# Inflation and Supersymmetry Breaking in Higgs-R2 Supergravity

Shuntaro Aoki (Chung-Ang University)

with Hyun Min Lee and Adriana G. Menkara Based on JHEP 10(2021)178



Seminar @ IBS 2022/2/23

### Contents

#### Introduction

Unitarity issue in Higgs inflation Higgs-R^2 model

#### Supergravity embedding

Unitarity in linear-sigma frame

Inflation, stabilization

Supersymmetry breaking

#### • Summary & Discussion

How to distinguish from non-susy model?

### Introduction

#### Inflation

- Rapid expansion of early universe
- theoretically & observationally established (solve flatness, horizon, monopole problems)
- Slow roll inflation (scalar field)

#### Higgs inflation

Inflaton = SM Higgs boson The original SM Higgs potential  $\times$ 



arXiv:0907.5424

### Higgs inflation

Non-minimal coupling

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{1 + \xi h^2}{2} R\left(g_J\right) - \frac{1}{2} \partial_\mu h \partial_\nu h g_J^{\mu\nu} - \frac{\lambda}{4} h^4 \right]$$

conformal trans.

$$g_E^{\mu\nu} = \Omega^2 g_J^{\mu\nu} \qquad \Omega^2 = 1 + \xi h^2$$
$$d\psi \equiv dh \sqrt{\frac{\Omega^2 + 6\xi^2 h^2}{\Omega^4}}$$

$$\mathcal{L}_E = \sqrt{-g_E} \left( \frac{1}{2} R(g_E) - \frac{1}{2} \partial_\mu \psi \partial_\nu \psi \, g_E^{\mu\nu} - U(\psi) \right)$$
  
For  $h \gg 1/\sqrt{\xi}$   $U(\psi) \approx \frac{\lambda}{4\xi^2} \left( 1 + e^{-\frac{2\psi}{\sqrt{6}}} \right)^{-2}$ 

### Higgs inflation F. L. Bezrukov, N

F. L. Bezrukov, M. Shaposhnikov, '08

$$r = 8\left(\frac{V'}{V}\right)^2, \ n_s - 1 = -3\left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V}$$



CMB observation  $\Rightarrow \xi^2 / \lambda \sim 10^{10}$  $\Rightarrow$  Large non-minimal coupling  $\xi \sim 10^4$ 

#### Unitarity problem in Higgs inflation

• Low cut off scale :  $\Lambda \sim M_{pl} / \xi \Rightarrow$  ?? perturbation Burgess, Lee, Trott, '09

Barbon, Espinosa, '09

c.f.,  $H_{inf} \sim \sqrt{\lambda} \Lambda$ ,  $h \gg M_{pl}/\sqrt{\xi} \gg \Lambda$ 

#### Unitarity problem in Higgs inflation

• Low cut off scale :  $\Lambda \sim M_{pl} / \xi \Rightarrow$  ?? perturbation Burgess, Lee, Trott, '09

Barbon, Espinosa, '09

c.f.,  $H_{inf} \sim \sqrt{\lambda} \Lambda$ ,  $h \gg M_{pl}/\sqrt{\xi} \gg \Lambda$ 

In Jordan frame

$$\sqrt{-g_J}\xi h^2 R_J \supset \frac{\xi}{M_p} h^2 \eta^{\mu\nu} \partial^2 h^J_{\mu\nu}$$

In Einstein frame

$$-g^{\mu\nu}\left[f(D_{\mu}\mathcal{H})^{\dagger}(D_{\nu}\mathcal{H})+\frac{3\,\xi^{2}f^{2}}{2\,M_{p}^{2}}\partial_{\mu}(\mathcal{H}^{\dagger}\,\mathcal{H})\,\partial_{\nu}(\mathcal{H}^{\dagger}\,\mathcal{H})\right]$$

 $f = \left[1 + 2\xi(\mathcal{H}^{\dagger}\mathcal{H})/M_{p}^{2}\right]^{-1}$ 

#### Unitarity problem in Higgs inflation

• However, cutoff scale depends on the inflaton field  $\Rightarrow \Lambda' \sim \sqrt{\xi}h$ 

Bezrukov, Magnin, Shaposhnikov, Sibiryakov' 10 Ferrara, Kallosh, Linde, Marrani, Van Proeyen' 10

 Unitarity problem again in preheating (Particles with momentum above the cutoff are generated)

Ema, Jinno, Mukaida, Nakayama' 17

#### Solution

One of solutions : Introduce a new d.o.f  $\sigma$  (like Higgs boson in SM)

$$\frac{\mathcal{L}_{J}}{\sqrt{-g_{J}}} = \frac{1}{2} \Big( \bar{M}^{2} + \underline{\xi} \bar{\sigma}^{2} + 2\zeta \mathcal{H}^{\dagger} \mathcal{H} \Big) R - \frac{1}{2} (\underline{\partial_{\mu} \bar{\sigma}})^{2} - |D_{\mu} \mathcal{H}|^{2} \\ - \frac{1}{4} \kappa \Big( \bar{\sigma}^{2} - \bar{\Lambda}^{2} - 2\alpha \mathcal{H}^{\dagger} \mathcal{H} \Big)^{2} - \lambda \Big( \mathcal{H}^{\dagger} \mathcal{H} - \frac{v^{2}}{2} \Big)^{2}. \qquad \text{Giudice, Lee '10} \\ \dots$$

cut off  $\Rightarrow M_{pl}$  $\sigma$ -field linearize Higgs kinetic term = UV completion of Higgs inflation

# R^2 (Starobinsky) model Starobinsky, '80, 83



## Higgs-R^2 model

Idea : scalaron for unitarizing Higgs inflation ?

$$\mathcal{L}_{J}/\sqrt{-g_{J}} = \frac{1}{2} \left( 1 + \xi h^{2} \right) R - \frac{1}{2} (\partial_{\mu} h)^{2} - \frac{\lambda}{4} h^{4} + \alpha R^{2}$$

✓ Yes, it works! Y. Ema' 17, D. Gorbunov and A. Tokareva'18,...

Scalaron pushes up to the unitarity scale to  $M_{\rm Pl}$ (I will discuss how it solves in susy case)

$$\sqrt{\alpha} \sim \xi^2$$
 emerges by RGE effect

A.Salvio and A. Mazumdar ' 15X. Calmet and I. Kuntz ' 16Y. Ema' 17,D. M. Ghilencea ' 18

### Phenomenology of Higgs-R^2 model

 $\checkmark$  inflation

A.Salvio and A. Mazumdar ' 15D. Gorbunov and A. Tokareva'18Y. Ema' 19,A. Gundhi and C. F. Steinwachs ' 20, ...

✓ (p)reheating

M. He, R. Jinno, K. Kamada, S. C. Park, A. A. Starobinsky, J. Yokoyama '18 F. Bezrukov and C. Shepherd, '20 M. He, '20,...

✓ PBH D. Y. Cheong, S. M. Lee, and S. C. Park , '19

✓ Dark energy H. M. Lee, A. G. Menkara '21

✓ Dark matter

SA, H. M. Lee, A. G. Menkara, K.Yamashita, in progress

#### S.A., H.M.Lee, A.G.Menkara, 2108.00222 = embed Higgs-R^2 inflation into supergravity

#### Supersymmetry

...

- solution of the Hierarchy problem
- unification of the forces
- · dark matter candidate





particle data group

SUSY embedding may give an UV completion of Higgs-R^2 inflation

#### S.A., H.M.Lee, A.G.Menkara, 2108.00222 = embed Higgs-R^2 inflation into supergravity

#### Questions

- Unitarity?
- maintain successful inflation?
- supersymmetry breaking and phenomenology?
- difference from non-SUSY model?



#### Higgs inflation in NMSSM

$$\mathcal{L} = -\frac{1}{6}\Omega R - \Omega_{I\bar{J}}\partial_{\mu}z^{\bar{I}}\partial^{\mu}\bar{z}^{\bar{J}} - V \qquad \Omega : \text{frame function}$$

$$K = -3\log\left(-\frac{\Omega}{3}\right)$$

$$Contents : z^{\bar{I}} = \{H_u, H_d, S\}$$

$$\begin{cases} \Omega = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \left(\frac{3}{2}\partial H_u \cdot H_d + \text{h.c.}\right) \\ W = \lambda SH_u \cdot H_d + \frac{\rho}{3}S^3 \end{cases}$$
Einhorn, Jones' 10
H.M.Lee' 10
Ferrara, Kallosh, Linde, Marrani, Van Proeyen' 10

## Higgs inflation in NMSSM

Truncate every field other than h

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \to \begin{pmatrix} 0 \\ h/2 \end{pmatrix}, H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \to \begin{pmatrix} h/2 \\ 0 \end{pmatrix}, S \to 0$$



Realize Higgs inflation with  $\chi/\lambda \sim 10^4$ 

CMB  $\Rightarrow \chi/\lambda \sim 10^4$   $\Rightarrow$  Large non-minimal coupling  $\chi$ 

R^2 (Starobinsky) inflation in supergravity

$$[\alpha \bar{\mathcal{R}} \mathcal{R}]_D = \alpha R^2 + \cdots$$

S. Cecotti' 87

$$\mathcal{R} = (X^0)^{-1}\Sigma(\bar{X}^{\bar{0}})$$
 : curvature superfield  
 $X^0$  : compensator  
 $\Sigma$  : chiral projection (~  $\overline{D}^2$ )

#### R^2 (Starobinsky) inflation in supergravity

Introduce auxiliary superfield T and C

$$[\alpha \bar{\mathcal{R}} \mathcal{R}]_{D} = [\alpha \bar{C}C]_{D} + [T(C - \mathcal{R})]_{F} \int d^{2}\theta$$

$$= [TC - \Sigma(T(X^{0})^{-1}\bar{X}^{\bar{0}})]_{F}$$

$$= [TC]_{F} - [T(X^{0})^{-1}\bar{X}^{\bar{0}} + c.c.]_{D}$$

$$\&$$

$$T \rightarrow T(X^{0})^{2} \text{ and } C \rightarrow CX^{0}$$

$$C \rightarrow C/\sqrt{\alpha},$$

$$= [|X^{0}|^{2}\Omega]_{D} + [(X^{0})^{3}W]_{F}$$

$$\Omega = |C|^{2} - (T + \bar{T})$$

$$W = \frac{1}{\sqrt{\alpha}}TC$$

$$R^{2} \text{ SUGRA = SUGRA + two chiral multiplets } (T, C)$$

#### R^2 (Starobinsky) inflation in supergravity

Kallosh, Linde' 13

 $\text{Im}T \to 0, C \to 0$ 



Realize R^2 inflation with ReT identified as inflaton (scalaron)



### Supergravity embedding

Higgs R^2 SUGRA =

NMSSM inflation in SUGRA + R^2 SUGRA ( $\Rightarrow$  dual scalars T, C)

$$K = -3 \log \left( -\frac{\Omega}{3} \right)$$
  

$$\Omega = -3 + |S|^2 + |H_u|^2 + |H_d|^2 + \left( \frac{3}{2} \chi H_u \cdot H_d + \text{h.c.} \right) + |C|^2 - (T + \bar{T})$$
  

$$W = \lambda S H_u \cdot H_d + \frac{\rho}{3} S^3 + \frac{1}{\sqrt{\alpha}} T C$$

✓ Fields : NMSSM singlet *S*, Higgs doublets  $H_u$ ,  $H_d$ , Dual scalars *T*, *C* ✓ Parameters :  $\chi$  (non-minimal coupling),  $\lambda$ ,  $\rho$ ,  $\alpha$  (coefficient of R^2) ↓ Large Large ✓ By construction, it has R^2 origin (*T*-sector is fixed by R^2 structure)





Conformal trans.  $g_{\mu\nu}^{J} = \left(1 + \frac{1}{\sqrt{6}}\sigma\right)^{2}g_{\mu\nu}$ 

Field redef.  $\hat{z}^i \equiv \left(1 + \frac{1}{\sqrt{6}}\sigma\right) z^i$ ,  $\hat{T} \equiv \left(1 + \frac{1}{\sqrt{6}}\sigma\right)^2 T$ ,  $z^i = \{S, H_u, H_d, C\}$ 

choose  $\sigma$  s.t.

$$\left(1 + \frac{1}{\sqrt{6}}\sigma\right)^2 + \left(-\frac{1}{2}\chi\hat{H}_u\cdot\hat{H}_d + \text{h.c.}\right) + \frac{2}{3}\text{Re}\hat{T} = 1 - \frac{1}{6}\sigma^2.$$
  
Convert Re $\hat{T}$  (scalaron)  $\rightarrow \sigma$ 

Conformal trans.  $g_{\mu\nu}^{J} = \left(1 + \frac{1}{\sqrt{6}}\sigma\right)^{2}g_{\mu\nu}$ 

Field redef.  $\hat{z}^i \equiv \left(1 + \frac{1}{\sqrt{6}}\sigma\right) z^i$ ,  $\hat{T} \equiv \left(1 + \frac{1}{\sqrt{6}}\sigma\right)^2 T$ ,  $z^i = \{S, H_u, H_d, C\}$ 

choose  $\sigma$  s.t.

$$\left(1+\frac{1}{\sqrt{6}}\sigma\right)^2 + \left(-\frac{1}{2}\chi\hat{H}_u\cdot\hat{H}_d + \text{h.c.}\right) + \frac{2}{3}\text{Re}\hat{T} = 1 - \frac{1}{6}\sigma^2.$$

Convert Re $\hat{T}$  (scalaron)  $\rightarrow \sigma$ 

Where  $\chi$ ,  $\alpha$  have gone?

$$\begin{split} V &= |\lambda \hat{H}_{u} \cdot \hat{H}_{d} + \rho \hat{S}^{2}|^{2} + \lambda^{2} |\hat{S}|^{2} (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) + \frac{1}{4\alpha} \left( \sigma^{2} + \sqrt{6}\sigma - \left( \frac{3}{2} \chi \hat{H}_{u} \cdot \hat{H}_{d} + \text{h.c.} \right) \right)^{2} \\ &+ \frac{1}{\alpha} (\text{Im} \hat{T})^{2} + \frac{3}{2} \frac{\chi \lambda}{\sqrt{\alpha}} (\hat{S} \bar{C} + \bar{S} \hat{C}) (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) \\ &+ \frac{1}{\alpha} |\hat{C}|^{2} \left\{ 3 + 2\sqrt{6}\sigma + \frac{3}{2}\sigma^{2} + \frac{9}{4} \chi^{2} (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) \right\} \\ &+ \frac{g'^{2}}{8} \left( |\hat{H}_{u}|^{2} - |\hat{H}_{d}|^{2} \right)^{2} + \frac{g^{2}}{8} \left( (\hat{H}_{u})^{\dagger} \vec{\tau} \hat{H}_{u} + (\hat{H}_{d})^{\dagger} \vec{\tau} \hat{H}_{d} \right)^{2}. \end{split}$$

Where  $\chi$ ,  $\alpha$  have gone?

$$V = |\lambda \hat{H}_{u} \cdot \hat{H}_{d} + \rho \hat{S}^{2}|^{2} + \lambda^{2} |\hat{S}|^{2} (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) + \frac{1}{4\alpha} \left( \sigma^{2} + \sqrt{6}\sigma - \left( \frac{3}{2} \chi \hat{H}_{u} \cdot \hat{H}_{d} + \text{h.c.} \right) \right)^{2} + \frac{1}{\alpha} (\text{Im} \hat{T})^{2} + \frac{3}{2} \frac{\chi \lambda}{\sqrt{\alpha}} (\hat{S} \hat{C} + \hat{S} \hat{C}) (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) + \frac{1}{\alpha} |\hat{C}|^{2} \left\{ 3 + 2\sqrt{6}\alpha + \frac{3}{2}\sigma^{2} + \frac{9}{4}\chi^{2} (|\hat{H}_{u}|^{2} + |\hat{H}_{d}|^{2}) + \frac{g'^{2}}{8} \left( |\hat{H}_{u}|^{2} - |\hat{H}_{d}|^{2} \right) + \frac{g^{2}}{8} \left( (\hat{H}_{u})^{\dagger} \vec{\tau} \hat{H}_{u} + (\hat{H}_{d})^{\dagger} \vec{\tau} \hat{H}_{d} \right)^{2}.$$

no unitary violation up to Planck scale even after susy extension

 $\chi/\sqrt{\alpha}$  can be order one

• perturbativity requires  $\frac{\chi}{\sqrt{\alpha}} < 1$ 

### Inflation

SUGRA Higgs-R^2 system 
$$\supset \sigma, h, S, C, ...$$
  
= non-susy Extra fields due to susy  
(should be stabilized)

Naïve embedding  $\rightarrow$  tachyonic instability in one of mixed state of S and C

Solution: 
$$\Delta \Omega = -\zeta_s |S|^4 - \zeta_c |C|^4$$
$$\downarrow$$
$$-[\zeta_c \alpha^2 |X^0|^{-2} (\bar{\mathcal{R}}\mathcal{R})^2]_D \text{ in dual side}$$

Note : doesn't destroy R^2 structure in component

# Stabilization of S and C

#### $\rho$ : cubic coupling of S



 $(\lambda, \xi, \alpha) = (4 \times 10^{-5}, 1, 10^2)$ 

#### Inflation

I.

EFT of  $\sigma$  and h = same as non-susy case

$$V_{\text{eff}}(\phi) \approx \begin{cases} \frac{9}{4\alpha} \left(1 - e^{-2\phi/\sqrt{6}}\right)^2, & \frac{\xi^2}{\alpha} \ll \lambda^2 & \frac{R^2}{\alpha} \gg \lambda^2 \end{cases}$$

$$k^2 = \frac{\frac{1}{\alpha}\sigma(\sigma + \sqrt{6}) \left(\sigma - 3\left(\xi + \frac{1}{6}\right)(\sigma - \sqrt{6})\right)}{\frac{\lambda^2}{4}(\sigma - \sqrt{6}) - \frac{3}{\alpha}\left(\xi + \frac{1}{6}\right)(\sigma - 3\left(\xi + \frac{1}{6}\right)(\sigma - \sqrt{6})}{\left(1 - 3\left(\xi + \frac{1}{6}\right)^2\right)^2}\right)^{-1} \qquad \xi \equiv -\frac{1}{6} + \frac{\chi}{4}$$

$$V_{\text{eff}}(\phi) \approx \begin{cases} \frac{9}{4\alpha} \left(1 - e^{-2\phi/\sqrt{6}}\right)^2, & \frac{\xi^2}{\alpha} \ll \lambda^2 & \frac{R^2}{\alpha} = \frac{1}{6} + \frac{\chi}{4} \end{cases}$$

#### Inflationary observables



# SUSY breaking

#### 1. SUSY breaking by higher curvature effects

I. Dalianis, F. Farakos, A. Kehagias, A. Riotto, R. von Unge, '14

$$\Omega = -3 + (T + \overline{T}) + |C|^{2} - \gamma_{c}(C + \overline{C}) - \zeta_{c}|C|^{4}$$

$$\operatorname{R^{2} SUGRA}$$

$$\operatorname{In \ dual \ picture,} -3 + \alpha |\mathcal{R}/X^{0}|^{2} - \gamma_{c}\alpha \left(\mathcal{R}/X^{0} + \mathrm{c.c.}\right) - \zeta_{c}\alpha^{2}|\mathcal{R}/X^{0}|^{4}$$

$$(T) \neq 0, \langle C \rangle \neq 0$$

$$\operatorname{SUSY} \sim M_{P}/\sqrt{\alpha} \gtrsim 10^{13} \ \mathrm{GeV} \qquad : \operatorname{High \ scale \ SUSY \ breaking}$$

$$\operatorname{perturbativity \ constraint}$$

## SUSY breaking

2. O'Raifeartaigh model

$$\Omega = -3 - (T + \overline{T}) + |C|^2 + |\Phi|^2 - \gamma |\Phi|^4,$$

$$W = \frac{1}{\sqrt{\alpha}} TC + \kappa \Phi + g \Phi C^2 + \lambda \Phi^3 + \kappa' C + g' \Phi^2 C + \lambda' C^3$$

$$\swarrow Z_{4R} \text{ R-symmetry with } R[\Phi] = R[C] = +2 \quad R[T] = 0$$

SUSY breaking scale ~  $F_{\Phi} = \kappa$  : adjustable

# Implication for phenomenology

 $\mu$ -term



Sequestered form

Randall, Sundrum' 99

vanishing soft mass at tree level



anomaly mediation



tachyonic slepton

$$\Omega_{\text{contact}} = C_{\bar{\alpha}\beta} X^{\dagger} X z_{\bar{\alpha}}^{\dagger} z_{\beta} + \text{c.c} \qquad X = C, \Phi,$$

# Summary

- embed Higgs-R^2 inflation into SUGRA
- interpolate NMSSM inflation and R^2 inflation in SUGRA
- Three frames (Jordan, Einstein, Linear sigma)
- no unitarity issue even after supersymmetrization
- successful slow-roll inflation with quartic couplings (higher curvature terms)
- two SUSY breaking mechanisms & transmission to visible sector

## **Discussion & Future**

No distinction in inflationary observables  $(n_s, r)$ of susy/non-susy models  $\Rightarrow$  How to distinguish?

- S, C contributions to inflation if they are light
- SUSY breaking effect on inflation
- go beyond standard analysis (e.g., Non-Gaussianity?)

Im*T* : relatively light  $\sim 2H$ 

• gravitino problem

 $T_r$  should be  $\leq 10^{8,9}$  GeV  $\Leftrightarrow$  naïve estimation  $\sim 10^{13}$  GeV For pure R^2 SUGRA  $\Rightarrow$  T. Terada, Y. Watanabe, Y. Yamada and J. Yokoyama, JHEP 02, 105 (2015)

#### Thank you for your attention !!

# Backup

Cut off depends on the background field value F. Bezrukov, A. Magnin, M. Shaposhnikov, S. Sibiryakov [arXiv:1008.5157]

$$\Lambda^{J}(\bar{\phi}) = \frac{M_{P}^{2} + \xi \bar{\phi}^{2} + 6\xi^{2} \bar{\phi}^{2}}{\xi \sqrt{M_{P}^{2} + \xi \bar{\phi}^{2}}}$$

 $\bar{\phi} \gg M_P/\sqrt{\xi}$ , large fields (inflationary period)  $\Lambda^J \simeq \sqrt{\xi} \bar{\phi}$ 





#### Jordan frame supergravity

S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, Phys. Rev. D 82, 045003 (2010)

$$\mathcal{L}/\sqrt{-g} = -\frac{1}{6}(X^0)^2 \Omega R - \Omega (\partial_\mu X^0)^2 - X^0 \partial^\mu X^0 \left(\Omega_I \partial_\mu z^I + \Omega_{\bar{I}} \partial_\mu \bar{z}^{\bar{I}}\right) + (X^0)^2 \Omega \mathcal{A}^2_\mu - (X^0)^2 \Omega_{I\bar{J}} \partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} - V, \qquad (2.16)$$

where

$$\mathcal{A}_{\mu} = -\frac{\mathrm{i}}{2\Omega} \left( \partial_{\mu} z^{I} \Omega_{I} - \partial_{\mu} \bar{z}^{\bar{I}} \Omega_{\bar{I}} \right).$$
(2.17)

$$V^{F} = \left(X^{0}\right)^{4} \left(\Omega_{I\bar{J}} - \frac{\Omega_{I}\Omega_{\bar{J}}}{\Omega}\right)^{-1} \left(W_{I} - \frac{3\Omega_{I}}{\Omega}W\right) \left(\bar{W}_{\bar{J}} - \frac{3\Omega_{\bar{J}}}{\Omega}\bar{W}\right) + \frac{9}{\Omega} \left(X^{0}\right)^{4} |W|^{2}$$
$$= \left(X^{0}\right)^{4} e^{\mathcal{K}/3} \left[\mathcal{K}^{I\bar{J}} \left(W_{I} + \mathcal{K}_{I}W\right) \left(\bar{W}_{\bar{J}} + \mathcal{K}_{\bar{J}}\bar{W}\right) - 3|W|^{2}\right], \qquad (2.18)$$

$$V^{D} = \frac{(X^{0})^{-1}}{2} (\operatorname{Re} f)^{-1AB} \Omega_{\alpha} k^{\alpha}_{A} \Omega_{\bar{\beta}} k^{\bar{\beta}}_{B}, \qquad (2.19)$$

### R<sup>2</sup> frame

 $S = [|X^0|^2 \tilde{\Omega}(z^\alpha, \bar{z}^{\bar{\beta}})]_D + [(X^0)^3 \tilde{W}(z^\alpha)]_F + [f_{AB}(z^\alpha) \bar{\mathcal{W}}^A \mathcal{W}^B]_F + [\alpha \bar{\mathcal{R}} \mathcal{R}]_D,$ 

 $\mathcal{R} = (X^0)^{-1} \Sigma(\bar{X}^{\bar{0}}),$   $\Sigma$  is a chiral projection operator

$$\mathcal{L}/\sqrt{-g} = -\tilde{\Omega}_{\alpha\bar{\beta}}\partial_{\mu}z^{\alpha}\partial^{\mu}\bar{z}^{\bar{\beta}} + (-i\tilde{\Omega}_{\alpha}\partial_{\mu}z^{\alpha}\mathcal{A}^{\mu} + \text{c.c.}) + \tilde{\Omega}(-\mathcal{A}^{2} + |F^{0}|^{2}) + (3F^{0}\tilde{W} + \text{c.c.})$$

$$+ \left(-\frac{\tilde{\Omega}}{6} + \frac{\alpha}{6}|F^{0}|^{2} + \frac{\alpha}{3}\mathcal{A}^{2}\right)R + \frac{\alpha}{36}R^{2} + \alpha\left(\mathcal{A}^{2} + |F^{0}|^{2}\right)^{2} + \alpha(\nabla_{\mu}\mathcal{A}^{\mu})^{2}$$

$$- \alpha\left|\partial_{\mu}F^{0} - 3i\mathcal{A}_{\mu}F^{0}\right|^{2} - \tilde{\Omega}^{\alpha\bar{\beta}}(\tilde{\Omega}_{\alpha}\bar{F}^{\bar{0}} + \tilde{W}_{\alpha})(\tilde{\Omega}_{\bar{\beta}}F^{0} + \bar{\tilde{W}}_{\bar{\beta}})$$

$$- \frac{1}{2}(\text{Re}f)^{-1AB}\tilde{\Omega}_{\alpha}k^{\alpha}_{A}\tilde{\Omega}_{\bar{\beta}}k^{\bar{\beta}}_{B}, \qquad (2.3)$$

### R^2 frame

$$\tilde{\Omega}(z^{\alpha}, \bar{z}^{\bar{\beta}}) = -3 + |S|^{2} + |H_{u}|^{2} + |H_{d}|^{2} + \left(\frac{3}{2}\chi H_{u} \cdot H_{d} + \text{h.c.}\right),$$

$$\tilde{W}(z^{\alpha}) = \lambda S H_{u} \cdot H_{d} + \frac{\rho}{3}S^{3},$$

$$\mathcal{L}/\sqrt{-g} = \left\{\frac{1}{2} - \frac{1}{6}|S|^{2} - \frac{1}{6}|H_{u}|^{2} - \frac{1}{6}|H_{d}|^{2} + \left(-\frac{1}{4}\chi H_{u} \cdot H_{d} + \text{h.c.}\right)\right\}R$$

$$- |\partial_{\mu}S|^{2} - |\partial_{\mu}H_{u}|^{2} - |\partial_{\mu}H_{d}|^{2} + \frac{\alpha}{36}R^{2} + \cdots,$$

### Unitarity issue for ImT

$$\begin{aligned} \mathcal{L}_{LS}/\sqrt{-g} &= \frac{1}{2} \left( 1 - \frac{1}{3} |\hat{S}|^2 - \frac{1}{3} |\hat{H}_u|^2 - \frac{1}{3} |\hat{H}_d|^2 - \frac{1}{3} |\hat{C}|^2 - \frac{1}{6} \sigma^2 \right) R \\ &- |\partial_\mu \hat{S}|^2 - |\partial_\mu \hat{H}_u|^2 - |\partial_\mu \hat{H}_d|^2 - |\partial_\mu \hat{C}|^2 - \frac{1}{2} (\partial_\mu \sigma)^2 + \Omega \mathcal{A}_\mu^2 - V_{LS}, \end{aligned}$$

$$\Omega \mathcal{A}^2_{\mu} = -\frac{1}{4\Omega} \left[ (X^0)^{-1} \left( (\bar{\hat{C}} \partial_{\mu} \hat{C} + \bar{\hat{S}} \partial_{\mu} \hat{S} + \bar{\hat{H}}_u \partial_{\mu} \hat{H}_u + \bar{\hat{H}}_d \partial_{\mu} \hat{H}_d - \text{c.c}) - 2i \partial_{\mu} b \right) -4ib \, \partial_{\mu} (X^0)^{-1} \right]^2. \qquad X^0 = 1 + \frac{1}{\sqrt{6}} \sigma,$$
$$\hat{\pi} = \bar{\hat{\sigma}} - \bar{\hat{\sigma}} -$$

$$\hat{T} - \bar{T} - \frac{3}{2}\chi(\hat{H}_u \cdot \hat{H}_d - \bar{H}_u \cdot \bar{H}_d) = 2ib, \text{ convert } \operatorname{Im} \widehat{T} \rightarrow b$$

$$V_{LS}^F \supset \frac{1}{\alpha} (\mathrm{Im}\hat{T})^2 = \frac{1}{\alpha} \left( b - \frac{3}{4} i\chi (\hat{H}_u \cdot \hat{H}_d - \bar{\hat{H}}_u \cdot \bar{\hat{H}}_d) \right)^2.$$

perturbative as long as

$$\frac{\chi}{\alpha} \lesssim 1, \qquad \frac{\chi^2}{\alpha} \lesssim 1.$$

#### Effective mu-term

$$\mathcal{K}(z,\bar{z}) = -3M_P^2 \log \left[ 1 - \frac{\phi^a \bar{\phi}_a}{3M_P^2} - \frac{J(\phi)}{3M_P^2} - \frac{\bar{J}(\bar{\phi})}{3M_P^2} - \dots \right] \qquad \qquad J(\phi) = -\chi \, C_{ab} \phi^a \phi^b.$$

$$\mathcal{K}(z,\bar{z}) = \phi^a \bar{\phi}_a + J(\phi) + \bar{J}(\bar{\phi}) + \dots$$

#### Kaehler transformation

$$\begin{split} W_{\rm eff} \to W \mathrm{e}^{J(\phi)/M_P^2} \approx W + \frac{\langle W_{\rm hid} \rangle}{M_P^2} J\left(\phi\right) \approx W + \frac{m_{3/2} J\left(\phi\right)}{m_{3/2}} \\ \left[ m_{3/2} = \mathrm{e}^{\frac{\kappa}{2M_P^2}} \frac{\langle W \rangle}{M_P^2} \approx \frac{\langle W_{\rm hid} \rangle}{M_P^2} \right] \end{split}$$

$$Z_M^R$$
-symmetry

M. Lee, S. Raby, M. Ratz, G. G. Ross, R. Schieren, K. Schmidt-Hoberg and P. K. S. Vaudrevange, Nucl. Phys. B 850, 1-30 (2011)

$$\mathscr{W} \to e^{2\pi i q_{\mathscr{W}}/M} \mathscr{W} \quad \text{with } q_{\mathscr{W}} = 2.$$
$$\theta \to e^{2\pi i/M} \theta$$
$$\Phi^{(f)} \to e^{2\pi i q^{(f)}/M} \Phi^{(f)}$$

Our case :

 $Z_{4R}$  R-symmetry with  $R[\Phi] = R[C] = +2$  R[T] = 0