

2021 Combinatorics Workshop **(2021 조합론학술대회)**

Report of Contributions

Contribution ID: 3

Type: **Contributed talk**

Detecting and Recovering Vote Manipulation on Produce X 101

Wednesday, 22 December 2021 11:30 (25 minutes)

This talk presents a method for detecting and restoring integer datasets that have been manipulated by operations involving non-integral real-number multiplication and rounding. Detecting and restoring such manipulated integer datasets is not straightforward, nor are there any known solutions. We introduce the manipulation process, which was motivated by an actual case of fraud on the TV program “Produce X 101”, and survey several areas of literature dealing with the possibility that manipulation may have happened or might occur.

From our mathematical analysis of the manipulation process, we can prove that the non-integral real number a used in the multiplication exists not as a single real number but as an interval containing infinitely many real numbers, any of which could have been used to produce the same manipulation result.

Based on these analytic findings, we provide an algorithm that can detect and restore manipulated integer datasets. To validate our algorithm, we applied it to 40,000 test datasets that were randomly generated using controllable parameters that matched the real fraud case. Our results indicated that the algorithm detected and perfectly restored all datasets for which the value of the non-integral real number was at least 16 and the number of data entries was at least 40.

This is joint work with Taejung Park (Duksung Women’s University) and Hyunjoo Song (Soongsil University).

Primary author: LEE, Sang June (Kyung Hee University)

Presenter: LEE, Sang June (Kyung Hee University)

Session Classification: Session

Contribution ID: 4

Type: **Contributed talk**

Bounds for the twin-width of graphs

Tuesday, 21 December 2021 11:00 (25 minutes)

Bonnet, Kim, Thomassé, and Watrigant (2020) introduced the twin-width of a graph. We show that the twin-width of an n -vertex graph is less than $(n + \sqrt{n \ln n} + \sqrt{n} + 2 \ln n)/2$, and the twin-width of an m -edge graph is less than $\sqrt{3m} + m^{1/4} \sqrt{\ln m} / (4 \cdot 3^{1/4}) + 3m^{1/4}/2$. Conference graphs of order n (when such graphs exist) have twin-width at least $(n - 1)/2$, and we show that Paley graphs achieve this lower bound. We also show that the twin-width of the Erdős-Rényi random graph $G(n, p)$ with $1/n \leq p \leq 1/2$ is larger than $2p(1 - p)n - (2\sqrt{2} + \varepsilon) \sqrt{p(1 - p)n \ln n}$ asymptotically almost surely for any positive ε . Lastly, we calculate the twin-width of random graphs $G(n, p)$ with $p \leq c/n$ for a constant $c < 1$, determining the thresholds at which the twin-width jumps from 0 to 1 and from 1 to 2.

Primary authors: Mr KIM, Donggyu (KAIST and IBS DIMAG); AHN, Jungho (KAIST and IBS DIMAG); Dr HENDREY, Kevin (IBS DIMAG); Prof. OUM, Sang-il (IBS DIMAG and KAIST)

Presenter: AHN, Jungho (KAIST and IBS DIMAG)

Session Classification: Session

Contribution ID: 5

Type: **Contributed talk**

Fractional Helly theorem for Cartesian products of convex sets

Tuesday, 21 December 2021 15:00 (25 minutes)

Helly's theorem and its variants asserts that for a family of convex sets in Euclidean space, local intersection patterns influence global intersection patterns. A classical result of Eckhoff in 1988 provided an optimal fractional Helly theorem for axis-parallel boxes, which are Cartesian products of line segments. Answering a question raised by Barany and Kalai, and independently by Lew, we generalize Eckhoff's result to Cartesian products of convex sets in all dimensions. Namely, we prove that, given $\alpha \in (1 - \frac{1}{t^d}, 1]$ and a finite family of Cartesian products of convex sets $\prod_{i \in [t]} A_i$ in \mathbb{R}^{td} with $A_i \subset \mathbb{R}^d$, if at least α -fraction of the $(d+1)$ -tuples in \mathcal{F} are intersecting, then at least $(1 - (t^d(1 - \alpha))^{1/(d+1)})$ -fraction of the sets in \mathcal{F} are intersecting.

Primary authors: Dr CHAKRABORTI, Debsoumya (IBS); KIM, Minki (IBS DIMAG); Prof. LIU, Hong (University of Warwick); Prof. KIM, Jaehoon (KAIST); Dr JINHA, Kim (IBS)

Presenter: KIM, Minki (IBS DIMAG)

Session Classification: Session

Contribution ID: 6

Type: **Contributed talk**

On multicolor extremal problems

Tuesday, 21 December 2021 15:30 (25 minutes)

We study a natural generalization of the well-studied Turán problems, known as multicolor Turán problems, which was first introduced and nurtured by Keevash, Saks, Sudakov, and Verstraëte. A simple k -coloring of a multigraph G is a decomposition of the edge multiset as a disjoint sum of k simple graphs which are referred as *colors*. A subgraph H of a multigraph G is called *multicolored* if all of its edges have distinct colors. The multicolor extremal number, $ex_k(n, H)$, is defined as the maximum number of edges in an n -vertex multigraph that has a simple k -coloring containing no multicolored copy of H .

Two natural constructions for this problem are as follows: When $k < e(H)$, it is clear that the unique extremal construction comes from k copies of the complete graph. Even when $k \geq e(H)$, one can consider the multigraph consisting of $e(H) - 1$ copies of the complete graph. A second natural construction is to take the sum of k copies of a fixed extremal H -free graph. Keevash, Saks, Sudakov, and Verstraëte showed that the multicolor extremal problem always admits one of the two natural constructions when H is a complete graph of any fixed order. Moreover, they conjectured the same for every color-critical graphs and proved it for 3-color-critical graphs.

We prove their conjecture for 4-color-critical graphs and for ‘most’ r -color-critical graphs when $r > 4$. Moreover, we show that for every non-color-critical non-bipartite graphs, none of the two natural constructions is extremal for certain values of k . This answers a question of Keevash, Saks, Sudakov, and Verstraëte.

Primary authors: CHAKRABORTI, Debsoumya (Institute for Basic Science); KIM, Jaehoon (KAIST); LEE, Hyunwoo (KAIST); LIU, Hong (University of Warwick); SEO, Jaehyeon (KAIST)

Presenter: SEO, Jaehyeon (KAIST)

Session Classification: Session

Contribution ID: 7

Type: **Contributed talk**

On 1-subdivision of transitive tournaments

Monday, 20 December 2021 17:30 (25 minutes)

The oriented Ramsey number $\vec{r}(H)$ for an acyclic digraph H is the minimum integer n such that any n -vertex tournament contains a copy of H as a subgraph. We prove that the 1-subdivision of the k -vertex transitive tournament H_k satisfies $\vec{r}(H_k) \leq O(k^2 \log \log k)$. This is tight up to multiplicative $\log \log k$ -term.

We also show that if T is an n -vertex tournament with $\Delta^+(T) - \delta^+(T) \leq O(n/k) - k^2$, then T contains a 1-subdivision of \vec{K}_k , a complete k -vertex digraph with all possible $k(k-1)$ arcs. This is also tight up to multiplicative constant.

Primary authors: LEE, Hyunwoo (KAIST); KIM, Jaehoon (KAIST); SEO, Jaehyeon (KAIST)

Presenter: LEE, Hyunwoo (KAIST)

Session Classification: Session

Contribution ID: 8

Type: **Contributed talk**

Combinatorics of Euclidean spaces over finite fields

Tuesday, 21 December 2021 10:30 (25 minutes)

q -analogues of quantities in mathematics involve perturbations of classical quantities using the parameter q , and revert to the original quantities when q goes to 1. An important example is the q -analogues of binomial coefficients, which give the number of k -dimensional subspaces in \mathbb{F}_q^n . When q goes to 1, this reverts to the binomial coefficients which measure the number of k -sets in $[n]$.

In this talk, we describe a formula of the number of quadratic subspaces of Euclidean type in $(\mathbb{F}_q^n, x_1^2 + \cdots + x_n^2)$, which can be described as the form of the analogue of binomial coefficients. The main goal of this talk is to explain this new analogue of binomial coefficients and to study their related combinatorics.

Primary author: YOO, Semin (KIAS)**Presenter:** YOO, Semin (KIAS)**Session Classification:** Session

Contribution ID: 9

Type: **Contributed talk**

On independent domination of cubic graphs without 4-cycles

Tuesday, 21 December 2021 18:00 (25 minutes)

A dominating set of a graph G is a set S of vertices such that each vertex not in S is adjacent to some vertex in S . The independent domination number of a graph G , denoted $i(G)$, is the minimum cardinality of a dominating set of G which is also independent. In 2018, Abrishami and Henning showed that $i(G) \leq \frac{4}{11}|V(G)|$ for every cubic graph G with girth at least 6.

In this talk, we present a result on the independent domination number of a cubic graph, which implies the aforementioned result. More precisely, we prove that if G is a cubic graph without 4-cycles, then $i(G) \leq \frac{5}{14}|V(G)|$, and the bound is tight. This is based on joint work with Eun-Kyung Cho, Ilkyoo Choi and Boram Park.

Primary authors: PARK, Boram; CHO, Eun-Kyoung; KWON, Hyemin (Ajou University); CHOI, Ilkyoo

Presenter: KWON, Hyemin (Ajou University)

Session Classification: Session

Contribution ID: 10

Type: **Contributed talk**

Large clique subdivisions in graphs without small dense subgraphs

Tuesday, 21 December 2021 17:35 (25 minutes)

What is the largest number $f(d)$ where every graph with average degree at least d contains a subdivision of $K_{f(d)}$? Mader asked this question in 1967 and $f(d) = \Theta(\sqrt{d})$ was proved by Bollobás and Thomason and independently by Komlós and Szemerédi. This is best possible by considering a disjoint union of $K_{d,d}$. However, this example contains a much smaller subgraph with the almost same average degree, for example, one copy of $K_{d,d}$.

In 2017, Liu and Montgomery proposed the study on the parameter $c_\varepsilon(G)$ which is the order of the smallest subgraph of G with average degree at least $\varepsilon d(G)$. In fact, they conjectured that for small enough $\varepsilon > 0$, every graph G of average degree d contains a clique subdivision of size $\Omega(\min\{d, \sqrt{\frac{c_\varepsilon(G)}{\log c_\varepsilon(G)}}\})$. We prove that this conjecture holds up to a multiplicative $\min\{(\log \log d)^6, (\log \log c_\varepsilon(G))^6\}$ -term.

As a corollary, for every graph F , we determine the minimum size of the largest clique subdivision in F -free graphs with average degree d up to multiplicative $\text{polylog}(d)$ -term.

This is joint work with Jaehoon Kim, Youngjin Kim, and Hong Liu.

Primary authors: LIU, Hong (University of Warwick); KIM, Jaehoon (KAIST); IM, Seonghyuk (KAIST); KIM, Younjin (Institute of Mathematical Science, Ewha Womans University)

Presenter: IM, Seonghyuk (KAIST)

Session Classification: Session

Contribution ID: 11

Type: **Contributed talk**

When all holes in a graph have the same length

Monday, 20 December 2021 17:00 (25 minutes)

We call an induced cycle of length at least four a hole. The parity of a hole is the parity of its length. Forbidding holes of certain types in a graph has deep structural implications. In 2006, Chudnovsky, Seymour, Robertson, and Thomas famously proved that a graph is perfect if and only if it does not contain an odd hole or a complement of an odd hole. In 2002, Conforti, Cornuejols, Kapoor, and Vušković provided a structural description of the class of even-hole-free graphs. I will describe the structure of all graphs that contain only holes of length ℓ for every $\ell \geq 7$ (joint work with Jake Horsfield, Myriam Preissmann, Paul Seymour, Ni Luh Dewi Sintiari, Cleophe Robin, Nicolas Trotignon, and Kristina Vušković).

Primary author: COOK, Linda (IBS DIMAG)**Presenter:** COOK, Linda (IBS DIMAG)**Session Classification:** Session

Contribution ID: 12

Type: **Contributed talk**

Isomorphism problem for even-cycle matroids

Monday, 20 December 2021 16:00 (25 minutes)

A signed graph is a pair (G, Σ) where G is a graph and Σ is a subset of edges of G . We say that a cycle C of G is even in (G, Σ) if $|C \cap \Sigma|$ is even; otherwise, C is odd. A matroid M is an even-cycle matroid if there exists a signed graph (G, Σ) such that the circuits of M precisely correspond to the even cycles or the unions of two odd cycles sharing at most one vertex. Isomorphism problem for even-cycle matroids is the problem of characterizing two signed graphs (G_1, Σ_1) and (G_2, Σ_2) representing the same even-cycle matroid. In this talk, I will give the structures for solving this problem when G_1 and G_2 are 4-connected.

This is joint work with Bertrand Guenin and Irene Pivotto.

Primary author: HEO, Cheolwon (SKKU AORC)

Co-authors: GUENIN, Bertrand (University of Waterloo); PIVOTTO, Irene

Presenter: HEO, Cheolwon (SKKU AORC)

Session Classification: Session

Contribution ID: 13

Type: **Contributed talk**

Mathematical Approach to the Pallet Loading Problem

Wednesday, 22 December 2021 11:00 (25 minutes)

The (manufacturer's) pallet loading problem has been a topic of wide interest for operational research and companies for more than 50 years. It is to find the optimal loading of identical rectangular boxes onto a single rectangular pallet (also allowing for 90 degree rotation).

It was already known that all box layouts in the given pallet are completely determined by the set of efficient partitions of the pallet's width and height in the box dimensions. The main interest of this talk is the enumeration of all efficient partitions.

We review the pallet loading problem and discuss a mathematical approach to this problem.

Primary author: Dr CHO, Jin-Hwan (National Institute for Mathematical Sciences)

Presenter: Dr CHO, Jin-Hwan (National Institute for Mathematical Sciences)

Session Classification: Session

Contribution ID: 14

Type: **Contributed talk**

Towards a dichotomy classification for list switch homomorphism problem for signed graphs

Monday, 20 December 2021 18:00 (25 minutes)

The list switch homomorphisms problem $LSwHom(H)$ is for a signed graph G with list if there is a switch homomorphism to H preserving lists. We present towards a structural characterisation of the signed graphs H for which the $LSwHom(H)$ problem is polynomial time solvable. We prove the characterisation in the case that the signed graph is reflexive.

Primary authors: KIM, Hyobin (Kyungpook National University); Prof. SIGGERS, Mark (Kyungpook National University)

Presenter: KIM, Hyobin (Kyungpook National University)

Session Classification: Session

Contribution ID: 15

Type: **Invited talk**

Classical statistics on permutations and inversion sequences

Tuesday, 21 December 2021 09:30 (50 minutes)

The study of patterns in inversion sequences was initiated by Corteel-Martinez-Savage-Weselcouch and Mansour-Shattuck independently. This talk introduces some investigation on several classical statistics on restricted inversion sequences that are either known or conjectured to be enumerated by Catalan, Large Schröder, Baxter and Euler numbers. This talk is based on joint work with Zhicong Lin [refined restricted inversion sequences, available at <https://doi.org/10.1007/s00026-021-00550-7>].

Presenter: Prof. KIM, Dongsu (KAIST)

Session Classification: Session

Contribution ID: 16

Type: **Invited talk**

Smooth toric Richardson varieties of Catalan type

Tuesday, 21 December 2021 16:30 (50 minutes)

We define an n -dimensional projective smooth toric variety associated with a triangulation of the $(n+2)$ -gon. Such a toric variety is called of Catalan type. We show that toric varieties of Catalan type are Fano Bott manifolds, and they appear in certain smooth toric Richardson varieties in the flag variety. We also see that toric varieties of Catalan type are classified up to isomorphism in terms of unordered binary trees.

Presenter: PARK, Seonjeong (Jeonju University)

Session Classification: Session

Contribution ID: 17

Type: **Invited talk**

χ -boundedness of graphs with no cycles with k chords

Tuesday, 21 December 2021 14:00 (50 minutes)

A family \mathcal{H} of graphs is said to be χ -bounded, if there is a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that for every graph $H \in \mathcal{H}$ the chromatic number $\chi(H)$ of H is at most $f(\omega)$, where ω is the clique number of H . We show that the family of graphs that do not have a cycle with exactly k chords is χ -bounded, for every large enough k . This proves a conjecture of Aboulker and Bousquet (2015) for sufficiently large k . Joint work with Shoham Letzter and Alexey Pokrovskiy.

Presenter: LEE, Joonkyung (Hanyang University)

Session Classification: Session

Contribution ID: 18

Type: **Invited talk**

Eigenvalues and factors in graphs

Monday, 20 December 2021 15:00 (50 minutes)

Let g, f be non-negative integer-valued functions on $V(G)$ such that $g(v) \leq f(v) \leq d_G(v)$ for all $v \in V(G)$.

A (g, f) -factor of G is a spanning subgraph H of G such that for every vertex $v \in V(G)$, $g(v) \leq d_H(v) \leq f(v)$. For g and f with $g(v) \equiv f(v) \pmod{2}$ for all $v \in V(G)$, a (g, f) -parity factor of G is a (g, f) -factor H such that $d_H(v) \equiv f(v) \pmod{2}$ for all $v \in V(G)$.

For integers a and b , an $[a, b]$ -factor of G is a (g, f) -factor such that $g(v) = a$ and $f(v) = b$ for all $v \in V(G)$,

and a k -factor is a $[k, k]$ -factor. For odd (or even, respectively) integers a and b , an odd (or even, respectively) $[a, b]$ -factor is an $[a, b]$ -factor H such that $d_H(v)$ is odd (or even, respectively). The eigenvalues of G are the eigenvalues of its adjacency matrix.

In this talk, we investigate eigenvalue conditions for a certain graph to have a k -factor, an (even or odd) $[a, b]$ -factor,

a (g, f) -parity factor, or a connected (even or odd) factor.

Presenter: Prof. O, Suil (SUNY Korea)

Session Classification: Session

Contribution ID: 19

Type: **Invited talk**

Exponential decay of intersection volume and applications

Wednesday, 22 December 2021 10:00 (50 minutes)

When two balls in a metric space has small intersection? We give some natural conditions to guarantee an exponential decay on the volume of such intersections. Our proof is conceptually simple, making use of concentration of measure on a “slice”. We will discuss a couple of applications of this volume estimate in coding theory. This is joint work with Jaehoon Kim and Tuan Tran.

Presenter: LIU, Hong (University of Warwick)

Session Classification: Session