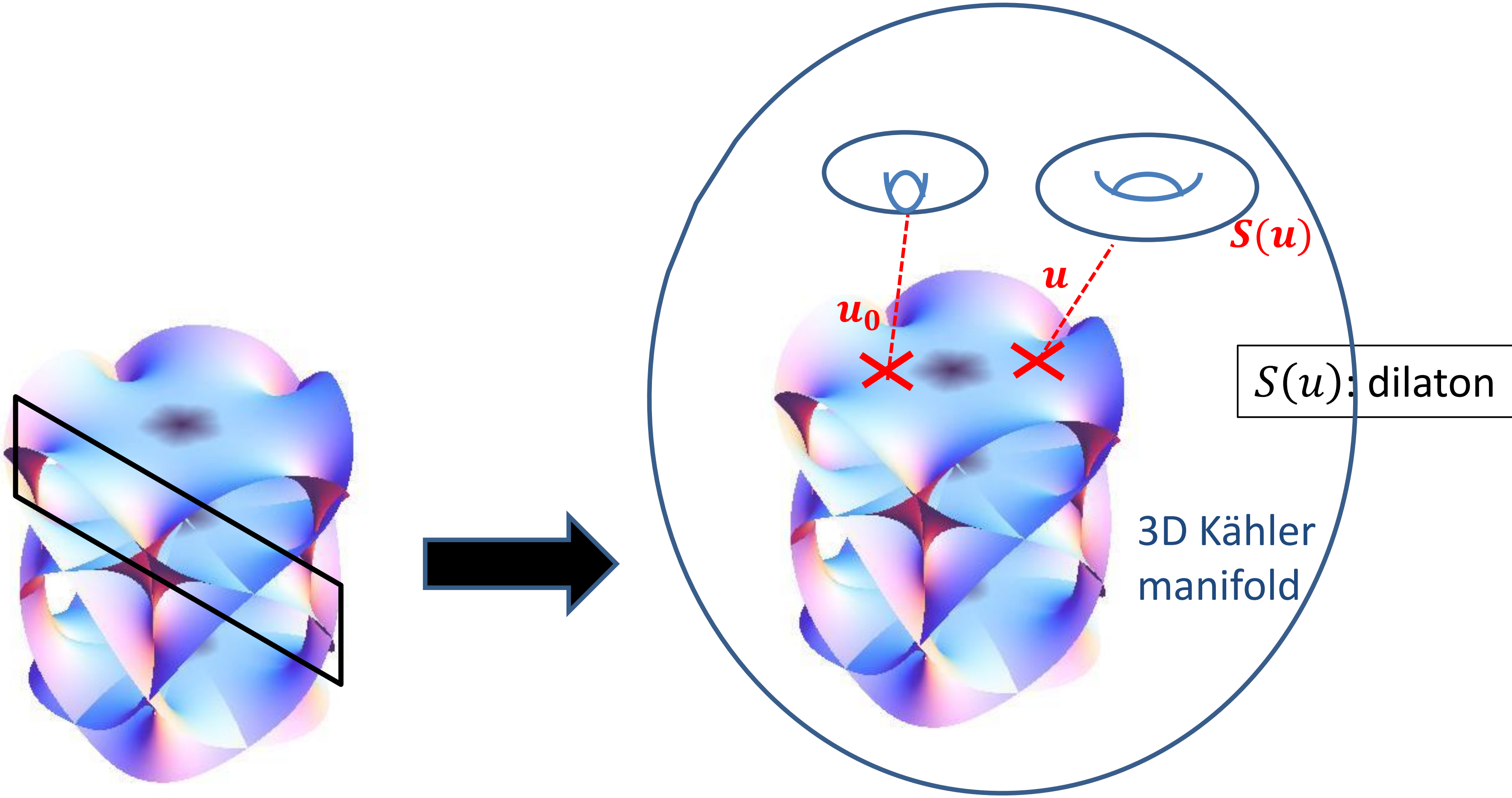
Open string moduli
(Brane moduli)



This talk :

Stabilization of both the open and closed string moduli in F-theory flux compactifications

F-theory geometrizes type IIB orientifolds with 7-branes



CY3+branes

Elliptically fibered CY4

Complex structure moduli of CY3
Dilaton
Open string (position) moduli

Complex structure moduli of CY4

F-theory flux compactifications on elliptically-fibered CY4

EFT: The no-scale 4D N=1 SUGRA

Kähler potential:

$$K = -\ln\left(\int_{\mathrm{CY4}} \Omega \wedge \bar{\Omega}\right) - 2\ln V$$

V : Volume of 3D Kähler base

Ω : holomorphic 4-form of CY4

Flux superpotential:

$$W_{\mathrm{flux}} = \int_{\mathrm{CY4}} G_4 \wedge \Omega$$

[Gukov-Vafa-Witten, '99]

No-scale scalar potential:

$$V = e^K \left(\sum_{I,J} K^{I\bar{J}} D_I W D_{\bar{J}} W \right)$$

I, J : CS moduli of CY4

$$D_I W = \partial_I W + W \partial_I K$$

Flux superpotential

○ G_3-flux superpotential + brane superpotential in type IIB

Period integrals :

$$\Pi = \int_\Gamma \Omega$$

Flux superpotential (if Γ has no boundary)

[Gukov-Vafa-Witten'99]

Brane superpotential (if Γ has boundary) [Witten'97]

(D7-Brane potential with magnetic flux F)

[Grimm-Ha-Klemm-Klevers '09]

○ G_4-flux superpotential in F-theory

Fourfold periods : $\Pi_A = \int_{\gamma_A} \Omega$

$$W_{\text{flux}} = \int_{\text{CY4}} G_4 \wedge \Omega \;\; = n_A \eta^{AB} \Pi_B$$

η^{AB} : Topological intersection matrix

$n_A = \int_{\gamma_A} G_4$: G_4-flux quanta

γ_A : Homology basis of $H_4^H(\text{CY4}, \mathbb{Z})$

Flux superpotential

- G_3-flux superpotential + brane superpotential in type IIB = G_4-flux superpotential in F-theory

[Grimm-Ha-Klemm-Klevers '09,…]

$$W_{\rm flux} = \int_{\rm CY4} G_4 \wedge \Omega \;\; = n_A \eta^{AB} \Pi_B$$

- Self-dual G_4-fluxes = ISD three-form fluxes in type IIB

$$G_4 = * \, G_4$$

$$i G_3 = * \, G_3$$

- Tadpole cancellation conditions

[Becker-Becker '96]

[Sethi-Vafa-Witten '96]

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2} \int_{CY4} G_4 \wedge G_4$$

χ: Euler number of CY4

n_{D3}: # of D3

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3. ***Small flux superpotential***

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Small flux superpotential in F-theory compactifications

Around large complex structure points of elliptically fibered CY4s

$$W_{\text{flux}}^{(\text{classical})} = C_0 + \tilde{C}_0 S + C_a z^a + \tilde{C}_a S z^a + \frac{1}{2} C_{ab} z^a z^b + \frac{1}{2} \tilde{C}_{ab} S z^a z^b + \frac{1}{3} C_{abc} z^a z^b z^c + \frac{1}{3} \tilde{C}_{abc} S z^a z^b z^c + \frac{1}{4} C_{abcd} z^a z^b z^c z^d$$

[Alim-Hecht-Jockers-Mayr-Mertens-Soroush '09
Grimm-Ha-Klemm-Klevers '09,...]

$\{C_0, \widetilde{C}_0, C_a, \widetilde{C}_a, C_{ab}, \widetilde{C}_{ab}, C_{abc}, \widetilde{C}_{abc}, C_{abcd}\}$: functions of G_4-flux quanta (n_A)

z^a : CS moduli ($a = 1,2, \ldots, h^{3,1}(CY_4) - 1$)

S : axio-dilaton (in Type IIB)

(enters only linearly in W)

Small flux superpotential in F-theory compactifications

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Only primitive (2,2)-components of background G_4-flux quanta

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$$W_{\rm flux}^{\rm (classical)} = \tilde{C}_a S z^a + \frac{1}{2} C_{ab} z^a z^b$$

SUSY conditions : $\partial_S W_{\rm flux}^{\rm (cl)} = \partial_{z^a} W_{\rm flux}^{\rm (cl)} = W_{\rm flux}^{\rm (cl)} = 0$

Small flux superpotential in F-theory compactifications

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One flat direction at the classical level : $z^a = S P^a$

$P^a \equiv -\frac{1}{2}(C^{-1})^{ab}\, \widetilde{C}_b$

with $\widetilde{C}_a P^a = 0$

lifted by quantum corrections, along with a small superpotential

Small flux superpotential on elliptically fibered CY4

Toric charge :

$l_1 = (0,-2,1,0,1,0,0,1,-1,0)$	$l_1 + l_3$: $\mathbb{CP}_{11169}$ modulus
$l_2 = (-6,1,0,0,0,2,3,0,0,0)$	l_2: $\mathbb{CP}_{11169}$ modulus
$l_3 = (0,-1,0,1,0,0,0,-1,1,0)$	l_3: brane deformation
$l_4 = (0,-1,0,-1,0,0,0,1,0,1)$	$l_3 + l_4$: base CP^1

Elliptically fibered CY4 (mirror dual to $\mathbb{CP}_{11169}$ over $\mathbb{CP}^1$)

[Grimm-Ha-Klemm-Klevers '09, Honma-Otsuka'21]

- $h^{3,1} = 4$ CS moduli ($z^{a=1,2,3}$ and S)
- From the mirror symmetry calculation with the toric charge, there exist 16 independent G_4-flux quanta (associated with 16 period integrals)

Small flux superpotential on elliptically fibered CY4

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⇓

primitive (2,2)-components of background G_4-flux

$n_1 = n_2 = n_3 = n_4 = n_5 = 0$

$n_{12} = n_{13} = n_{14} = n_{15} = n_{16} = 0$

6 G_4-flux quanta $\{n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$

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$\{\tilde{C}_a, C_{ab}\}$: functions of G_4-flux quanta $\{n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$

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Flat direction at SUSY vauum : $z^a = S P^a$

$\{\tilde{C}_a, C_{ab}\}$: functions of G_4-flux quanta $\{n_6, n_7, n_8, n_9, n_{10}, n_{11}\}$

Small flux superpotential on elliptically fibered CY4

- By utilizing algebraic methods of the toric geometry, non-perturbative corrections are determined by solving the Picard-Fuchs equations

- Relevant period integrals (including leading quantum corrections):

$$\widetilde{\Pi}_6 = \Pi_6 + \frac{e^{2\pi i z_1}}{4\pi^2} \qquad \widetilde{\Pi}_9 = \Pi_9 - \frac{1545 e^{2\pi i z_2}}{\pi^2}$$

$$\widetilde{\Pi}_7 = \Pi_7 - \frac{e^{2\pi i S}}{2\pi^2} \qquad \widetilde{\Pi}_{10} = \Pi_{10} - \frac{825 e^{2\pi i z_2}}{\pi^2}$$

$$\widetilde{\Pi}_8 = \Pi_8 - \frac{e^{2\pi i z_3}}{2\pi^2} \qquad \widetilde{\Pi}_{11} = \Pi_{11} - \frac{915 e^{2\pi i z_2}}{\pi^2}$$

No α' corrections

- We analyze the vacuum structure under the following range of background fluxes :

$$-20 \leq n_6, n_7, n_8, n_9, n_{10}, n_{11} \leq 20 \qquad 0 \leq n_{D3} \leq 10$$

$$n_1 = n_2 = n_3 = n_4 = n_5 = n_{12} = n_{13} = n_{14} = n_{15} = n_{16} = 0$$

Tadpole cancellation condition :

$$\frac{\chi}{24} = n_{D3} + \frac{1}{2}\int_{\mathrm{CY4}} G_4 \wedge G_4 \qquad \chi = 16848$$

Small flux superpotential on elliptically fibered CY4

$$-20 \leq n_6, n_7, n_8, n_9, n_{10}, n_{11} \leq 20 \quad 0 \leq n_{D3} \leq 10$$

Vacuum	Set of fluxes $(n_6, n_7, n_8, n_9, n_{10}, n_{11})$	n_{D3}	z_1	z_2	z_3	S	$\lvert W_0 \rvert$
A	$(-10, -8, 12, 7, 15, -8)$	2	$1.95i$	$2.60i$	$5.86i$	$4.56i$	6.75×10^{-9}
B	$(-9, -8, 14, 0, 11, -11)$	5	$1.35i$	$1.97i$	$4.06i$	$4.06i$	6.11×10^{-7}
C	$(-15, 8, 6, 20, -4, -8)$	6	$2.41i$	$1.81i$	$1.20i$	$2.71i$	2.50×10^{-6}

$$W_0 \equiv \langle e^{K/2} W \rangle$$

Vacuum A :

- Flat direction at the classical level

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \frac{S}{7} \begin{pmatrix} 3 \\ 4 \\ 9 \end{pmatrix}$$

- Non-perturbative superpotential (along the flat locus)

$$W = \frac{1}{4\pi^2} \left[-44 e^{\frac{6\pi i S}{7}} + 2040 e^{\frac{8\pi i S}{7}} + 49 e^{2\pi i S} - 68 e^{\frac{18\pi i S}{7}} \right]$$

Racetrack

- Mass squareds of canonically normalized moduli

Vacuum	Eigenvalues of mass matrix $\partial_I \partial_J V \times \mathcal{V}^2$
A	$(24.7,\ 24.7,\ 4.86,\ 4.86,\ 0.634,\ 0.634,\ 9.79 \times 10^{-14},\ 9.65 \times 10^{-14})$
B	$(42.5,\ 42.5,\ 8.76,\ 8.76,\ 1.33,\ 1.33,\ 4.68 \times 10^{-10},\ 4.56 \times 10^{-10})$
C	$(61.9,\ 61.9,\ 15.2,\ 15.2,\ 0.765,\ 0.765,\ 1.30 \times 10^{-8},\ 1.27 \times 10^{-8})$

- Stable against the next instanton corrections

Conclusion and Discussions

- We explicitly demonstrate an exponentially small flux superpotential in F-theory flux compactifications on Calabi-Yau fourfolds

- Generalizing a simple but broadly applicable method in Type IIB into F-theory compactifications, we clarified G_4-flux components

  1. Perturbatively flat direction at the SUSY minima
  2. Leading-instanton corrections lift the flat direction (by utilizing the mirror symmetry techniques)

  - Impact on the realization of KKLT construction in a broad range of F-theory frameworks

- Comprehensive study about global structure of CY moduli space ? (for instance, conifold region)

- Kähler moduli stabilization in F-theory ?