

# Fractional Helly theorem for Cartesian products of convex sets

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Helly's theorem and its variants asserts that for a family of convex sets in Euclidean space, local intersection patterns influence global intersection patterns. A classical result of Eckhoff in 1988 provided an optimal fractional Helly theorem for axis-parallel boxes, which are Cartesian products of line segments. Answering a question raised by Barany and Kalai, and independently by Lew, we generalize Eckhoff's result to Cartesian products of convex sets in all dimensions. Namely, we prove that, given  $\alpha \in (1 - \frac{1}{t^d}, 1]$  and a finite family of Cartesian products of convex sets  $\prod_{i \in [t]} A_i$  in  $\mathbb{R}^{td}$  with  $A_i \subset \mathbb{R}^d$ , if at least  $\alpha$ -fraction of the  $(d+1)$ -tuples in  $\mathcal{F}$  are intersecting, then at least  $(1 - (t^d(1 - \alpha))^{1/(d+1)})$ -fraction of the sets in  $\mathcal{F}$  are intersecting.

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