Refined de Sitter Conjectures in No-Scale Supergravity Models

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Outline

Introduction: Refined Swampland de Sitter Conjectures (RdSC) and TCC

Obied, Ooguri, Spodyneiko and Vafa [1806.08362]:

Given a potential, V, depending on scalar fields: **no de Sitter minima are allowed, only maxima are allowed**

$$|\nabla V| \geq \frac{c}{M_{\rm P}}\, V\,,$$

or

$$\min\left(\nabla_i \nabla_j V\right) \leq -\frac{c'}{M_{\rm P}^2}\, V\,.$$

Problems:

1998: *Discovery of the accelerating expansion of the Universe due to a non-vanishing vacuum energy*

2018: *Observational support for inflationary cosmology, according to which the Universe underwent an early epoch of near-exponential quasi-de Sitter expansion driven by an inflaton field energy*

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Given a potential, V, depending on scalar fields: **no de Sitter minima are allowed, only maxima are allowed**

$$|\nabla V| \geq \frac{c}{M_{\rm P}} \, V \, , \qquad \text{1st. RdSC}$$

or

$$\min \left(\nabla_i \nabla_j V \right) \leq -\frac{c'}{M_{\rm P}^2} \, V \qquad \text{2nd. RdSC.}$$

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Bedroya and Vafa [1909.11063]: O.k. we need an explanation based on fundamental aspects of quantum gravity [Trans-Planckian Conjecture]:

- In an expanding universe sub-Planckian *quantum fluctuations should remain quantum* and can never become larger than the Hubble horizon
- A meta-stable dS point is compatible with TCC as long as its lifetime τ is bounded by

$$\tau < \frac{1}{H_f} \ln \frac{M_{\mathrm{P}}}{H_f},$$

where H is the Hubble parameter and is related to the cosmological constant by

$$V = \Lambda = \frac{(d-1)(d-2)}{2} H^2, \quad d \text{ dimensions.}$$

Inflation successfully allows field theory computations without relying on any trans-Planckian physics.
But there is a problem:
Trans-Planckian problem:

- If macroscopic fluctuations trace back to trans-Planckian wavelenghts during inflation, the evolution of fluctuations cannot be reliably extracted from the effective field theory.
- When sub-Planckian quantum fluctuations become larger than the Hubble horizon $1/H$, they can become classical and freeze, which would lead to the classical observation of a sub-Planckian quantum mode.

TCC: These questions never arise in a consistent quantum gravitational theory: **Sub-Planckian quantum fluctuations should remain quantum.**

But if our universe is stuck in a metastable dS minimum, it cannot be for an infinite amount of time

$$V = \Lambda \approx 2.9 \times 10^{-122} \xrightarrow{\text{TCC}} \tau_{\mathrm{U}} < 2.4 \times 10^{12} \text{ yr.}$$

As we know $\tau_{\mathrm{U}} \sim 1.4 \times 10^{10}$ yr.

This is not a very useful bound (prediction) so we need to construct models which give us more insight into the fundamental physics of them.

De Sitter No-Scale Supergravity Models

Ellis, Kounnas and Nanopoulos, Nucl. Phys. B, 247, 1984

$$
\begin{aligned}
K &= -3 \sum_{i=1}^{N} \alpha_i \ln(\phi_i + \bar{\phi}_i), \\
W &= a \left(\prod_{i=1}^{N} \phi_i^{n_{i+}} - \prod_{i=1}^{N} \phi_i^{n_{i-}} \right),
\end{aligned}
$$

where i runs over N no-scale chiral superfields, $\alpha_i > 0$, a is an arbitrary constant and

$$
n_{i\pm} = \frac{3}{2}\left(\alpha_i \pm \frac{r_i}{s}\right) \quad \text{for} \quad \sum_{i=1}^{N} r_i^2 = 1, \quad s^2 = \sum_{j=1}^{N} \frac{r_j^2}{\alpha_j}.
$$

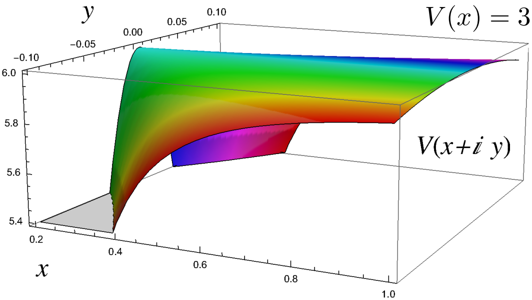
Example with one field

$$
\begin{aligned}
G &= K + \ln W + \ln \overline{W}, \\
X &= G_i \, K^{i\bar{\jmath}} \, G_{\bar{\jmath}}, \\
V &= e^G \,(X-3) = a^2 \; \left(\phi + \bar{\phi}\right)^{-3\,\alpha} \left(\phi\bar{\phi}\right)^{n_-} \left|\phi^{n_+} - \phi^{n_-}\right|^2 (X-3)\,,
\end{aligned}
$$

$$
\begin{aligned}
\partial_\phi V &= -3\left[a^2(-1+3\alpha)\phi^{-2-3\,(\sqrt{\alpha}+\alpha)/2}\bar{\phi}^{-1-3\,(\sqrt{\alpha}2+\alpha)/2}(\phi-\bar{\phi}) \right. \\
&\qquad \left. (\phi+\bar{\phi})^{1-3\alpha}\right],
\end{aligned}
$$

$$\Rightarrow \quad \phi = \bar{\phi} \quad \text{extremum value}$$

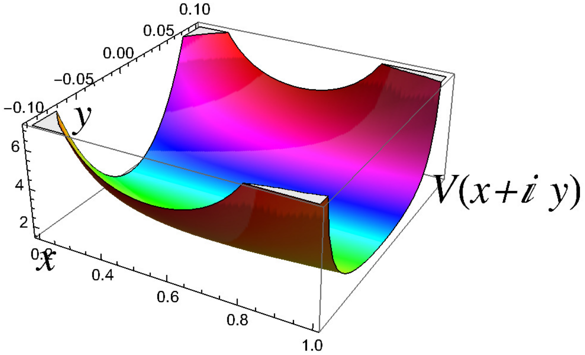
$$V(x) = 3\,a^2\,2^{2-3\sum_{i=1}^{N}\,\alpha_i}$$



Phenomenologist point of view: Stabilize and apply (Inflation)

Stabilize the imaginary component with ([2009.01709], Ellis, et. al.)

$$K = -3 \sum_{i=1}^{N} \alpha_i \ln(T + \overline{T} - b\,(T - \overline{T})^4),$$



Apply it to inflation, T modulus, Φ inflaton ([2009.01709], Ellis, et. al.):

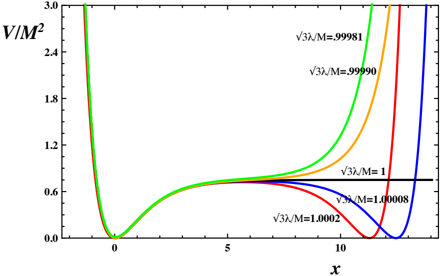
$$K = -3 \sum_{i=1}^{N} \alpha_i \ln(T + \overline{T} - |\phi|^2/3),$$

$$W = \frac{M}{2}\,\phi^2 - \frac{\lambda}{3}\phi^3 + a\,(T^{n_{i+}} - T^{n_{i-}})$$

$$V(\phi, T = \text{fixed}) = M^2 \frac{|\phi|^2\,|-\lambda\,\phi/M|^2}{(1-|\phi|^2/3)^2}$$

$$\Downarrow$$

Starobinsky Model



Great consistency with Planck and BICEP data

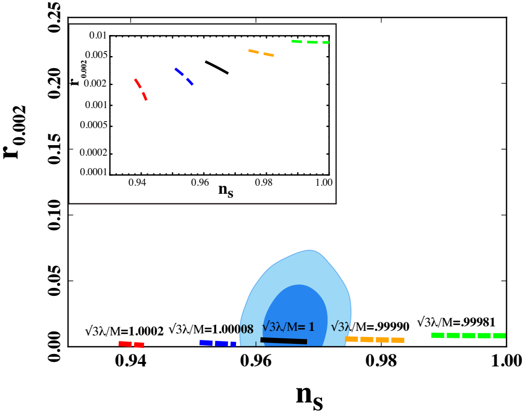


Figure: Predictions for the tilt n_s in the spectrum of scalar perturbations and for the tensor- to-scalar ratio r plane, compared with the 68 and 95% CL regions from Planck data combined with BICEP2/Keck results. From 2009.01709, Ellis, J. et al

Refined Swampland de Sitter Conjectures in No-Scale: allow those maxima

$$\begin{aligned} K &= -3\,\alpha\,\ln(\phi+\bar{\phi}), \\ W &= a\left(\phi^{n_+}-\phi^{n_-}\right), \end{aligned}$$

Evaluate the second criterion (2nd. RdSC)

$$\min\left(\nabla_i\nabla_j V\right) \leq -\frac{c'}{M_{\rm P}}\,V\,.$$

$$\Rightarrow \mathcal{H} = \begin{bmatrix} \nabla_i\nabla_{\bar{j}}V & \nabla_i\nabla_j V \\ \nabla_{\bar{j}}\nabla_{\bar{\imath}}V & \nabla_{\bar{\imath}}\nabla_j V \end{bmatrix},$$

The second criterion in [1806.08362] should be evaluated for canonical fields, but for one field, this is equivalent to include in the Hessian the inverse of the Kähler metric, for this case we get

$$m^2_{{\rm Im}\;\phi} = \frac{2^{2-3\,\alpha}a^2}{\alpha}\,\phi^{-3\sqrt{\alpha}}\left[\alpha\left(-1+\;\phi^{3\,\sqrt{\alpha}}\right)^2-\left(1+\phi^{3\,\sqrt{\alpha}}\right)^2\right].$$

Equivalently for a canonical normalized field, with a Kähler potential

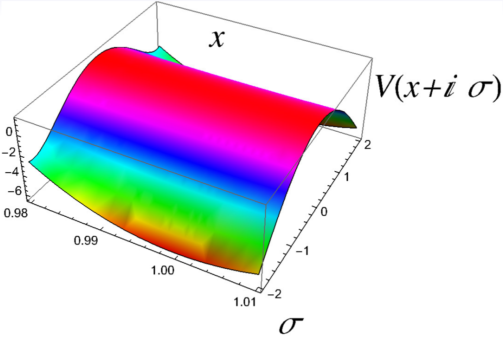
$$K = -3\,\alpha\,\ln\left[\phi+\bar{\phi}+\frac{\left(\phi+\bar{\phi}-2\,b\right)^{4}}{L}\right],$$

Which fixes the real term to a desired value

$$\phi=\bar{\phi}=b$$

and leaves theory with one canonical field. Then by defining $\Phi=\frac{\chi+i\,\sigma}{\sqrt{2}}=\sqrt{\frac{3\,\alpha}{4\,b^{2}}}\,y$, we can express the squared mass of the imaginary direction when the real part is fixed

$$m^2_{\mathrm{Im}\,[\Phi]}\Big|_{\chi=\bar{\chi}=\sqrt{3\alpha}/2}=C\;\;\alpha\left[-\left(\frac{3\,\alpha}{4\,b^{2}}\right)^{3\,\sqrt{\alpha}}+\Phi^{3\,\sqrt{\alpha}}\right]^{2}-\left[\left(\frac{3\,\alpha}{4\,b^{2}}\right)^{3\,\sqrt{\alpha}}+\Phi^{3\,\sqrt{\alpha}}\right]^{2}\Bigg|_{\chi=\bar{\chi}=\sqrt{3\alpha}/2}$$



$$\alpha = 25/9$$

$$\left[\frac{\sqrt{\alpha}-1}{1+\sqrt{\alpha}}\right]^{\frac{1}{3\sqrt{\alpha}}} < \phi < \left[\frac{1+\sqrt{\alpha}}{\sqrt{\alpha}-1}\right]^{\frac{1}{3\sqrt{\alpha}}} .$$

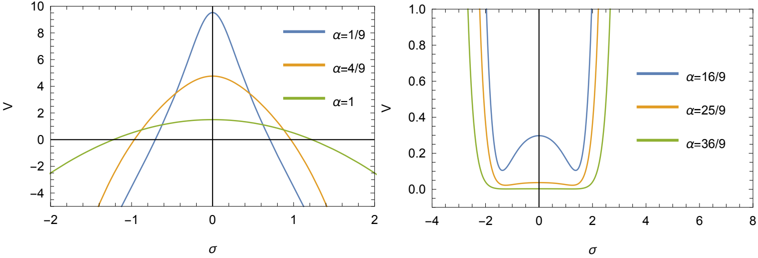


Figure: The scalar potential, $V(\sigma)$, with the Kähler potential of that fixes the real direction, $\langle \mathrm{Re}\,[\phi] \rangle = b = 1$, as a function of $\sigma = \sqrt{3\,\alpha/2}\;y/b$, $\phi = x + iy$. We have chosen values of α which are perfect squares modulo 9. Left: V" < 0, right V" > 0.

Trans-Planckian Conjecture in No-Scale dS

In Bedroya and Vafa [1909.11063] the TCC can be evaluated for two regimes

1. Long-range field, asymptotic behaviour
2. Small-range, local behaviour

Starting point: Friedmann equation

$$\frac{(d-1)(d-2)}{2}\,H^2 = \frac{1}{2}\,\dot{\phi}^2 + V$$

For unstable dS maxima, this translates into

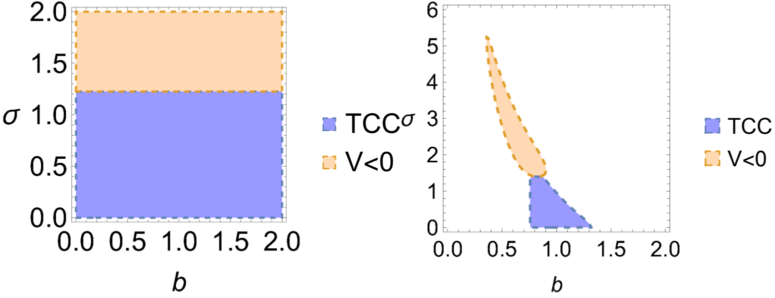
$$\frac{|V''|_{\max}}{V_{\min}} \geq \frac{2}{3}\left(\log\sqrt{\frac{3}{V_{\min}}}\right)^{-2}$$

where

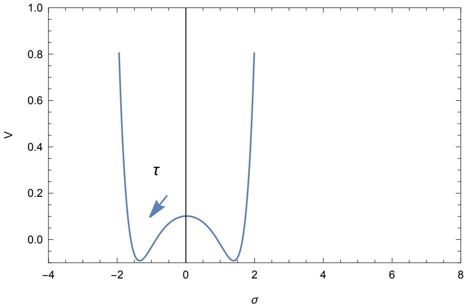
$$0 < \phi < \Delta\phi = \phi_f - \phi_i, \quad |V''| \leq |V''|_{\max}$$

We show that in the range of ϕ that the refined de Sitter conjecture is satisfied TCC is also satisfied for certain values:

$$V''(b)|_{\sigma=0} = 4a^2 2^{-3\alpha} b^{-3\sqrt{\alpha}} \left[(-1 + b^{3\sqrt{\alpha}})^2 - \frac{1}{\alpha}(1 + b^{3\sqrt{\alpha}})^2 \right]$$



Computing a smaller lifetime The TCC allows metastable dS because the only allowed minima are AdS



- This kind of potentials can shed light in the physics of such configurations, because we expect $\tau \ll 10^{12}$ yr.
- We are also trying to reproduce Cow-boy like potentials with a de-Sitter minimum to calculate the tunneling time to the AdS vacua.

Adding Rolling Dynamics

Ferrara, Tournoy and Van Proeyen [1912.06626]:

For dS potentials, $\widehat{V} > 0$, the first criterion of [1806.08362] becomes

$$2\,\frac{K^{I\bar{J}}\,\partial_I \widehat{V}\,\partial_{\bar{J}}\widehat{V}}{\widehat{V}^2} \geq c^2,$$

[1912.06626]: Idea add to a theory rolling dynamics to be in agreement with this criterion.

Our Minimal Model: One de Sitter field plus one rolling field

$$\begin{aligned} \widehat{K} &= K - q\,\ln(\chi+\bar{\chi}) = -3\,\alpha\,\ln(\phi+\bar{\phi}) - q\,\ln(\chi+\bar{\chi}), \\ \widehat{W} &= W\,\chi^{-p/2} = a\,(\phi^{n_+} - \phi^{n_-})\,\chi^{-p/2}, \end{aligned}$$

where α, $q > 0$, $p < 0$, a is an arbitrary constant and $n_\pm = 3/2(\alpha \pm \sqrt{\alpha})$.

The full scalar potential, $\widehat{V}$, can be written nicely in terms of the quantities of the original theory

$$\begin{aligned}\widehat{V} &= e^{\widehat{G}}\left(\sum_{I=1,\bar{J}=1}^{N+M,N+M}\widehat{G}_I\,\widehat{K}^{I\bar{J}}\,\widehat{G}_{\bar{J}}-3\right)\\ &= e^{\widetilde{G}}\left(V+\widehat{X}\,\mathrm{e}^{G}\right), \quad \widetilde{G}=\ln\left[(2\mathrm{Re}\,[\chi])^{-q}\,|\chi|^{-p}\right],\\ &\qquad\qquad\qquad\qquad \widehat{X}\equiv\widehat{G}_m\,\widehat{K}^{m\bar{n}}\,\widehat{G}_{\bar{n}}=\widetilde{G}_m\,\widehat{K}^{m\bar{n}}\,\widetilde{G}_{\bar{n}},\end{aligned}$$

At a frozen value of the rolling parameter

$$\gamma=\widehat{X}\Big|_{\widehat{V}_{\min}},$$

we can study the theory with only the new degrees of freedom.

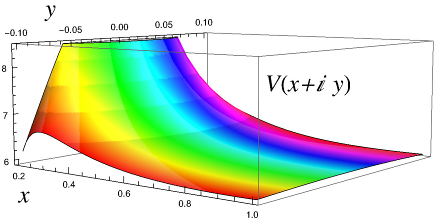
Now we check that whether the modified no-scale model obeys one of the RdSC.

$$\gamma = \widehat{X}\Big|_{\widehat{V}_{\rm min}} = G_\chi \, K^{\chi\bar{\chi}} \, G_{\bar{\chi}}\Big|_{{\rm Im}\chi=0} = \frac{(p+q)^2}{q},$$

$$c^2 \leq \left[2 \, \frac{\widehat{K}^{i\bar{j}}\partial_i \, \widehat{V}\partial_{\bar{j}}\widehat{V}}{V^2} + 2 \, \frac{\widehat{K}^{m\bar{n}}\partial_m \, \widehat{V}\partial_{\bar{n}}\widehat{V}}{\widehat{V}^2}\right]\Bigg|_{\widehat{\rm V}_{\rm min}} = 2\,\gamma,$$

χ is fixed along the real direction

$$e^{\widetilde{G}} = |\chi\,\bar{\chi}|^{-p/2}(\chi+\bar{\chi})^{-q} = ({\rm Re}\,\chi)^{-(p+q)} = e^{\sqrt{2\gamma}}\,\chi^c$$



$\widehat{V} = V\,e^{\sqrt{2\gamma}}\,\chi^c$, γ can be smaller than 1

2 +1 Model: One de Sitter field plus two rolling fields

$$W = (\phi^{n_+} - \phi^{n_-})(a_1\, \chi^{-p_1/2} + a_2\, \chi^{-p_2/2}).$$

$$\widehat{V} = V\, e^{\sqrt{2\gamma}\, \chi_1^c} + V\, e^{\sqrt{2\gamma}\, \chi_2^c}$$

Application: Quintessence models

Copeland, Liddle and Wands [gr-qc/9711068] Scalar potentials with scalar fields, which are a form of dark energy and can explain an accelerating rate of expansion of the universe [Ratra and Peebles, 1988.]

$$\widehat{V} = V\, e^{\lambda_1\, \chi_1^c} + V\, e^{\lambda_2\, \chi_2^c}, \quad V \text{ constant}$$

$$\Rightarrow \quad W = a_1\, (\phi^{n_{1+}} - \phi^{n_{1-}})\, \chi^{-p_1/2} + a_2\, (\phi^{n_{2+}} - \phi^{n_{2-}})\, \chi^{-p_2/2}).$$

Quintessence models

1. Astrophysical observations constrain the ratio of the dark energy density to the critical density ($\Omega_\phi(z)$) and the equation of state ($w(z)$) of dark energy as a function of redshift (z).
2. 2nd. RdSC gives at its boundary a value of V' of the same order as V. This relationship means that current experiments already impose bounds on the value of c in Criterion 2 and future experiments have the possibility of significantly tightening those bounds.

   Agrawal, Prateek, Obied, Steinhardt, and Vafa [1806.09718]:
   Quintessence models with specific values of γ satisfy TCC.

Agrawal, Prateek, Obied, Steinhardt, and Vafa [1806.09718]:
Have shown that the potential with ($\gamma \rightarrow \lambda$)

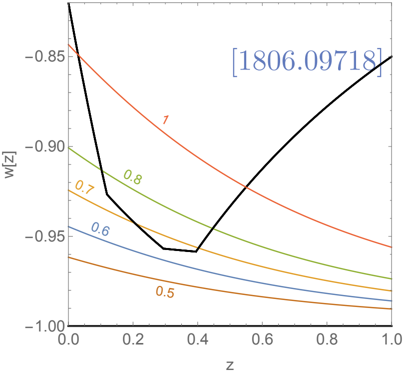
$$\widehat{V} = V\, e^{\lambda_1\, \chi_1^c} + V\, e^{\lambda_2\, \chi_2^c}$$

with $\lambda \approx \lambda_1 >> \sqrt{3}$ in the early universe and then switches to $\lambda \approx \lambda_2 = c = 0.6$ at some recent point in the past. Together these two stages approximate the boundary trajectory that the dark energy equation of state.
Equation of State (EOS)

$$\omega(a) = \omega_0 + \omega\, a(1-a),$$

where a is the scale factor normalized as $a = 1$ today, needs to satisfy in order to be in agreement with SNeIa, CMB and BAO and the RdSC, Predicted EOS, $w(z)$, from Quintessence models as a function of the redshift, z, for different values of $\gamma = \lambda$, constrained by (black curve):

1. $1 + \omega(z) \ll 2/3, \;\text{for}\quad z < 1$
2. $\Omega_\phi(z = 0) = 0.7$
3. $\Omega_\phi(z > 1) \ll 1.$
   To avoid suppression for large-scale structure formation



Here the constants λ_i are constrained to $\lambda_1 \gg 1$ and $\lambda_2 \lesssim 1$. In the early matter era, the potential is approximated by the first term in $\widehat{V}$ while at late times the second term.

The black curve shows the current observational 2σ bound on $w(z)$ for $0 < z < 1$ based on SNeIa, CMB and BAO. This is compared with the predicted $w(z)$ for exponential quintessence potentials with different values of λ.

- In our models we can choose two de Sitter fields and one rolling field to emulate the early universe with one rolling field rendering the potential

$$\widehat{V}_E = V\, e^{\lambda_1\, \chi_1^c}, \quad \lambda_1 \gg \sqrt{3},$$

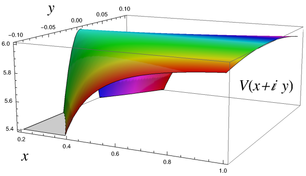
and then switch to the solution with another rolling field

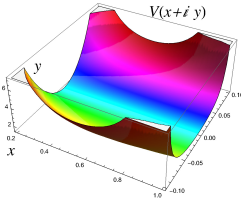
$$\widehat{V} = V\, e^{\lambda_2\, \chi_2^c}, \quad \lambda_1 \sim 0.6$$

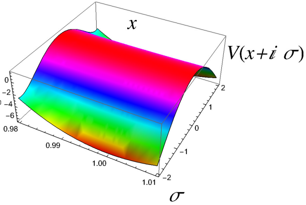
- But we have not thought about couplings to matter.

Conclusions

- No-Scale Supergravity dS (NSdS) models provide a great scenario to test RdSC and TCC
- We explored the minimal example of NSdS and shown that there are regions satisfying both the 2nd. RdSC and the TCC
    - We look forward to understand implications out of this scenario







- Computing a lifetime which can give us info and just not satisfies the bound.
- When adding rolling dynamics the models can be effectively used as Quintessence models.
  - Brax and Martin [hep-th/0612208]: Supergravity models + Quintessence: either trivial or not possible to couple to rest of matter