INSTITUT POLYTECHNIQUE DE PARIS

SLOW AND SAFE GRAVITINOS

Based on :

E.D., M.A.G.Garcia, Y.Mambrini, K.A.Olive, M.Peloso and S.Verner, Phys. Rev. **D103** (2021), 123519 [arXiv:2104.03749 [hep-th]]

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Outline

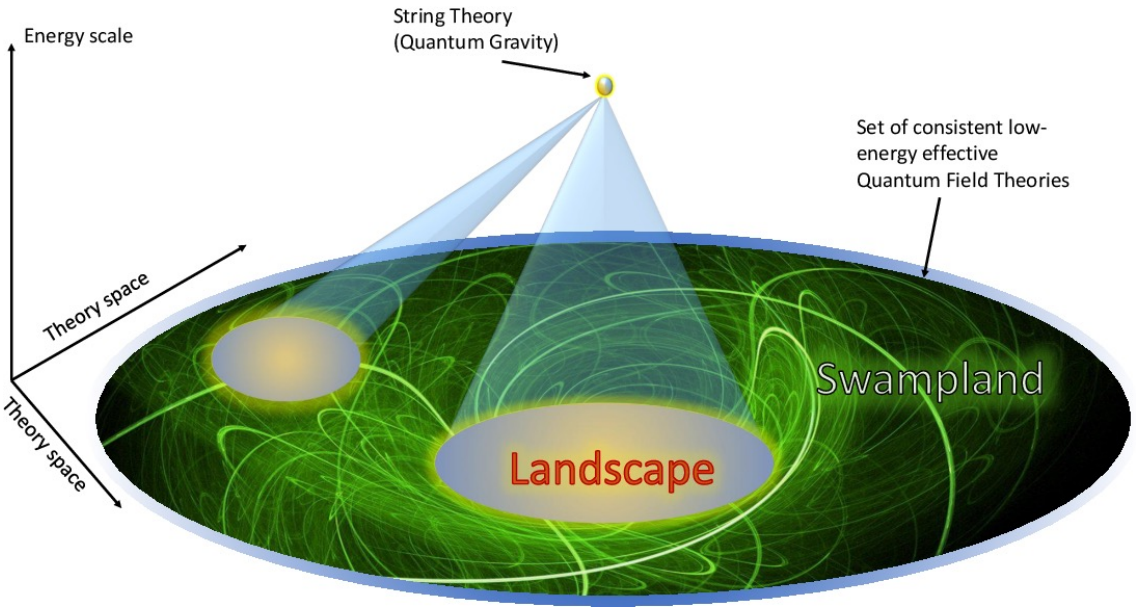
1) The swampland program
- Spin 3/2, potential problems
2) Gravitino sound speed in supergravity
3) Complete eqs. for the longitudinal gravitino
4) Results, gravitino swampland conjecture
5) Conclusions

1) The swampland program

Are all consistent Quantum Field Theories obtainable from a Quantum Gravity Theory (ex. String Theory) ?

Probably NO

Swampland = the set of consistent QFT with no consistent coupling to Quantum Gravity (Vafa,2005)



(taken from E. Palti, « The Swampland: Introduction and Review », [arXiv:1903.06239 [hep-th]])

Final goal swampland program ?

Supplement rules of effective QFT with additional constraints, which would guide Beyond the Standard Model and cosmology constructions.

On the other hand, the consistency of low-energy actions for the spin 3/2 Rarita-Schwinger field has a long history :

- 1941: Rarita-Schwinger action
- 1969: Velo-Zwanziger pointed out potential acausal propagation for a charged gravitino in an em background
- 1977: Deser-Zumino proved that gravitino propagation in minimal supergravity is causal
- 2001: Deser-Waldron proved that gravitino propagation in gauged supergravities is causal

  .....
- 2021 – Gravitino swampland conjecture, gravitino distance conjecture

History of the subject strongly suggest that usual supergravities should have no problems with gravitino propagation.

Recently, simple nonlinear SUSY/SUGRA models were constructed. More minimal inflationary models, fewer fields.

Even possible to construct models with only:

- graviton
- massive gravitino
- inflaton (real scalar)

Example constrained superfields (talks F. Quevedo,T. Terada)

- Volkov-Akulov action can be constructed in superspace (Rocek,78) introducing a constrained, nilpotent superfield

$$X^2 = 0$$

whose solution is (no fundamental scalar)

$$X = \frac{GG}{2F_X} + \sqrt{2}\theta G + \theta^2 F_X$$

The full VA action is

$$\mathcal{L}_{VA} = \left[X\overline{X}\right]_D + \left[fX + h.c.\right]_F$$

Analogy with the sigma model :

- O(N) linear sigma model

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a - \lambda(\phi_a \phi_a - v^2)^2.$$

has 1 massive (« Higgs ») and N-1 goldstone bosons,

versus the

- O(N)/ O(N-1) nonlinear sigma model ($\lambda \to \infty$ limit)

$$\mathcal{L} = \partial_m \phi_a \partial^m \phi_a$$

+ constraint $\phi_a \phi_a = v^2$, describes self-interactions of the N-1 goldstone's. O(N) symmetry is nonlinearly realized.

My talk deals with the propagation (« speed of sound » c_s) of gravitino in supergravity, in an inflationary setup.

Usually $0 < c_s \leq 1$

Recently, two potential problematic behaviours were discussed:

- $c_s = 0$ at particular points on the inflationary trajectory

  ⟹ Large (catastrophic) production of gravitinos

- $c_s > 1$ acausal behaviour at particular points on the inflationary trajectory

2) Gravitino sound speed in supergravity

Supergravity = Supersymmetry + Gravity

It contains :

- gravity multiplet: Graviton $g_{\mu\nu}$, gravitino ψ_μ

- « matter » fields: (complex) Scalars , Weyl Fermions
  chiral superfields
  Φ_i ϕ_i ψ_i

- In supergravity, the gravitino Ψ_μ becomes massive by absorbing the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

and its mass is $m_{3/2} = e^{\frac{K}{2}} |W|$

The sound speed c_s is defined from the dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 + a^2 m^2$$

The transverse spin 3/2 component in a FRW background has a standard dispersion relation with $c_s = 1$

$$(\gamma^0 \partial_0 + i\gamma^i k_i + a m_{3/2}) \Psi_{3/2,\mathbf{k}} = 0$$

scale factor

The longitudinal (goldstino) component satisfies a more involved equation

$$(\gamma^0\partial_0 - i\gamma^i k_i \frac{\alpha_1+\gamma^0\alpha_2}{\alpha} + am_{3/2})\Psi_{1/2,\mathbf{k}} = 0$$

with $\alpha_1, \alpha_2, \alpha$ specific functions of scalar fields in SUGRA, with the sound speed depending generically on time

$$c_s^2 = \frac{|\alpha_1|^2+|\alpha_2|^2}{\alpha^2}$$

$c_s < 1 \Longrightarrow$ Slow gravitino (Benakli,Darmé, Oz, 2014)

A general expression for longitudinal gravitino speed sound is

pressure

time-derivative

$$c_s^2 = \frac{\left(p - 3m_{3/2}^2\right)^2}{\left(\rho + 3m_{3/2}^2\right)^2} + \frac{4\dot{m}_{3/2}^2}{\left(\rho + 3m_{3/2}^2\right)^2}$$

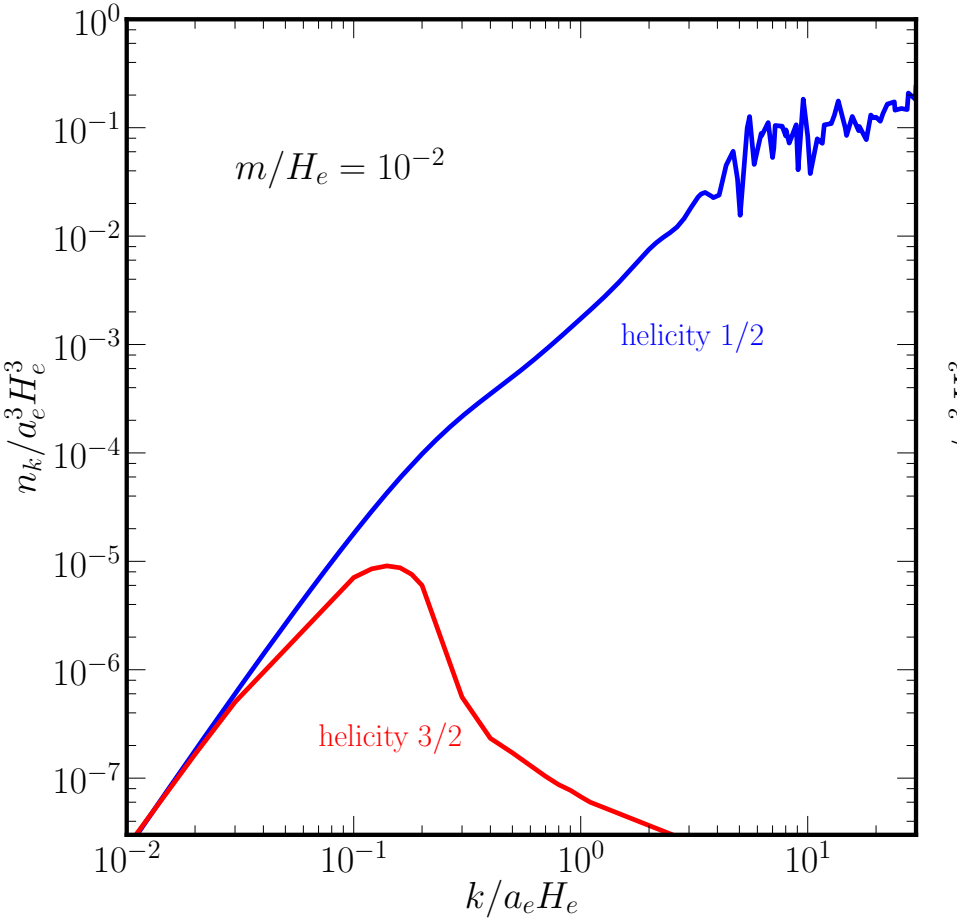
energy density

$c_s = 0$ is possible if $m_{3/2}$ is const. and $p = 3m_{3/2}^2$

In this case, there would be a catastrophic production of gravitinos during inflation

(Hasegawa, Terada et al, 2017; Kolb, Long, McDonough, 2021).

The problem was argued to arise for $m_{3/2} < H$. If the problem is generic ⟹ potential issue for low-energy SUSY models.



(taken from Kolb et al, 2021)

The explicit formula in SUGRA is

$$c_s^2 = 1 - \frac{4}{\left(|\dot{\varphi}|^2 + |F|^2\right)^2} \left\{ |\dot{\varphi}|^2 |F|^2 - |\dot{\varphi} \cdot F^*|^2 \right\}$$

where $F^i \equiv \mathrm{e}^{K/2} K^{ij^*} D_{j^*} W^*$ in standard SUGRA ,

$$D_i W \equiv \frac{\partial W}{\partial \varphi^i} + \frac{\partial K}{\partial \varphi^i} W$$

and we used the compact notation $|\dot{\varphi}|^2 = \dot{\varphi}^i K_{ij^*} \dot{\varphi}^{j*}$,etc

Obs: Cauchy-Schwarz inequality ⟹ causality $c_s \leq 1$ respected in all standard SUGRA's

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3) Complete eqs. for the longitudinal gravitino

In an expanding background, the longitudinal gravitino θ is coupled to another fermion, the inflatino

$$\Upsilon = K_{ij^*}\left(\chi^i \partial_0 \varphi^{j^*} + \chi^{j^*}\partial_0 \varphi^i\right)$$

(Kallosh,Kofman,Linde, Van Proeyen,2000; Nilles,Peloso,Sorbo, 2001)

θ and Υ are coupled via

$$\left(\gamma^0 \partial_0 + i\gamma^i k_i N + M\right) X = 0 \quad , \quad X = \begin{pmatrix} \tilde{\theta} \\ \tilde{\Upsilon} \end{pmatrix}$$

where the « sound speed matrix »

$$N = \begin{pmatrix} -\frac{\alpha_1}{\alpha} - \gamma^0 \frac{\alpha_2}{\alpha} & -\gamma^0 \Delta \\ -\gamma^0 \Delta & -\frac{\alpha_1}{\alpha} + \gamma^0 \frac{\alpha_2}{\alpha} \end{pmatrix}$$

with $\Delta = \sqrt{1 - c_s^2}$, is now the key to the « slow gravitino » problem.

When $c_s = 0$, then $N = \begin{pmatrix} 0 & -\gamma^0 \\ -\gamma^0 & 0 \end{pmatrix}$

is nonsingular, leading to a nonvanishing sound speed for the physical eigenstates.

(DGMOPV,2021; see also Antoniadis,Benakli, Ke, 2021)

4) Results, gravitino swampland conjecture

For the (large) majority of SUGRA models we investigated , we found no problems, $0 < c_s^i \leq 1$:

- standard SUGRA models with two chiral superfields (inflaton+SUSY breaking): general statement

- SUGRA models with nilpotent SUSY breaking field

$$S^2 = 0$$

The only models with problems we found is with the « orthogonal constraint » for the inflaton multiplet Φ

$$S(\Phi - \overline{\Phi}) = 0$$

Only $Re\ \phi$ is a dynamical degree of freedom.
$Im\ \phi$, the inflatino ψ_ϕ and the auxiliary field F_ϕ are determined by the constraint*.

In particular F_ϕ is a bilinear in fermions and does not appear in the scalar potential : $F^\Phi \neq e^{K/2} K^{\Phi \bar{i}} D_{i^*} W^*$

* See Terada (2021) for a minimal inflation model in SUGRA with no gravitino sound speed problem, using a cubic constraint.

Consequences:

- There is no inflatino $\Rightarrow$ $\Upsilon = 0$, the gravitino sound speed problem can reappear (model-dependent)

- The Cauchy-Schwarz argument for $c_s \leq 1$ not valid. We found examples with $c_s > 1$!

On the other hand, the UV origin of the orthogonal constraint is not clear (dall'Agata,E.D.,Farakos, 2006)

$\Rightarrow$ Potential pathological behaviour reminiscent of the swampland program !

Interesting to contemplate a « gravitino swampland conjecture »

« In all 4d effective field theories that are low-energy limits of quantum gravity, at all points
in moduli space and for all initial conditions, the sound speed of the gravitino(s) must be non-vanishing $c_s > 0$ »
(Kolb,Long,McDonough)

refined version

« In all 4d effective field theories that are low- energy limits of quantum gravity, at all points
in moduli space and for all initial conditions, all eigenvalues of the sound speed matrix for fermions must be non-vanishing and subluminal $0 < c_s^i \leq 1$ »

Conclusions

- Gravitino production constraints important for viability of SUGRA models.

- Very often, the inflatino is produced, alleviating the gravitino problem.

- Important to check and impose sound speed

$$0 < c_s \leq 1 \Longrightarrow$$ gravitino swampland conjecture

- We found that most SUGRA models do satisfy it, except peculiar models with orthogonal constraint. Interesting to test inflation against other swampland conjectures.