"The weak scale hierarchy problem"

Let's consider the SM lagranigian.

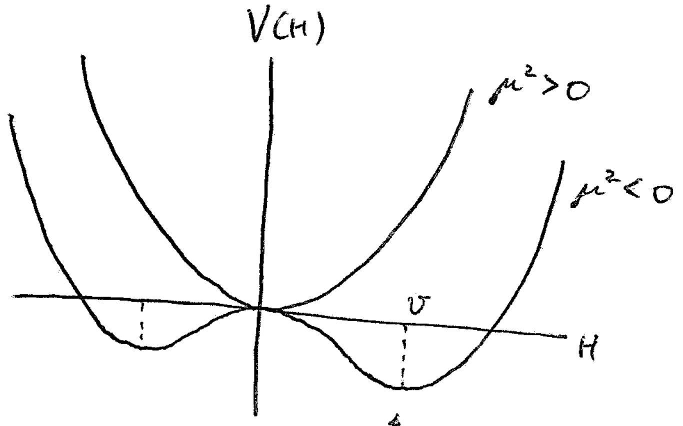
$$\mathcal{L}_{SM} = \sum_{\psi_i = \{Q_i,\, u^c_{Ri},\, d^c_{Ri},\, L_i,\, e^c_{Ri}\}} \psi_i^\dagger \bar{\sigma}^\mu D_\mu \psi_i + \sum_{ij} y_{u_{ij}} Q_i u^c_{Rj} H + \sum_{ij} y_{d_{ij}} Q_i d^c_{Rj} H^*$$

$i = 1, 2, 3$ generation (flavor) index

$$+ \sum_{ij} y_{e_{ij}} L_i e^c_{Rj} H^*$$

$$-\frac{1}{4} \sum_{F = \{G,\, W,\, B\}} F^{\mu\nu a} F^a_{\mu\nu} + (D_\mu H)^\dagger D^\mu H \underbrace{- \mu^2 |H|^2 - \lambda |H|^4}_{-V(H)}$$

$SO(3)_c$ $SO(2)_L$ $U(1)_Y$



Non-zero Vacuum expectation Value (VEV) of the Higgs field

(spontaneous gauge symmetry breaking)

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4$$

$$= \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

$H = \frac{1}{\sqrt{2}} h\, e^{i\theta_h(x)}$

Complex scalar field

two real scalar fields

$$\frac{\partial V}{\partial h} = 0 \Rightarrow v = \langle h \rangle = \sqrt{\frac{-\mu^2}{\lambda}}$$

Expand the Higgs field around the vacuum $H = \frac{1}{\sqrt{2}} (v + \hat{h}) e^{i\theta_h}$

$$\Rightarrow V(H) = \frac{1}{2} m_h^2 \hat{h}^2 + \lambda v \hat{h}^3 + \frac{\lambda}{4} \hat{h}^4 - \frac{\lambda}{4} v^4$$

$$m_h^2 = 2|\mu^2| = 2\lambda v^2$$

$$(D_\mu H)^\dagger (D^\mu H) \xRightarrow{\text{gives rise to}} \text{gauge boson masses}$$

$$D_\mu H = (\partial_\mu + i g_2 W_\mu^a \underbrace{\tau^a}_{\to \frac{\sigma^a}{2}} + i g_1 B_\mu \underbrace{Y_H}_{\to \frac{1}{2}}) H$$

$$\Rightarrow \quad (D_\mu H)^\dagger (D^\mu H) = -\left(\frac{g_2 v}{2}\right)^2 W^{+\mu} W^-_\mu - \frac{1}{2}\left(\frac{g_2 v}{2\cos\theta_W}\right)^2 Z^\mu Z_\mu$$

$$+ \cdots$$

where $W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$

$$Z_\mu \equiv \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$$

$$A_\mu \equiv \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$$

$$\cos\theta_W \equiv \frac{g_2}{\sqrt{g_1^2+g_2^2}} \quad ; \text{ Weinberg angle}$$

Thus $m_W = \dfrac{g_2 v}{2}$, $\quad m_Z = \dfrac{m_W}{\cos\theta_W}$

Measured values

$$\left.\begin{aligned} m_W &\doteqdot 80.4 \text{ GeV} \\ m_Z &\doteqdot 91.2 \text{ GeV} \end{aligned}\right\} \Rightarrow \begin{aligned} \cos\theta_W &\doteqdot 0.882 \\ \sin^2\theta_W &\doteqdot 0.223 \end{aligned}$$

$$e = g_2 \sin\theta_W \text{ (electric charge)} \quad \Rightarrow g_2 \xRightarrow{m_W} \boxed{v \simeq 246 \text{ GeV}}$$

$$\frac{e^2}{4\pi} \doteqdot \frac{1}{137}$$

$\sim m_W \sim m_Z \sim m_t$

$\sim 10^2$ GeV

"Weak Scale"

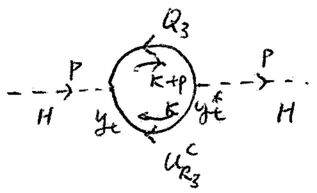
The weak scale is originated from the Higgs mass scale.

$(\mu^2 \sim v^2)$

Is the weak scale quantum mechanically "natural"?

Let's examine the loop correction to the Higgs mass parameter μ^2 from the top Yukawa coupling.

$$y_{u_{ij}} Q_i u^c_{R_j} H \Rightarrow y_t Q_3 u^c_{R_3} H$$



$$= (\Delta\mu^2)_t |H|^2$$

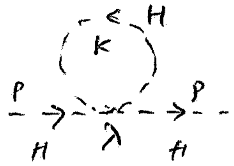
$$(\Delta\mu^2)_t = |y_t|^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\frac{i}{\not{p}+\not{k}-m_t}\frac{i}{\not{k}-m_t}\right]$$

$$\underset{(p\to 0)}{\simeq} -|y_t|^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\frac{(\not{k}+m_t)^2}{(k^2-m_t^2)^2}\right]$$

$$= -4|y_t|^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k^2-m_t^2} + \frac{2m_t^2}{(k^2-m_t^2)^2}\right]$$

$$= -\frac{|y_t|^2}{8\pi^2}\left[\underbrace{\Lambda^2}_{\text{quadratic divergence}} - 3m_t^2 \ln\frac{\Lambda^2}{m_t^2} + \cdots\right] \times \underbrace{3}_{\text{color}}$$

Also $\lambda |H|^4$



$$= (\Delta\mu^2)_H |H|^2$$

$$(\Delta\mu^2)_H = 4\lambda \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m_h^2}$$

$$= \frac{\lambda}{4\pi^2}\left[\Lambda^2 - m_h^2 \ln\frac{\Lambda^2}{m_h^2} + \cdots\right]$$

Λ : high energy scale (UV cutoff)
above which the SM description of the nature
is no longer valid.

If the SM is valid up to the Planck scale M_{pl},

$\Lambda \sim M_{pl}$

$\hookrightarrow \Delta\mu^2 \sim M_{pl}^2$

QM correction

$$\mu^2 \left(= -\frac{m_h^2}{2}\right) = \mu_0^2 + \Delta\mu^2 \ll \Delta\mu^2 \sim M_{pl}^2$$

$\sim m_W^2$ bare Higgs mass parameter $\sim (10^{18}\,\mathrm{GeV})^2$

$\sim (10^2\,\mathrm{GeV})^2$

Severe fine-cancellation between μ_0^2 and $\Delta\mu^2$
is needed to explain the observed value of μ^2
$\sim (10^2\,\mathrm{GeV})^2$.

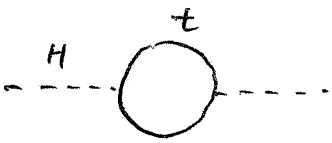
$\Rightarrow$ "Weak scale hierarchy problem"
(historically also named "gauge hierarchy problem")

The Dimensional Regularization may change the conclusion ?

$$\int^{\Lambda} \frac{d^4k}{(2\pi)^4} \quad \text{(momentum cutoff regularization)}$$

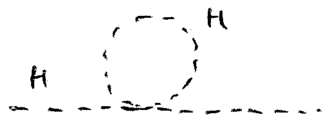
$$\Rightarrow \int \frac{d^Dk}{(2\pi)^4} \quad \text{(dimensional regularization)}$$

$$D = 4 - \epsilon$$



$$\sim -\frac{|y_t|^2}{8\pi^2} m_t^2 \left[\frac{1}{\epsilon} - \gamma + 1 + \ln \frac{\mu_R^2}{m_t^2} + \cdots \right]$$

renormalization scale

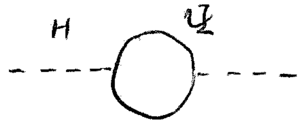


$$\sim \frac{\lambda}{8\pi^2} m_H^2 \left[\frac{1}{\epsilon} - \gamma + 1 + \ln \frac{\mu_R^2}{m_H^2} + \cdots \right]$$

No quadratic divergence $\Rightarrow$ No hierarchy problem ?
(due to the absence of
any dimensionful parameter in the regularization scheme)

Suppose that the Higgs couples to some unknown heavy ptls

e.g. $y_\Psi H \Psi_L \Psi_R^c$
$\lambda_S |H|^2 |S|^2$ where $m_\Psi, m_S \gg m_W$



$$\sim -\frac{|y_\Psi|^2}{8\pi^2} m_\Psi^2 \ln \frac{\Lambda^2}{m_\Psi^2}$$

$$\sim \frac{\lambda_S}{8\pi^2} m_S^2 \ln \frac{\Lambda^2}{m_S^2}$$

i.e. $\Delta\mu^2 \sim \frac{k}{8\pi^2} m_{new}^2 \ln \frac{\Lambda^2}{m_{new}^2}$

Therefore, if $m_{new} \gg 4\pi m_W \sim 1$ TeV, the hierarchy problem still exists. (i.e. Higg mass is very sensitive to UV physics regardless of regularization schemes.)

* Conventional approach to the weak scale hierarchy problem

To make $\Delta\mu^2 \sim m_W^2$ by symmetries, new strong dynamics, or extra dimensions.

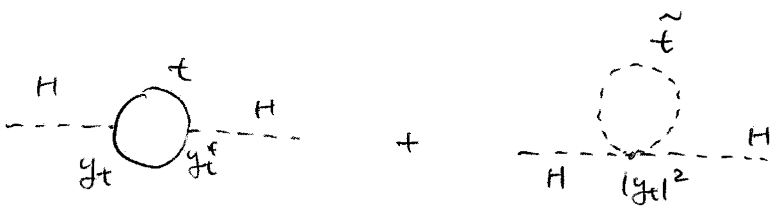
SUPERSYMMETRY

introduces superpartner ptls of spin 1/2 difference.

$\psi_{SM} \longleftrightarrow \Phi_{SUSY}$ $(q \leftrightarrow \tilde{q}, \ l \leftrightarrow \tilde{l})$

$A_{\mu SM} \longleftrightarrow \lambda_{SUSY}$

$H_{SM} \longleftrightarrow \tilde{H}_{SUSY}$



$$= \frac{|y_t|^2}{8\pi^2}\left[-\Lambda^2 + \Lambda^2 + 3(m_{\tilde{t}}^2 - m_t^2)\ln\frac{\Lambda^2}{m_{\tilde{t}}^2} - 3m_t^2 \ln\frac{m_{\tilde{t}}^2}{m_t^2} + \cdots\right] \times 3$$

$$\Rightarrow \quad \Delta\mu^2 \sim \frac{|y|^2}{8\pi^2} m_{SUSY}^2 \ln\frac{\Lambda^2}{m_{SUSY}^2}$$

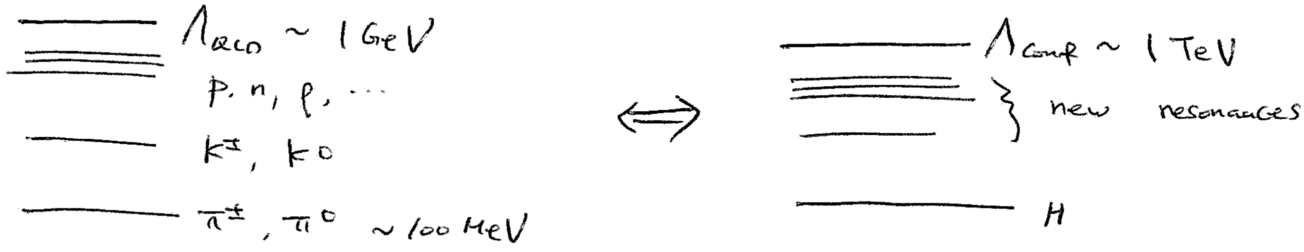
If $m_{SUSY} \lesssim 4\pi m_W \sim \text{TeV}$, $\Delta\mu^2 \lesssim m_W^2$

no fine-tuning for the observed weak scale.

$\Rightarrow$ Naturalness argument for "TeV scale SUSY"

Composite Higgs

Higgs as a composite pseudo-Nambu-Goldstone boson (i.e. QCD pion-like ptl) of a new QCD-like confining force



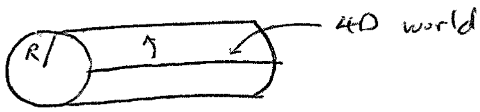
$\Lambda \sim \Lambda_{conf}$ (confinement scale of the new force)

If $\Lambda_{conf} \lesssim \mathrm{TeV}$, $\Delta\mu^2 \lesssim m_W^2$

It predicts new resonances (new baryons & mesons) around TeV scale.

Extra dimensions

introduces compact extra spatial dimensions where gravity can propagate.

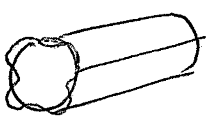


$$2\pi R M_5^3 = M_{Pl}^2$$

$$M_5^3 = \frac{M_{Pl}^2}{2\pi R} \rightarrow \text{small for a large } R \text{ (i.e. strong gravity)}$$

$\Rightarrow$ $\Lambda \sim M_5$ higher dimensional Planck scale

$$M_5 \lesssim \mathrm{TeV} \Rightarrow \Delta\mu^2 \lesssim m_W^2$$



$m_n \sim \frac{n}{R}$ It predicts Kaluza-Klein excitations (KK ptls) below TeV scale.

$$\frac{1}{2}(\partial_M \Phi)^2 = \frac{1}{2}(\partial_\mu \Phi)^2 + \underbrace{\frac{1}{2}(\partial_y \Phi)^2}_{m_n^2 \Phi^2}$$

In summary, conventional approach :

$$\Delta\mu^2 \lesssim m_W^2 \quad \text{with} \quad \Lambda \sim \text{TeV}$$

generally predicts new ptls of TeV scale which sizably couple to SM ptls.

However, the LHC experiment has not yet discovered any strong evidence for the existence of such new ptls.

$\Rightarrow$ $\Lambda \gtrsim$ a few TeV.

Consequently, it is very difficult to completely solve the hierarchy problem by the conventional ideas.

(At least 1~10 % fine-tuning of the Higgs mass squared parameters is mostly unavoidable.)

"Little hierarchy problem"

* A new approach : dynamical relaxation of the Higgs mass

"P. Graham, D.E. Kaplan, S. Rajendran 1504.07551

$\Delta\mu^2 \sim \Lambda^2$ can be $\gg (\mathrm{TeV})^2$.

Yet $\mu^2 = \mu_0^2 + \Delta\mu^2 \rightarrow m_W^2 \ll \Lambda^2$ by a cosmological dynamics of the "relaxion" ϕ.

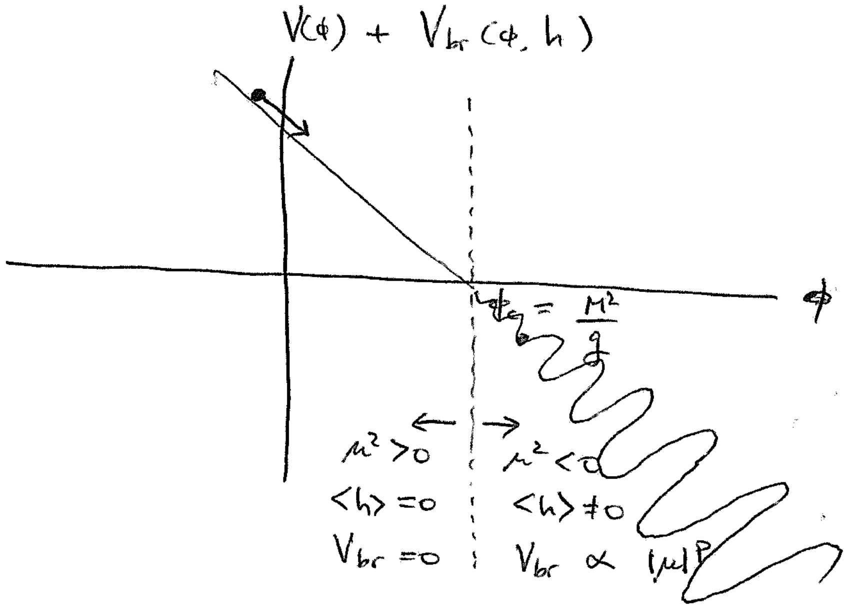
↓ promote it to be a dynamical field: $\mu_0^2(\phi)$

$$\mathcal{L} = \underbrace{(-M^2 + g\phi)}_{-\mu^2(\phi)} |h|^2 + V(\phi) + \underbrace{V_{br}(\phi, h)}_{\propto \langle h \rangle^p}$$

$[g] = 1$

$\langle h \rangle^p$: Higgs VEV

V_{br}: backreaction potential



$\Rightarrow$ ϕ will be stabilized near $\phi_c = \dfrac{M^2}{g}$ where $\langle h \rangle = 0$.

$$\Rightarrow \quad \langle h \rangle \ll M^2 \sim \frac{\Lambda^2}{16\pi^2}$$