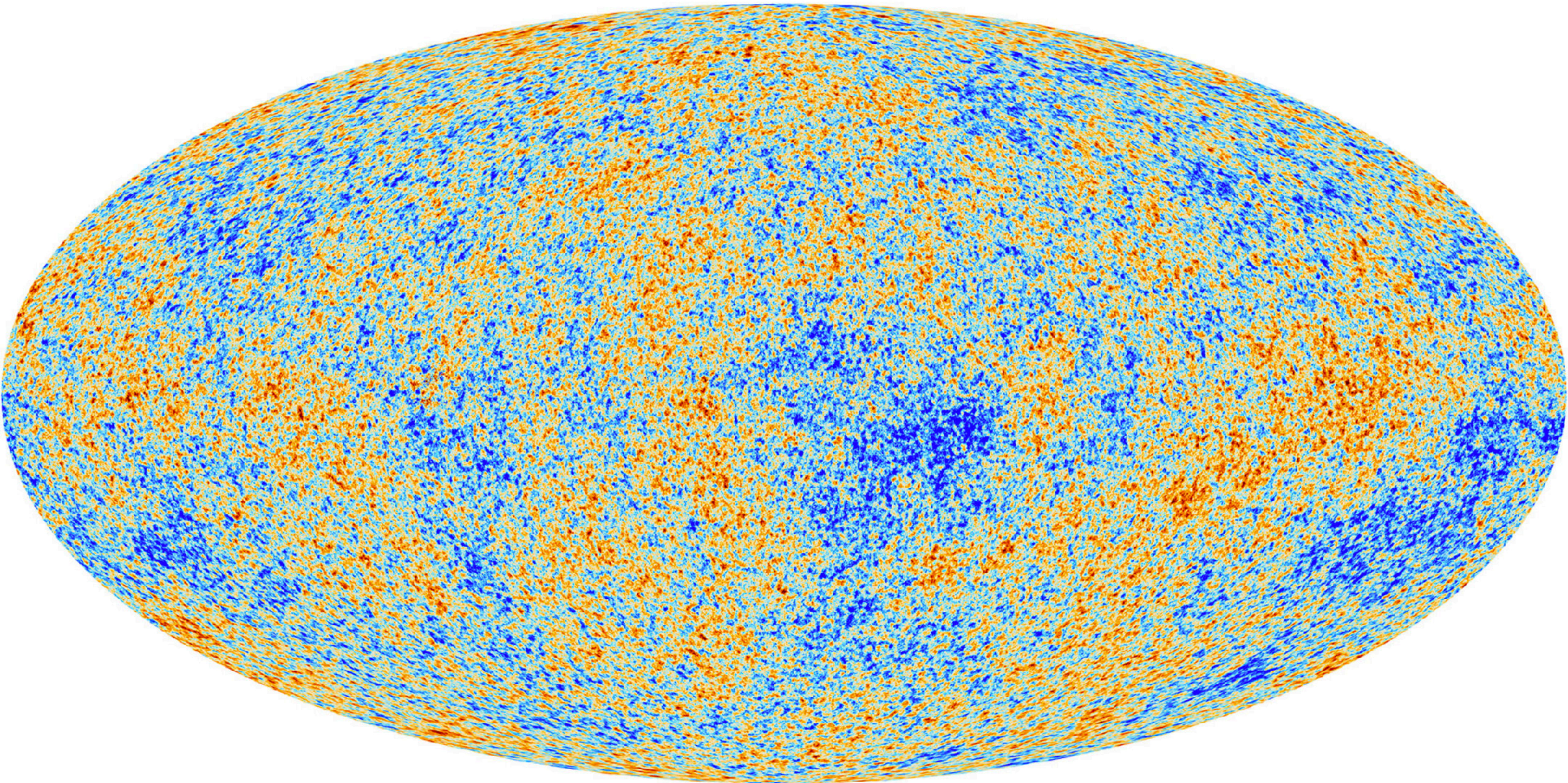


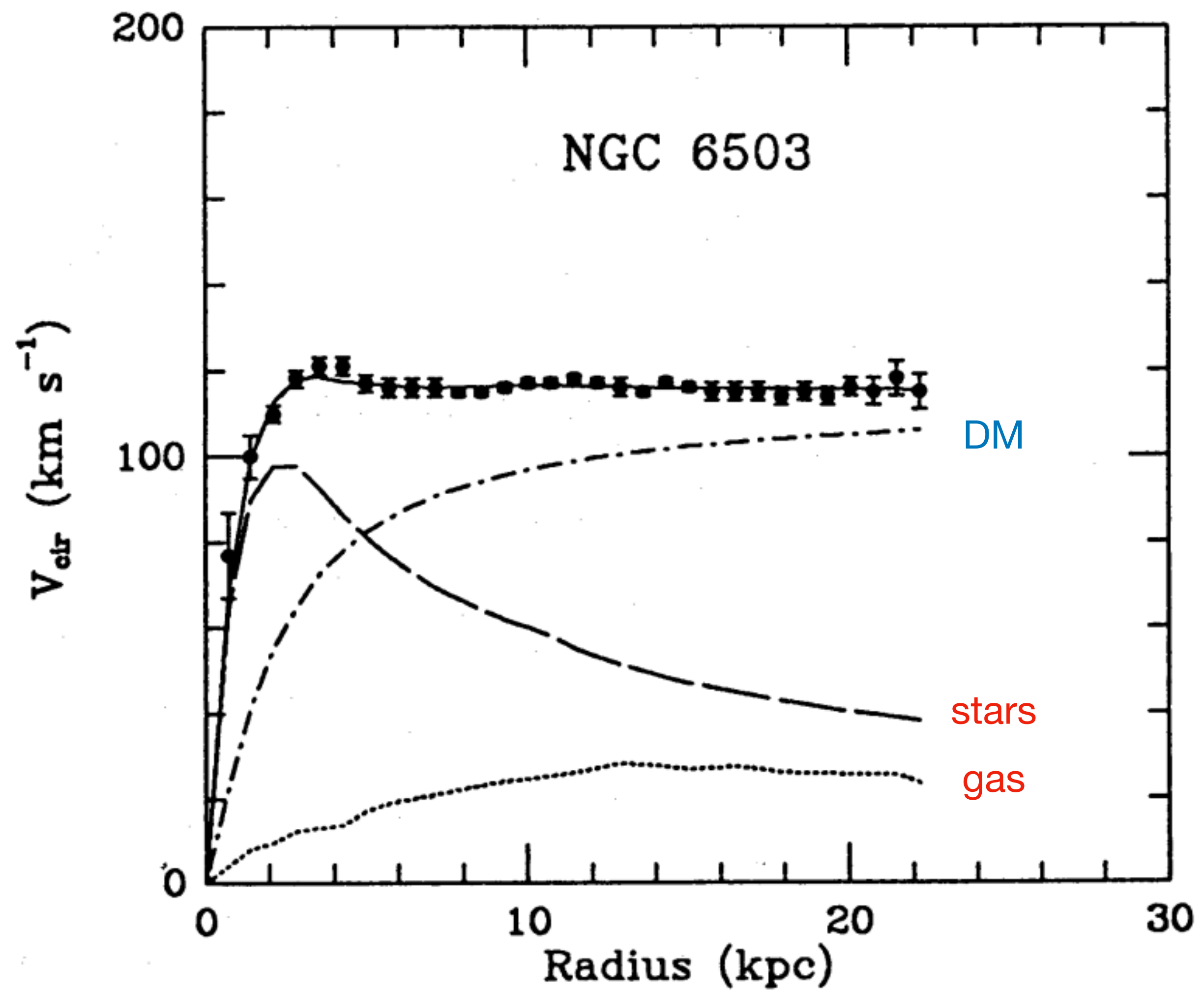
Wave dark matter around the solar system

Hyungjin Kim

DESY



Credit: ESA and the Planck Collaboration



[Begeman et al 91]

What's its mass?

How does it interaction with SM?

What's its mass?

ULDM

Black holes

10^{-22} eV

10^{57} GeV

Anything in between is possible

What's its mass?

ULDM

Black holes

10^{-22} eV

10^{57} GeV

Anything in between is possible

Countless ways for DM to interact with SM particles

What's its mass?

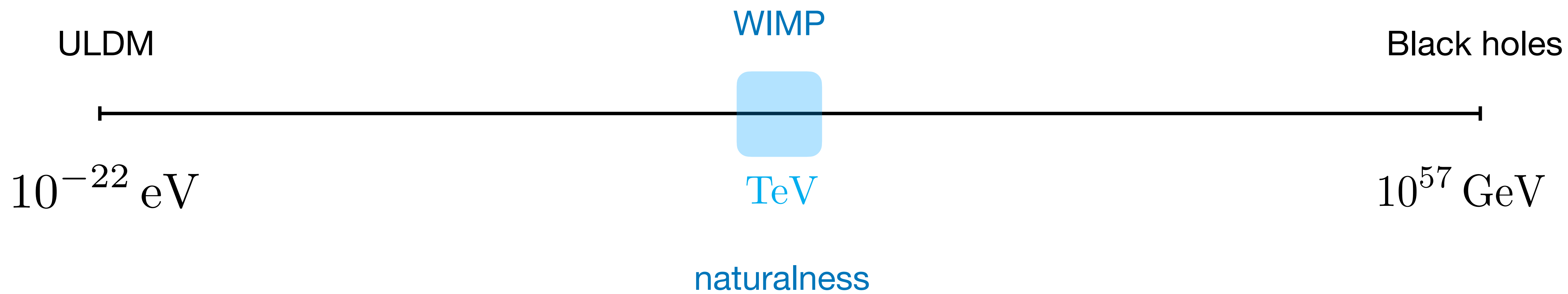
ULDM

Black holes

10^{-22} eV

10^{57} GeV

This constitutes *N-dimensional DM parameter space* ($N \gg 1$)



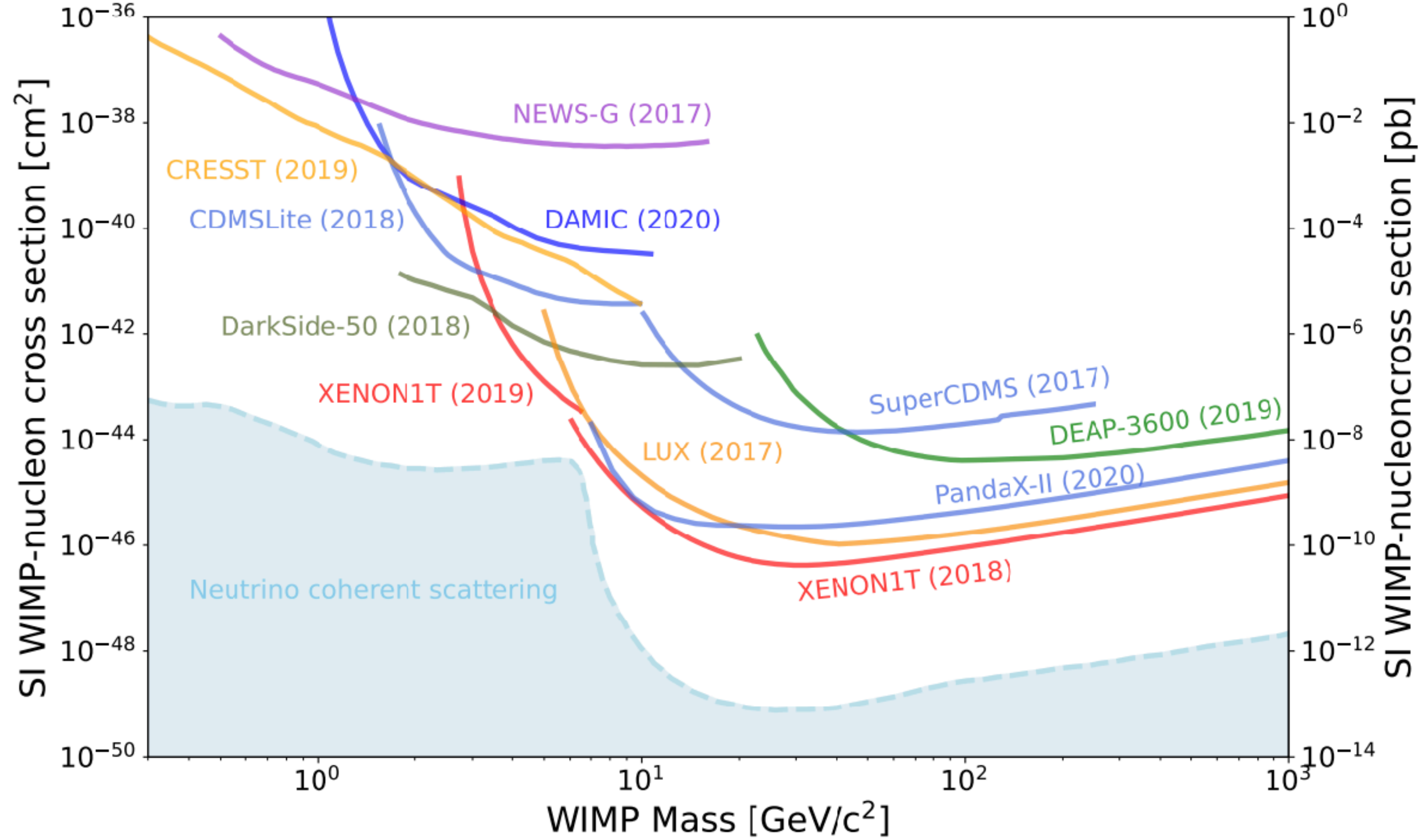
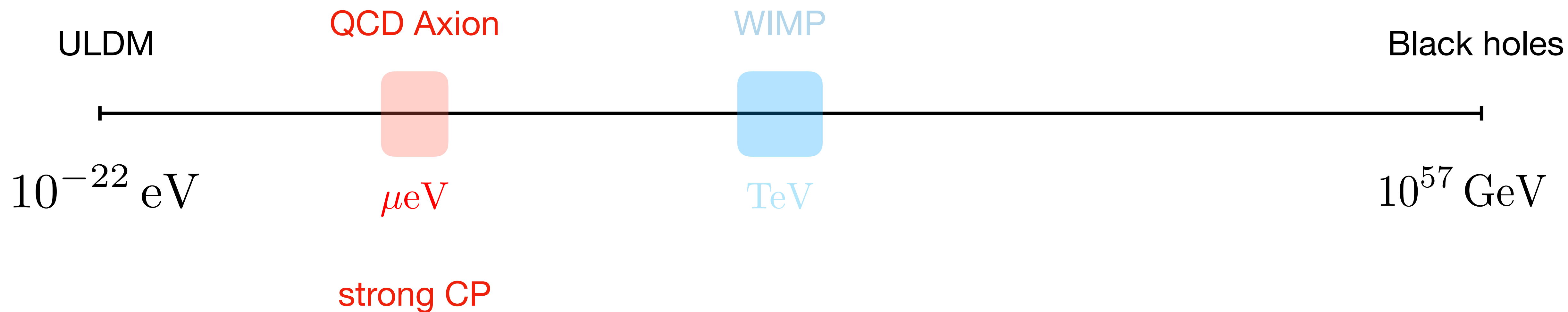
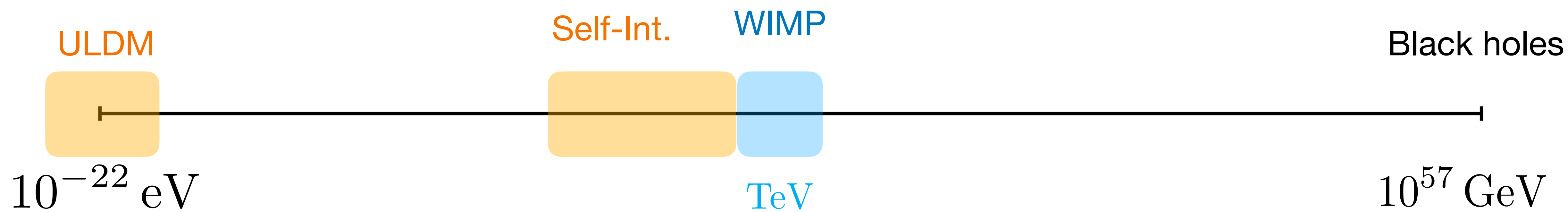
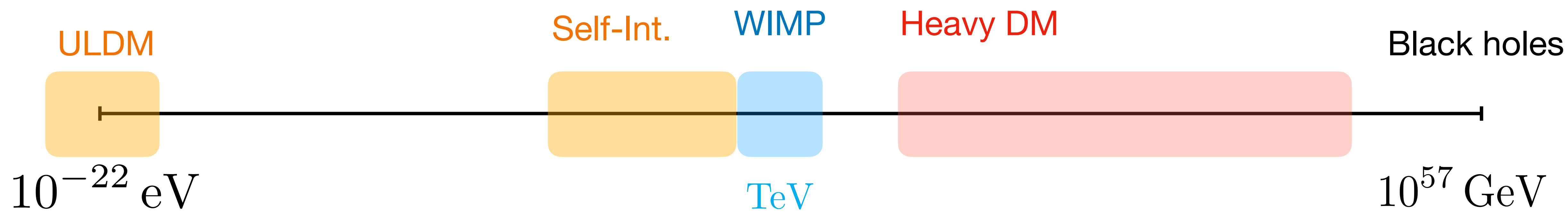


Figure 27.1: Upper limits on the SI DM-nucleon cross section as a function of DM mass.





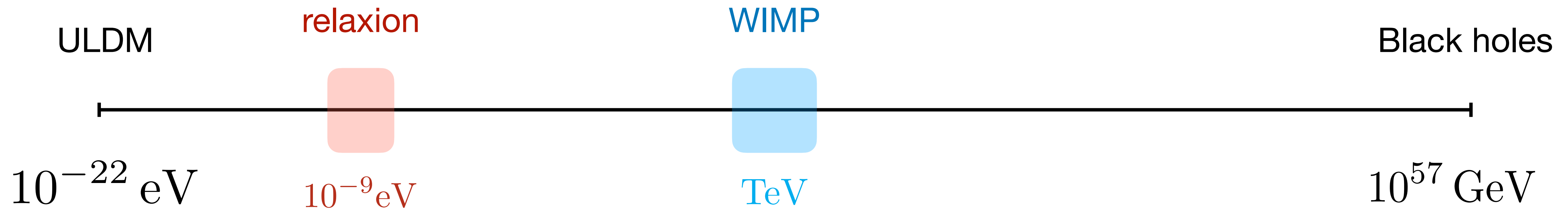
small scale issues in LCDM



small scale issues in LCDM

Recent developments of *dynamical solutions to hierarchy problem*

also provide some insights on DM

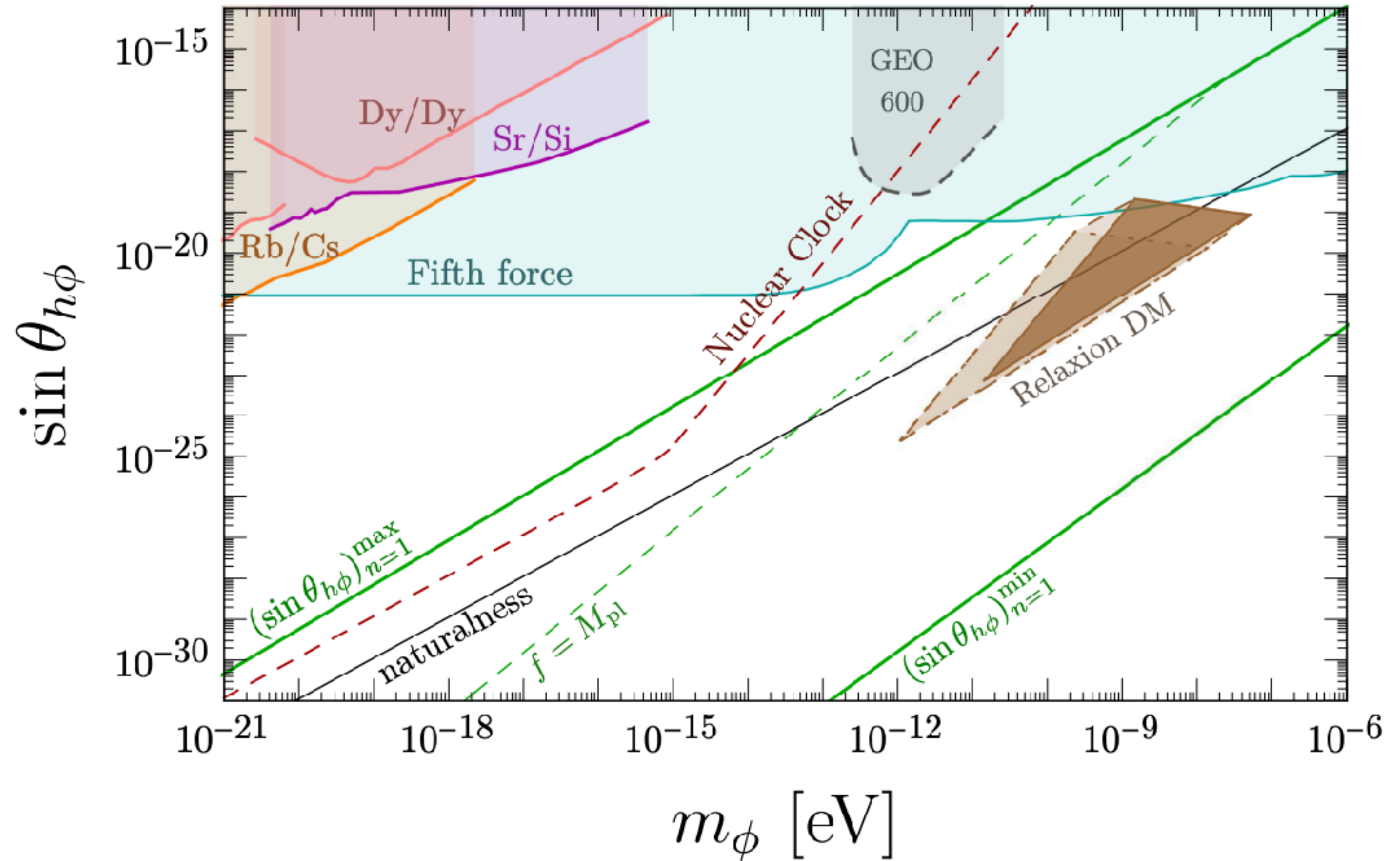
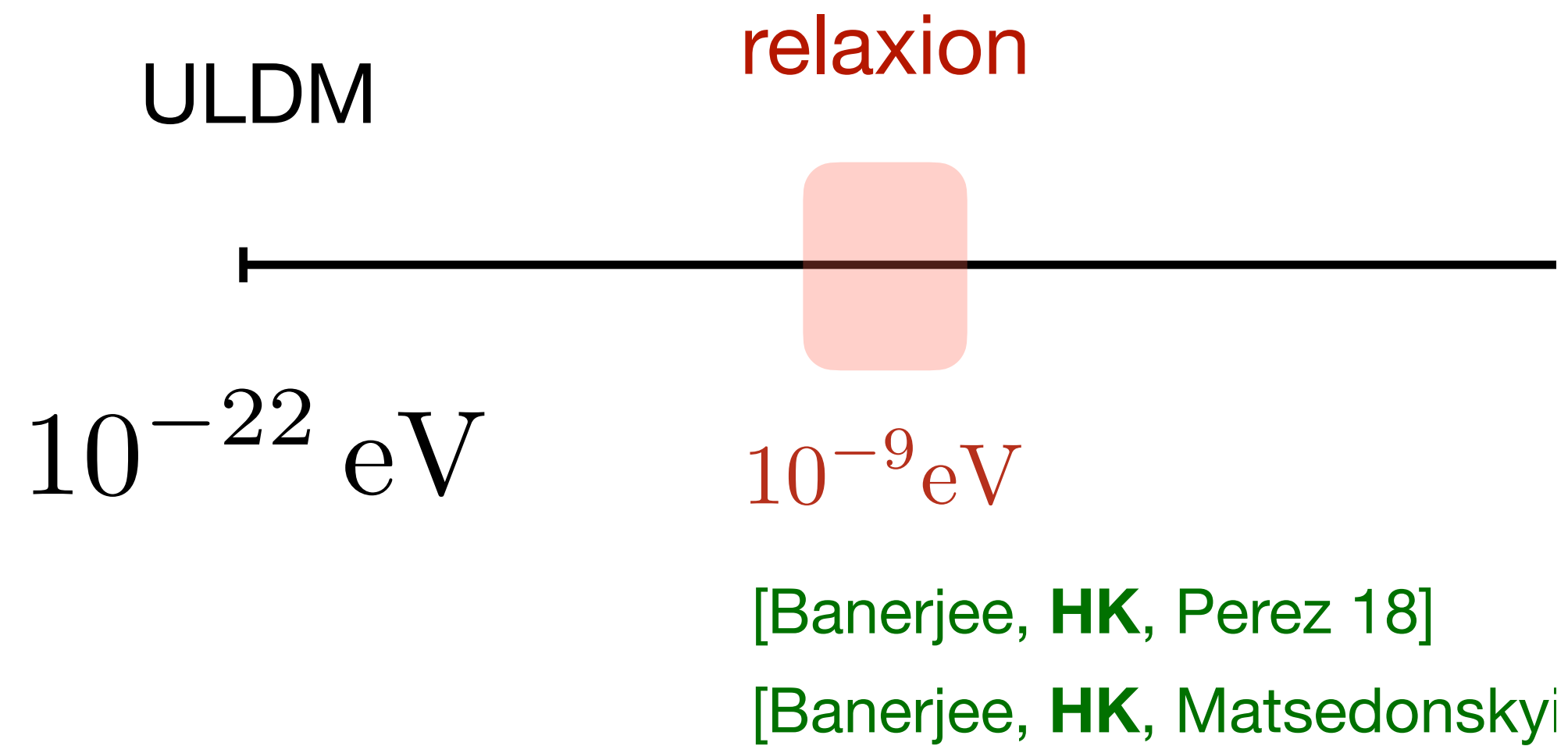


[Banerjee, **HK**, Perez 18]

[Banerjee, **HK**, Matsedonskyi, Perez, Safronova 20]

Recent developments of *dynamical solutions to hierarchy problem*

also provide some insights on DM



Recent developments of *dynamical solutions to naturalness problem*

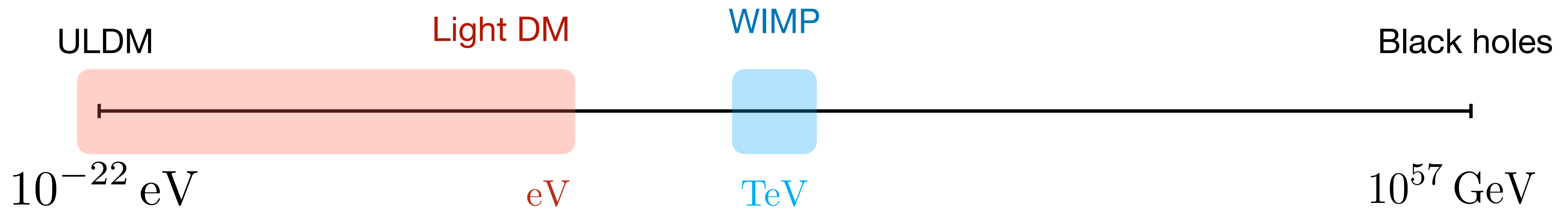
also provide some insights on DM

[Graham, Kaplan, Rajendran 15]

[Arvanitaki et al 16]

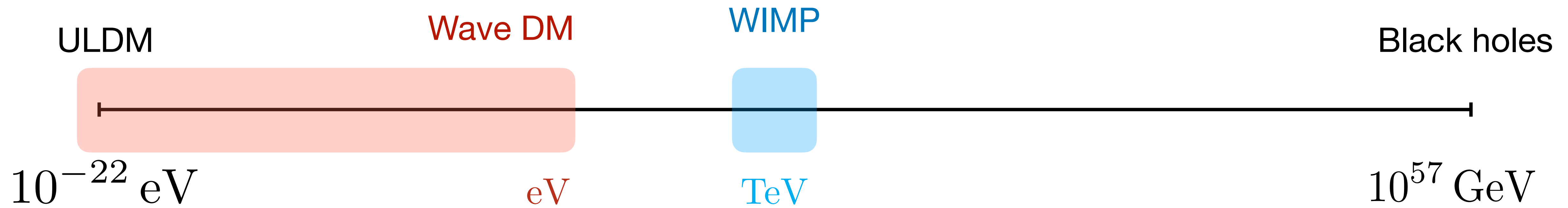
[Arkani-Hamed, D'Agnolo, Kim 20]

[D'Agnolo, Teresi 21]



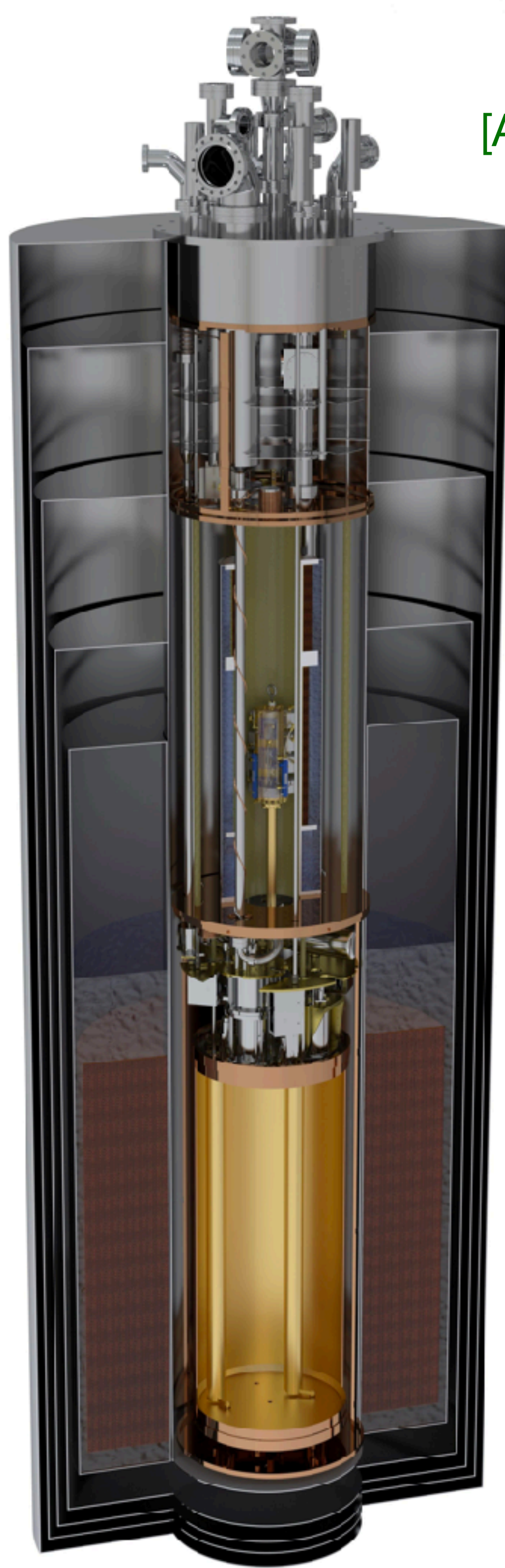
A light bosonic DM ($m < eV$) is ubiquitous in many BSM scenarios

Since they behave like a classical wave
we call it *wave dark matter*

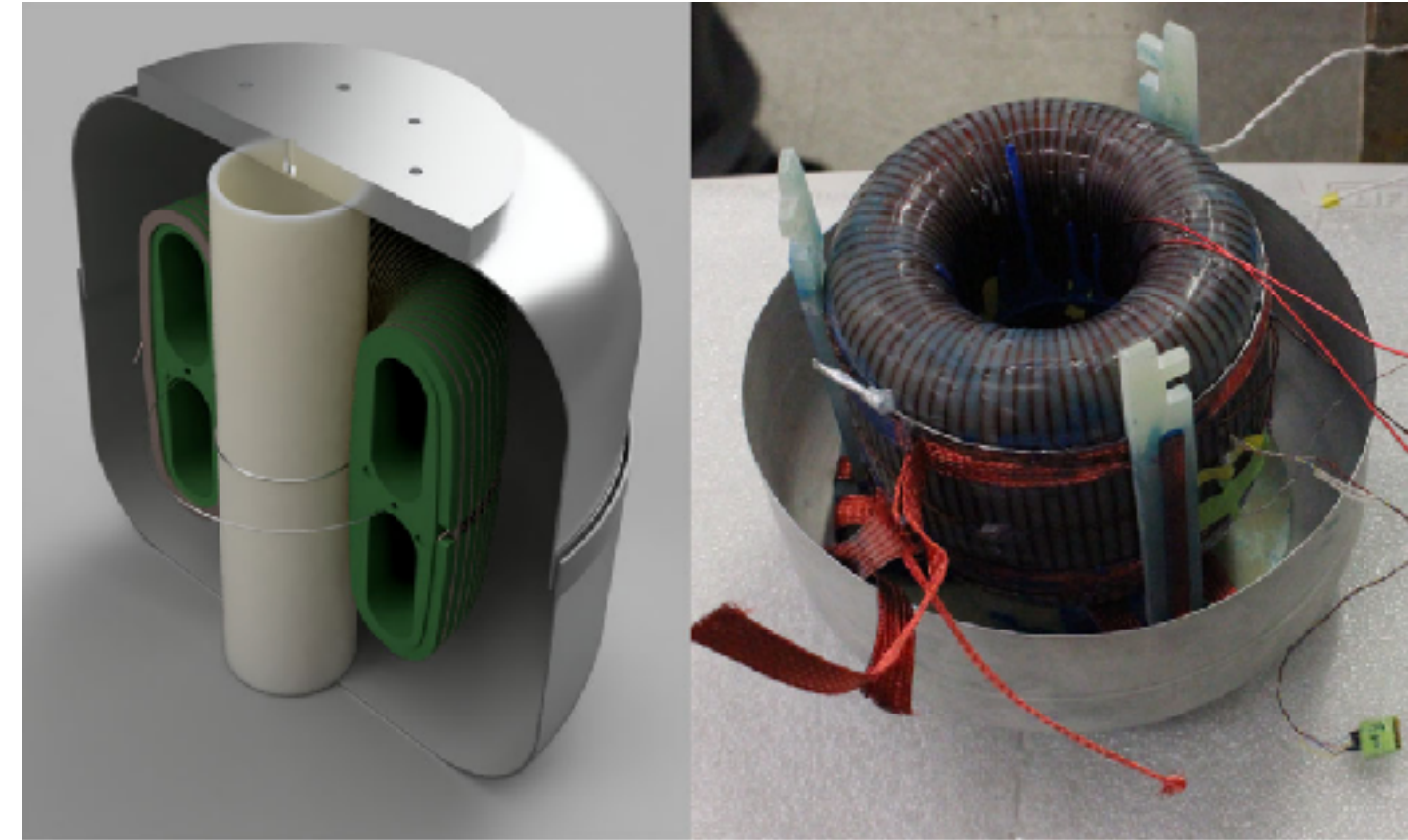
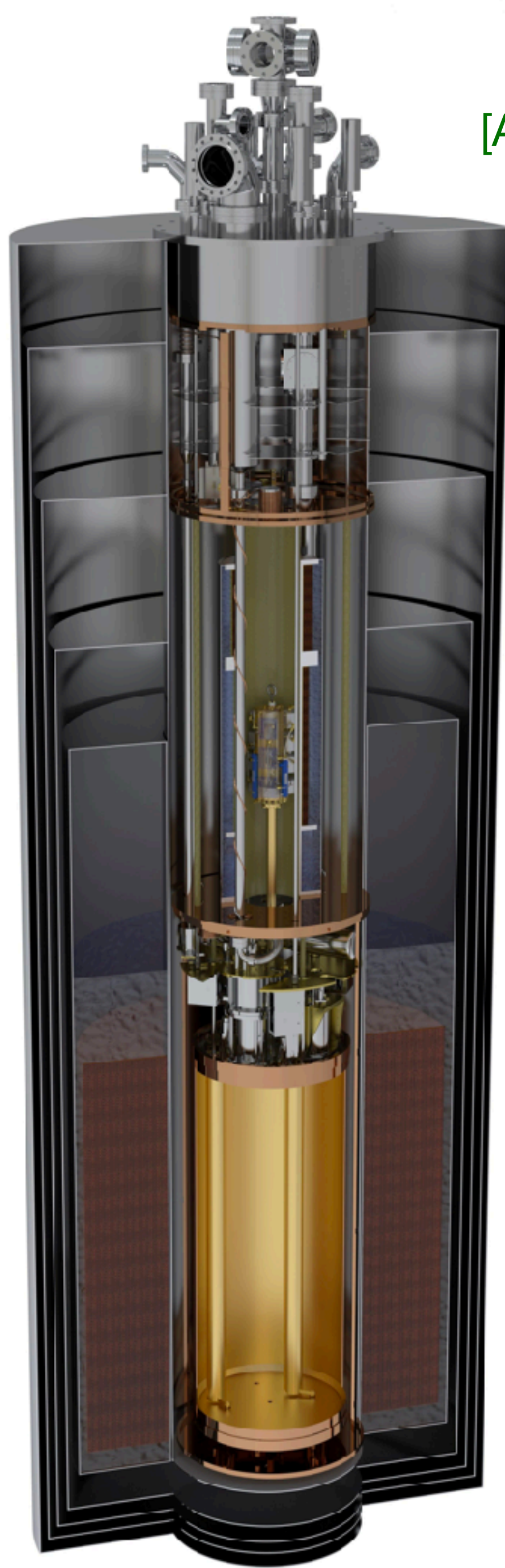


How to search for wave DM?

[ADMX Collaboration]

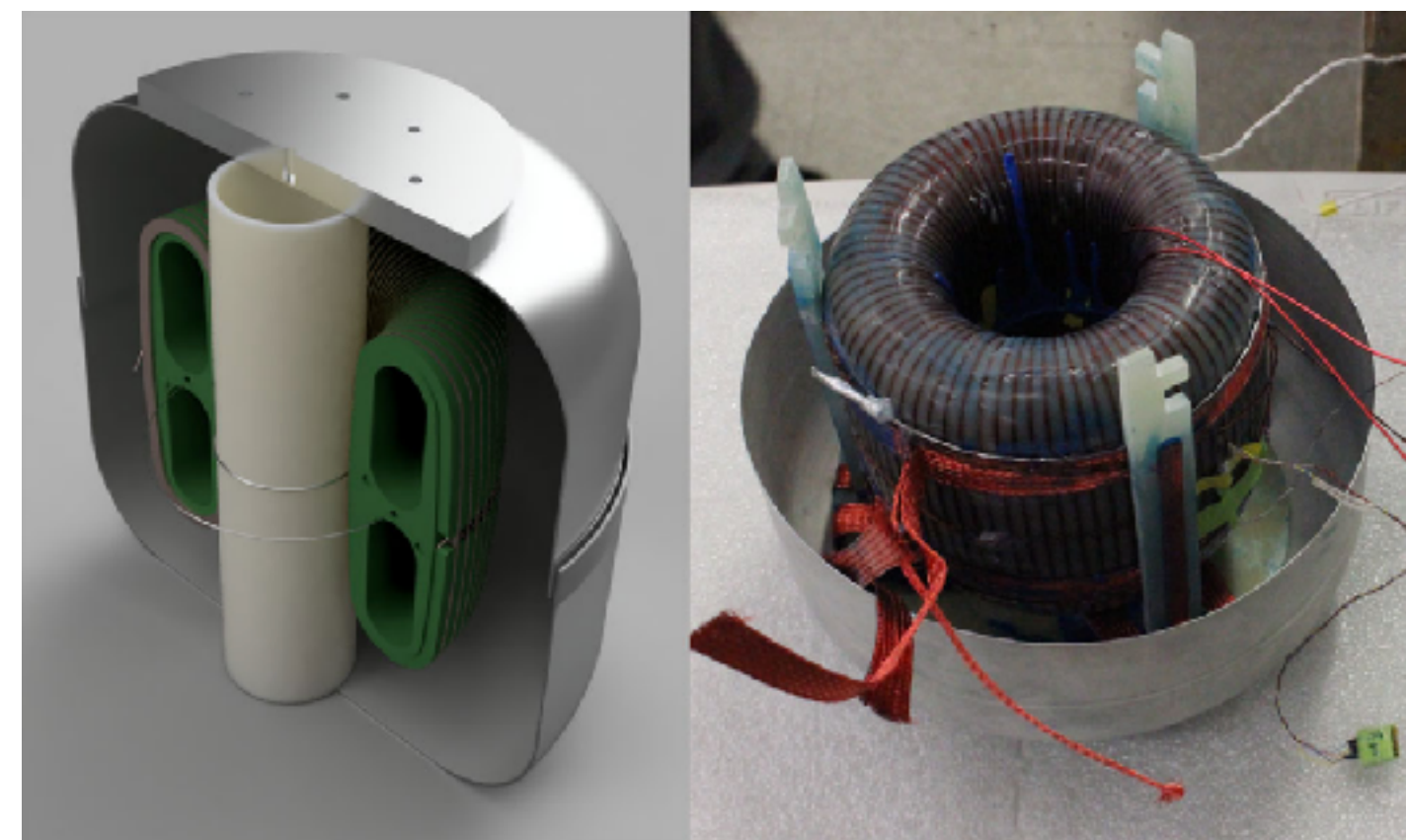
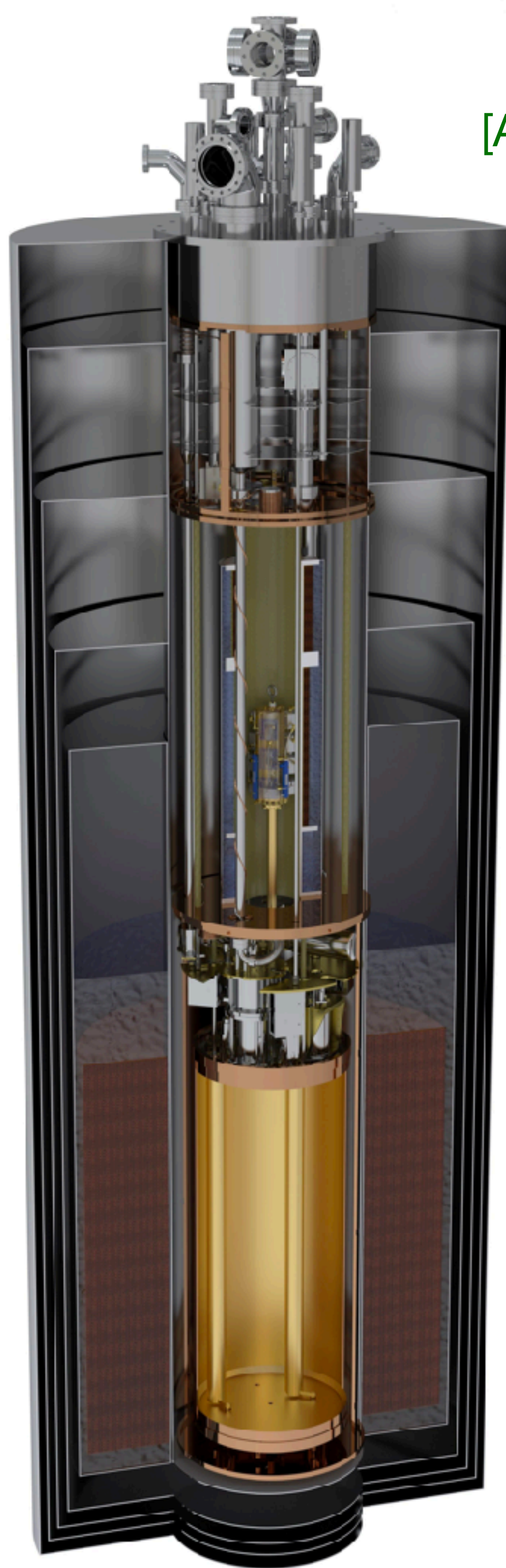


[ADMX Collaboration]

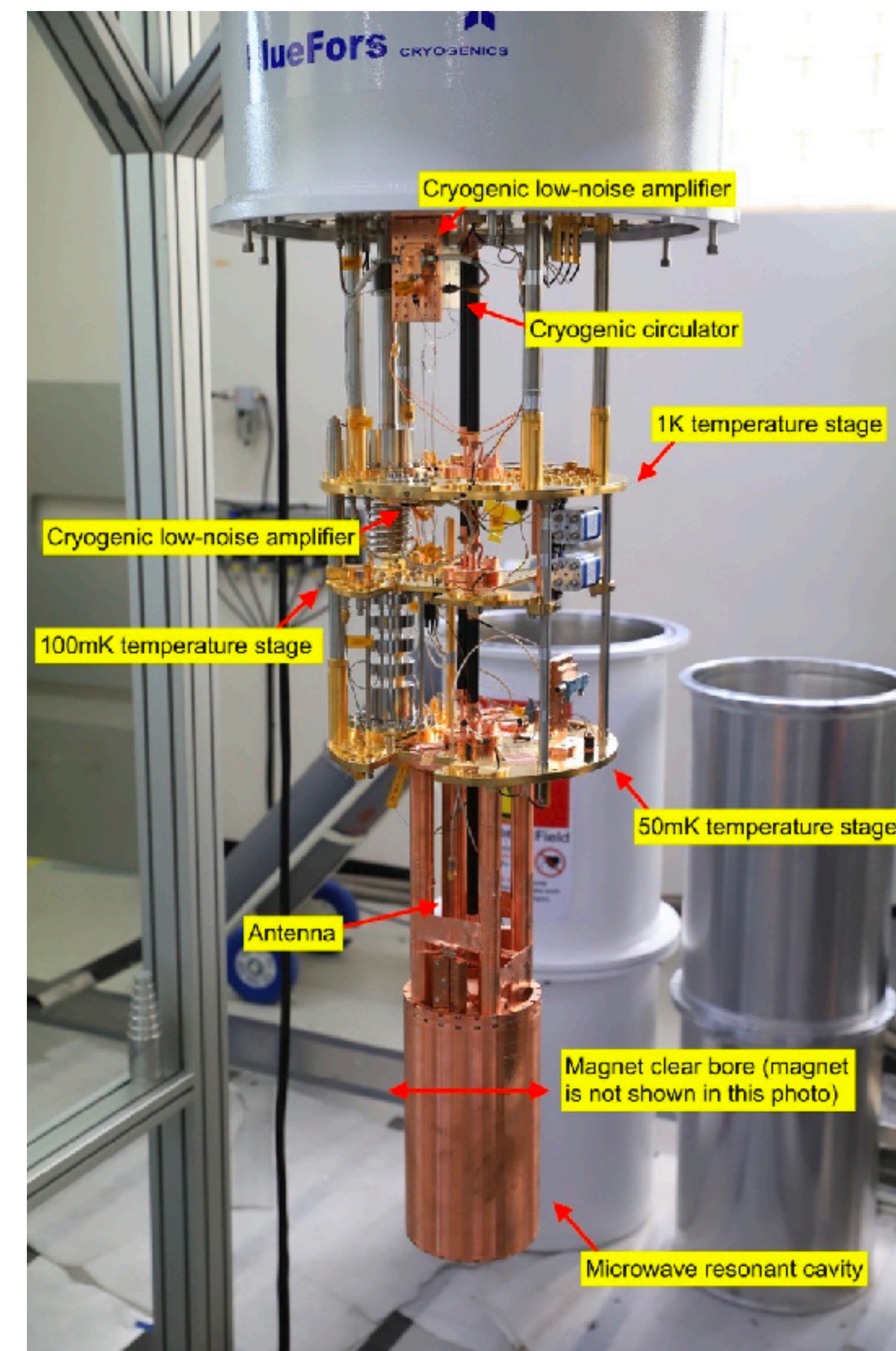


[ABRACADABRA Collaboration]

[ADMX Collaboration]



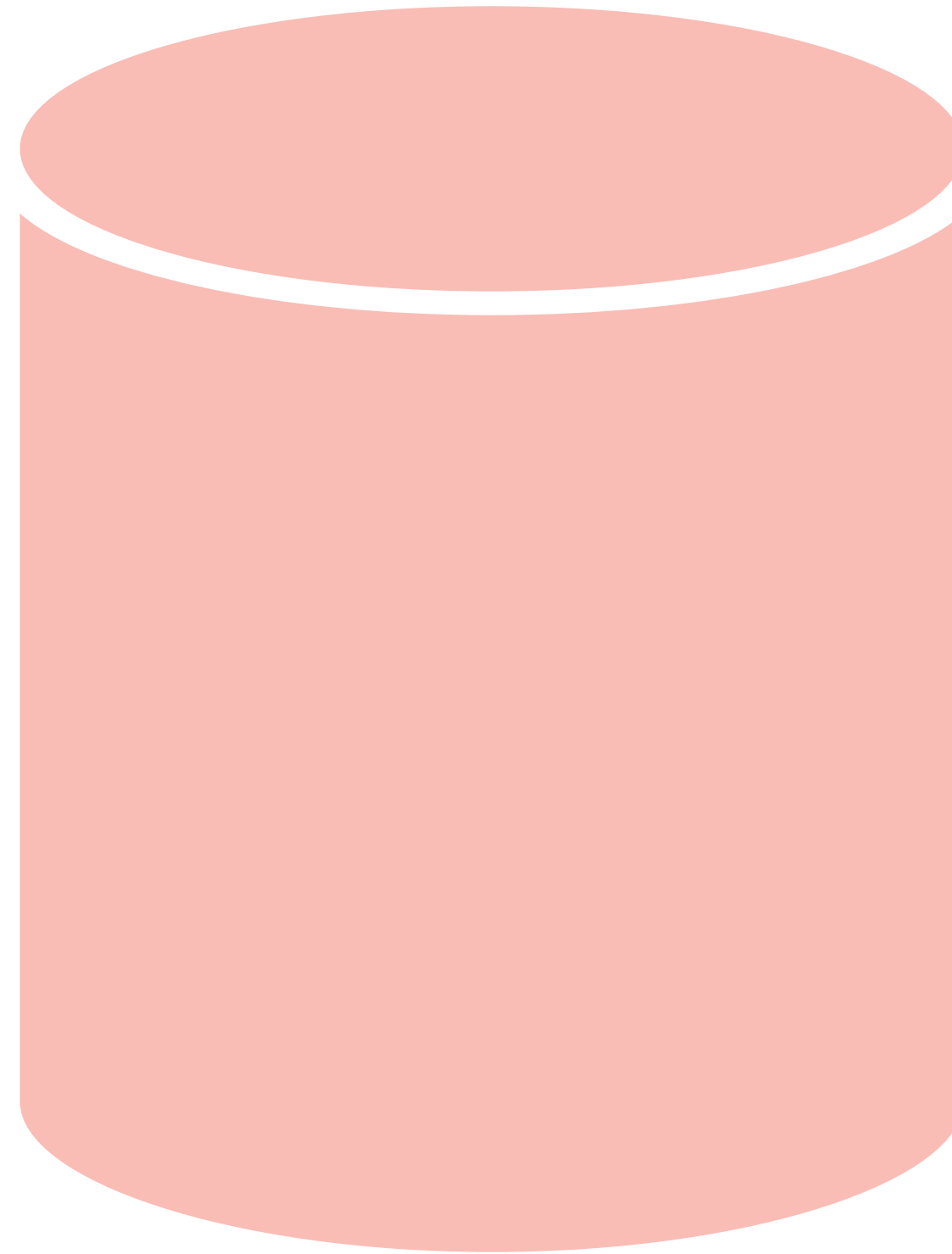
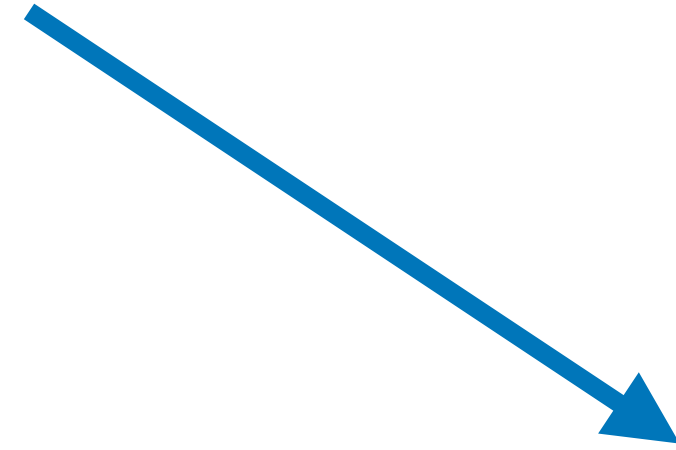
[ABRACADABRA Collaboration]



[IBS-CAPP Collaboration]

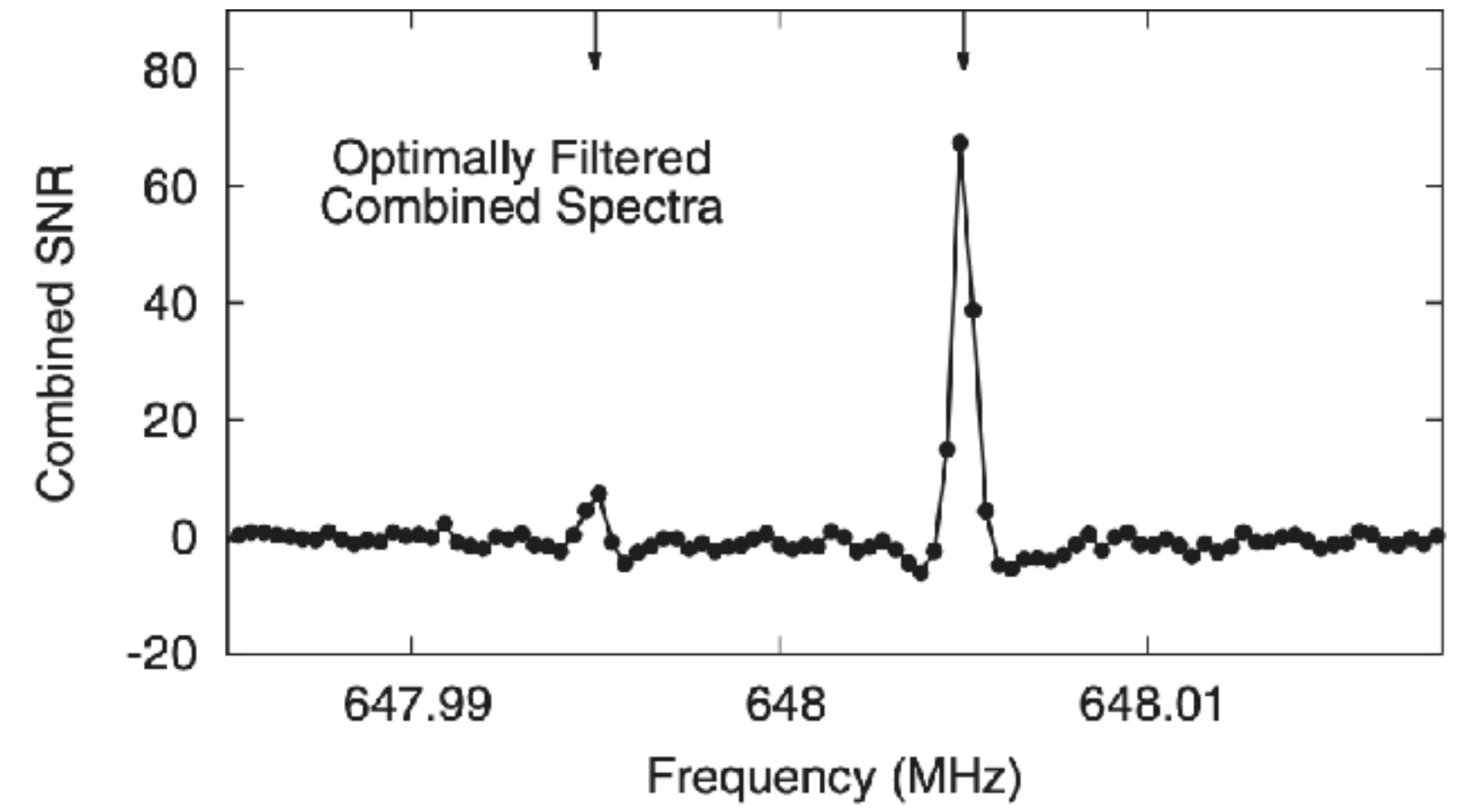
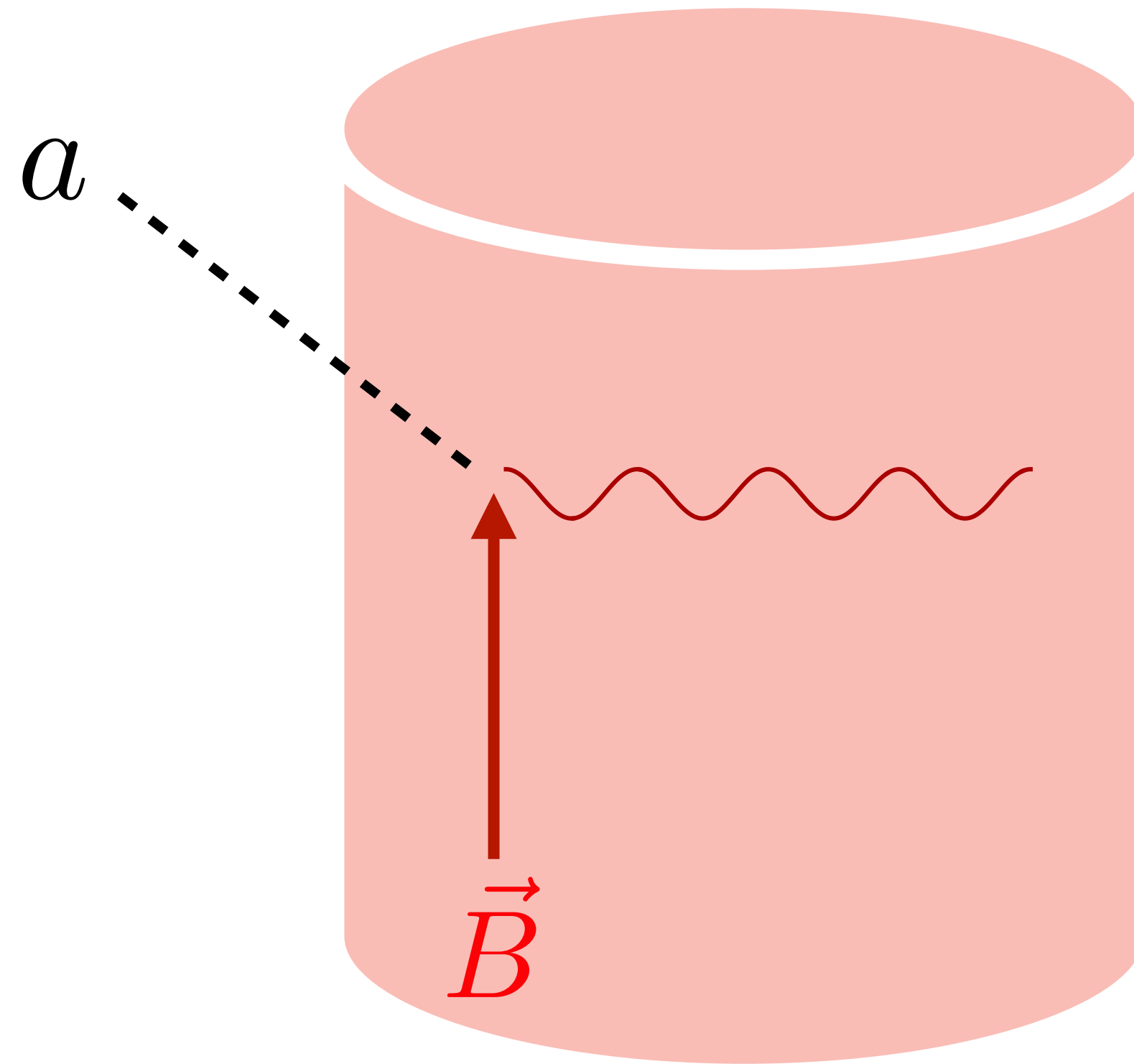
$$\mathcal{L} \supset \phi \mathcal{O}_{\text{SM}}$$

DM



signals (event rate, spectrum, etc)

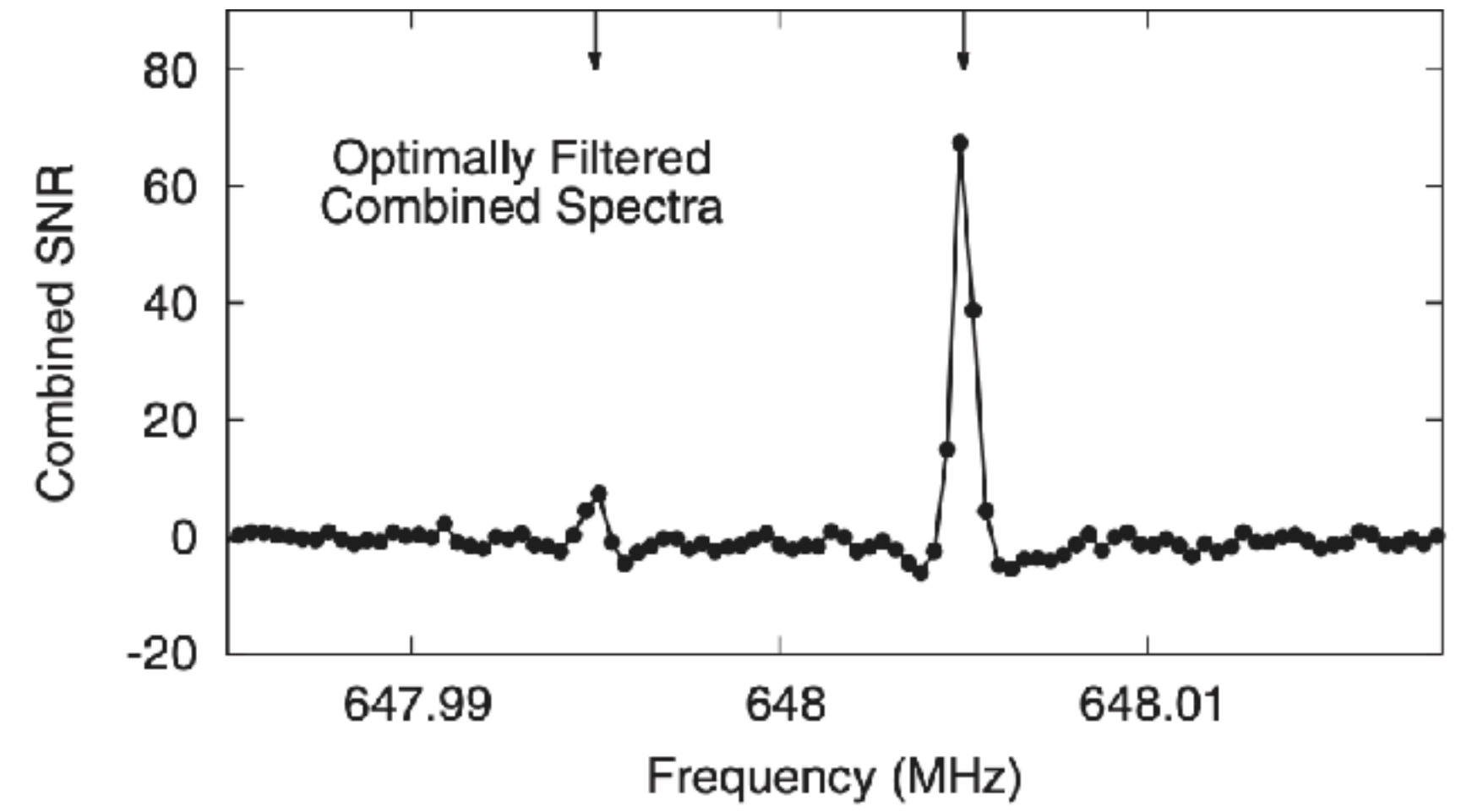
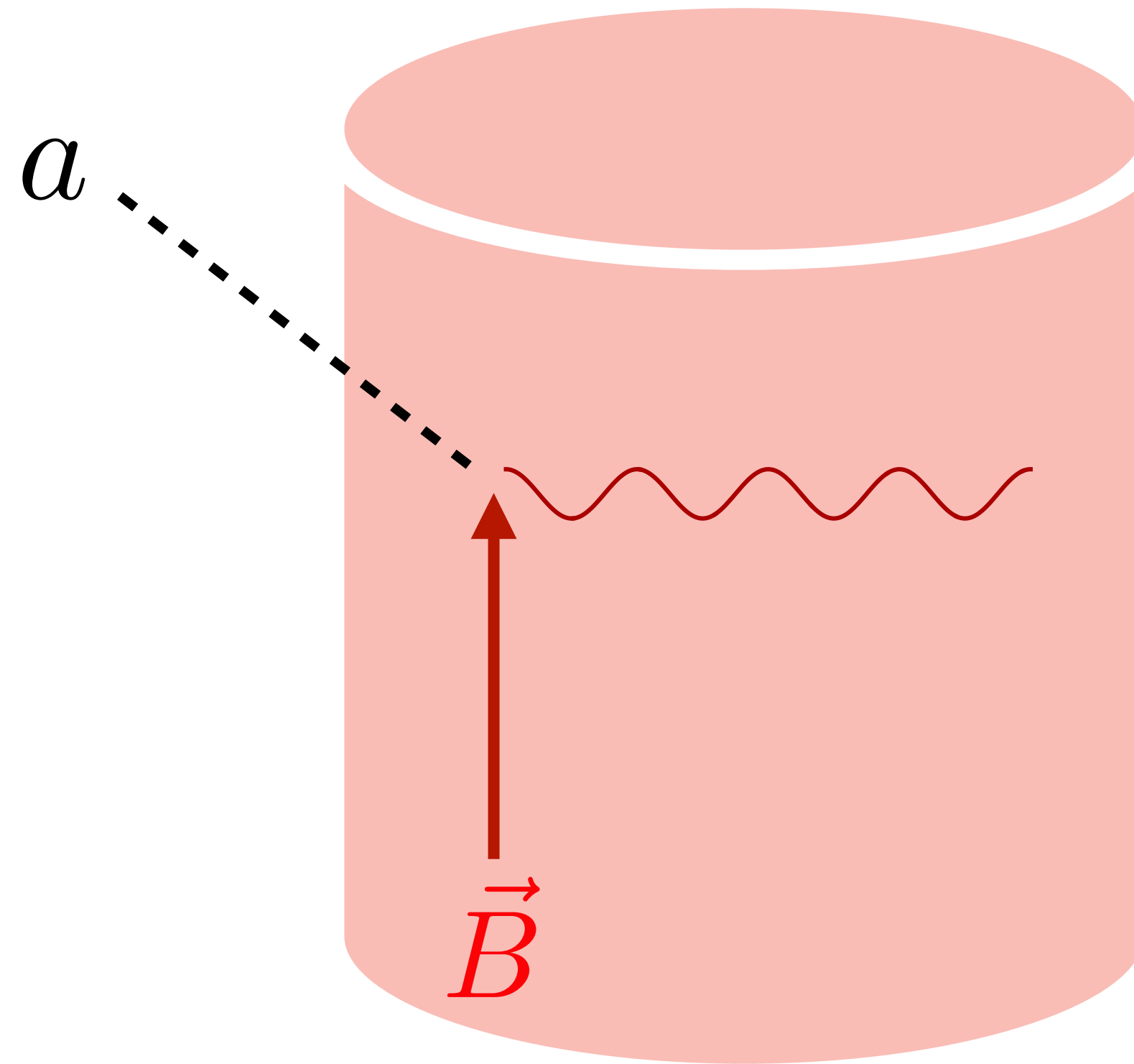
$$\mathcal{L} = aF\tilde{F}$$



ADMX Collaboration 2018

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\text{DM}} f(v)}{v}$$

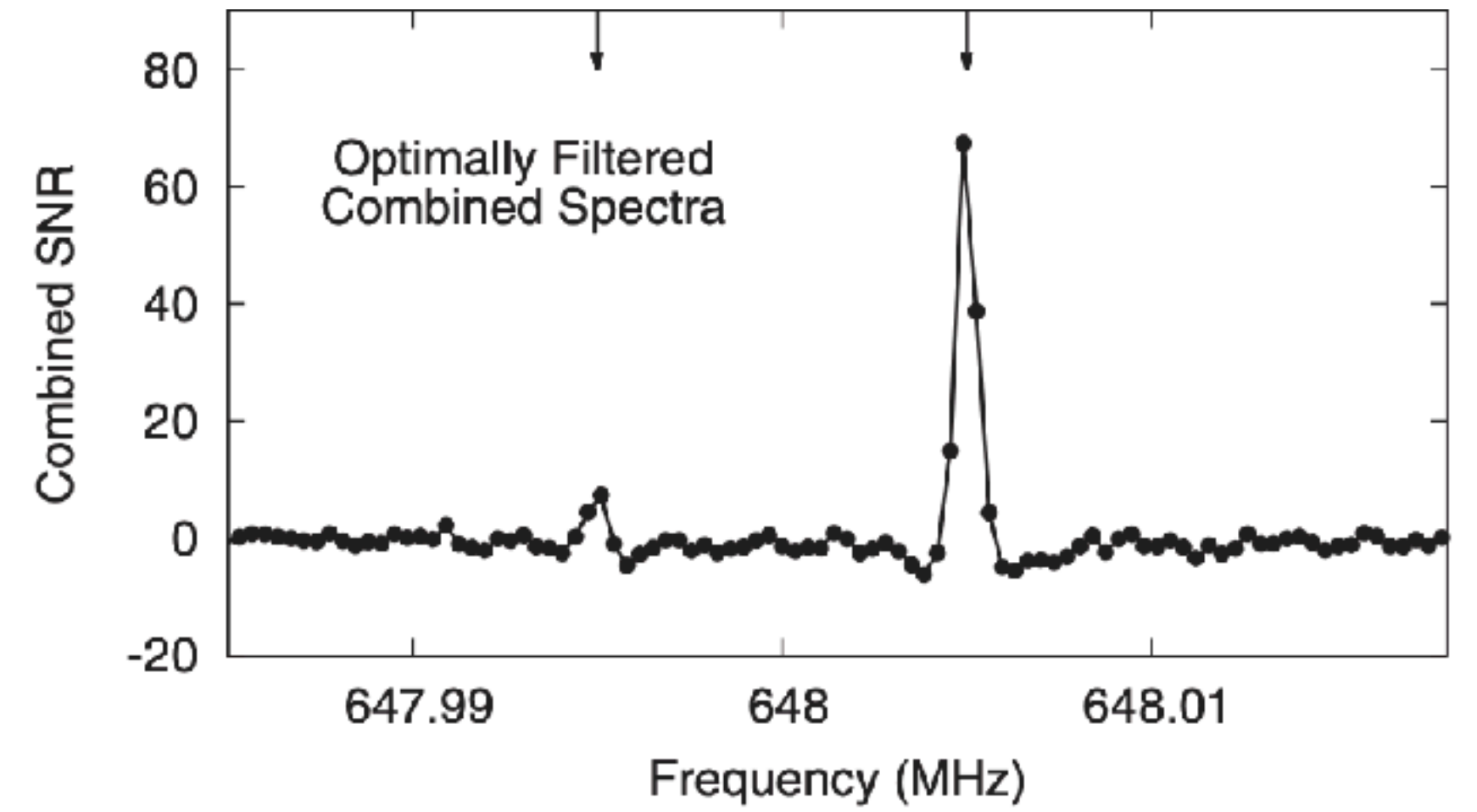
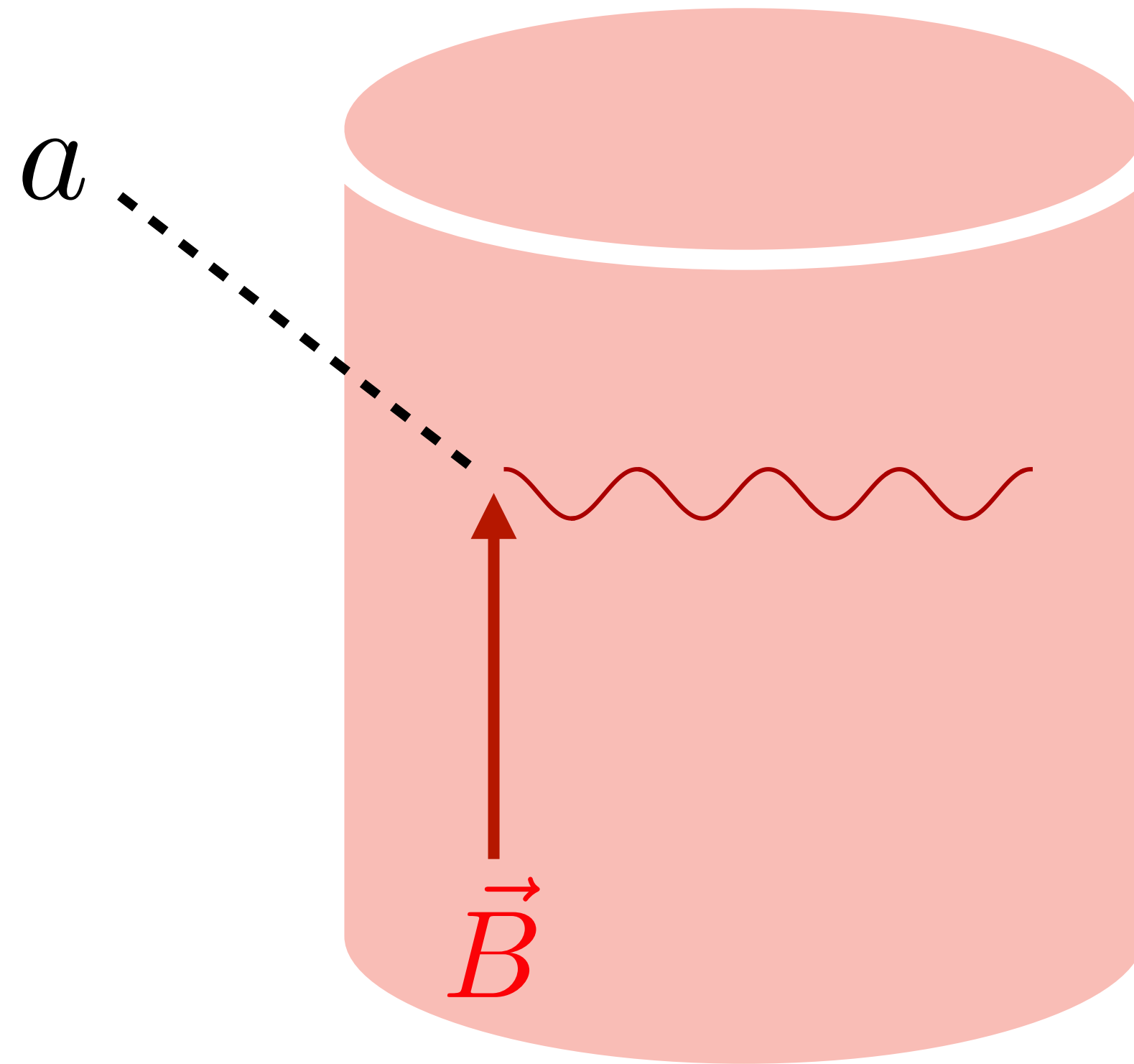
$$\mathcal{L} = aF\tilde{F}$$



ADMX Collaboration 2018

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\text{DM}} f(v)}{v}$$

$$\mathcal{L} = aF\tilde{F}$$



ADMX Collaboration 2018

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\text{DM}} f(v)}{v}$$

A correct modeling of local DM will be crucial
for interpretation of experimental results.

The Standard Halo Model

(isothermal sphere with isotropic velocity distribution)

$$f(\vec{v}) = \frac{1}{(2\pi)^{3/2}} \exp\left[-\frac{v^2}{2\sigma^2}\right]$$

$$v_c(R) = \sqrt{2}\sigma$$

It's *simple*

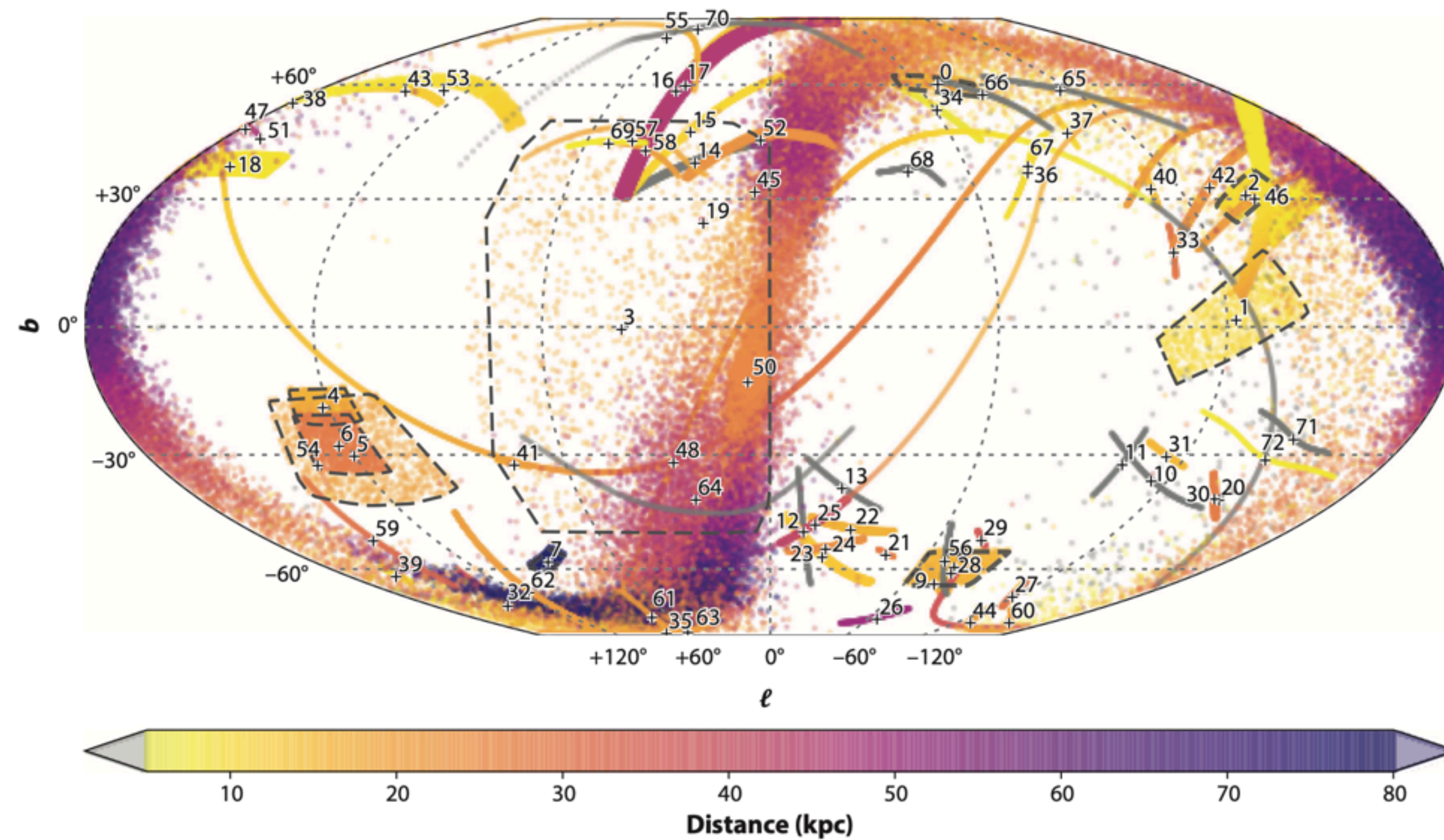
and *a reasonable approx. for the bulk halo*

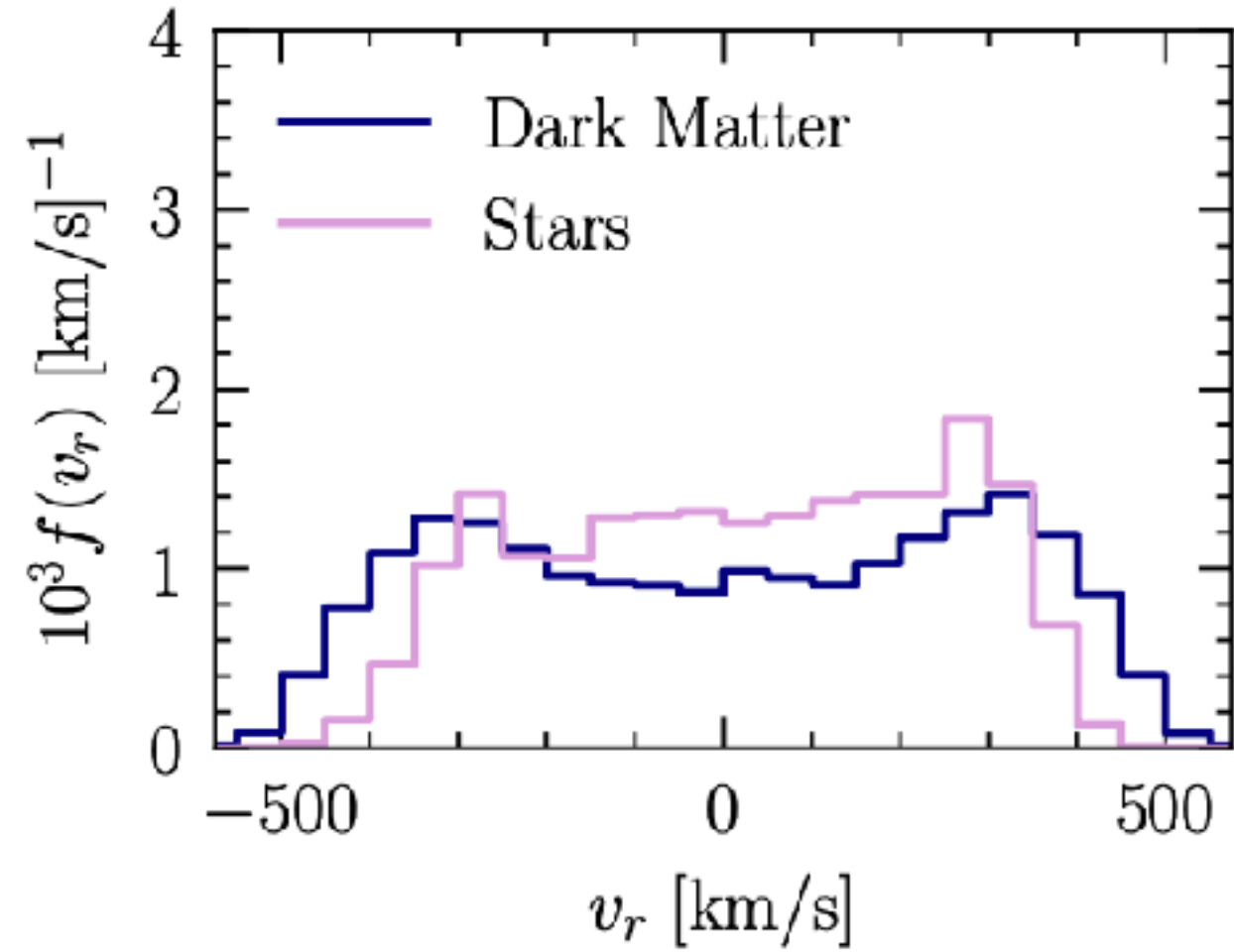
But it does not fully reflect reality:

1. Dark matter substructures
2. Gravitational deformation due to the sun

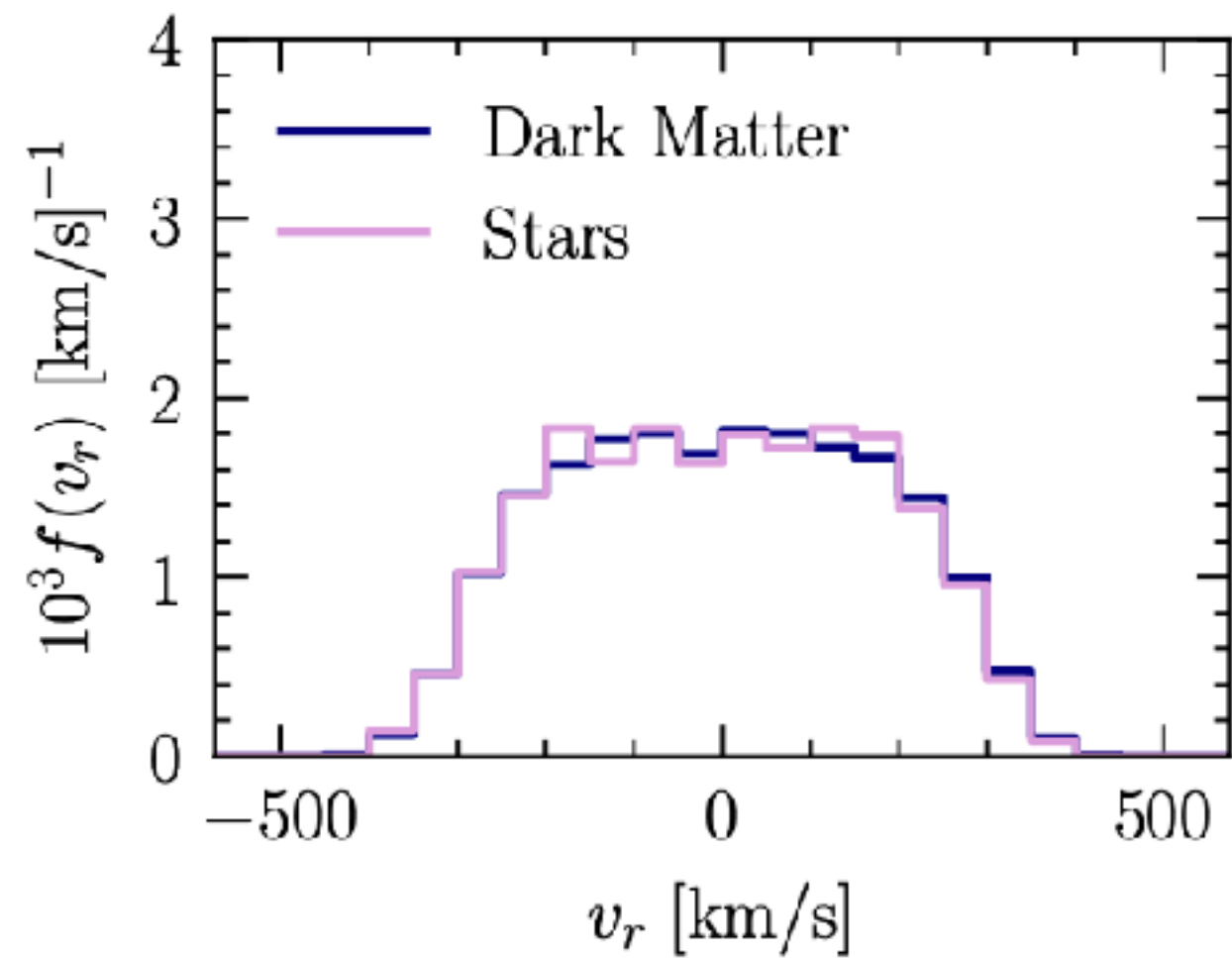
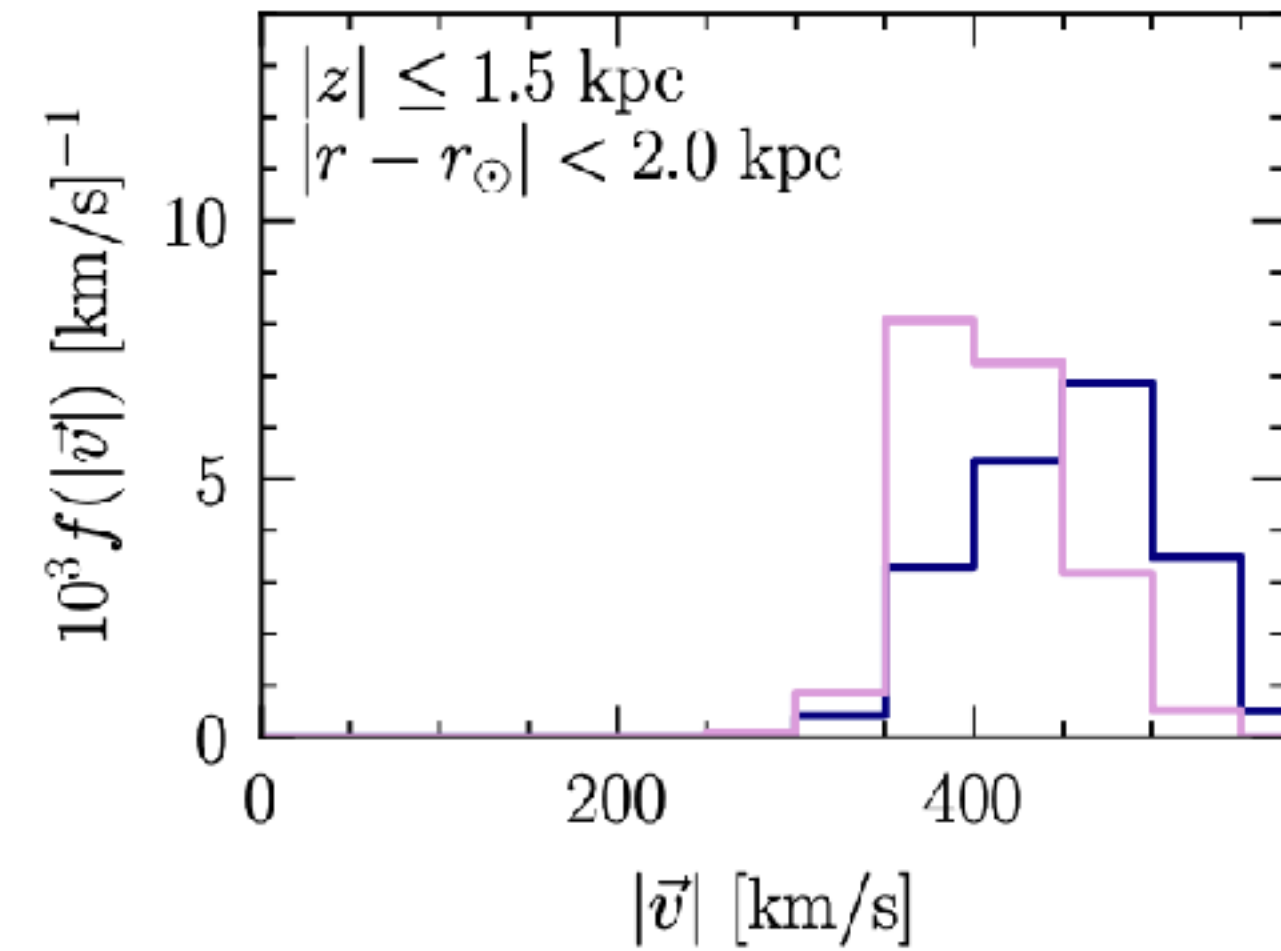
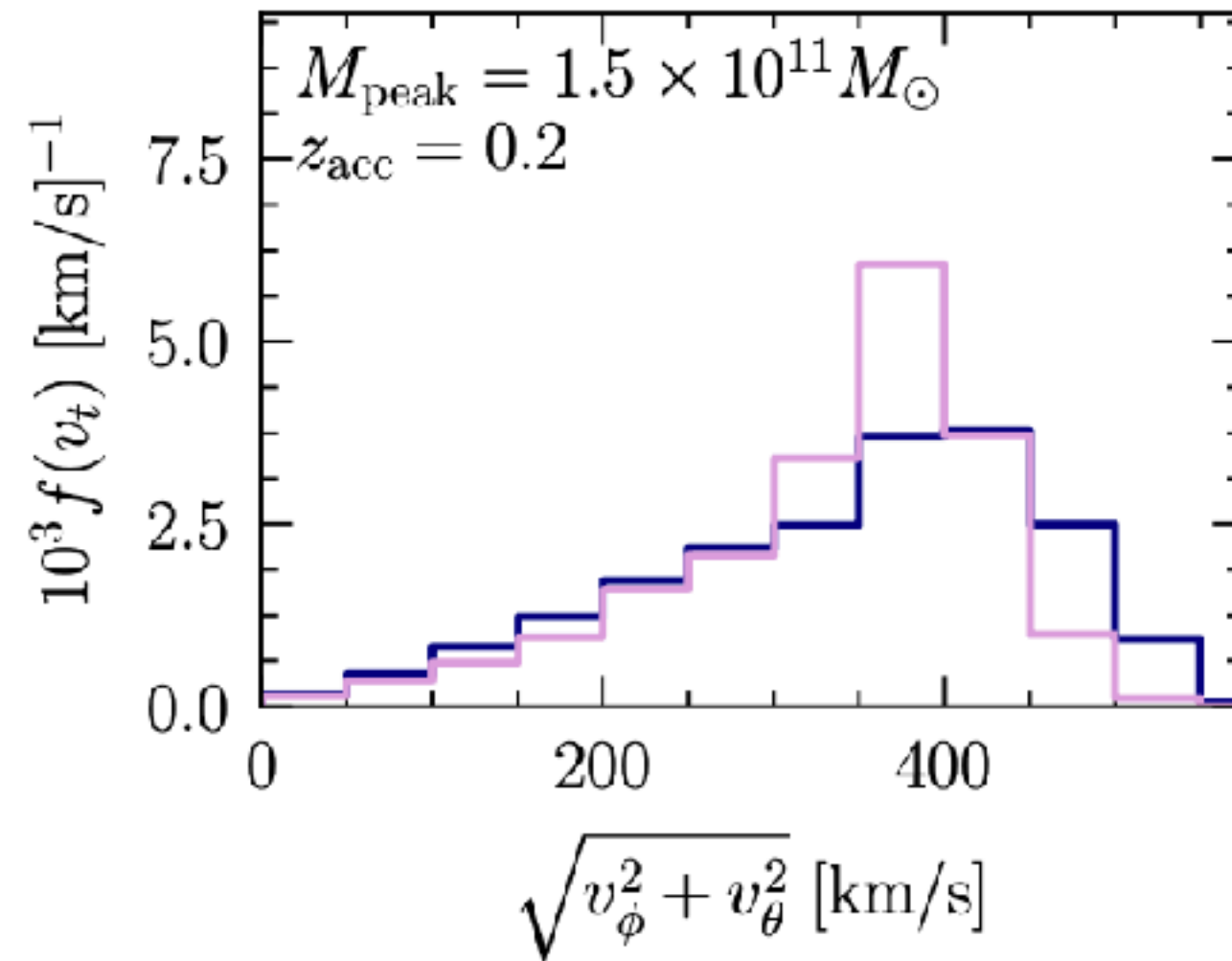
Stellar (and DM) substructures

0: VOD/VSS	1: Monoceros	2: EBS	3: Her-Aq	4: PAndAS	5: Tri-And	6: Tri-And2	7: PiscesOv	8: EriPhe
9: Phoenix	10: WG1	11: WG2	12: WG3	13: WG4	14: Acheron	15: Cocytos	16: Lethe	17: Styx
18: ACS	19: Pal15	20: Eridanus	21: Tucana III	22: Indus	23: Jhelum	24: Ravi	25: Chenab	26: Elqui
27: Aliqa Uma	28: Turbio	29: Willka Yaku	30: Turranburra	31: Wambelong	32: Palca	33: Jet	34: Gaia-1	35: Gaia-2
36: Gaia-3	37: Gaia-4	38: Gaia-5	39: PS1-A	40: PS1-B	41: PS1-C	42: PS1-D	43: PS1-E	44: ATLAS
45: Ophiucus	46: Sangarius	47: Scamander	48: Corvus	50: Sgr-L10	51: Orphan	52: Pal5	53: GD-1	54: Tri/Pis
55: NGC5466	56: Alpheus	57: Hermus	58: Hyllus	59: Cetus	60: Kwando	61: Molonglo	62: Murrumbidgee	63: Orinoco
64: Phlegethon	65: Slidr	66: Sylgr	67: Ylgr	68: Fimbulthul	69: Svol	70: Fjorm	71: Gjoll	72: Leiptr

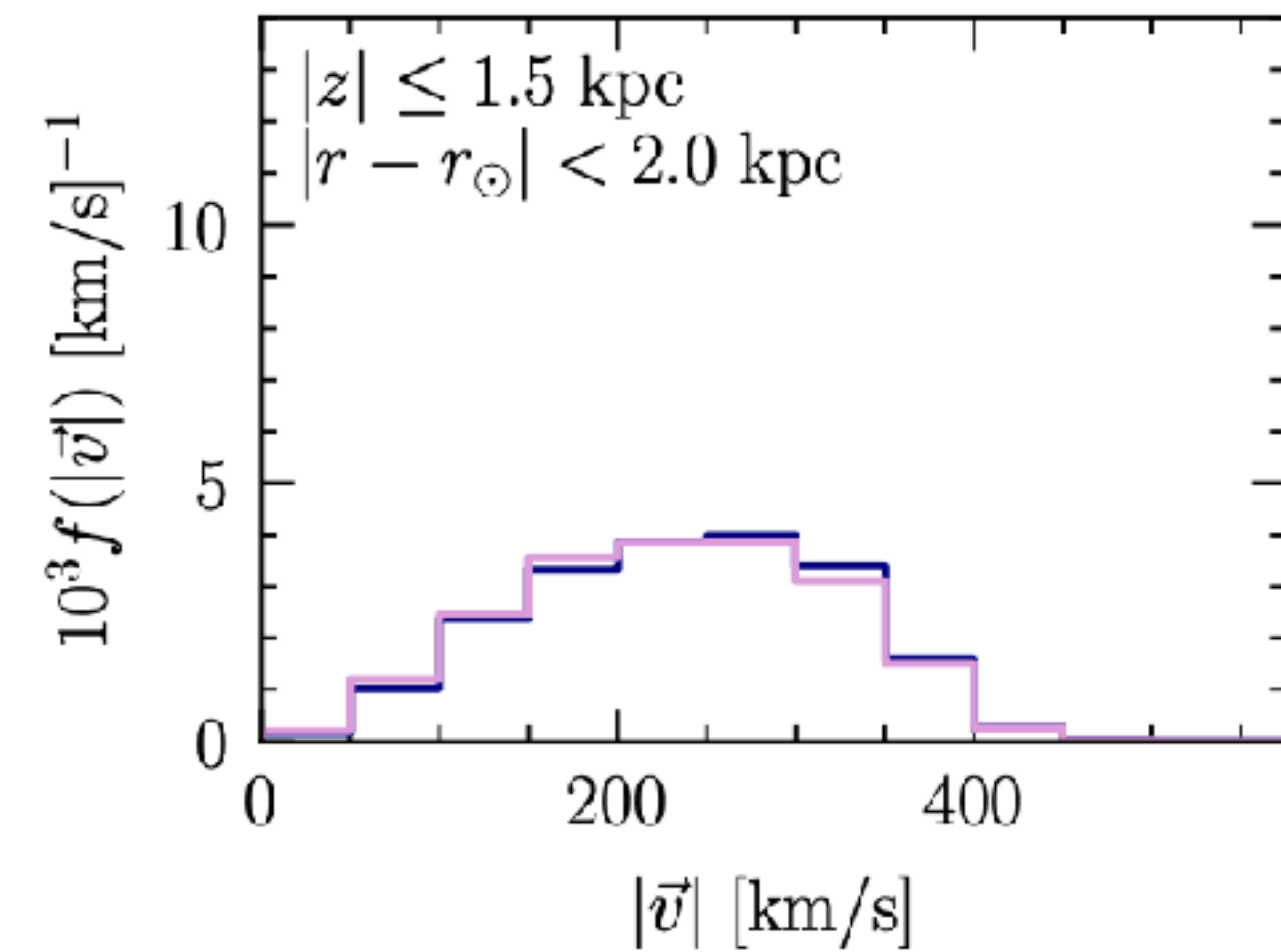
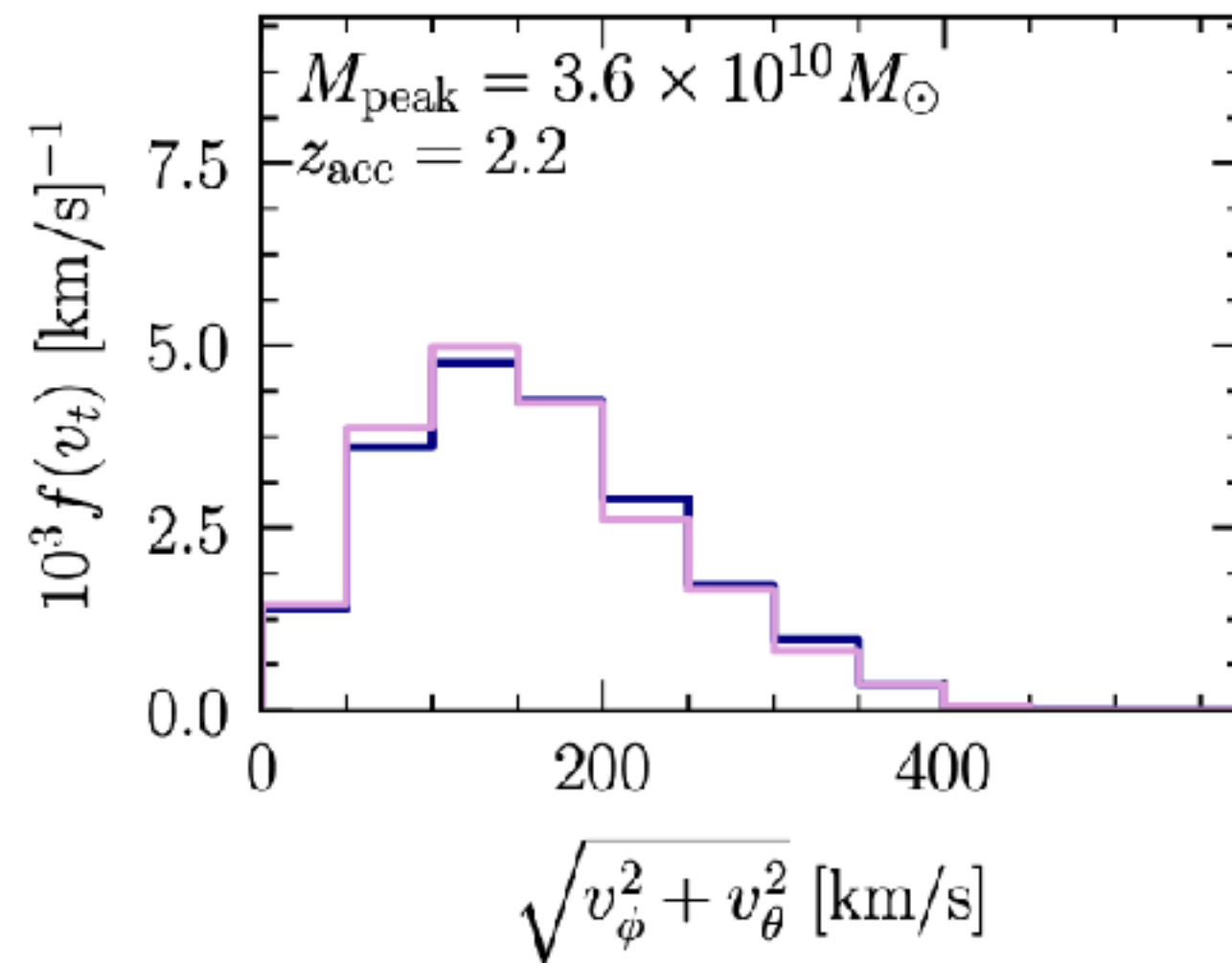




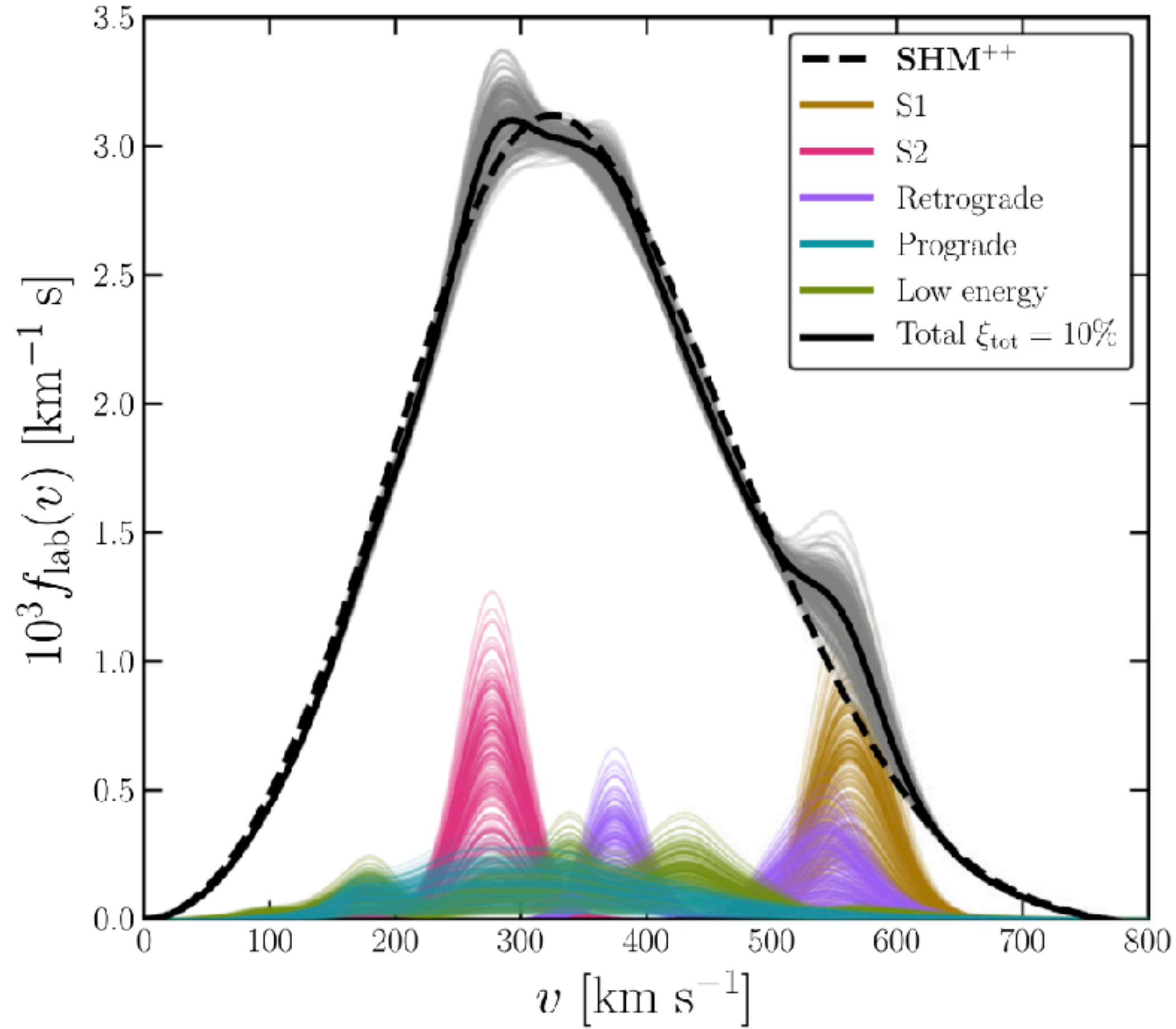
FIRE Host Halo m12f Merger I



FIRE Host Halo m12i Merger II



And many other substructures near solar neighborhood



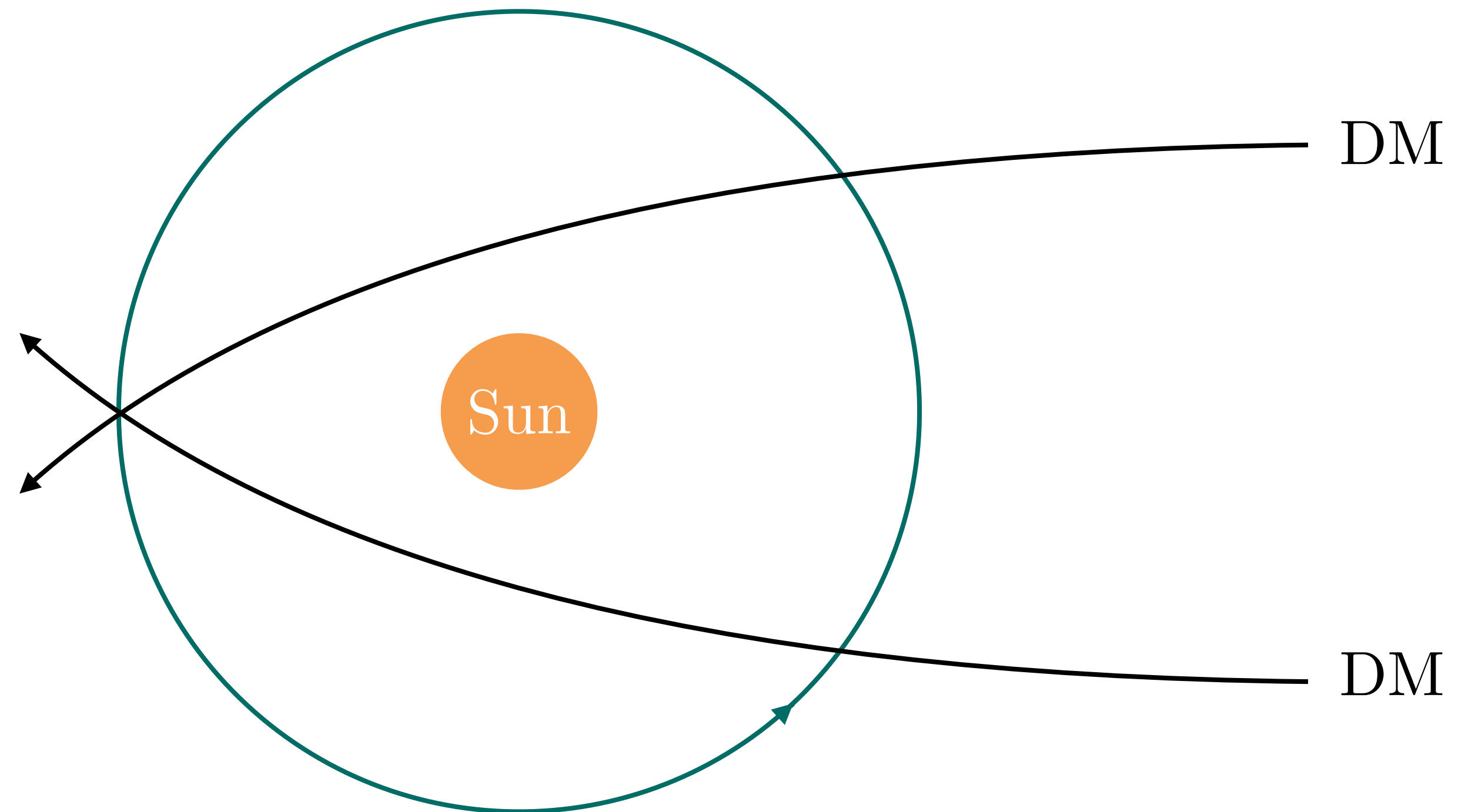
Name	Number of stars	(X, Y, Z) kpc	($\Delta X, \Delta Y, \Delta Z$) kpc	(v_R, v_ϕ, v_z) km s ⁻¹	($\sigma_R, \sigma_\phi, \sigma_z$) km s ⁻¹	$\langle [Fe/H] \rangle$	$P(\mathbf{x}_\odot)$ (σ)
S1	28	(8.4, 0.6, 2.6)	(0.7, 1.8, 2.2)	(-34.2, -306.3, -64.4)	(81.9, 46.3, 62.9)	-1.9 ± 0.3	1.2
S2	a	(8.7, 0.4, 0.1)	(0.7, 1.2, 6.9)	(5.8, 163.6, -250.4)	(45.9, 13.8, 26.8)	-2.0 ± 0.2	0.3
	b	(10.1, 0.2, 3.3)	(4.9, 0.7, 1.4)	(-50.6, 138.5, 183.1)	(90.8, 25.0, 43.8)	-2.0 ± 0.3	1.5
Retrograde	Rg2	(8.9, 0.3, 4.4)	(0.8, 2.1, 2.7)	(44.5, -248.4, 185.2)	(105.9, 23.1, 63.5)	-1.6 ± 0.2	1.4
	Rg5a	(8.4, 0.8, 1.1)	(1.0, 1.3, 3.3)	(6.4, -74.5, -159.5)	(32.4, 17.5, 31.7)	-2.2 ± 0.3	0.7
	Rg5b	(8.1, -0.2, 2.2)	(1.1, 1.2, 2.4)	(-37.6, -83.8, 178.1)	(47.5, 16.8, 31.1)	-2.1 ± 0.3	0.9
	Rg6a	(8.3, 0.2, 3.3)	(1.8, 1.4, 2.0)	(105.1, -230.2, 202.4)	(73.7, 16.8, 86.6)	-1.6 ± 0.2	1.1
	Rg6b	(8.5, 0.9, 3.2)	(1.5, 1.5, 2.2)	(-233.2, -221.8, 51.6)	(32.7, 14.4, 115.7)	-1.7 ± 0.3	0.6
	Rg7a	(8.2, 0.5, 3.3)	(2.1, 1.5, 3.3)	(309.0, -191.3, -83.4)	(66.7, 17.1, 102.7)	-1.5 ± 0.1	1.1
	Rg7b	(8.9, -0.0, 5.1)	(1.9, 1.3, 2.0)	(-288.7, -158.1, -105.5)	(78.7, 65.8, 111.8)	-1.5 ± 0.3	1.8
	Prograde	Cand8a	(9.9, -0.1, 2.4)	(2.1, 2.5, 4.4)	(-6.7, 207.7, -186.4)	(114.6, 20.8, 73.5)	-1.8 ± 0.4
Cand8b		(8.4, 0.6, 1.1)	(1.5, 2.2, 3.6)	(33.6, 213.9, 214.1)	(96.5, 22.7, 37.7)	-1.8 ± 0.2	0.1
Cand9		(9.2, -0.2, 1.7)	(1.1, 1.4, 3.4)	(11.0, 177.5, -251.4)	(120.6, 13.9, 132.2)	-1.8 ± 0.2	0.5
Cand10		(8.6, -0.0, 2.0)	(1.7, 1.3, 2.5)	(-37.4, 20.0, 192.3)	(161.5, 18.2, 195.0)	-2.0 ± 0.2	0.2
Cand11a		(9.1, -0.3, 2.7)	(2.5, 1.4, 3.8)	(36.8, 116.5, -271.5)	(96.1, 27.9, 95.4)	-2.1 ± 0.3	0.3
Cand11b		(9.0, -0.1, 2.4)	(1.9, 1.1, 2.8)	(-152.7, 80.2, 258.2)	(122.1, 21.0, 38.9)	-2.0 ± 0.3	0.5
Cand12		(9.6, -0.8, 3.7)	(2.0, 2.4, 4.2)	(-43.3, 102.4, 50.0)	(172.8, 21.2, 197.8)	-1.6 ± 0.2	0.6
Cand13		(9.1, 1.0, 3.1)	(2.5, 2.0, 4.1)	(-2.1, -13.2, 202.2)	(215.7, 28.1, 215.9)	-1.4 ± 0.2	0.4
Cand14a		(11.9, 0.2, 1.8)	(1.8, 1.7, 3.6)	(-168.0, 166.7, -25.1)	(29.1, 27.9, 82.7)	-1.4 ± 0.2	1.2
Cand14b		(10.7, 0.3, 1.4)	(1.8, 2.1, 3.5)	(193.6, 202.9, -5.7)	(14.3, 13.5, 51.8)	-1.5 ± 0.1	0.7
Cand15a		(10.5, 1.4, 4.0)	(1.9, 2.1, 3.9)	(-297.4, 220.0, -49.9)	(29.6, 23.5, 79.3)	-1.5 ± 0.1	1.2
Cand15b		(10.3, -0.3, 2.4)	(1.8, 2.3, 5.9)	(291.3, 207.3, 48.3)	(20.2, 10.4, 68.7)	-1.4 ± 0.1	0.5
Cand16a		(8.7, 0.5, 3.9)	(1.6, 1.5, 3.9)	(315.2, 109.2, -12.5)	(30.9, 4.6, 67.2)	-1.4 ± 0.2	0.7
Cand16b		(8.9, 2.8, -1.3)	(1.3, 2.1, 3.2)	(-360.7, 147.5, 81.7)	(26.7, 9.2, 76.3)	-1.4 ± 0.1	0.9
Cand17		(9.5, -0.4, 2.0)	(1.0, 0.9, 2.5)	(127.6, 68.0, 339.4)	(157.4, 8.0, 54.8)	-2.1 ± 0.2	0.7

This suggests

$$f(\vec{v}) \rightarrow \sum_i f_i(\vec{v})$$

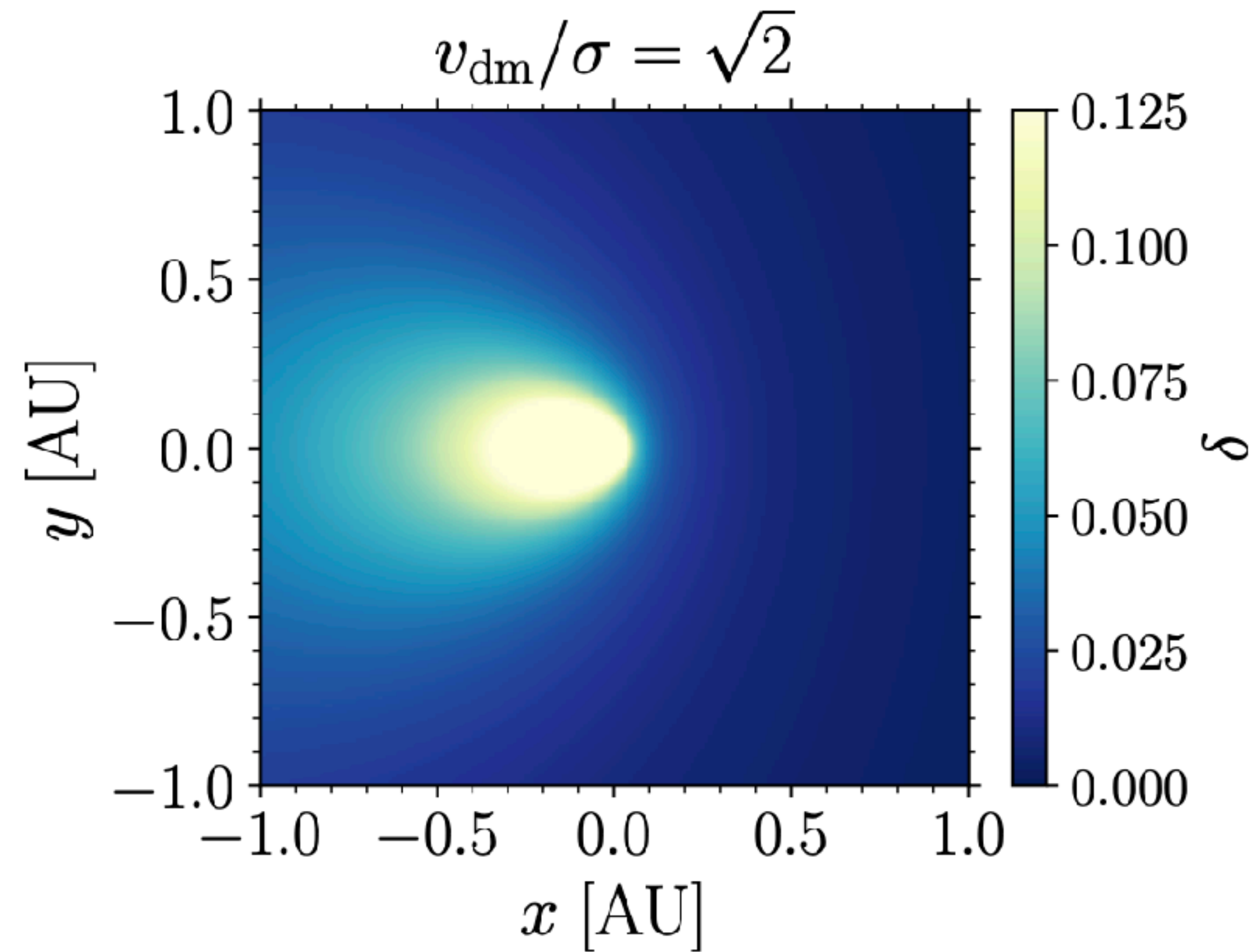
sum over all possible substructures in solar neighborhood

Gravitational influence of the sun



Gravity distorts a local map of DM

Gravitational influence of the sun



The sun *gravitationally focus* dark matter particles

This suggests

$$f_i(\vec{v}) \rightarrow \tilde{f}_i(\vec{x}, \vec{v})$$

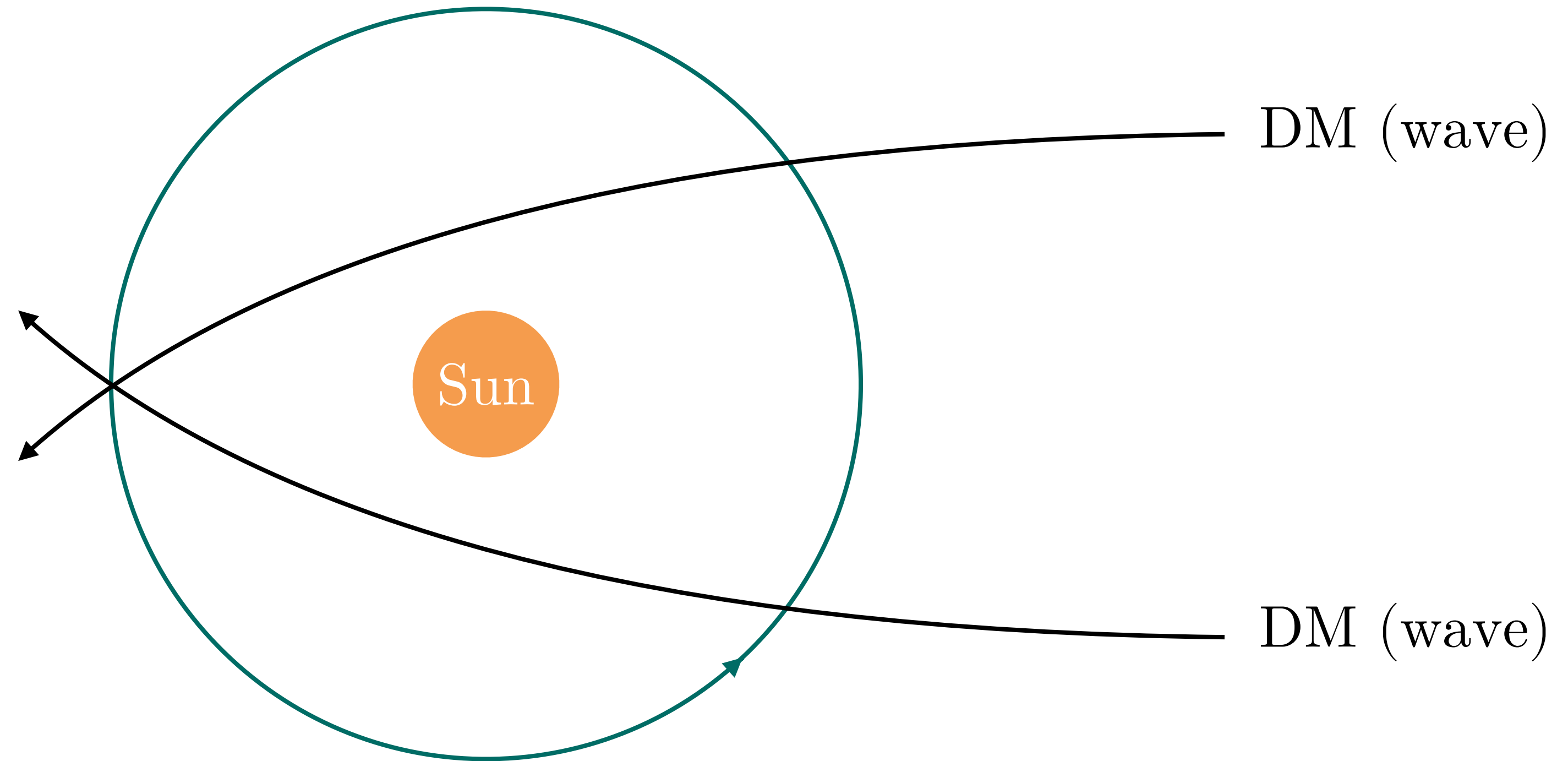
gravitational focusing deforms the local DM distribution

to which degree

these effects would modify signals in DM detectors?

HK, Alessandro Lenoci [2112.05718]

An action for the following situation?



Action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$$

Lagrangian for wave dark matter

Action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right]$$

Lagrangian for wave dark matter

$$ds^2 = (1 + 2\Phi) dt^2 - (1 - 2\Phi) dx^2$$

Gravitational potential

$$\Phi = -\frac{GM}{r}$$

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

$$(\square + m^2)\phi = 0 \quad \rightarrow \quad i\dot{\psi} = \left[-\frac{\nabla^2}{2m} - \frac{GMm}{r} \right] \psi$$

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

Gravitational Response

$$i\dot{\psi} = \left[-\frac{\nabla^2}{2m} - \frac{GMm}{r} \right] \psi$$

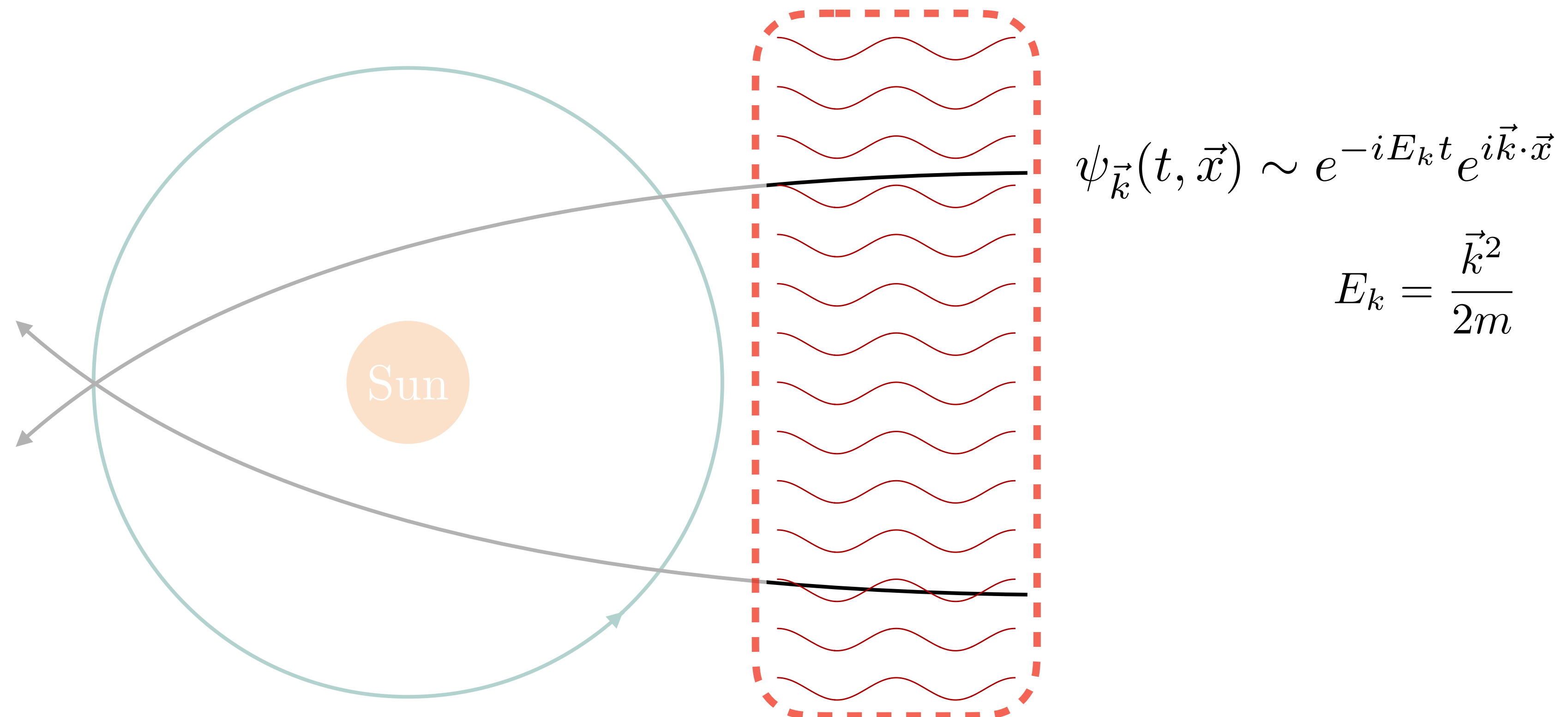
a solution is known

$$\psi_k(t, x) = e^{i(kx - Et)} \psi_k(0) M$$

Remarks on the solution

$$\psi_{\vec{k}}(t, \vec{x}) = e^{-iE_k t} e^{i\vec{k} \cdot \vec{x}} \psi_{\vec{k}}(0) M[i\alpha_G/v, 1, ikr(1 - \hat{k} \cdot \hat{x})]$$

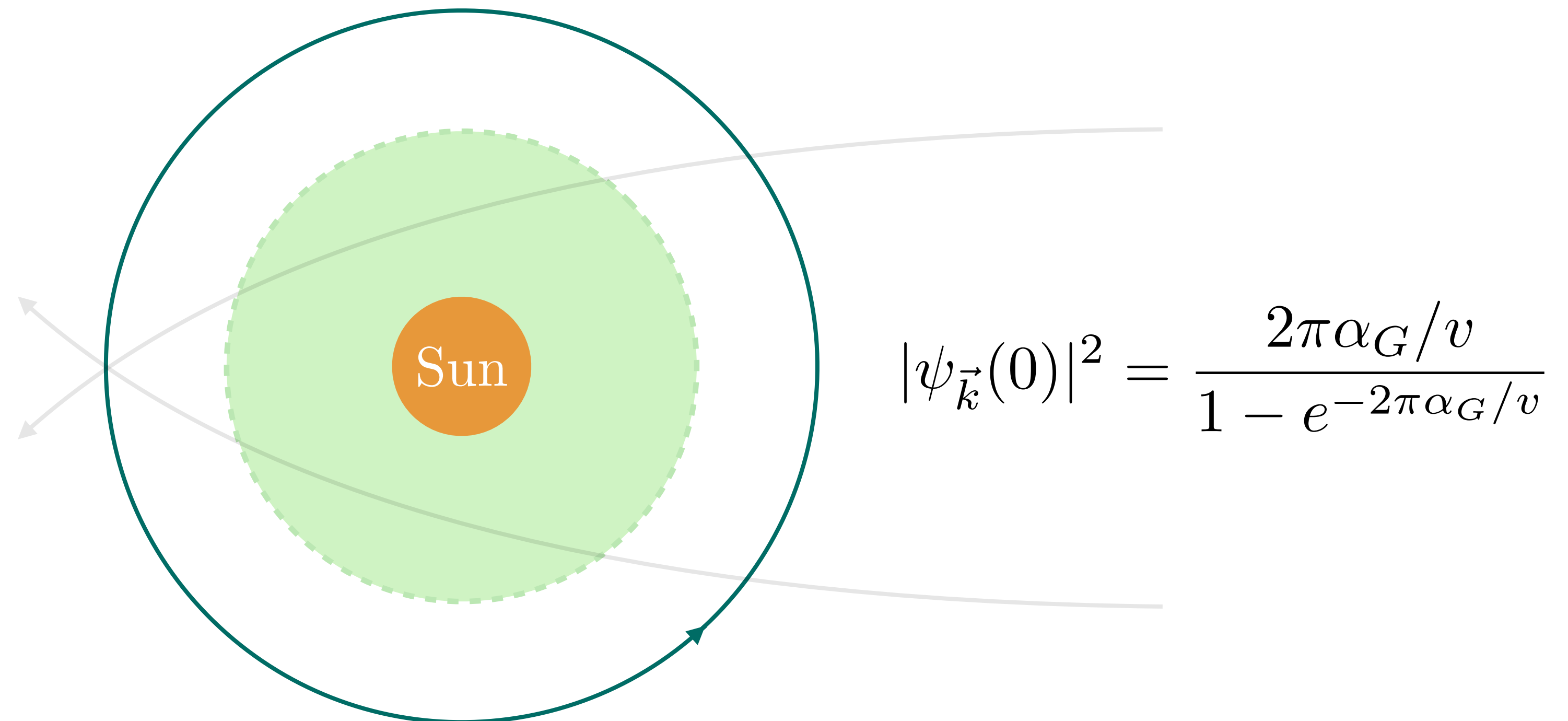
(i) it's a plane wave asymptotically far away from the sun

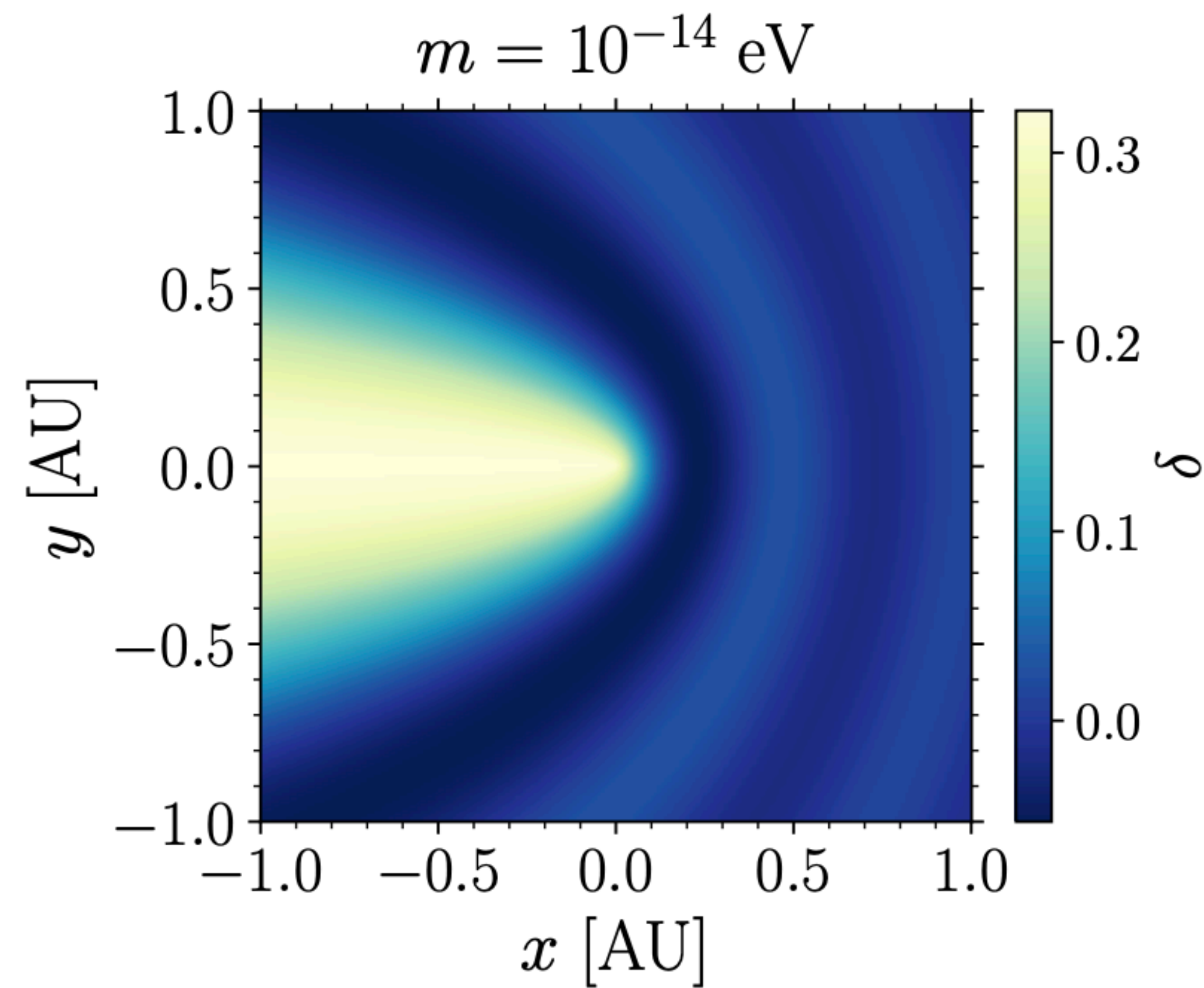
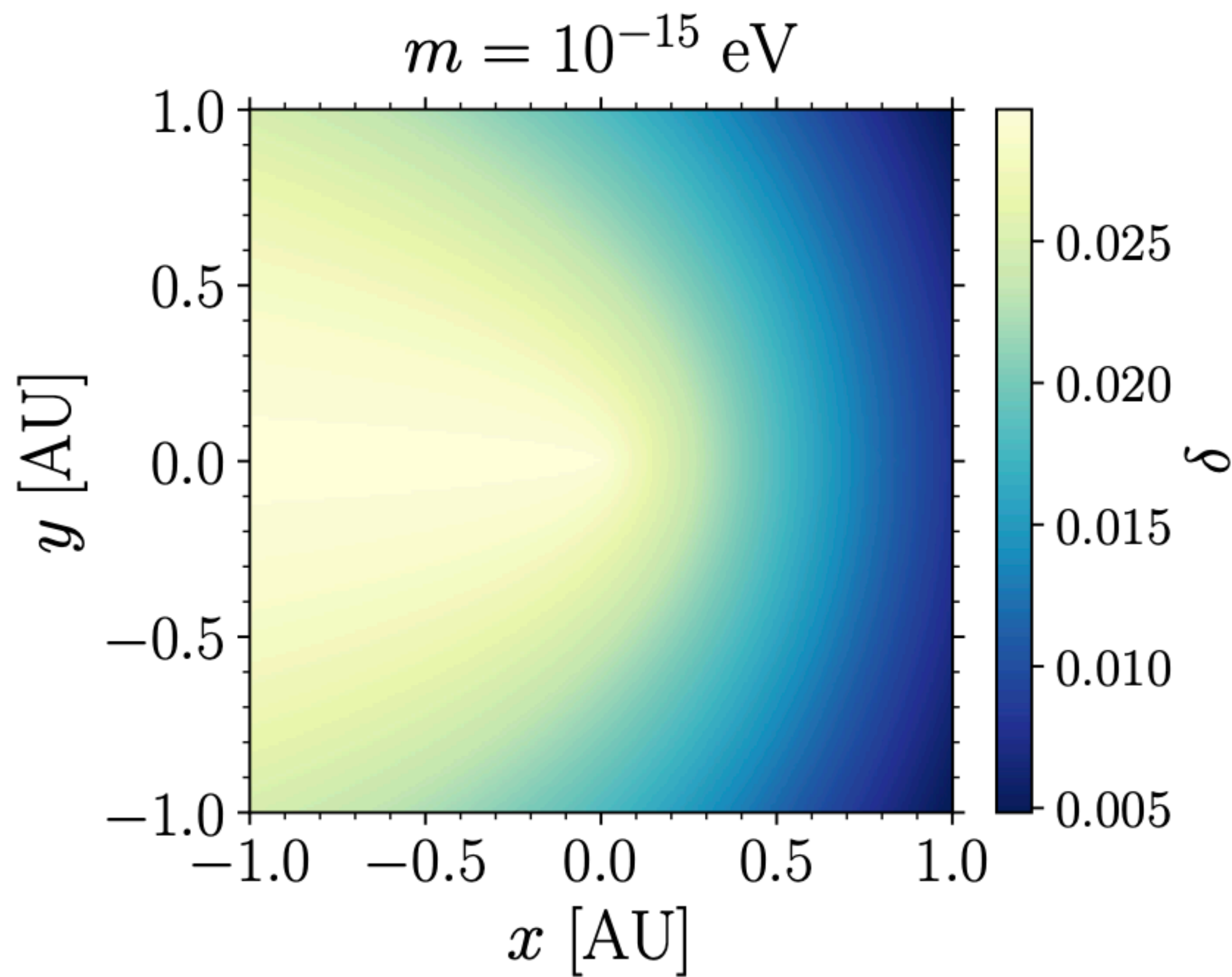


Remarks on the solution

$$\psi_{\vec{k}}(t, \vec{x}) = e^{-iE_k t} e^{i\vec{k} \cdot \vec{x}} \psi_{\vec{k}}(0) M[i\alpha_G/v, 1, ikr(1 - \hat{k} \cdot \hat{x})]$$

(ii) within de Broglie wavelength the wave function saturates





$$1 + \delta = \frac{\langle \phi^2 \rangle}{\phi_0^2}$$

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

density operator must be specified
to determine statistical properties

$$\langle \hat{O} \rangle = \text{Tr} (\hat{\rho} \hat{O})$$

we take

$$\hat{\rho} = \frac{1}{1 + \langle n \rangle} \sum_n \left(\frac{\langle n \rangle}{1 + \langle n \rangle} \right)^n |n\rangle \langle n|$$

we take

$$\hat{\rho} = \frac{1}{1 + \langle n \rangle} \sum_n \left(\frac{\langle n \rangle}{1 + \langle n \rangle} \right)^n |n\rangle \langle n|$$

- this holds for canonical ensemble $\hat{\rho} \propto e^{-\beta \hat{H}}$

$$\langle n \rangle = \frac{1}{e^{\beta \omega} - 1}$$

we take

$$\hat{\rho} = \frac{1}{1 + \langle n \rangle} \sum_n \left(\frac{\langle n \rangle}{1 + \langle n \rangle} \right)^n |n\rangle \langle n|$$

- this holds for canonical ensemble $\hat{\rho} \propto e^{-\beta \hat{H}}$
- this also holds for a wide range of system
(maximizes the *entropy* for a given mean occupation number)

This form is not quite convenient for our analysis
in particular when the mean occupation number is very large

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$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$\alpha \in \mathbb{C}$$

In this case

$$\hat{\rho} = \int d^2\alpha P(\alpha) |\alpha\rangle \langle\alpha|$$

$$P(\alpha) = \frac{1}{\pi \langle n \rangle} \exp \left[-\frac{|\alpha|^2}{\langle n \rangle} \right]$$

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Derevianko 16

Foster et al 18

Centers et al 19

Gramolin et al 21

$$\langle a^\dagger a \rangle = \langle n \rangle$$

$$\langle a a^\dagger \rangle = \langle n \rangle + 1$$

corrections arising from commutation relation are $\mathcal{O}(1/\langle n \rangle)$

$$\langle \mathcal{O}(\hat{a}, \hat{a}^\dagger) \rangle \approx \int d^2\alpha P(\alpha) \mathcal{O}(\alpha, \alpha^*)$$

to the leading order in the occupation number

we may replace all operator with a complex random number

$$\hat{\phi}(t, \vec{x}) \rightarrow \phi(t, \vec{x}) = \sum_i \frac{1}{\sqrt{2mV}} [\alpha_i \psi_i e^{-imt} + \text{h.c.}]$$

arbitrary moments of fields and derivatives can be computed with

$$P(\alpha_i) = \frac{1}{\pi \langle n_i \rangle} \exp \left[-\frac{|\alpha_i|^2}{\langle n_i \rangle} \right]$$

Expansion

$$\hat{\phi}(x) = \sum_i \frac{1}{\sqrt{2mV}} \left[\hat{a}_i \psi_i(x) e^{-imt} + \hat{a}_i^\dagger \psi_i^*(x) e^{imt} \right]$$

(i) *gravitational response* to the external potential

(ii) *stochastic properties* of wave dark matter

How would this affect signals in a detector?

Axion (and scalar) dark matter searches based on

$$\mathcal{L} \supset g\phi\mathcal{O}, \quad g\partial_\mu\phi\mathcal{O}^\mu, \quad \dots$$

signals can be generally written as

$$s(t) = \xi\phi(t, \vec{x}_{\text{dec}})$$

coupling constants + detector specific parameters

the power and spectrum of signal is proportional to that of field

density contrast for wave dark matter

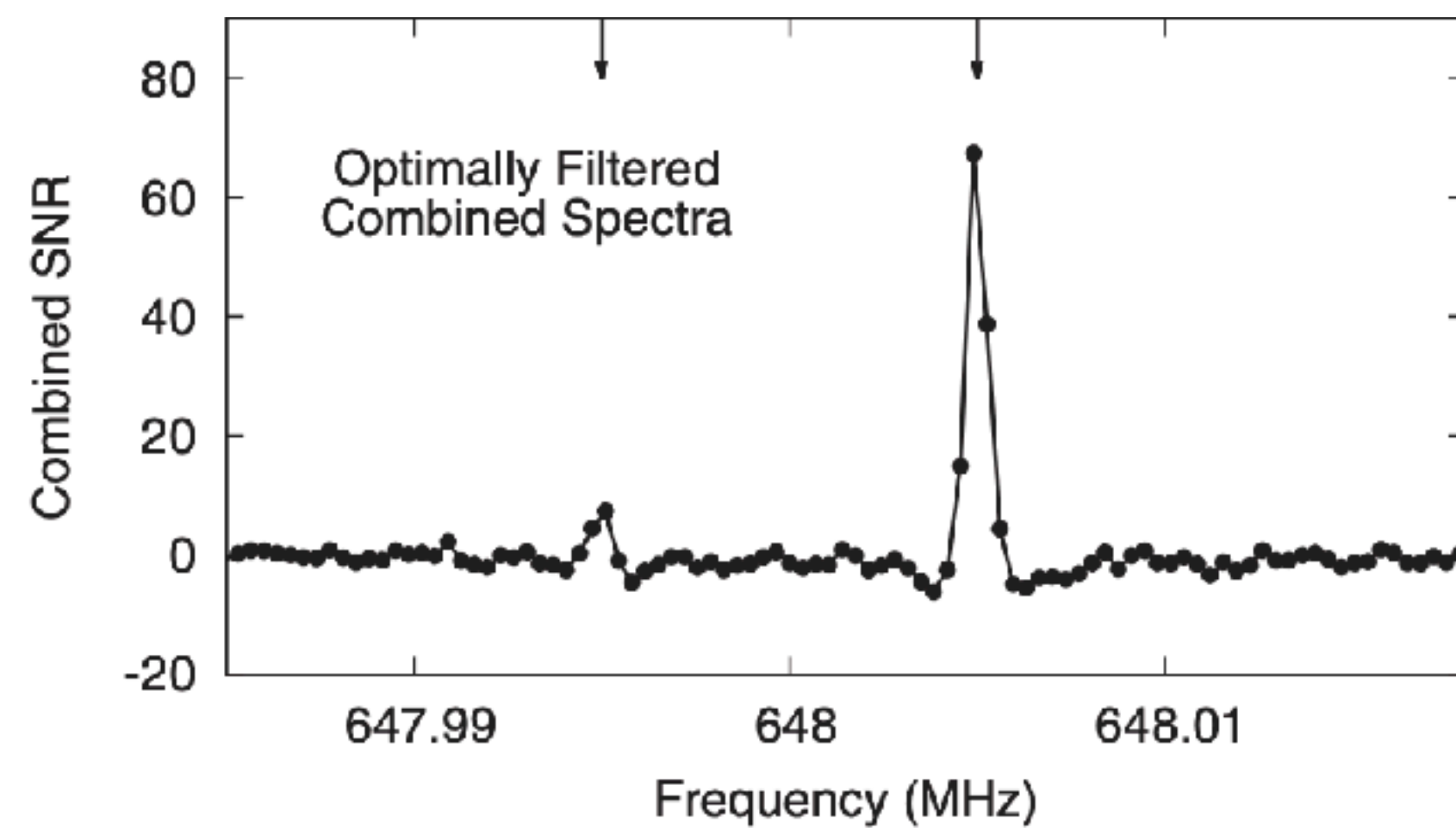
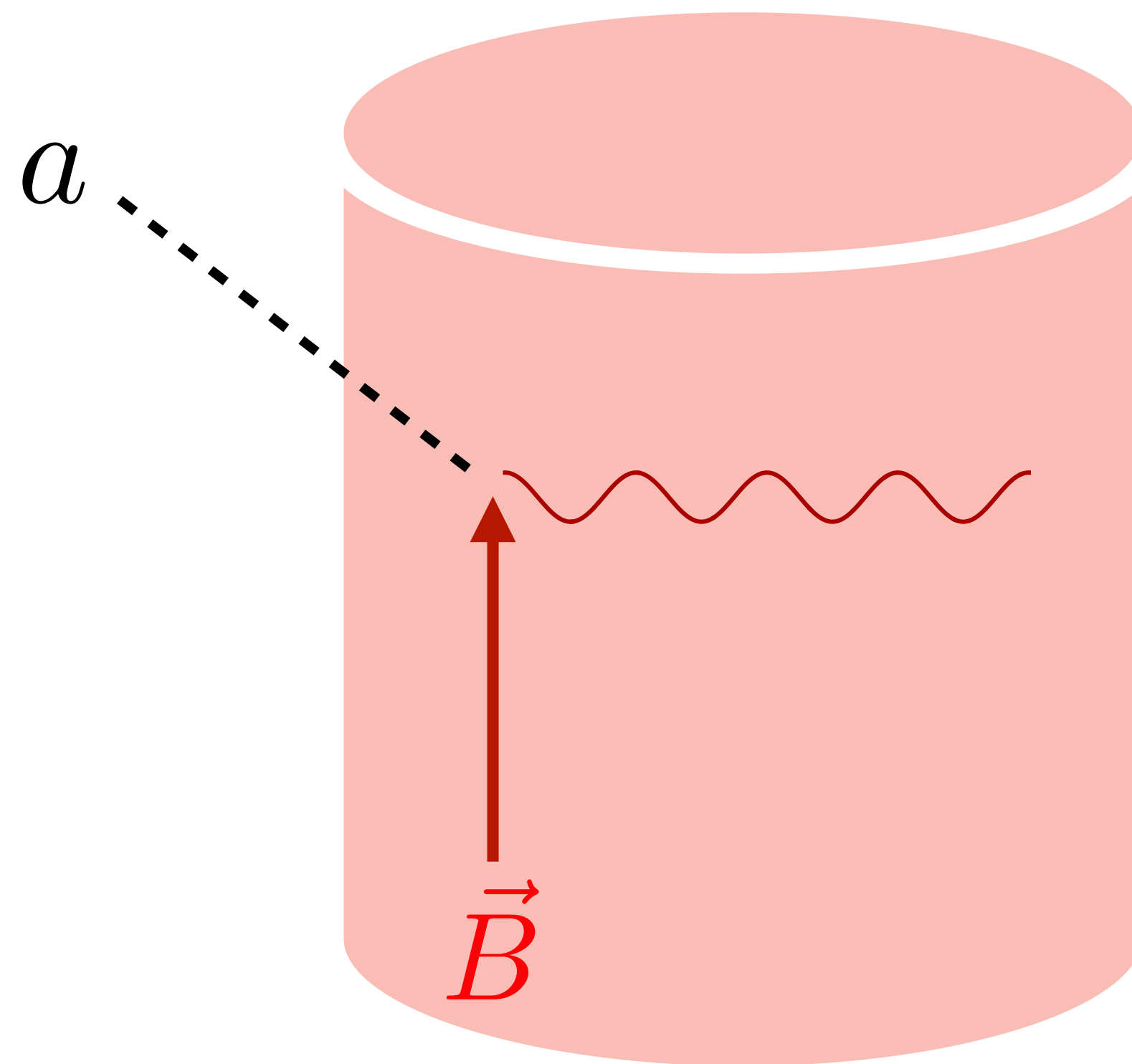
$$1 + \delta = \frac{\langle \phi^2 \rangle}{\phi_0^2} = \int d^3v f(\vec{v}) |\psi_{\vec{v}}|^2$$

density contrast for wave dark matter

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DM speed distribution

$$f(v) \rightarrow \tilde{f}(v) = v^2 \int d\Omega f(\vec{v}) |\psi_{\vec{v}}|^2$$



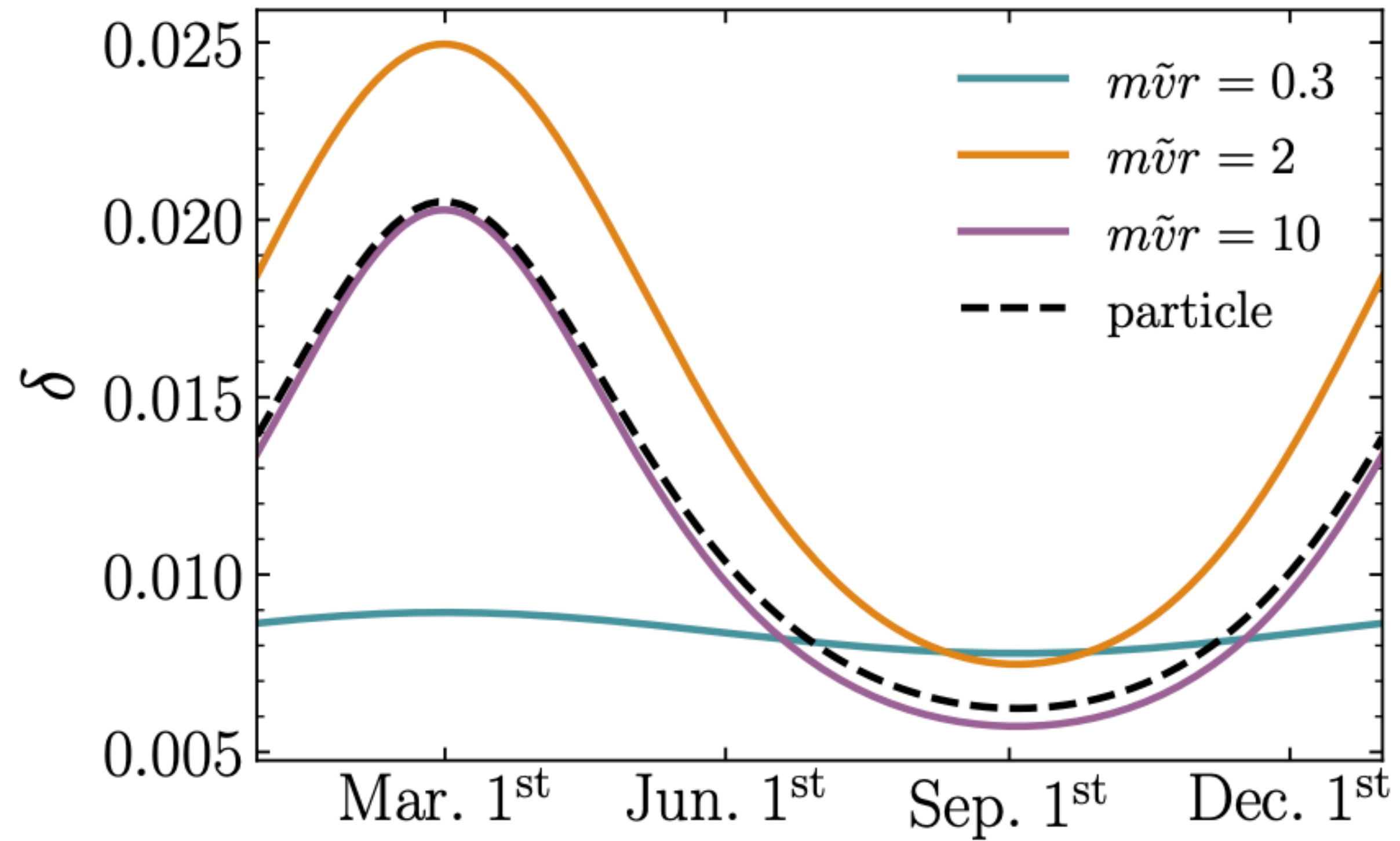
ADMX Collaboration 2018

$$S(\omega) \propto \frac{\xi^2}{m} \frac{\rho_{\text{DM}} f(v)}{v} \rightarrow \tilde{f}(v)$$

Applications:

Gravitational response of local DM substructures

Halo DM

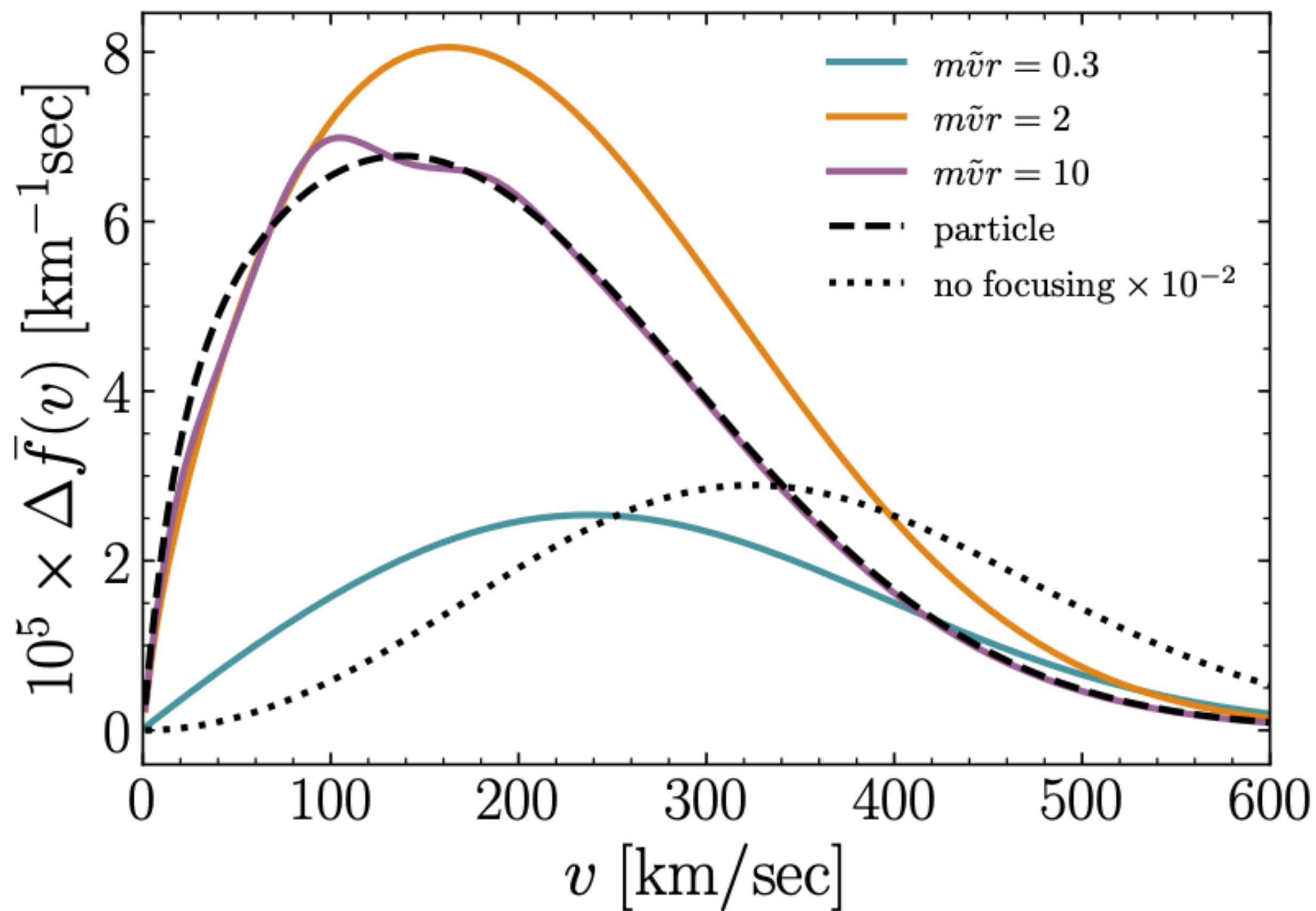


$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\vec{v} - \vec{v}_{\text{dm}})^2}{2\sigma^2}\right]$$

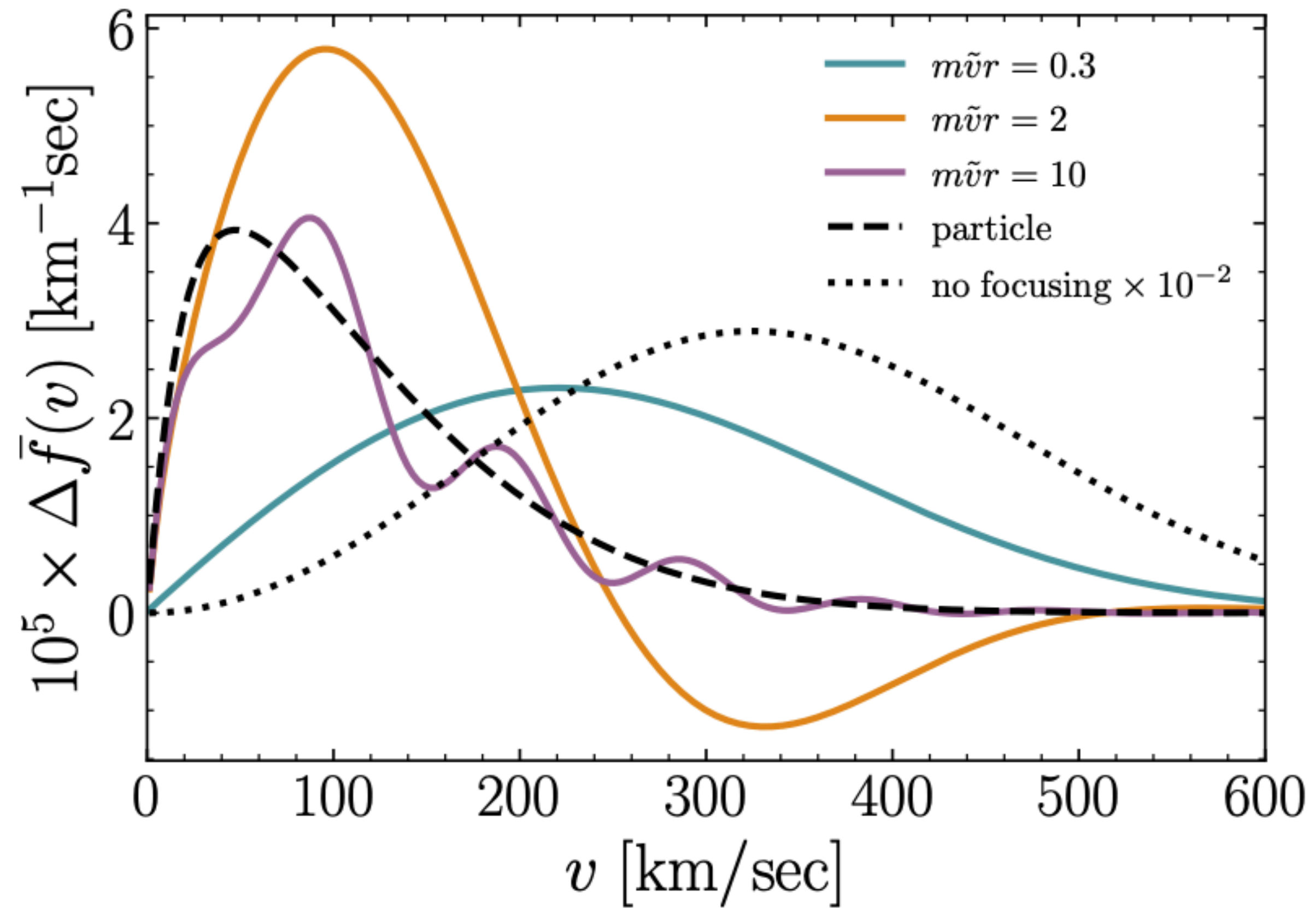
$$v_{\text{dm}} \simeq 230 \text{ km/sec}$$

$$\sigma \simeq 160 \text{ km/sec}$$

Halo DM, Mar. 1st



Halo DM, Sep. 1st

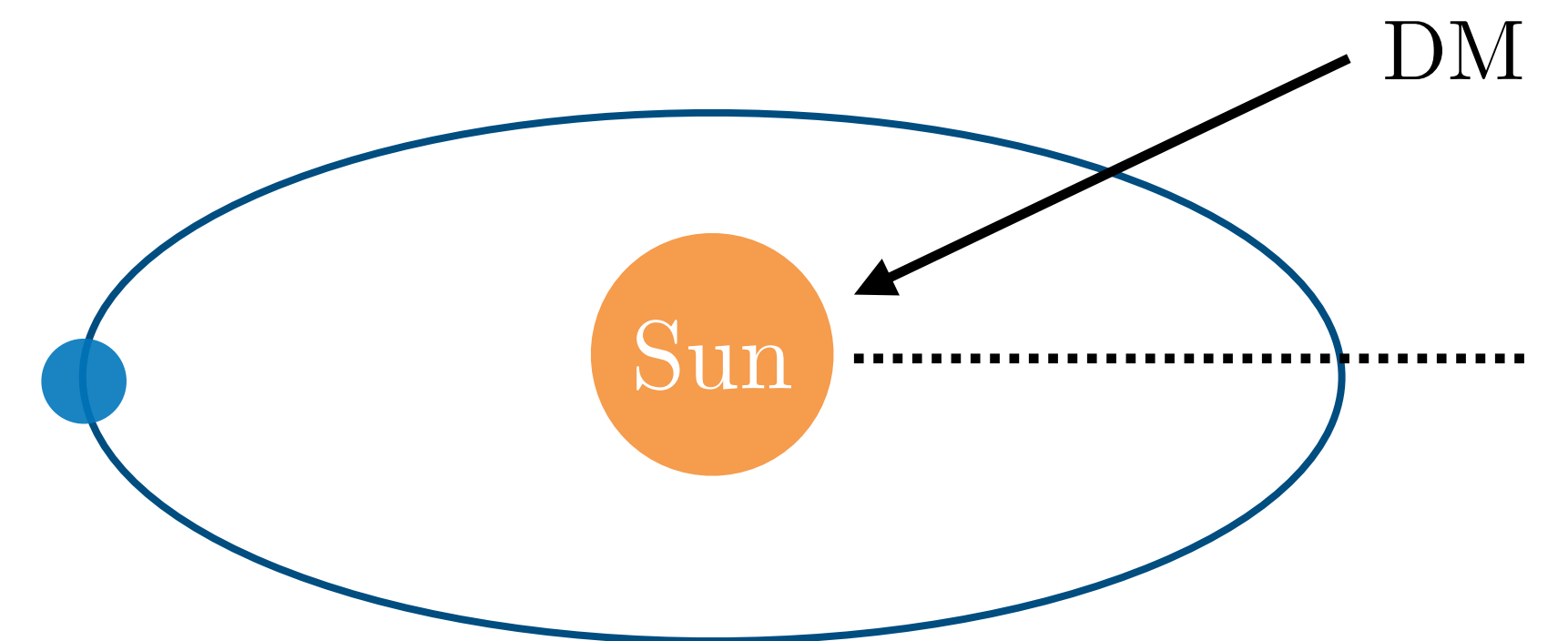
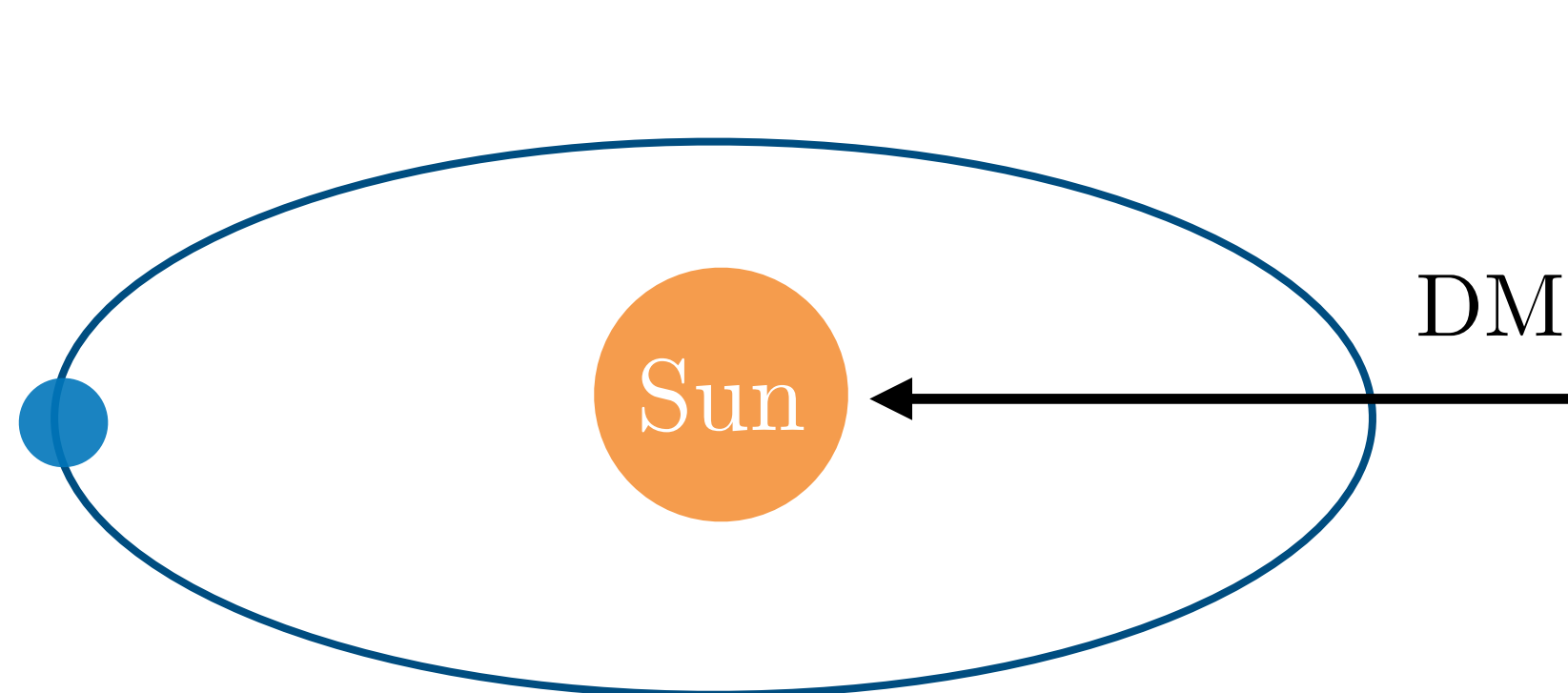
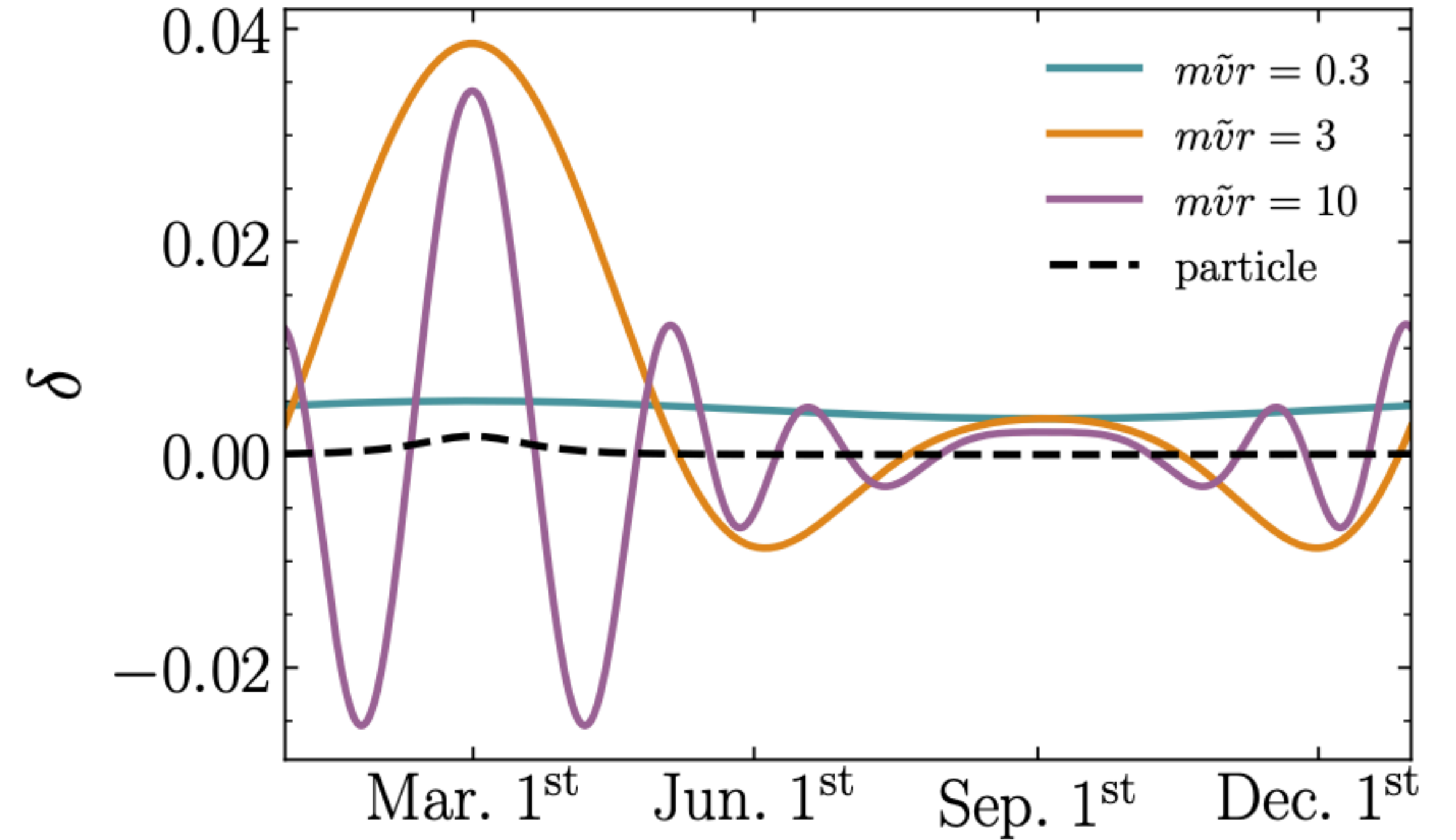
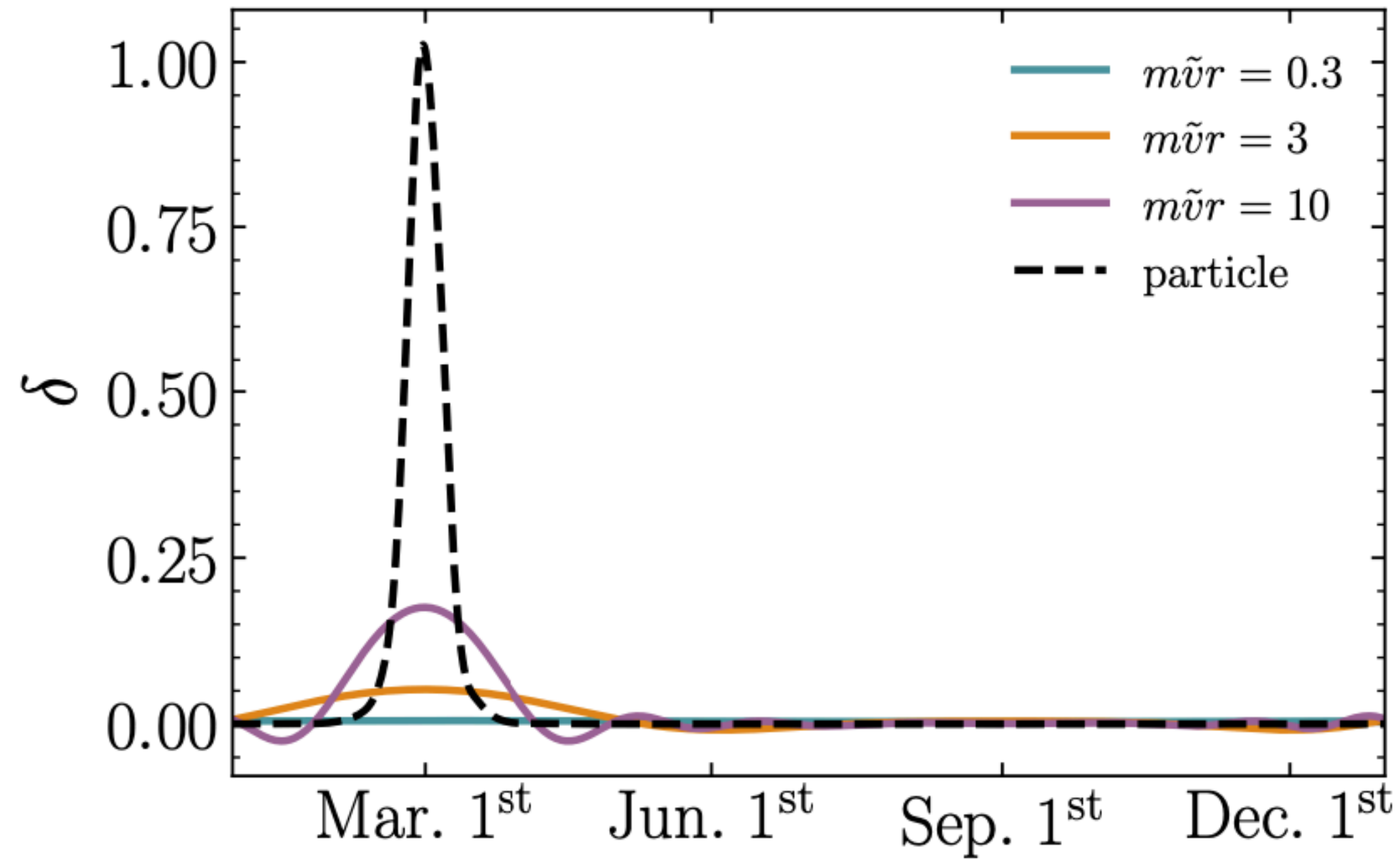


$$v_{\text{dm}} = 400 \text{ km/sec}$$

$$\sigma = 30 \text{ km/sec}$$

Stream DM, $\theta_{\text{dm}} = 0$

Stream DM, $\theta_{\text{dm}} = \pi/6$



Conclusions

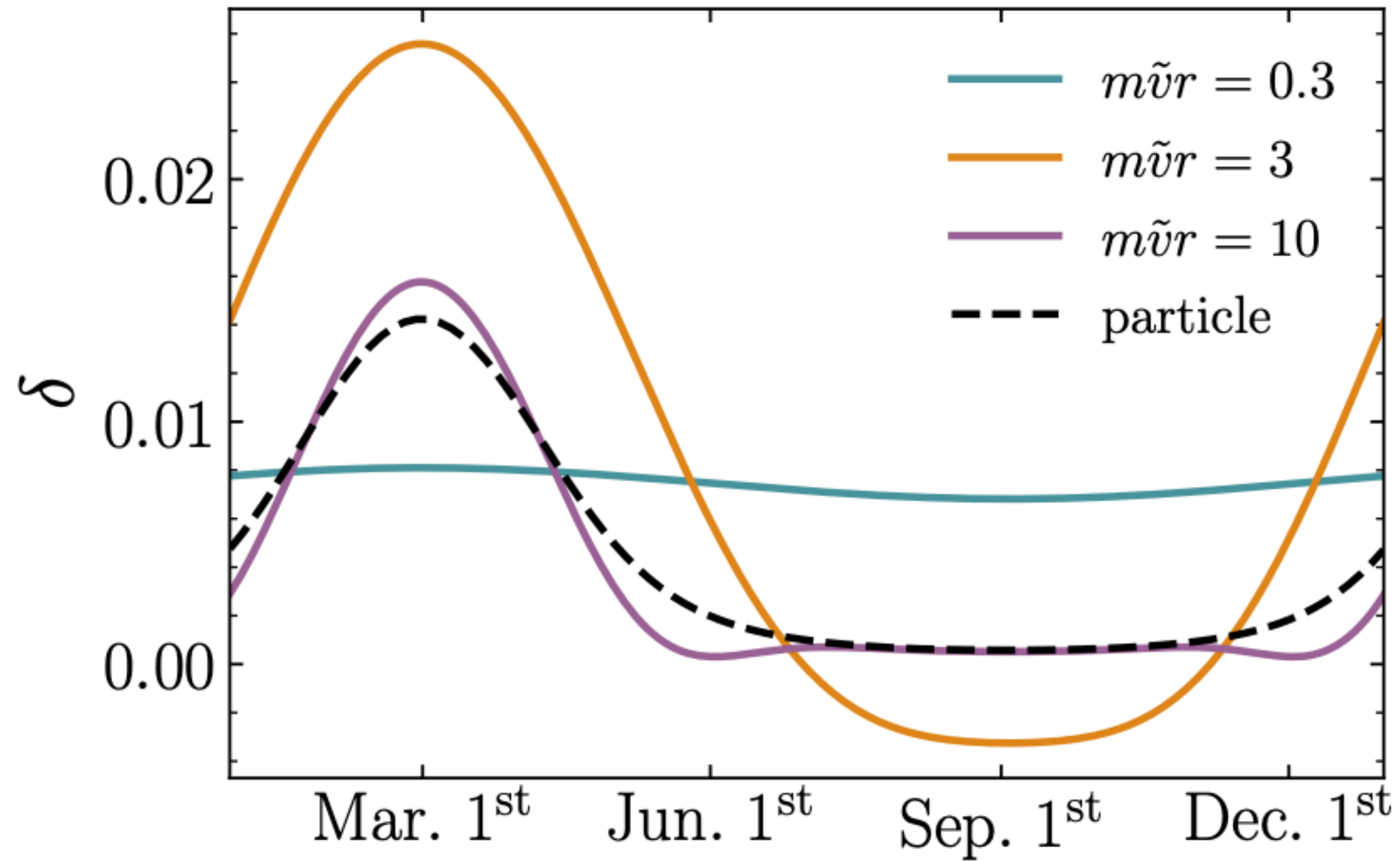
- Gravitational focusing / stochasticity are considered
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- A few percent level effects for the bulk halo dark matter
- More dramatic effects for local DM substructures
- Important for the correct modeling of local DM structure
- Important for constructing a local map of DM (upon detection)

Back-up

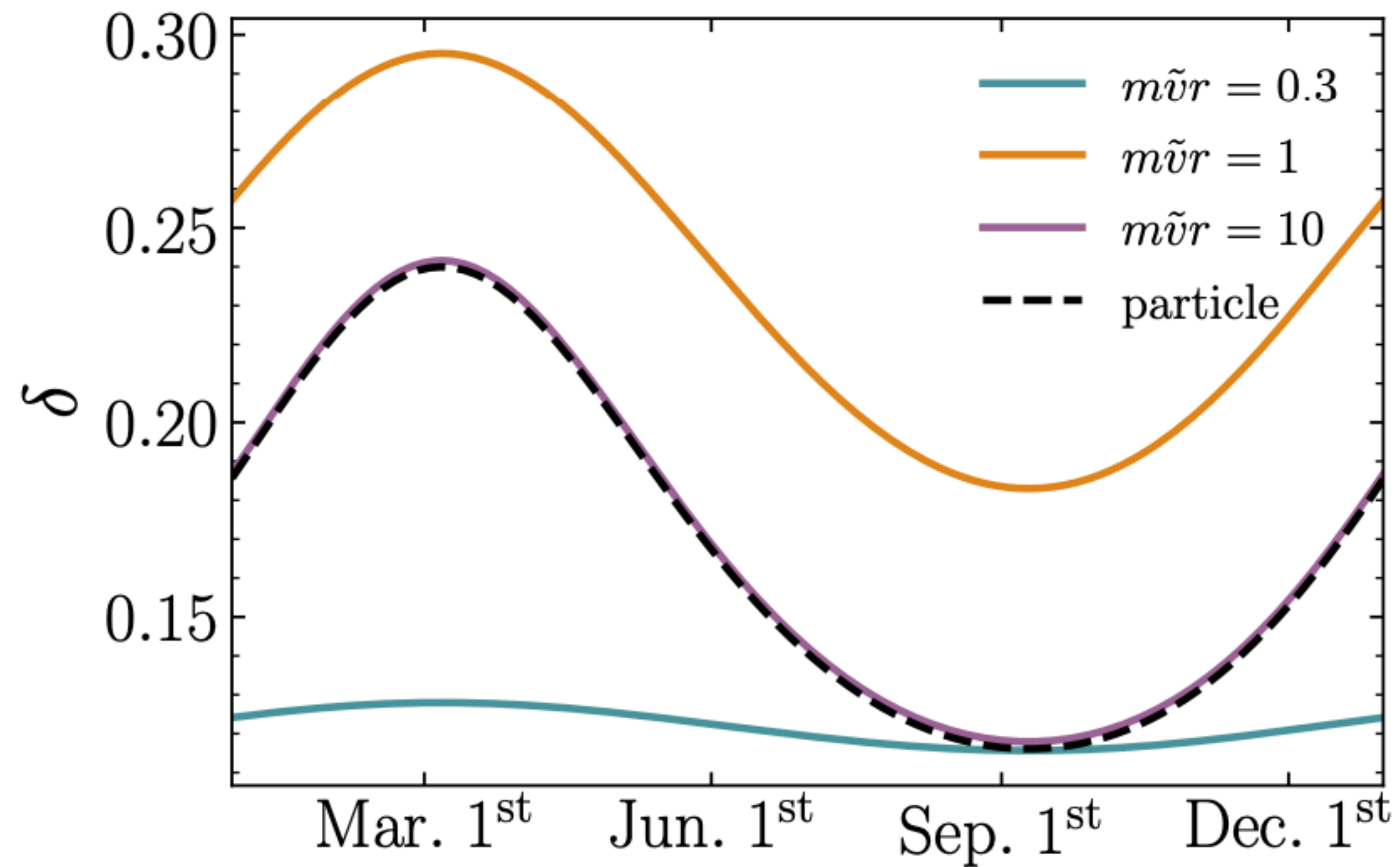
Sausage DM



$$f(\vec{v}) = \frac{1}{(2\pi)^{3/2} \sqrt{\det \Sigma}} \exp \left[-\frac{1}{2} (\vec{v} - \vec{v}_{\text{dm}}) \cdot \Sigma^{-1} \cdot (\vec{v} - \vec{v}_{\text{dm}}) \right]$$

$$\sigma_r = 256 \text{ km/sec}, \quad \sigma_\theta = \sigma_\phi = 81 \text{ km/sec},$$

Disk DM



$$f(\vec{v}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left[-\frac{(\vec{v} - \vec{v}_{\text{dm}})^2}{2\sigma^2}\right]$$

$$v_{\text{dm}} = \sigma \simeq 50 \text{ km/sec}$$

An example from QM

consider a harmonic oscillator

$$\hat{H} = \omega a^\dagger a$$

density matrix for canonical ensemble is

$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\text{tr}(e^{-\beta \hat{H}})}$$

$$\begin{aligned}\hat{\rho} &= \frac{e^{-\beta\hat{H}}}{\text{tr}(e^{-\beta\hat{H}})} \\ &= (1 - e^{-\beta\omega}) \sum e^{-n\beta\omega} |n\rangle\langle n| \\ &= \frac{1}{1 + \langle n \rangle} \sum_n \left(\frac{\langle n \rangle}{\langle n \rangle + 1} \right)^n |n\rangle\langle n|\end{aligned}$$

$$\text{with } \langle n \rangle = \frac{1}{e^{\beta\omega} - 1}$$

Equation of motion

For scalar field

$$(\square + m^2)\phi = 0$$

For potential

$$\nabla^2\Phi = 4\pi G(\rho_{\text{ext}} + \rho_\phi)$$

Assumption / approximation

(i) self-gravity due to DM can be ignored

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$$\Phi \approx -GM/r$$

(iii) DM is non-relativistic

$$\phi(t, x) \rightarrow \psi(t, x)e^{-imt} + \text{h.c.}$$