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On multicolor extremal problems

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We study a natural generalization of the well-studied Tur\'an problems, known as multicolor Tur\'an problems, which was first introduced and nurtured by Keevash, Saks, Sudakov, and Verstra\"{e}te. A simple k-coloring of a multigraph G is a decomposition of the edge multiset as a disjoint sum of k simple graphs which are referred as \emph{colors}. A subgraph H of a multigraph G is called \emph{multicolored} if all of its edges have distinct colors. The multicolor extremal number, $\mathrm{ex}_k(n, H)$, is defined as the maximum number of edges in an n-vertex multigraph that has a simple k-coloring containing no multicolored copy of H.
Two natural constructions for this problem are as follows: When $k < e(H)$, it is clear that the unique extremal construction comes from k copies of the complete graph. Even when $k \geq e(H)$, one can consider the multigraph consisting of $e(H) - 1$ copies of the complete graph. A second natural construction is to take the sum of k copies of a fixed extremal H-free graph. Keevash, Saks, Sudakov, and Verstra\"{e}te showed that the multicolor extremal problem always admits one of the two natural constructions when H is a complete graph of any fixed order. Moreover, they conjectured the same for every color-critical graphs and proved it for 3-color-critical graphs.
We prove their conjecture for 4-color-critical graphs and for 'most' r-color-critical graphs when $r > 4$. Moreover, we show that for every non-color-critical non-bipartite graphs, none of the two natural constructions is extremal for certain values of k. This answers a question of Keevash, Saks, Sudakov, and Verstra\"{e}te.

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