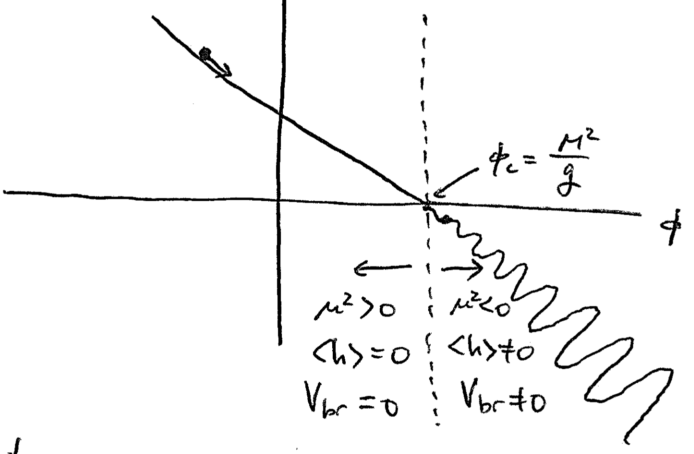
* Relaxion lagrangian : structure

$$\mathcal{L} = \underbrace{(-M^2 + g\phi)}_{-\mu^2(\phi)} |h|^2 - V(\phi) - \underbrace{V_{br}(\phi, h)}_{\propto \langle h \rangle^P \cos\frac{\phi}{f}}$$

$$M^2 \sim \frac{\Lambda^2}{16\pi^2}$$

$$V_{tot}(\phi) = V(\phi) + V_{br}(\phi, h)$$



ϕ is stabilized near $\phi_c = \frac{M^2}{g}$ where $\mu^2(\phi) \ll M^2$.

$$\Rightarrow \quad \langle h \rangle \ll M^2 \sim \frac{\Lambda^2}{16\pi^2}$$

* The rolling potential $V(\phi)$?

For $V_{br}(\propto \langle h \rangle^P \ll \Lambda^4)$ to stop the relaxion,

$V(\phi)$ has to be fairly small.

Assume the global shift symmetry of ϕ :

$$\phi \to \phi + c$$

which is explicitly broken only by a small non-zero g

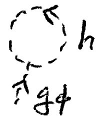
& some non-perturbative effects.

$$(-M^2 + \underset{\sim\sim}{g\phi})|h|^2 + c_1 g M^2 \phi + c_2 g^2 \phi^2 + \cdots$$

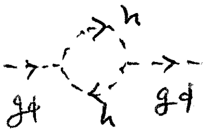
radiative corrections

$c_1 \sim \mathcal{O}(1) > 0$

$c_2 \sim \mathcal{O}(\frac{1}{16\pi^2})$



$\sim \frac{\Lambda^2}{16\pi^2} g\phi \sim M^2 g\phi$



Thus $V(\phi) \simeq c_1 g M^2 \phi$

Note that $V(\phi) \propto g$ since $\mathcal{L}$ is invariant under $\phi \to \phi + c$ if $g=0$.

(g can be naturally small against radiative correction ;

"Technical Naturalness" ; t'Hooft)

* The backreaction potential $V_{br}(\phi, h) \propto \langle h \rangle^p \cos\frac{\phi}{f}$?

A simple model : "QCD relaxion"

Consider the operator $\frac{g_s^2}{32\pi^2}\frac{\phi}{f}\underbrace{G^{\mu\nu a}}_{\text{QCD gluon field strength}}\tilde{G}_{\mu\nu}{}^{a}$ $\left[\tilde{G}^{\mu\nu a} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}{}^{a}\right.$

$G^{\mu\nu a}\tilde{G}_{\mu\nu}{}^{a} = \partial_\mu J^\mu_{CS}$ $\quad J^\mu_{CS} \equiv 2\epsilon^{\mu\nu\rho\sigma}\,\mathrm{tr}\left(A_\nu G_{\rho\sigma} + \frac{2}{3}ig_s A_\nu A_\rho A_\sigma\right)$

Chern-Simon current

Since $G\tilde{G}$ is a total divergence, the operator doesn't break the shift symmetry of ϕ by perturbative physics.
$(\phi \to \phi + c)$

However, by non-perturbative processes, called "instantons", it does beak ~~[illegible]~~ $(\phi \to \phi + c)$ and gives rise to a potential for ϕ.

$$\frac{g_s^2}{32\pi^2}\frac{\phi}{f}G\tilde{G} \underset{\substack{\text{QCD}\\ \text{instantons}}}{\Rightarrow} -T^4 e^{-S_{ins}(T)}\cos\frac{\phi}{f}$$

T: temperature of QCD thermal bath

where $S_{ins} = \frac{8\pi^2}{g_s^2(T)}$: QCD instanton action

Note that the potential is highly suppressed at high energies due to the asymptotic freedom $g_s^2(T) \to 0$ as $T \to \infty$

$\nearrow \frac{8\pi^2}{g_s^2(\Lambda_{QCD})} \sim \mathcal{O}(1)$

When $T \lesssim \Lambda_{QCD}$, the QCD confinement happens, and quarks also play an important role.

$$\frac{g_s^2}{32\pi^2}\frac{\phi}{f}G\tilde{G} \quad + \left(y_u h u_L u_R^c + h.c.\right)$$

$$\Downarrow \text{ QCD confinement} \qquad \langle u_L u_R^c\rangle \simeq \Lambda_{QCD}^3 e^{i\underset{\substack{\downarrow \\ \eta' \text{ meson field}}}{\underset{\sim}{\eta'}}/\overset{\substack{\eta' \text{ decay constant} \\ \uparrow}}{f_{\eta'}}}$$

$$V(\phi,\eta') \simeq -\Lambda_{QCD}^4\cos\left(\frac{\phi}{f} + \underbrace{\frac{\eta'}{f_{\eta'}}}_{\text{Chiral anomaly (see footnote)}}\right) - y_u h\Lambda_{QCD}^3\cos\frac{\eta'}{f_{\eta'}}$$

Since $f_{\eta'} \sim \Lambda_{QCD} \sim \text{GeV}$, $m_{\eta'} \sim \Lambda_{QCD}$.

So at low energies below Λ_{QCD}, we can integrate out η'.

$$\frac{\partial V(\phi,\eta')}{\partial\eta'} = 0 \quad \Rightarrow \quad \frac{\eta'}{f_{\eta'}} \simeq -\frac{\phi}{f} \quad (\because \ \Lambda_{QCD}^4 \gg y_u\langle h\rangle\Lambda_{QCD}^3)$$

$$\Rightarrow \qquad V_{eff}(\phi) \simeq -y_u\Lambda_{QCD}^3\,\underset{\sim}{h}\,\cos\frac{\phi}{f}$$

More precisely, including d, s quarks (whose masses are below Λ_{QCD})

$$V_{eff}(\phi) \simeq -\frac{f_a^2 m_a^2}{m_u+m_d}\frac{h}{v}\sqrt{m_u^2+m_d^2+2m_u m_d\cos\frac{\phi}{f}}$$

$$\times\left[1 + \mathcal{O}\left(\frac{m_{u,d}}{m_s}, \frac{m_{u,d}}{\Lambda_{QCD}}\right)\right]$$

Footnote - Chiral anomaly -

Chiral transformation $\begin{cases} u_L = e^{i\alpha}\tilde{u}_L \\ u_R^c = e^{i\alpha}\tilde{u}_R^c \end{cases}$ $\left(\Leftrightarrow u = e^{i\alpha\gamma_5}\tilde{u}\ ;\ u = \begin{pmatrix} u_L \\ u_R^{c\dagger}\end{pmatrix}\right)$

$$Z = \int Du_L Du_R^c\, e^{iS[u_L,\, u_R^c]}$$

$$= \int D\tilde{u}_L D\tilde{u}_R^c\, e^{iS[e^{i\alpha}\tilde{u}_L,\, e^{i\alpha}\tilde{u}_R^c] + \underbrace{i\int d^4x \frac{g_s^2}{32\pi^2}(2\alpha)G\tilde{G}}_{\text{Originated from } D\tilde{u}_L D\tilde{u}_R^c \neq Du_L Du_R^c}}$$

The chiral transf. $\begin{pmatrix} u_L \\ u_R^c \end{pmatrix} \to e^{i\alpha} \begin{pmatrix} u_L \\ u_R^c \end{pmatrix}$ corresponds to $\frac{\eta'}{f_{\eta'}} \to \frac{\eta'}{f_{\eta'}} + 2\alpha$

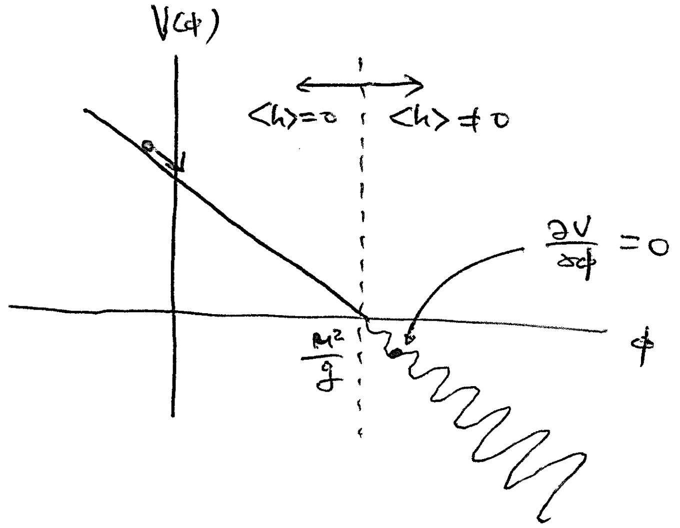
because $\langle u_L u_R^c \rangle \simeq \Lambda_{QCD}^3 \, e^{i\eta'/f_{\eta'}}$.

Since the phase of the QCD instanton potential has to be shifted by 2α due to the chiral transf., $\frac{\eta'}{f_{\eta'}}$ must be there.

* The QCD relaxion model : summary

$$\mathcal{L} \supset \underbrace{(-M^2 + g\phi)}_{-\mu^2(\phi)} |h|^2 + c_1 g M^2 \phi + \underbrace{\frac{g_s^2}{32\pi^2} \frac{\phi}{f} G\tilde{G}}_{\big\downarrow}$$

$$\underbrace{y_u \Lambda_{QCD}^3}_{\Lambda_{br}^4(h)} \, h \cos\frac{\phi}{f}$$



$$\frac{\partial V}{\partial \phi} \simeq -c_1 g M^2 + \frac{\Lambda_{br}^4(h)}{f} \sin\frac{\phi}{f} = 0$$

$$\Rightarrow \quad g \sim \frac{\Lambda_{br}^4(h)}{f M^2}$$

If the free parameter g is given by

$$g \sim \frac{y_u \Lambda_{QCD}^3 v}{f M^2} \ll v$$

The QCD relaxion can explain the weak scale hierarchy by the naturally small parameter g.

* Requirements for a Consistent cosmological scenario

cf) "K. Choi, S. H. Im, 1610.00680"

(I) Strong friction for dissipation of relaxion K.E.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \qquad \text{Eq. of motion of } \phi$$

$$H = \frac{\dot{a}}{a} \qquad \text{Hubble expansion parameter}$$

By the Hubble friction, $\dot{\phi} \to -\frac{V'}{3H} \sim \frac{gM^2}{H}$

To stop the relaxion, $\dot{\phi} < \Lambda_{br}^2 \to \frac{gM^2}{H} < \Lambda_{br}^2$

$$\to \boxed{\frac{\Lambda_{br}^2}{f} \sim m_\phi < H}$$

$$\left(gM^2 \sim \frac{\Lambda_{br}^4}{f} \text{ by } \frac{\partial V}{\partial \phi} = 0 \right)$$

(II) Long enough period of strong friction

A simple realization may be a long lasting inflation.

$$\underset{\substack{\text{number of}\\ \text{e-folds}}}{N_I} \times \frac{1}{H_I} \gtrsim \frac{\Delta\phi}{\dot{\phi}} \sim \frac{M^2/g}{gM^2/H_I} \sim \frac{H_I}{g^2}$$

$$\Rightarrow \quad N_I \gtrsim \frac{H_I^2}{g^2} \sim \frac{f^2 M^4 H_I^2}{\Lambda_{br}^8} \underset{\uparrow (I)}{>} \left(\frac{M}{\Lambda_{br}} \right)^4 \sim 10^{12} \quad \text{for } M \sim \text{TeV}, \ \Lambda_{br} \sim \Lambda_{QCD}$$

(III) The relaxion dynamics doesn't affect the inflation.

$$H_I > \frac{\sqrt{\Delta\rho_\phi}}{M_{pl}} \sim \frac{\sqrt{gM^2\Delta\phi}}{M_{pl}} \sim \frac{M^2}{M_{pl}}$$

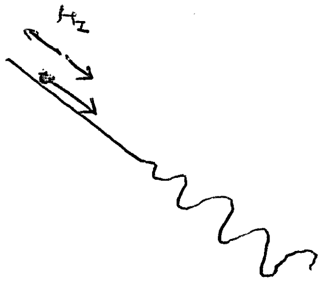
$$\Rightarrow \boxed{M \left(\sim \frac{\Lambda}{16\pi^2} \right) < \sqrt{H_I M_{pl}}}$$

; Upper bound for the UV cutoff Λ that the simple relaxion model allows.

(IV) The relaxion dynamics is to be dominated by classical motion.

$$(\Delta\phi)_{\text{Hubble}} \sim H_I < \frac{\dot{\phi}}{H_I} \sim \frac{V'}{H_I^2}$$

$\uparrow$
quantum fluctuation
over a Hubble time during inflation



$$\Rightarrow \quad H_I < (V')^{1/3} \sim (gM^2)^{1/3} \sim \Lambda_{br}\left(\frac{\Lambda_{br}}{f}\right)^{1/3}$$

(V) V_{br} has to be turned on during inflation. (Weaker than (IV))
for $f > \Lambda_{br}$

$$H_I < \Lambda_{br} \quad (\sim \Lambda_{QCD} \text{ for the QCD relaxion })$$

$$T_{\text{de Sitter}} \sim H_I \qquad \Lambda_{br}^4 \propto e^{-\frac{8\pi^2}{g^2(T)}}$$

(III) & (IV) $\rightarrow$

$$M < \sqrt{\Lambda_{br} M_{pl}}\left(\frac{\Lambda_{br}}{f}\right)^{1/6} \left[\sim 10^7 \text{ GeV} \left(\frac{10^9 \text{GeV}}{f}\right)^{1/6} \text{ for } \Lambda_{br} \sim \Lambda_{QCD}\right]$$

$$\lesssim 10^{10} \text{ GeV} \left(\frac{v}{f}\right)^{1/6}$$

$\uparrow$
$\boxed{\Lambda_{br} \lesssim v}$ to be discussed

The simple relaxion model cannot extend the SM all the way up to the Planck scale.

The simple QCD relaxion model is actually ruled out by "the strong CP problem".

$$\frac{y_u \Lambda_{QCD}^3 \overset{\swarrow \langle h\rangle}{v}}{f} \sin\frac{\phi}{f} \simeq g M^2$$ at the point where the relaxion is stabilized.

Since $\langle h\rangle$ is very finely scanned by ϕ,

the above condition is first satisfied at the point where $\sin\frac{\phi}{f} \simeq 1$.

$$\left(\Delta\langle h\rangle^2 \sim \Delta\mu^2 \sim g\Delta\phi \sim gf \sim \frac{y_u \Lambda_{QCD}^3 v}{M^2} \Rightarrow \Delta\langle h\rangle \sim y_u \frac{\Lambda_{QCD}^3}{M^2} \ll v \right)$$

$$\frac{g_s^2}{32\pi^2}\frac{\langle\phi\rangle}{f} G\tilde{G} = \frac{g_s^2}{32\pi^2}\underbrace{\theta_{eff}}_{\sim O(1)} G\tilde{G} \rightarrow \text{CP violation}$$

$$d_n \approx \frac{e m_\pi}{8\pi^2}\theta_{eff}$$

Nucleon EDM

By the non-observation of the neutron EDM,

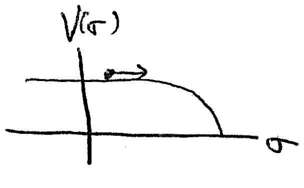
$$d_n \lesssim 10^{-26}\ e\cdot cm \quad (90\%\ \text{C.L.})$$

$$\Rightarrow \quad \theta_{eff} \lesssim 10^{-10}$$

Sol. 1) $V'(\phi)$ becomes smaller after inflation.

(ex) relaxion-inflaton coupling

$\mathcal{L} \supset k\,\sigma^2\phi^2$ $\quad$ σ: inflaton



$$\Rightarrow \text{Explicit breaking of} \begin{cases} \phi \to \phi + c \\ \sigma \to \sigma + c' \end{cases}$$

If the shift-symmetries are dominantly broken by k, k can be small.

Assuming almost constant $\langle\sigma\rangle > M_{pl}$ during inflation,

$$k\sigma^2\phi^2 = \tilde{g}^2\phi^2 \qquad \tilde{g}^2 \equiv k\sigma^2$$

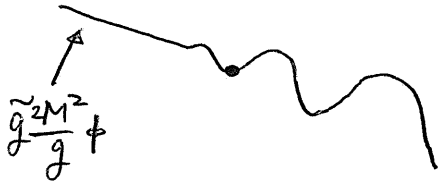
$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + gM^2\phi + \underbrace{\tilde{g}^2\phi^2}_{\tilde{g}^2 \frac{M^2}{g}\phi \text{ around } \phi_c = \frac{M^2}{g}} + \frac{g_s^2}{32\pi^2}\frac{\phi}{f}G\tilde{G}$$

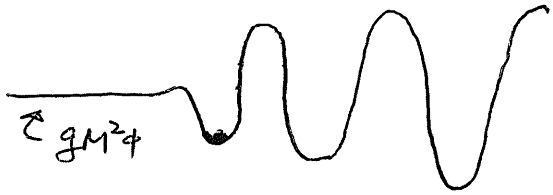
Suppose that $\tilde{g} \gg g$ during inflation and

$\tilde{g} \ll g$ after inflation.

During inflation, $\tilde{g}^2\frac{M^2}{g} \sim \frac{\Lambda_{br}^4}{f}\sin\frac{\phi}{f} \Rightarrow \langle\frac{\phi}{f}\rangle \sim \mathcal{O}(1)$

After inflation, $gM^2 \sim \frac{\Lambda_{br}^4}{f}\sin\frac{\phi}{f} \Rightarrow \langle\frac{\phi}{f}\rangle \sim \frac{g^2}{\tilde{g}^2} \lesssim 10^{-10}$





$$\theta_{QCD} \sim \frac{g^2}{\tilde{g}^2} \lesssim 10^{-10}$$

Consequently $\mathcal{N}_I \gtrsim \theta_{QCD}\frac{H_I^2}{g^2}$, $H_I \gtrsim \frac{M^2}{M_{pl}\sqrt{\theta_{QCD}}}$

cosmological requirement (VI) (III)

$$\Rightarrow \quad M \lesssim \sqrt{M_{pl}H_I}\,\theta_{QCD}^{1/4} < 30\,\text{TeV} \times \left(\frac{10^9\,\text{GeV}}{f}\right)^{1/6}\left(\frac{\theta_{QCD}}{10^{-10}}\right)^{1/4}$$

$H_I < \Lambda_{QCD}\left(\frac{\Lambda_{QCD}}{f}\right)^{1/3}$

(Requirement IV)

Sol. 2) Non-QCD relaxion

$$\frac{g_{hid}^2}{32\pi^2}\frac{\phi}{f}G_{hid}\tilde{G}_{hid} \Rightarrow -T^4 e^{-\frac{8\pi^2}{g_{hid}^2(T)}}\cos\frac{\phi}{f}$$

In order to get a higgs-dependent potential,

introduce a hidden-color charged $^\vee$Dirac fermion whose mass is proportional to the higgs VEV.

$$\frac{1}{\Lambda_N}|h|^2\left(\underset{\uparrow}{N}N^c + h.c.\right)$$

SM-neutral, vector-like under the hidden gauge group ex) $SU(N)_{hid}$

$$\underset{\square}{N} \quad \underset{\bar{\square}}{N^c}$$

At low energies below the hidden confinement scale,

$$\langle NN^c\rangle = \Lambda_{hid}^3 e^{i\eta'_N/f_N} \qquad 4\pi f_N \sim \Lambda_{hid}$$

$$V(\phi,\eta') = -\frac{1}{\Lambda_N}|h|^2\Lambda_{hid}^3\cos\frac{\eta'_N}{f_N} - \Lambda_{hid}^4\cos\left(\frac{\phi}{f}+\underbrace{\frac{\eta'_N}{f_N}}_{\text{Chiral anomaly}}\right)$$

$$\underset{\overset{\uparrow}{\frac{\partial V}{\partial \eta'_N}=0}}{\simeq} \underbrace{-\frac{1}{\Lambda_N}|h|^2\Lambda_{hid}^3}_{\Lambda_{br}^4(h)\,\propto\,|h|^2}\cos\frac{\phi}{f}$$

Non-QCD relaxion model

$$\frac{g_{hid}^2}{32\pi^2}\frac{\phi}{f}G_{hid}\tilde{G}_{hid} + \underbrace{\frac{1}{\Lambda_N}|h|^2 N N^c}_{\text{Non-renormalizable}}$$

$\swarrow$ UV completion ?

$$\mathcal{L} \supset \underset{\substack{\uparrow \\ \text{hidden-colored } SU(2)_L\text{-doublet}}}{m_L L}L^c + y h L N^c + \tilde{y} h^+ L^c N + \text{h.c.}$$

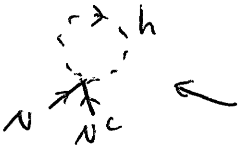
If $m_L > \Lambda_{hid}$, $L+L^c$ can be integrated out at low energies below Λ_{hid}.

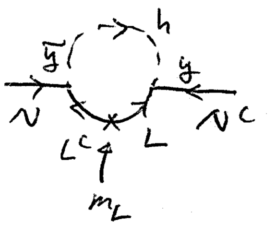
$$\frac{\partial\mathcal{L}}{\partial L} = m_L L^c + y h N^c = 0$$

$$L^c = -\frac{y h N^c}{m_L} \qquad L = -\frac{\tilde{y} h^+ N}{m_L}$$

$$\mathcal{L}_{eff} \supset -\frac{y\tilde{y}}{m_L}|h|^2 N N^c + \text{h.c.} \qquad \Lambda_N = \frac{\boxed{m_L}}{y\tilde{y}}$$

Quantum correction $\quad \dfrac{y\tilde{y}}{m_L}|h|^2 N N^c \longrightarrow \dfrac{y\tilde{y}}{16\pi^2} m_L \ln\dfrac{\Lambda^2}{m_L^2} N N^c$





So $$V_{br} = -\frac{y\tilde{y}}{m_L}\Lambda_{hid}^3\left(|h|^2 + \underset{\sim\sim\sim\sim}{\frac{m_L^2}{16\pi^2}\ln\frac{\Lambda^2}{m_L^2}}\right)\cos\frac{\phi}{f}$$

For the higgs VEV backreaction to work,

$$\frac{m_L^2}{16\pi^2}\ln\frac{\Lambda^2}{m_L^2} < v^2 \Rightarrow m_L\sqrt{\ln\frac{\Lambda^2}{m_L^2}} < 4\pi v \sim 1\ \text{TeV}$$

On the other hand, recall $m_L > \Lambda_{hid}$, and

$$m_N = \frac{y\tilde{y}}{m_L} v^2 < \Lambda_{hid}$$

So

$$\boxed{m_N = \frac{y\tilde{y}}{m_L} v^2 < \Lambda_{hid} < \underset{\sim\sim\sim}{m_L} < \frac{4\pi v}{\sqrt{\ln \Lambda^2/m_L^2}}}$$

It predicts a light doublet Dirac fermion below TeV.

Also it shows

$$\Lambda_{br}^4 \sim \frac{y\tilde{y}}{m_L} \Lambda_{hid}^3 v^2 \lesssim \mathcal{O}(v^4)$$

$$\Rightarrow \quad M < \sqrt{\Lambda_{br} M_{Pl}} \left(\frac{\Lambda_{br}}{f}\right)^{1/6} \lesssim 10^{10}\,\text{GeV} \times \left(\frac{v}{f}\right)^{1/6}$$

The simple relaxion models require UV completion above the intermediate scale $\sim 10^{10}$ GeV.

* A brief overview of the "Clockwork"

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + gM^2\phi + \Lambda_{br}^4(h)\cos\frac{\phi}{f}$$

$\underline{\phi \to \phi + c}$ is broken only by "g" & non-perturbative effects.

↙

UV origin of such a good symmetry?

If ϕ is an axion-like ptl, global $U(1)$

$$\Phi = \frac{1}{\sqrt{2}}(f_\phi + \rho)e^{i\phi/f_\phi} \quad \text{where} \quad \langle\Phi\rangle = \frac{1}{\sqrt{2}}f_\phi$$

$$U(1)_{PQ} : \quad \Phi \to \Phi e^{i\alpha} \quad \Leftrightarrow \quad \phi \to \phi + c$$

BUT then ϕ is a periodic field with a symmetry:

$$\frac{\phi}{f_\phi} \to \frac{\phi}{f_\phi} + 2\pi n \quad (n \in \mathbb{Z})$$ which cannot be broken

So $$g\phi|h|^2 \longrightarrow \left(M_1^2 \cos\left(\frac{\phi}{F}+\delta\right)\right)|h|^2 \sim \frac{M_1^2}{F}\phi|h|^2$$ long periodic potential

$$\Rightarrow \mathcal{L} \supset \left(-M^2 + M^2\cos\left(\frac{\phi}{F}+\delta\right)\right)|h|^2 + M^4\cos\left(\frac{\phi}{F}+\delta\right) + \Lambda_{br}^4(h)\cos\frac{\phi}{f}$$

$$\frac{\partial V}{\partial \phi} = 0 \quad \Rightarrow \quad \frac{F}{f} \sim \left(\frac{M}{\Lambda_{br}}\right)^4 \gtrsim 10^4$$

$\left(\begin{array}{l} M \gtrsim \text{TeV} \\ \Lambda_{br} \lesssim v \end{array}\right.$

If $$\frac{\phi}{F} \cong \frac{\phi}{F} + 2\pi$$

$$\cos\frac{\phi}{f} = \cos n\frac{\phi}{F} \quad \text{where} \quad n \in \mathbb{Z} \qquad \frac{F}{f} = n \gtrsim 10^4$$

How to obtain such a large integer number from a sensible theory?

Multiple axion mixing

$$V = -\Lambda_1^4 \cos\left(\frac{\phi_1}{f} + n_1\frac{\phi_2}{f}\right) - \Lambda_2^4 \cos\left(\frac{\phi_2}{f} + n_2\frac{\phi_3}{f}\right)$$

$$- \cdots - \Lambda_{N-1}^4 \cos\left(\frac{\phi_{N-1}}{f} + n_N\frac{\phi_N}{f}\right)$$

N axions & $N-1$ potentials

$\Rightarrow$ 1 massless axion ϕ

$$\phi = \sum_{i=1}^{N} c_i \phi_i \Leftrightarrow \phi_i = c_i \phi + \text{heavy axions}$$

$$\left.\begin{array}{l} c_1 + n_1 c_2 = 0 \\ c_2 + n_2 c_3 = 0 \\ \vdots \\ c_{N-1} + n_N c_N = 0 \end{array}\right\} \quad \phi \propto \phi_1 - \frac{1}{n_1}\phi_2 + \frac{1}{n_1 n_2}\phi_3 - \cdots (-1)^N \frac{1}{\underbrace{n_1 n_2 \cdots n_N}_{n^N \text{ if } n_i = n \ \forall i}} \phi_N$$

$$\Delta V = -M_1^4 \cos\frac{\phi_N}{f} - M_2^4 \cos\frac{\phi_1}{f}$$

$$\simeq -M_1^4 \cos\frac{\phi}{\boxed{n^N} f} - M_2^4 \cos\frac{\phi}{f}$$

After integrating out heavy modes

a large integer number from N-axion mixing

$$\Rightarrow \frac{F}{f} \sim n^N \text{ for } N \text{ axions}$$

"Clockwork mechanism"

K. Choi, H. Kim, S. Yun 1404.6209

K. Choi, S.H. Im 1511.00132

D.E. Kaplan, R. Rattazzi 1511.01827