Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics

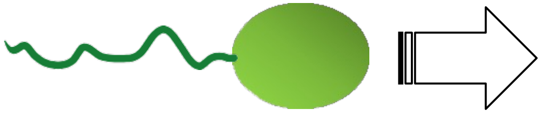
- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences

- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production

Active Colloids / Micro-Organisms

Self propulsion via local drive mechanism
steady-state entropy production

Examples: **Bacteria**



Autophoretic colloids

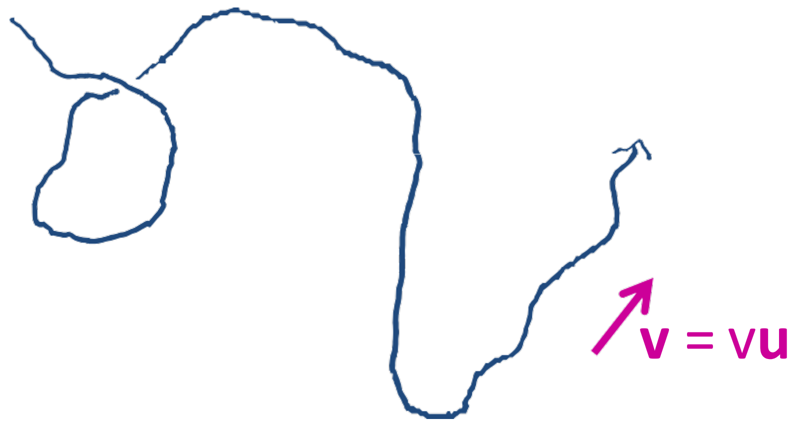
Pt-coated Janus particles bathed in fuel (H_2O_2)
JR Howse et al, PRL 99 048102 (2007)

Janus particles in binary solvent + laser heating
I Buttinoni et al, PRL 110 238301 (2013)

Each particle surfs a gradient of its own production
Cross-interactions:
R Golestanian PRL 108 038303 (2012) et seq

Simplest Model of Active Motion: ABPs

single particle



persistent Brownian motion

speed $v = v_0$, rotational diffusivity D_r

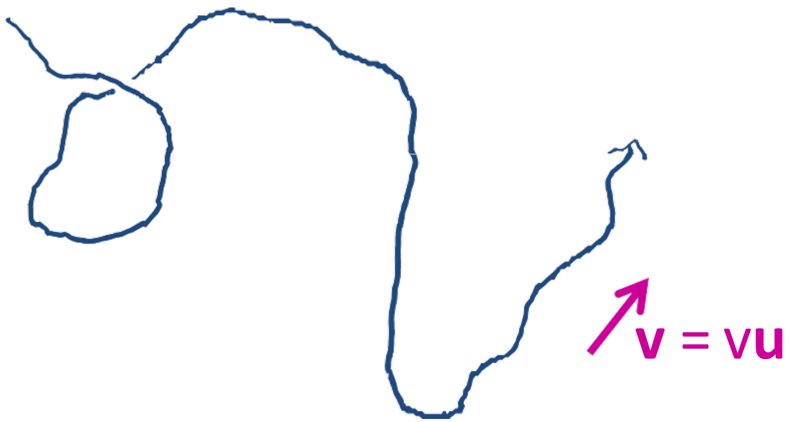
orientation **u**

Alternatives:

- Run-and-Tumble Particles (**u** reassigns with rate α)
- Active Ornstein-Uhlenbeck Particles
- various lattice models

Simplest Model of Active Motion: ABPs

single particle



persistent Brownian motion

speed $v = v_0$, rotational diffusivity D_r

orientation **u**

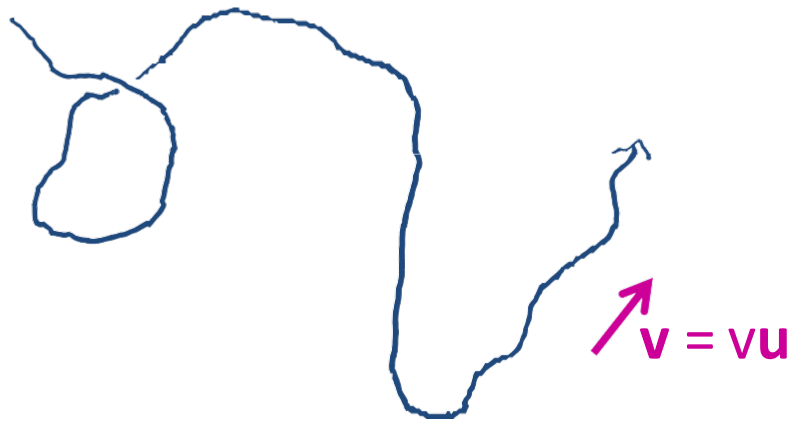
persistence length $\gamma = v_0 \tau$

rotational relaxation time: $\tau = ((d-1)D_r)^{-1}$

defined via $\langle \mathbf{u}(t).\mathbf{u}(t') \rangle = \exp[-|t-t'|/\tau]$

Simplest Model of Active Motion: ABPs

single particle



persistent Brownian motion

speed v = v_0, rotational diffusivity D_r

orientation **u**

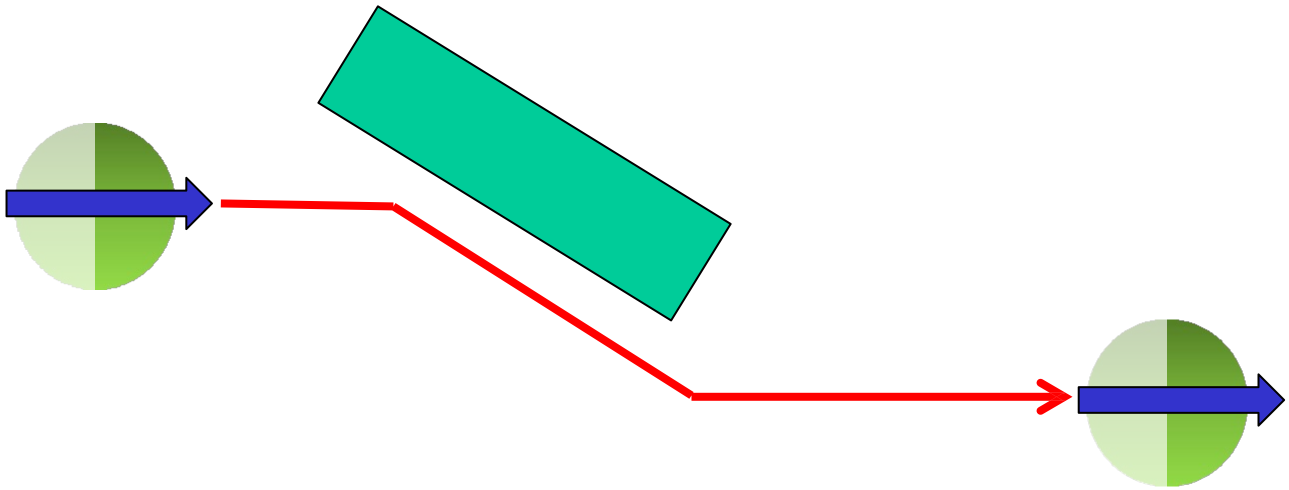
Coarse Grain to large scales ⇒ Random walk

$$D = \frac{v^2 \tau}{d}$$

≅ simple Brownian motion with TRS **if** v , τ constant

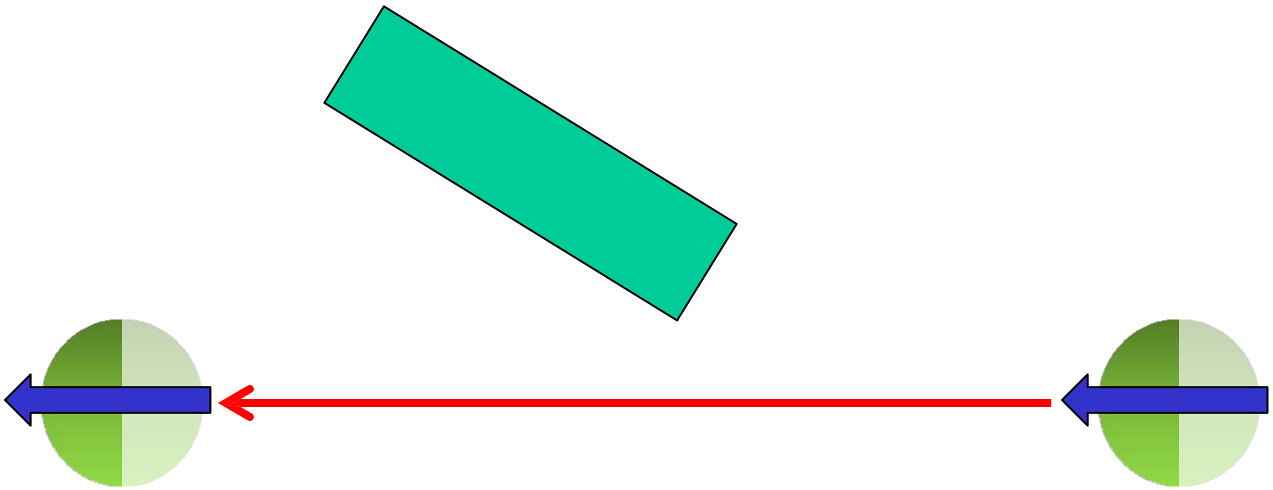
TRS-Breaking Made Manifest

1. Interaction with obstacles

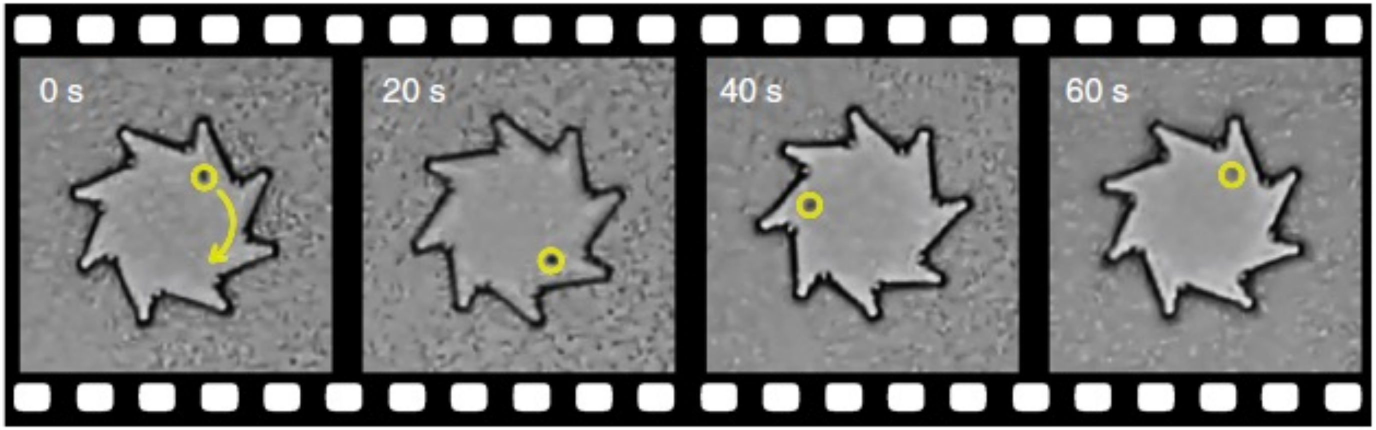


TRS-Breaking Made Manifest

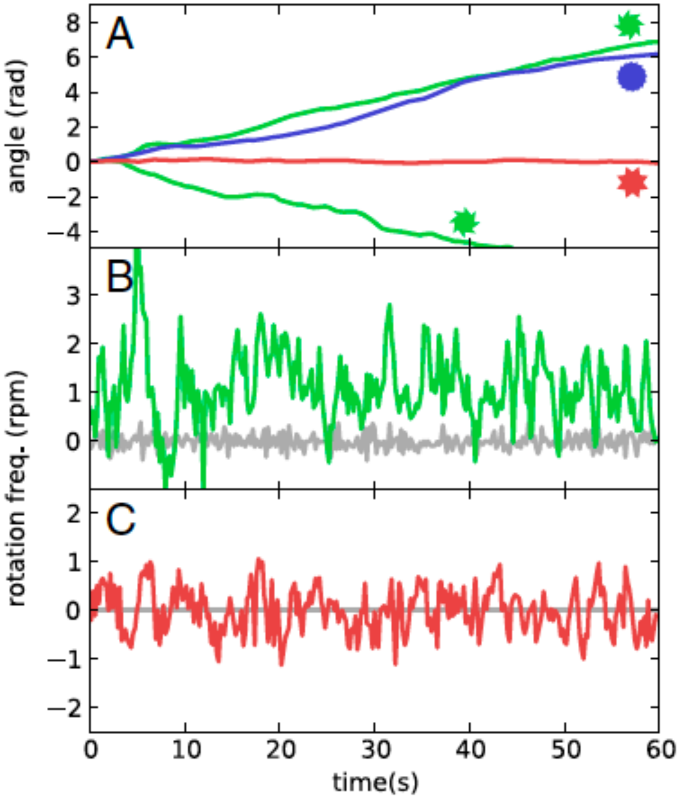
1. Interaction with obstacles



Manifestations of TRS Breaking



Passive obstacle:
rotor in bacterial bath

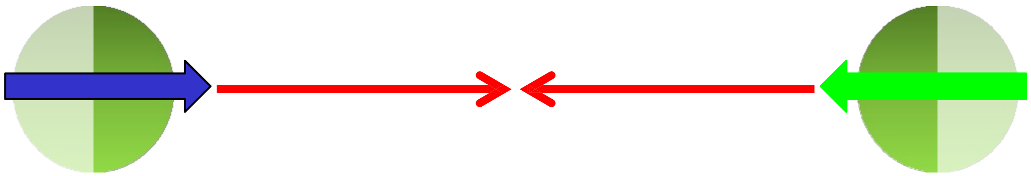


R. di Leonardo et al, PNAS 2009

TRS-Breaking Made Manifest

1. Interaction with obstacles

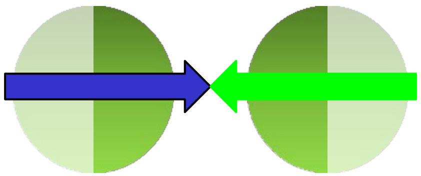
1. Interactions among particles



TRS-Breaking Made Manifest

1. Interaction with obstacles

1. Interactions among particles

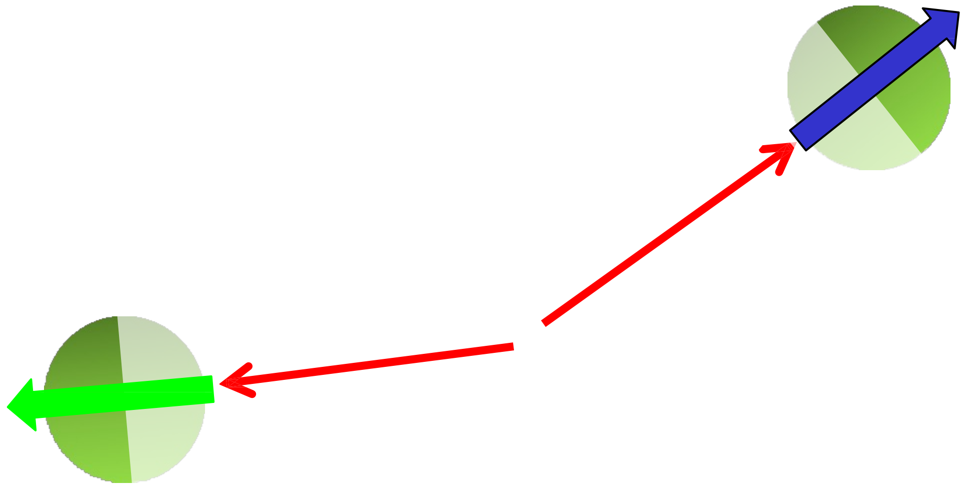


long pause
to rotate

TRS-Breaking Made Manifest

1. Interaction with obstacles

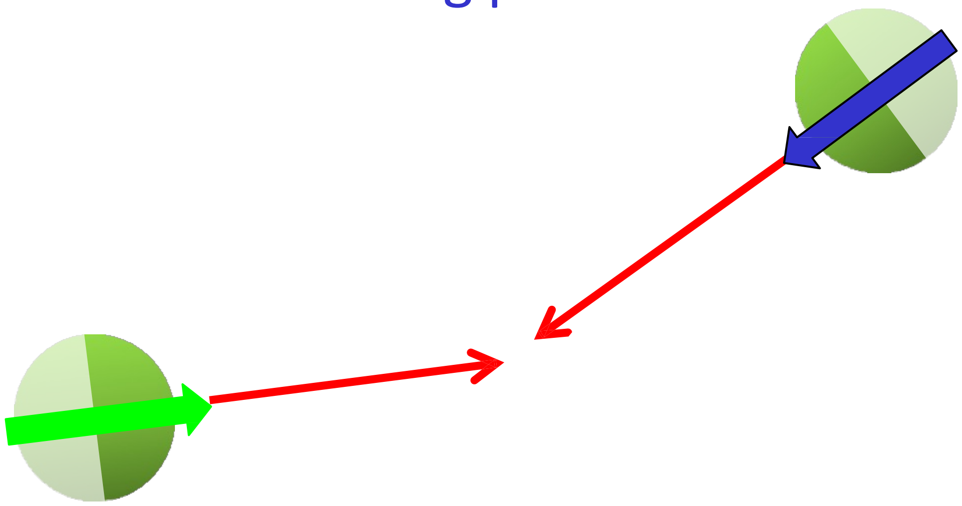
1. Interactions among particles



TRS-Breaking Made Manifest

1. Interaction with obstacles

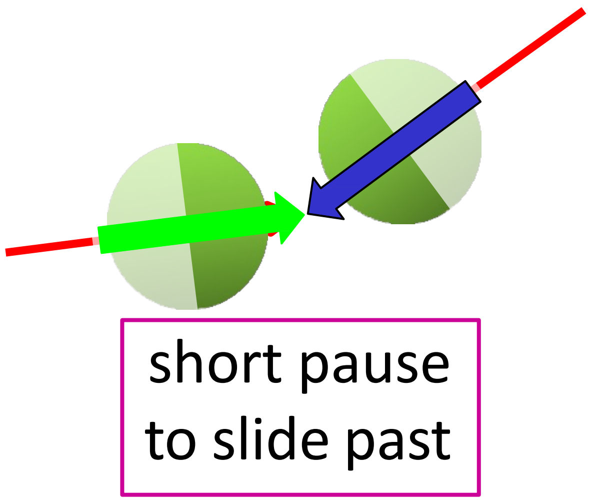
1. Interactions among particles



TRS-Breaking Made Manifest

1. Interaction with obstacles

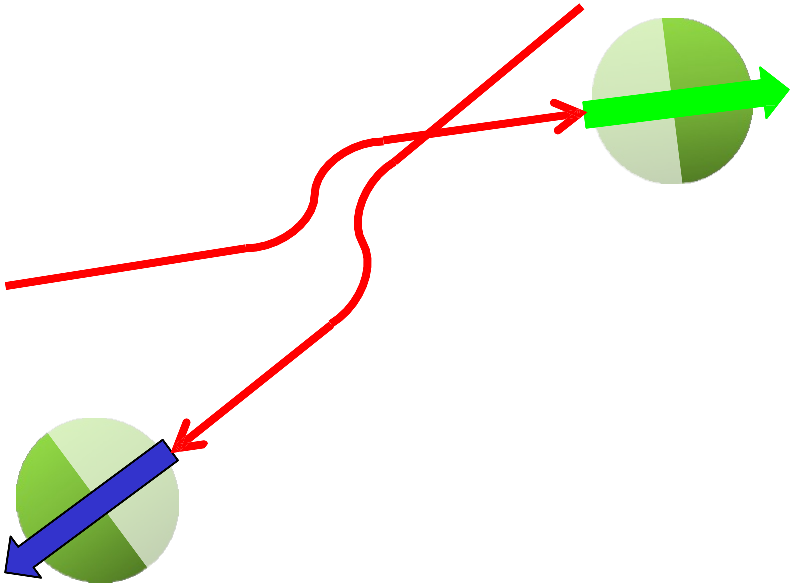
1. Interactions among particles



TRS-Breaking Made Manifest

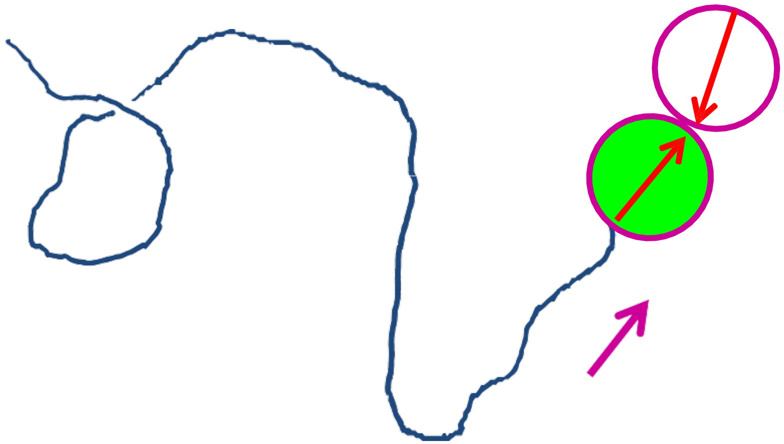
1. Interaction with obstacles

1. Interactions among particles



Simplest Model of Active Motion: ABPs

many particles



$\langle \mathbf{v} \rangle = v\mathbf{u}$

persistent Brownian motion

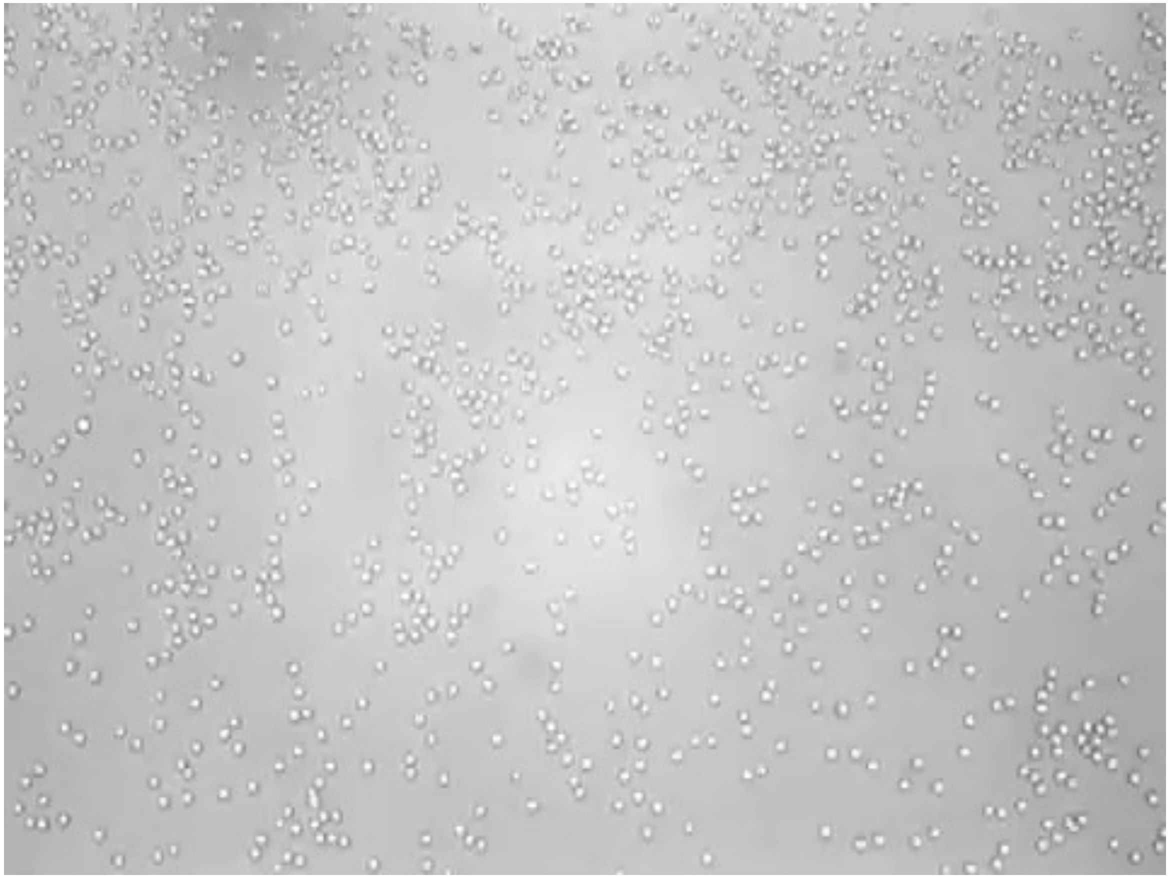
speed $v \neq v_0$, rotational diffusivity D_r orientation $\mathbf{u}$

Main ABP physics: Collisional slowing down

$\approx$ density-dependent swim speed $v[\rho] < v_0$ = 'Quorum Sensing'

Y Fily and M C Marchetti PRL 108 235702 (2012) MEC and J Tailleur EPL 101 20010 (2013)

Manifestations of TRS Breaking

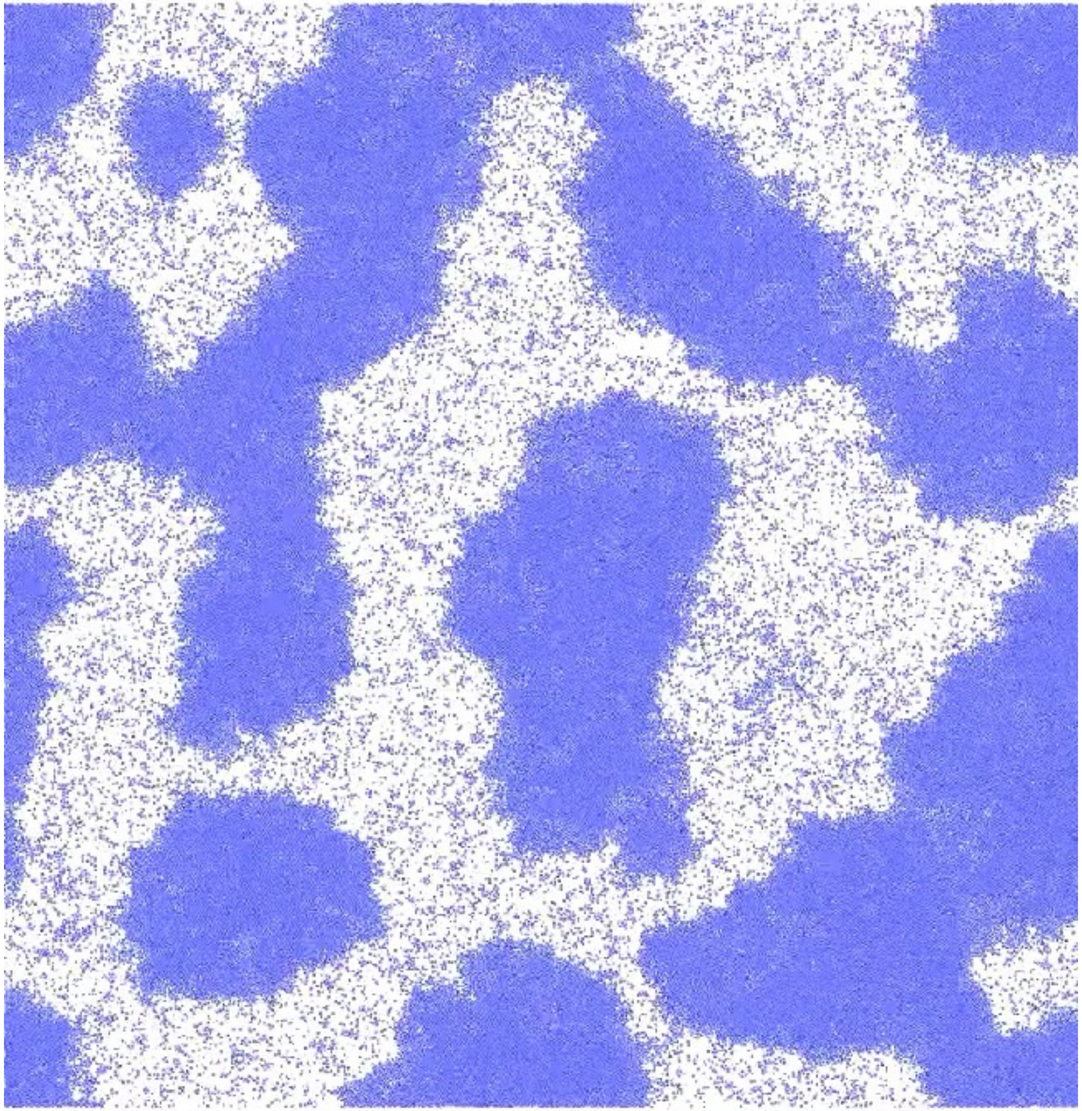


Motility-Induced Phase Separation (MIPS)

Janus particles in peroxide, light activated catalysis

J Palacci et al, Science 2013

Motility-Induced Phase Separation (MIPS)



Motility-Induced Phase Separation (MIPS)

purely repulsive ABPs

MEC + J Tailleur, Ann Rev CMP 2015

movie: J Stenhammar

MIPS: Phase Separation without TRS

Particles accumulate where they move slowly

Move slowly where they are dense: +ve feedback

Confirmed by simulations of repulsive ABPs

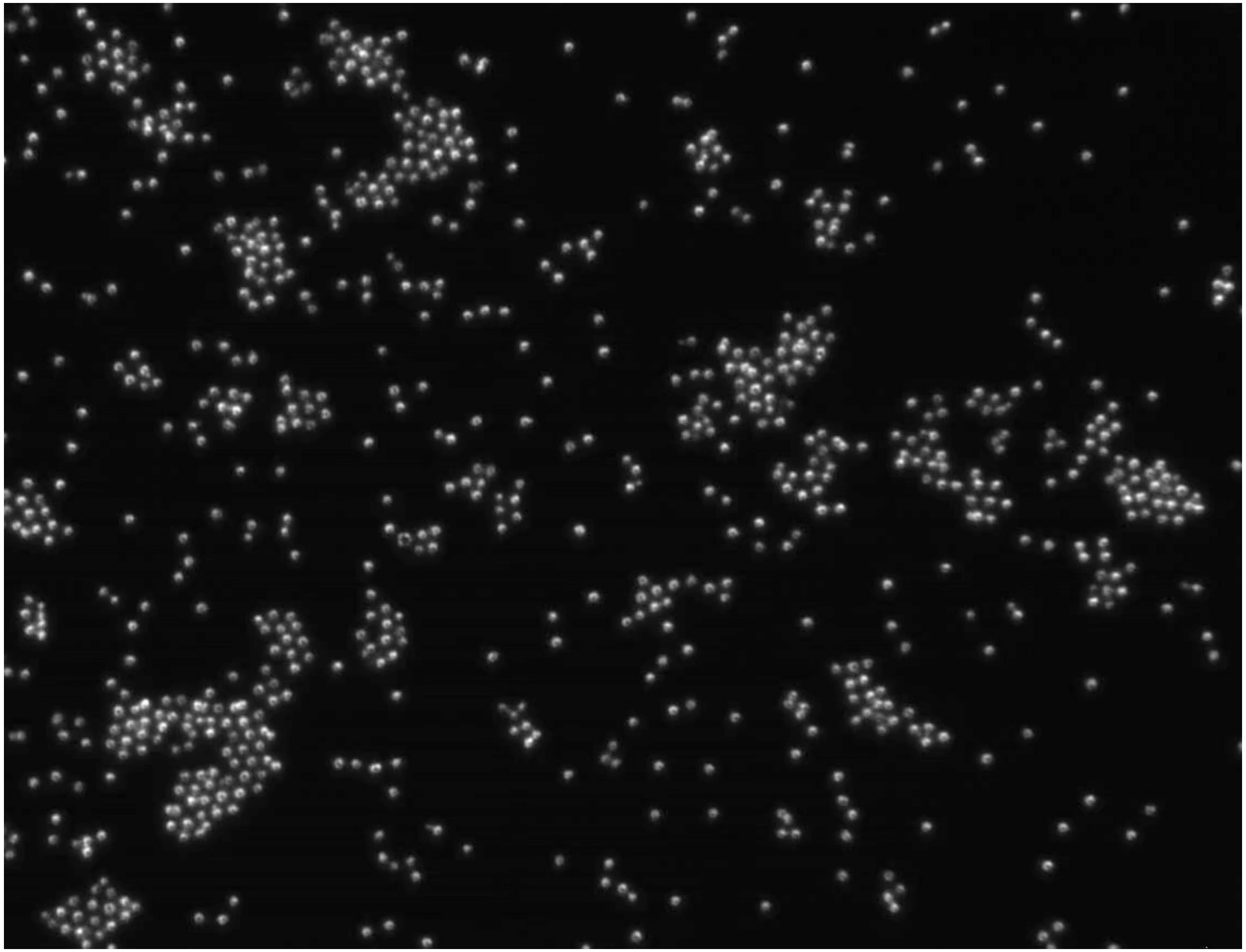
Experiments:

MIPS seen, but complicated by phoretic, hydrodynamic... interactions

Also seen: microphase separation = **cluster phases** in steady state

MEC + J Tailleur PRL 2008, EPL 2013, Y Fily et al PRL 2012
J Stenhammar et al PRL 2014, I Theurkauff et al PRL 2012
I Buttinoni et al, PRL 2013, J Palacci et al, Science 2013
J Schwarz-Linek et al, PNAS 2012

Cluster Phases



Movie: F Ginot et al, Nat Comms 2018

Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics

- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences

- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production

Field Theory of Phase Separation

$$\dot{\phi} = -\nabla.\mathbf{J}$$

$$\mathbf{J} = -\nabla\mu + \sqrt{2D}\boldsymbol{\Lambda}$$

$$\mu = a\phi + b\phi^3 - \kappa\nabla^2\phi$$

Λ = unit white noise
$D = k_B T M$
M = 1 mobility

MODEL B

Model B has Time Reversal Symmetry:
Forward and backward movies are statistically identical once steady state is achieved

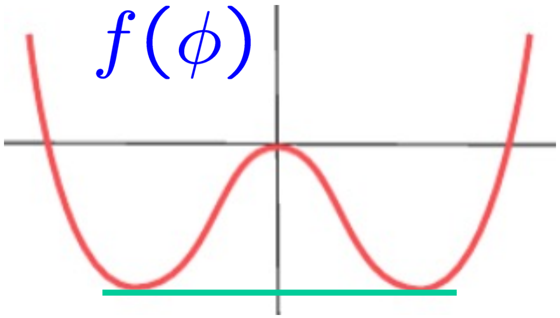
Field Theory of Phase Separation

$$\dot{\phi} = -\nabla.\mathbf{J}$$
$$\mathbf{J} = -\nabla\mu + \sqrt{2D}\boldsymbol{\Lambda}$$
$$\mu = a\phi + b\phi^3 - \kappa\nabla^2\phi$$

Λ = unit white noise
$D = k_B T M$
$M = 1$ mobility

MODEL B

$$\mu = \delta\mathcal{F}/\delta\phi$$
$$\mathcal{F} = \int dV[f(\phi) + \frac{\kappa}{2}(\nabla\phi)^2]$$
$$f(\phi) = \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4$$



phase equilibria:
common tangent

$$\mu_1 = \mu_2$$
$$P_1 = P_2 \qquad \text{where } P = \mu\phi - f$$

Active Field Theory of Phase Separation

$$\dot{\phi} = -\nabla.\mathbf{J}$$
$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda}$$
$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; +\lambda(\nabla\phi)^2$$

ACTIVE MODEL B

$\lambda(\nabla\phi)^2 \neq \delta\mathcal{F}/\delta\phi$ for any $\mathcal{F}$

= **minimal** violation of TRS

This form of TRS violation can be derived microscopically...
e.g. via $v = v(\rho_{local})$

E Tjhung et al PRX 2018

R Wittkowski et al, Nat Comms 2014

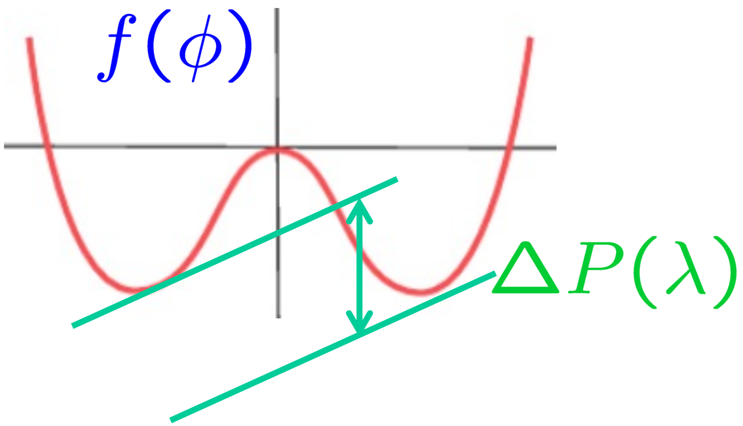
Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics
- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences
- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production

Active Field Theory of Phase Separation

MIPS: uncommon tangent construction

$$\widehat{\mu}_1 = \widehat{\mu}_2$$
$$\widehat{\mu}_1\phi_1 - f_1 \neq \widehat{\mu}_2\phi_2 - f_2$$



ACTIVE MODEL B

Calculation Method:

(a) 1D interface

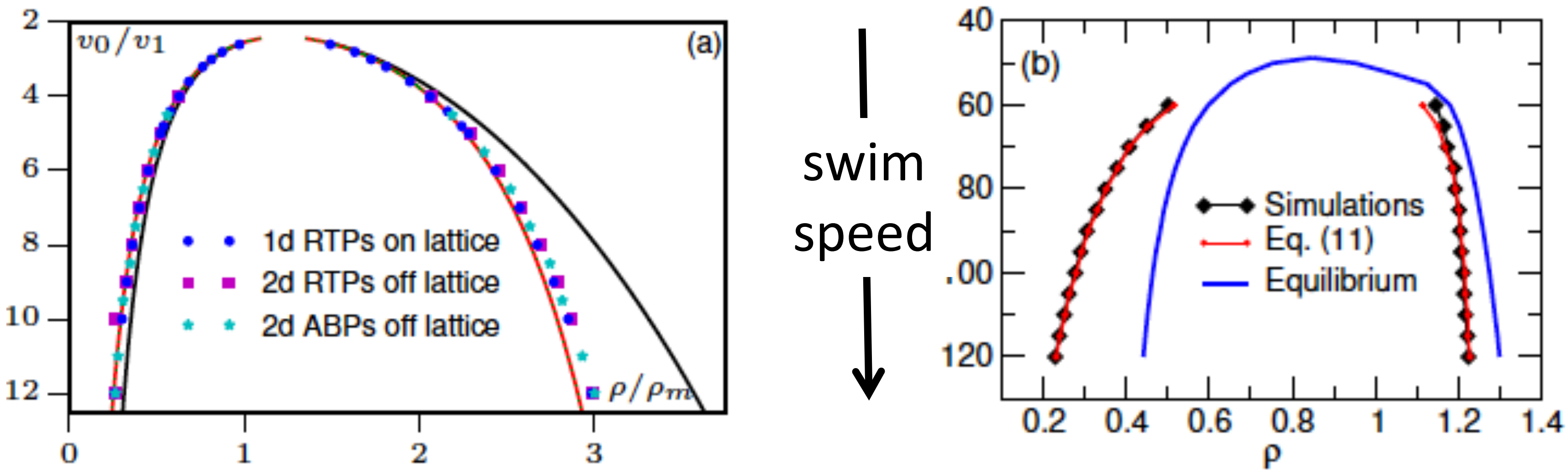
(b) J = 0 requires $\partial_x \widehat{\mu} = 0 \Rightarrow \widehat{\mu}_1 = \widehat{\mu}_2$

(c) seek J = 0 solutions with $\partial_x \phi = 0$ at $x = \pm\infty$

R Wittkowski et al, Nat Comms 2014

A Solon et al, PRE 2018

Active Field Theory of Phase Separation



Anomalous Phase Coexistence: seen for particle-based models

v(ρ) model: density-dependent swim speed

ABPs with hard-core collisions: similar outcomes, different analysis

A Solon et al, PRE 2018

Active Model B

$$\dot{\phi} = -\nabla.\mathbf{J}$$

$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\boldsymbol{\Lambda}$$

$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; +\lambda(\nabla\phi)^2$$

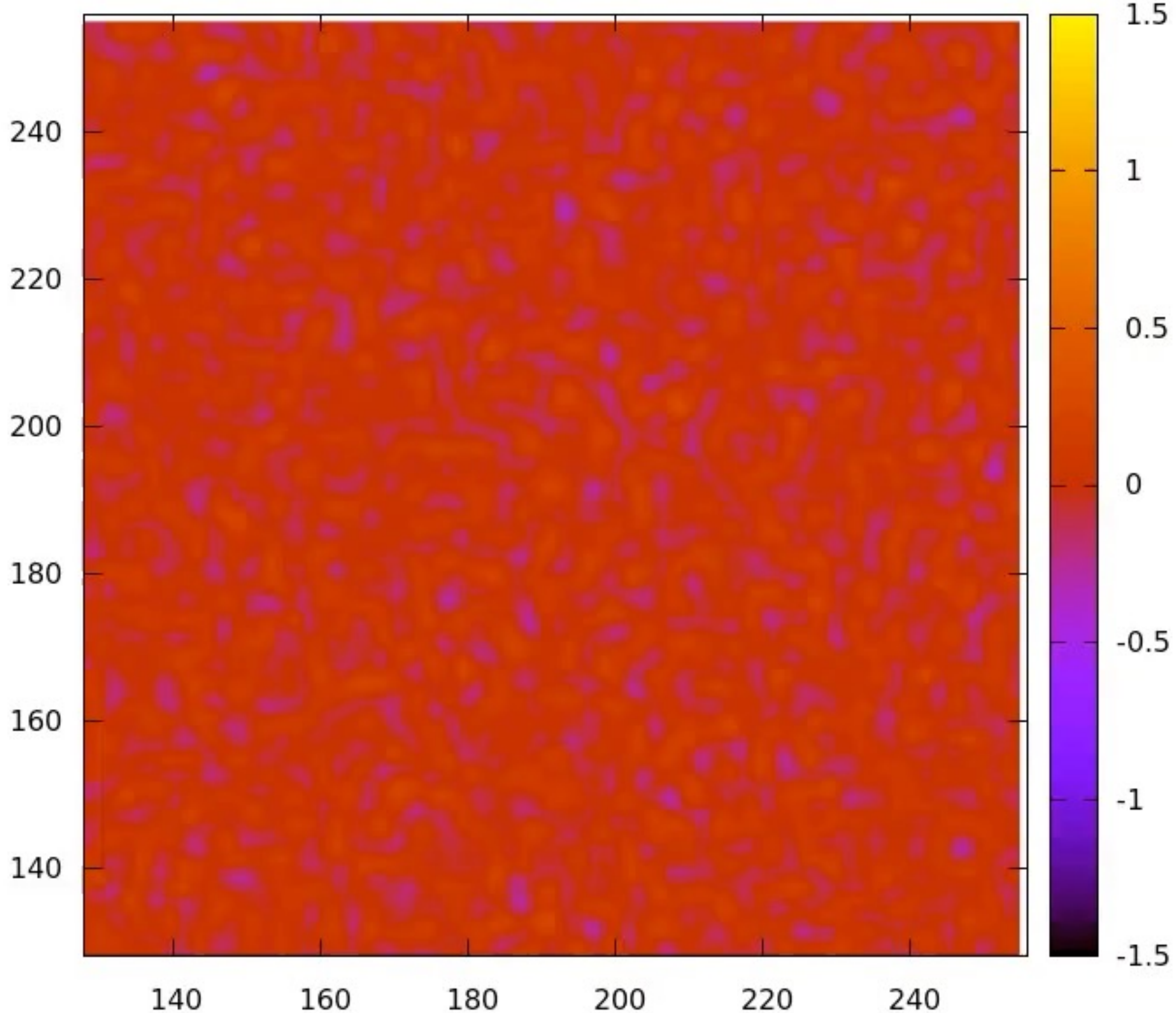
Anomalous Phase Coexistence ✓

Otherwise similar to passive phase separation

- Spinodal, Nucleation+ Growth regimes
- Late stage coarsening law $L(t) \propto t^{1/3}$

Phase Separation Kinetics: Active Model B

Spinodal decomposition



Active Model B

$$\dot{\phi} = -\nabla.\mathbf{J}$$
$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda}$$
$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; +\lambda(\nabla\phi)^2$$

Anomalous Phase Coexistence ✓

Otherwise similar to passive phase separation

- Spinodal, Nucleation+ Growth regimes
- Late stage coarsening law $L(t) \propto t^{1/3}$

Absent features: bubble / cluster phases
TRS violation in stationary state

Active Model B

$$\dot{\phi} = -\nabla.\mathbf{J}$$
$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda}$$
$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; +\lambda(\nabla\phi)^2$$

Anomalous Phase Coexistence ✓

What's missing?

Otherwise similar to passive phase separation

- Spinodal, Nucleation+ Growth regimes
- Late stage coarsening law $L(t) \propto t^{1/3}$

Absent features: bubble / cluster phases
TRS violation in stationary state

Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics
- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences
- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production

Active Model B+

$$\dot{\phi} = -\nabla.\mathbf{J}$$
$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda} + \zeta[(\nabla^2\phi)\nabla\phi - \tfrac{1}{2}\nabla(\nabla\phi)^2]$$
$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \ \ + \lambda(\nabla\phi)^2$$

Expansion of active currents to order ∇^3, ϕ^2

ζ and λ terms same order, separately break TRS

ζ term vanishes for flat interface: no effect on bulk coexistence

What does the ζ term do?

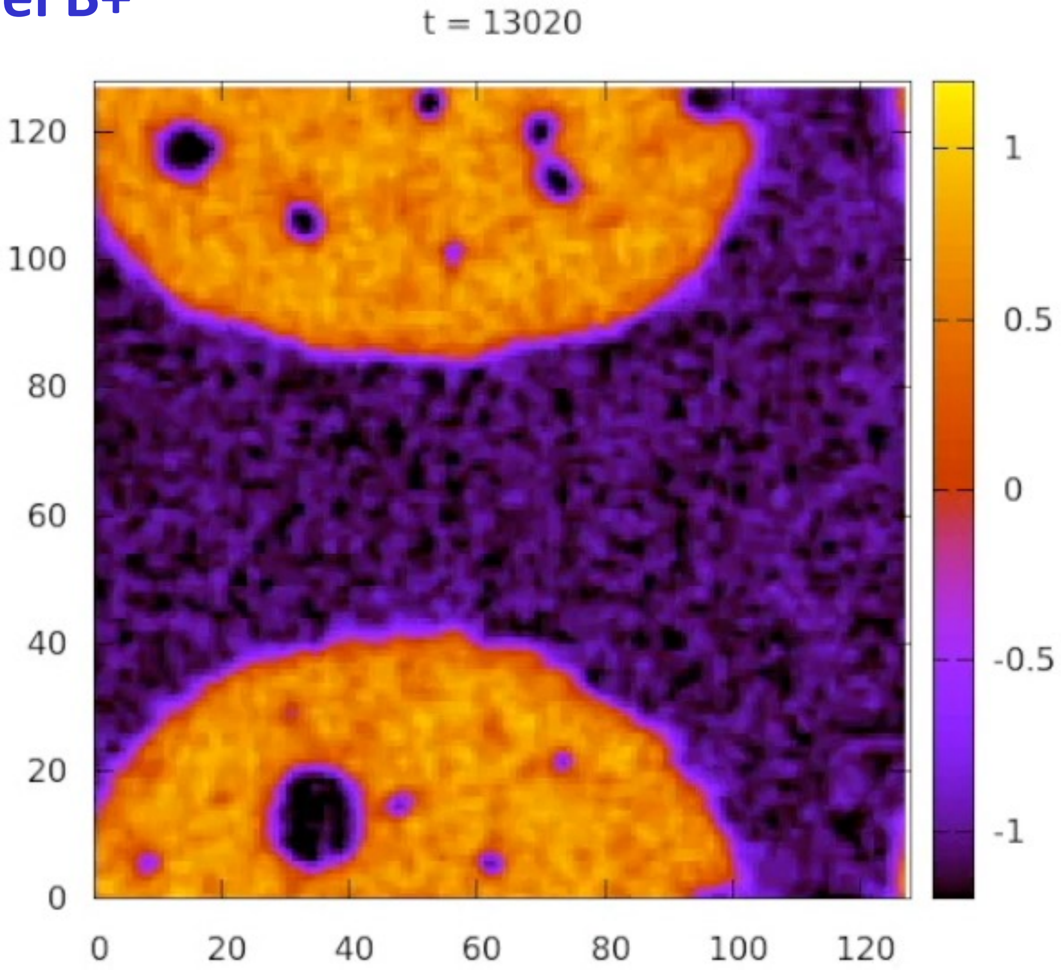
C. Nardini et al, PRX 2017

E. Tjhung et al, PRX 2018

Bubbles and Clusters

Active Model B+

$\zeta > 0$



E. Tjhung et al, PRX 2018

Active Model B+

$$\dot{\phi} = -\nabla.\mathbf{J}$$

$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda} + \zeta[(\nabla^2\phi)\nabla\phi - \frac{1}{2}\nabla(\nabla\phi)^2]$$

$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; + \lambda(\nabla\phi)^2$$

C. Nardini et al, PRX 2017

E. Tjhung et al, PRX 2018

Active Model B+

$$\dot{\phi} = -\nabla.\mathbf{J}$$

$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\mathbf{\Lambda} + \mathbf{J}_\zeta$$

$$\hat{\mu} = a\phi + b\phi^3 - \kappa\nabla^2\phi \;\; + \lambda(\nabla\phi)^2$$

Dealing with the ζ current:

$$\mathbf{J}_\zeta = -\nabla\mu_\zeta + \nabla \times \mathbf{H}$$

invisible for
ϕ dynamics

Leaves nonlocal chemical potential:

$$\mu_\zeta(\mathbf{r}) = -\int d\mathbf{r}' \frac{\nabla \cdot \mathbf{J}_\zeta}{4\pi|\mathbf{r} - \mathbf{r}'|}$$

Coulomb integral:
dipole density
$\propto$ curvature

E. Tjhung et al, PRX 2018

Active Model B+

$$\dot{\phi} = -\nabla.\mathbf{J}$$

$$\mathbf{J} = -\nabla\hat{\mu} + \sqrt{2D}\boldsymbol{\Lambda} + \mathbf{J}_\zeta$$

- ζ term changes the way chemical potential varies with interfacial curvature

- dramatic consequences for phase separation... whenever droplets involved

Deal

Leav

ϕ dynamics

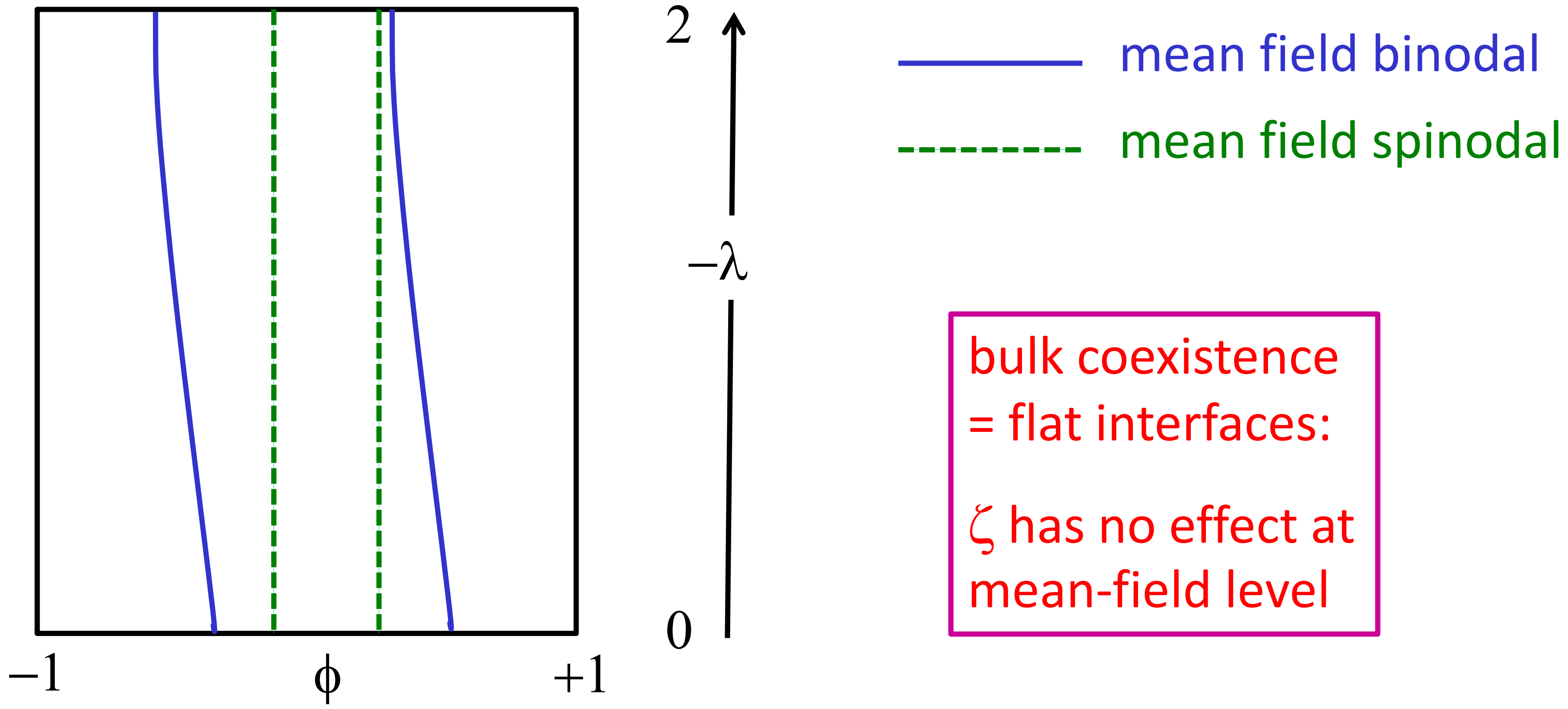
$$\mu_\zeta(\mathbf{r}) = -\int d\mathbf{r}' \frac{\nabla \cdot \mathbf{J}_\zeta}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Coulomb integral: dipole density $\propto$ curvature

E. Tjhung et al, PRX 2018

Bubbles and Clusters

Active Model B+ schematic phase diagram at fixed a,b,κ,D



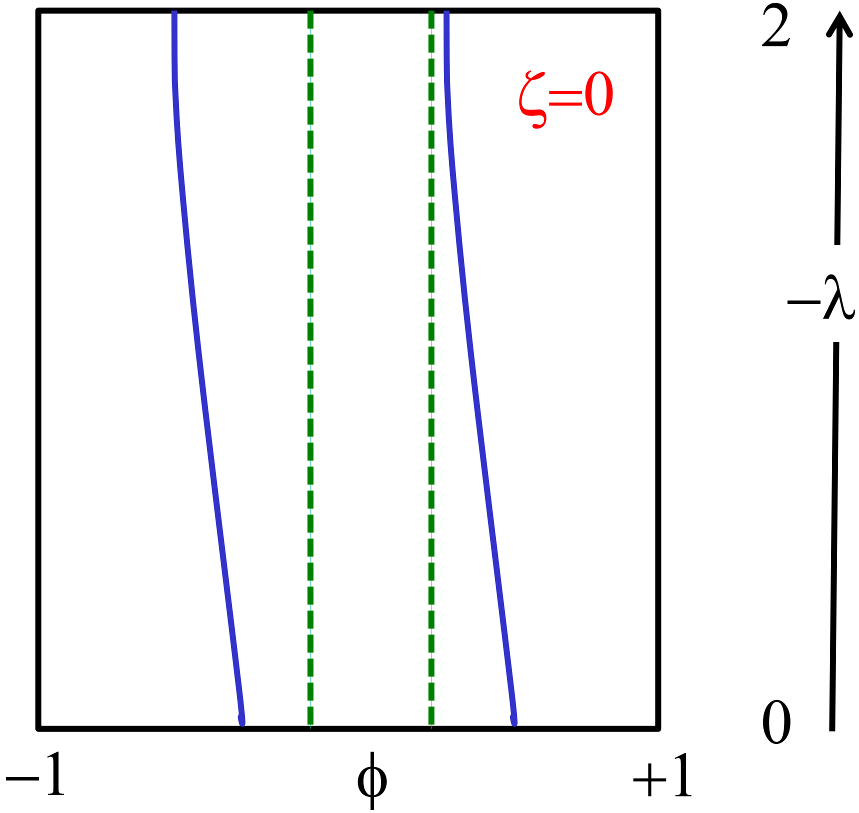
bulk coexistence
= flat interfaces:

ζ has no effect at
mean-field level

E. Tjhung et al., PRX 8, 031080 (2018)

Bubbles and Clusters

Active Model B+ schematic phase diagram at fixed a,b,κ,D



mean field binodal

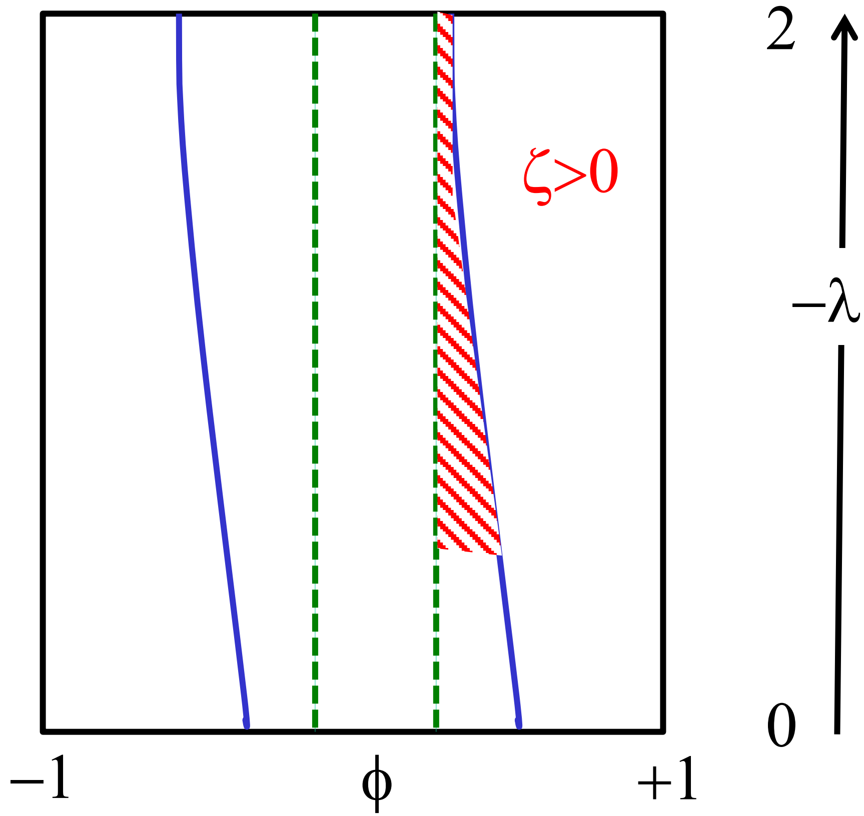
mean field spinodal

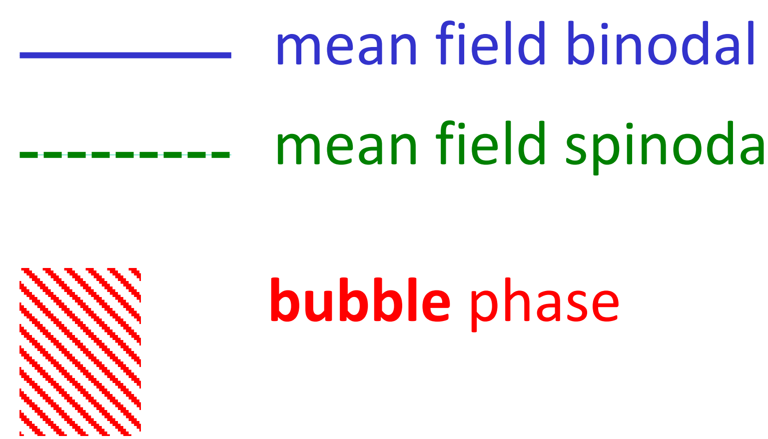
simulations for ζ = 0 confirm MF results

E. Tjhung et al., PRX 8, 031080 (2018)

Bubbles and Clusters

Active Model B+ schematic phase diagram at fixed a,b,κ,D



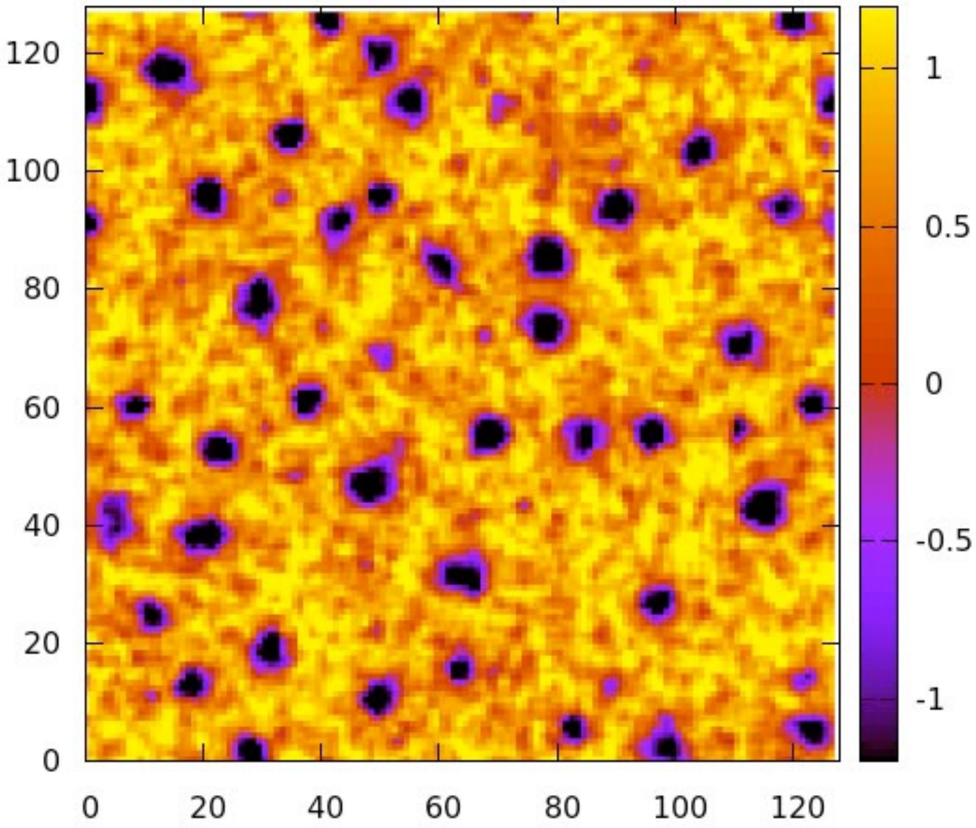


simulations for $\zeta > 0$
show non-MF effects

E. Tjhung et al., PRX 8, 031080 (2018)

Bubbles and Clusters

Active Model B+

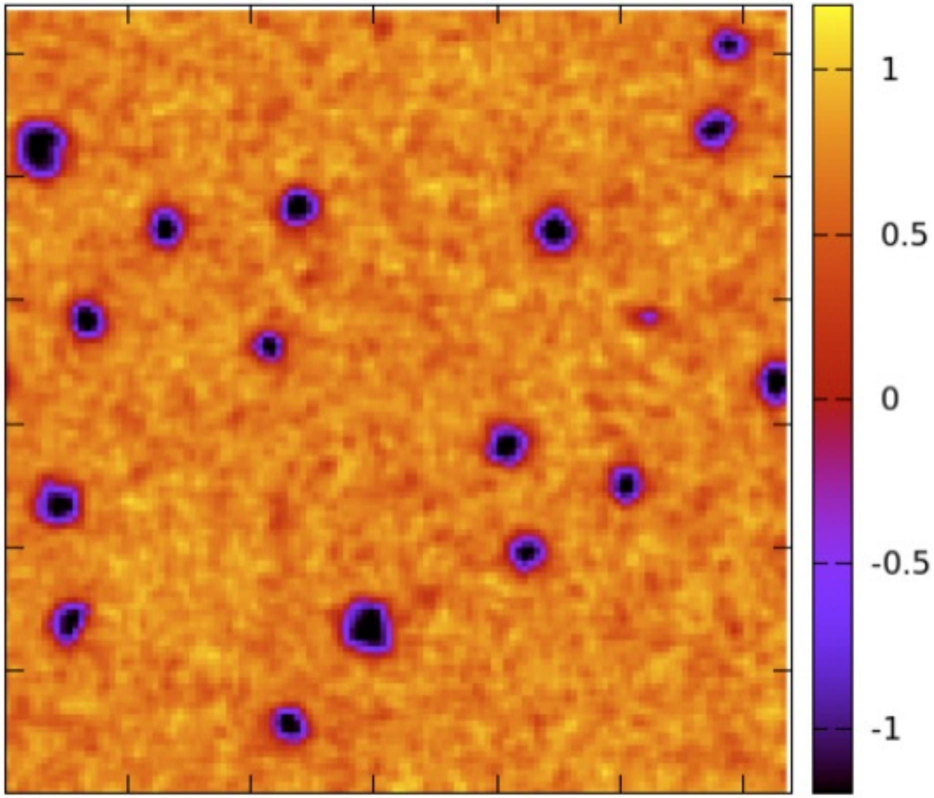


bubble phase

E. Tjhung et al, PRX 2018

Bubbles and Clusters

Active Model B+



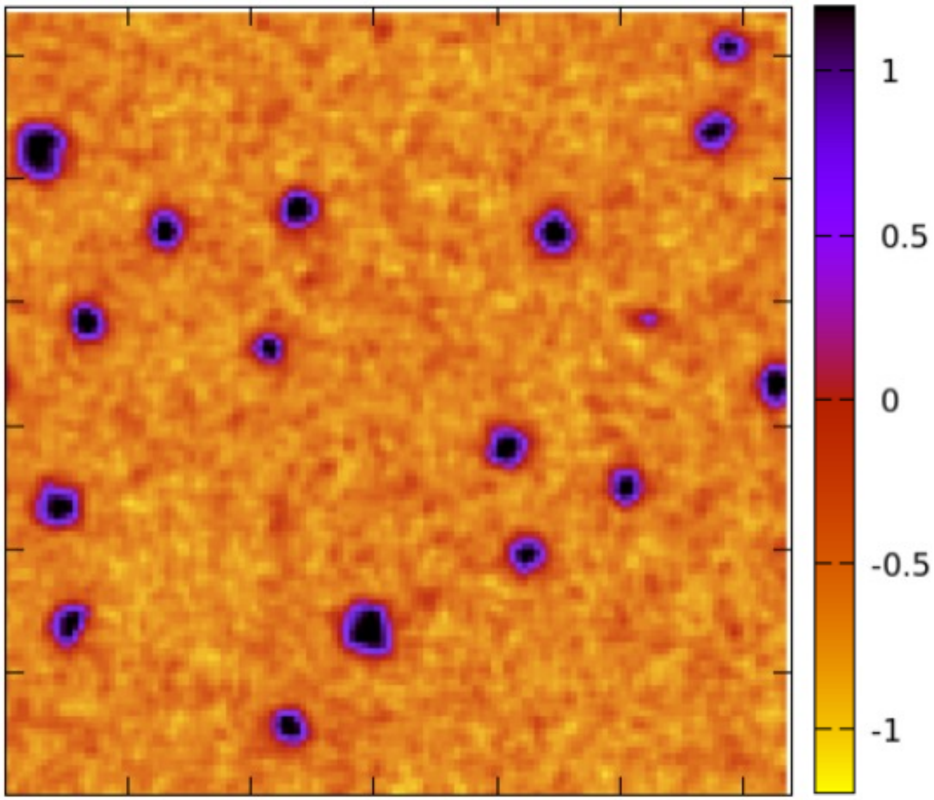
bubble phase

symmetry of model:
$(\zeta,\lambda,\phi) \rightarrow -(\zeta,\lambda,\phi)$
bubbles $\rightarrow$ clusters

E. Tjhung et al, PRX 2018

Bubbles and Clusters

Active Model B+



E. Tjhung et al, PRX 2018

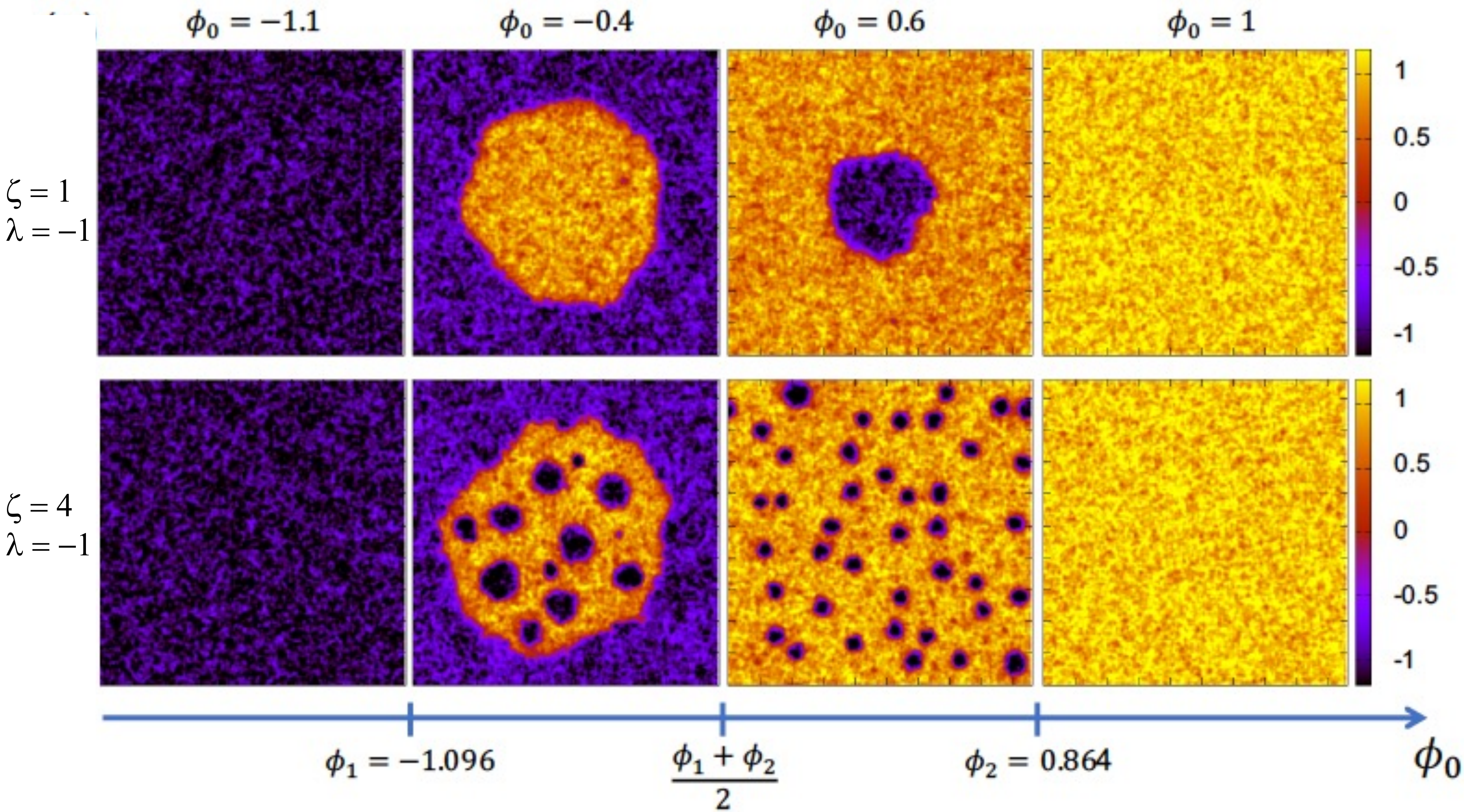
cluster phase

symmetry of model:
$(\zeta,\lambda,\phi) \to -(\zeta,\lambda,\phi)$
bubbles $\to$ clusters

ABPs: bubbles ($\zeta > 0$)

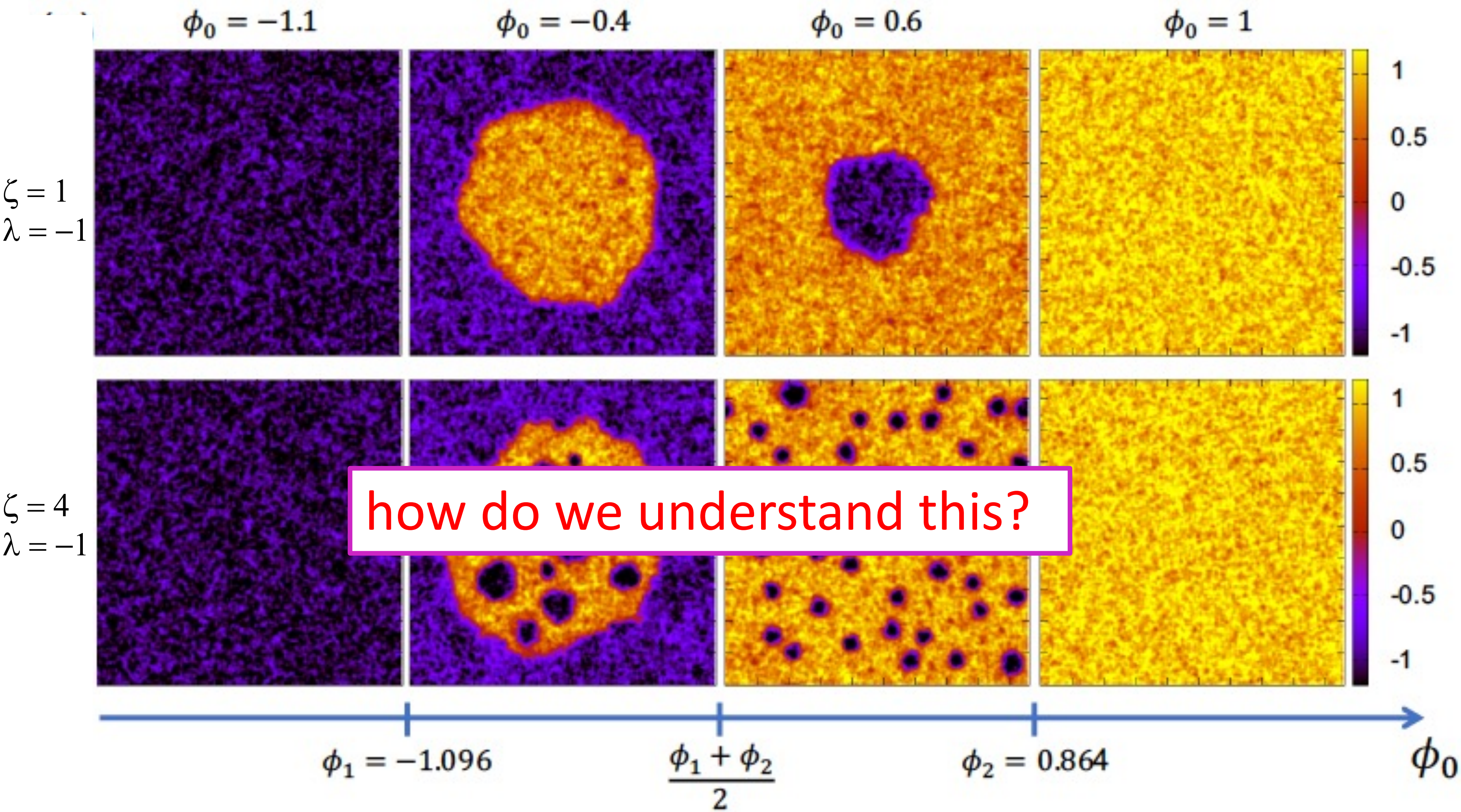
Colloid experiments:
clusters ($\zeta < 0$)

Phase Diagram: Microphase Separation



E. Tjhung et al, PRX 2018

Phase Diagram: Microphase Separation



E. Tjhung et al, PRX 2018

Ostwald Ripening: Passive Case

μ raised at curved interface:

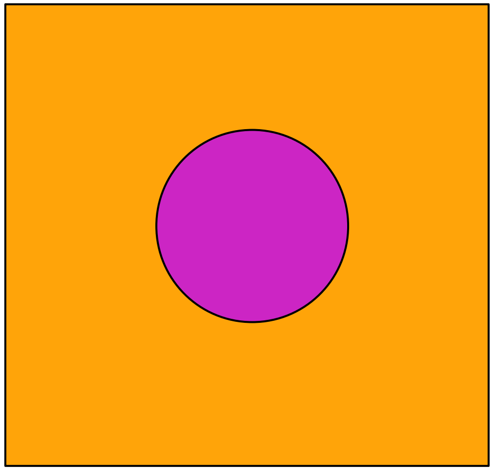
$$\mu_I(R) = \frac{\gamma(d-1)}{\phi_B R}$$

(γ = tension, ϕ_B = binodal, R = radius)

Current at surface:

$$J(R) = -\nabla\mu(R) = \frac{\mu_\infty - \mu_I(R)}{R^2}$$

Single droplet coexists with vapour at $\mu_\infty = \mu_I$ (R) in finite system



A. Bray, Adv. in Phys. 1994

Ostwald Ripening: Passive Case

μ raised at curved interface: $$\mu_I(R) = \frac{\gamma(d-1)}{\phi_B R}$$

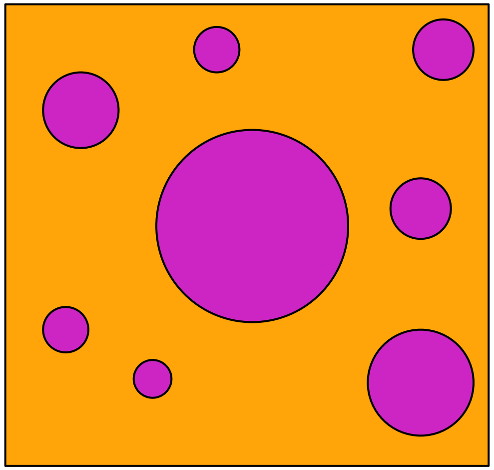
(γ = tension, ϕ_B = binodal, R = radius)

Current at surface: $$J(R) = -\nabla\mu(R) = \frac{\mu_\infty - \mu_I(R)}{R^2}$$

Single droplet coexists with vapour at $\mu_\infty = \mu_I$ (R) in finite system

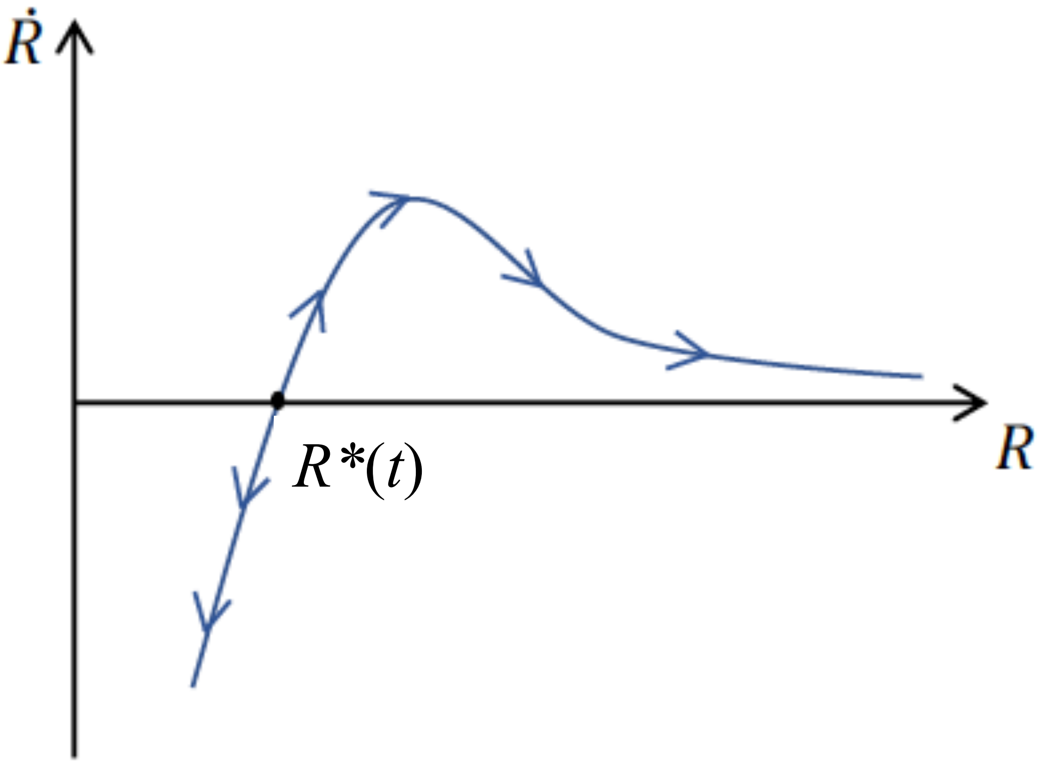
Many drops: μ_∞ set by mean size R*

$$\dot{R} = -\frac{J(R)}{2\phi_B} \propto \frac{\gamma}{R}\left[\frac{1}{R^*(t)} - \frac{1}{R}\right]$$



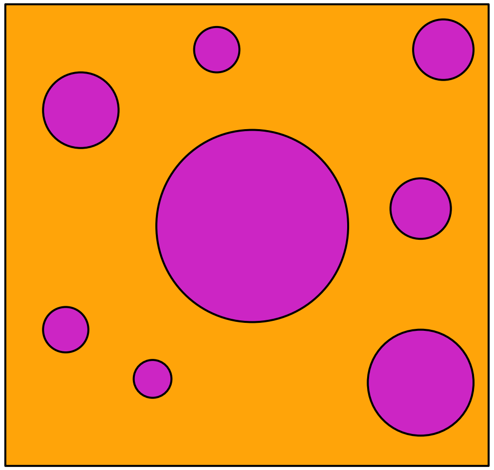
A. Bray, Adv. in Phys. 1994

Ostwald Ripening: Passive Case



- large droplets always grow at cost of small
- phase separation always completes
- noise is irrelevant (T=0 fixed point)

$$\dot{R} = -\frac{J(R)}{2\phi_B} \propto \frac{\gamma}{R}\left[\frac{1}{R^*(t)} - \frac{1}{R}\right]$$



A. Bray, Adv. in Phys. 1994

Ostwald Ripening: Active Effects

at curved interface $\mu_I(R) = \frac{\gamma(d-1)}{\phi_B R} + \mu_\zeta(R^+)$

Nonlocal ζ contribution:

$$\mu_\zeta(\mathbf{r}) = -\zeta \int d\mathbf{r}' \frac{\nabla \cdot [(\nabla^2\phi)\nabla\phi - \frac{1}{2}\nabla(\nabla\phi)^2]}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

Discontinuous across curved surface

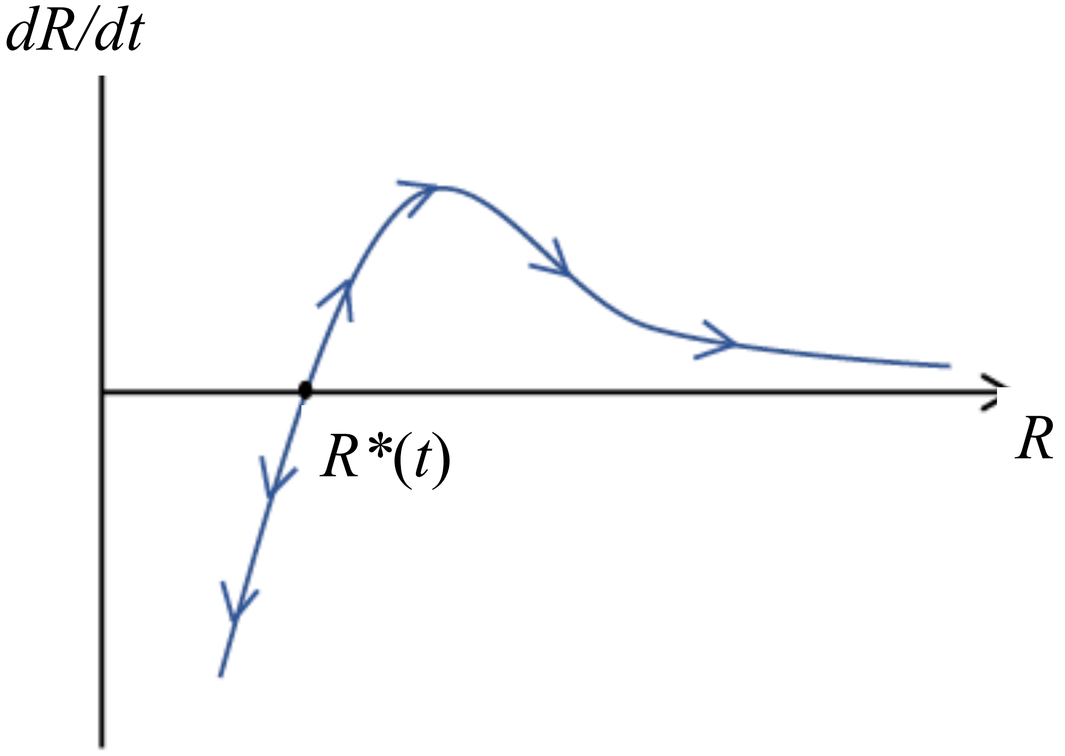
$$\mu_\zeta(R^+) \propto \frac{\zeta}{R}$$

Sign of effective tension set by ζ

Reverse Ostwald regime emerges at high activity

E. Tjhung et al, PRX 2018

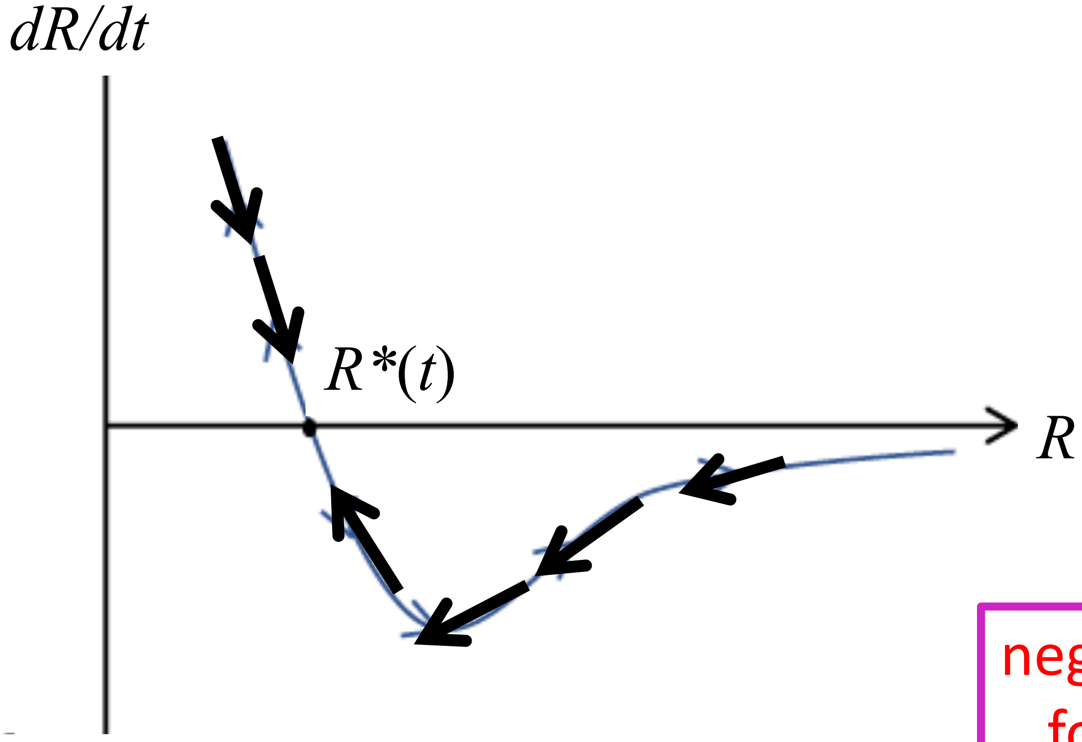
Ostwald Ripening: Active Effects



Normal Ostwald regime

A. Bray, Adv. in Phys. 1994

Ostwald Ripening: Active Effects



Reverse Ostwald regime

negative interfacial tension
...for Ostwald purposes only

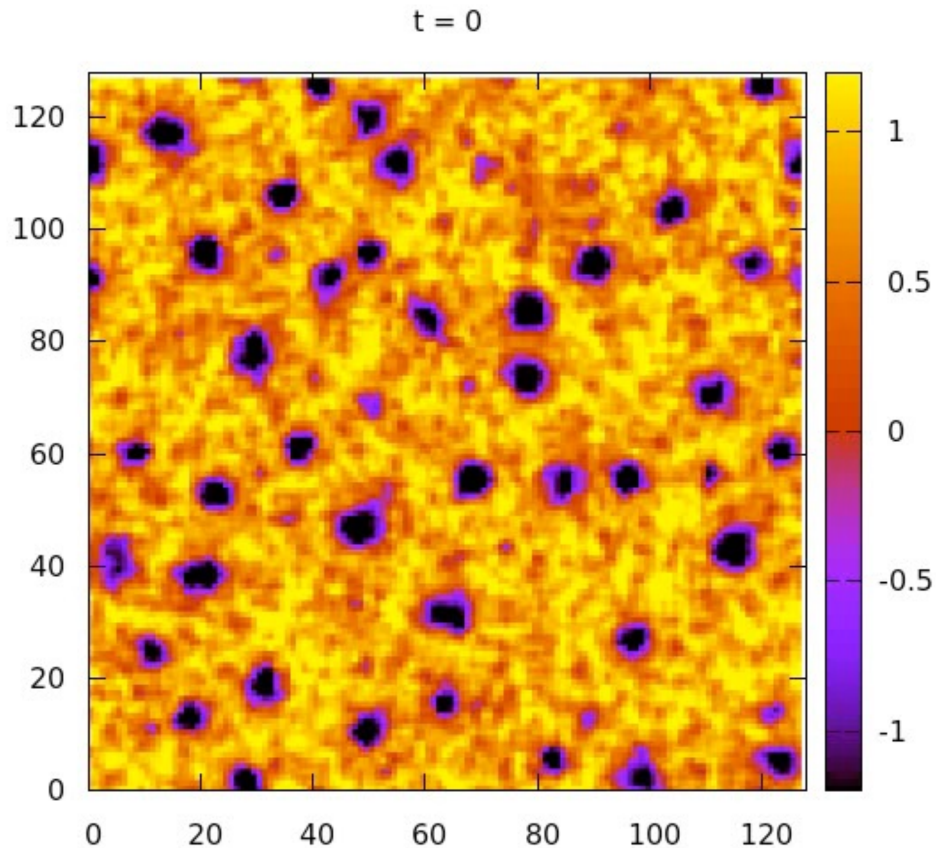
single stable fixed point at R*

E. Tjhung et al., PRX 2018

Reverse Ostwald Regime

Without noise: any initial size distribution converges to its own R*

With noise:



E. Tjhung et al, PRX 2018

Reverse Ostwald Regime

Without noise: any initial size distribution converges to its own R^*

With noise:

- Droplets form via nucleation
- Destroyed by coalescence
- Reverse Ostwald pulls towards uniform size
- Balance of these 3 fixes mean size
- Noise (T) no longer irrelevant
- Steady-state life cycle manifestly breaks TRS

E. Tjhung et al, PRX 2018

Reverse Ostwald Regime: Role of Interfacial Tension

The 'Ostwald tension' γ_O is defined via

$$\dot{R} = \frac{J}{2\phi_B} = \frac{\gamma_o}{2\phi_B^2}\left[\frac{1}{R^*} - \frac{1}{R}\right]$$

Microphase separation arises for $\gamma_O < 0$

In systems with DB, the Ostwald tension coincides with

- Capillary tension: $\gamma < 0 \Rightarrow$ divergent fluctuations $\langle |h_{\mathbf{q}}|^2 \rangle = \frac{k_B T}{\gamma q^2}$
- Excess free energy per unit area: $\gamma < 0 \Rightarrow$ phases remix

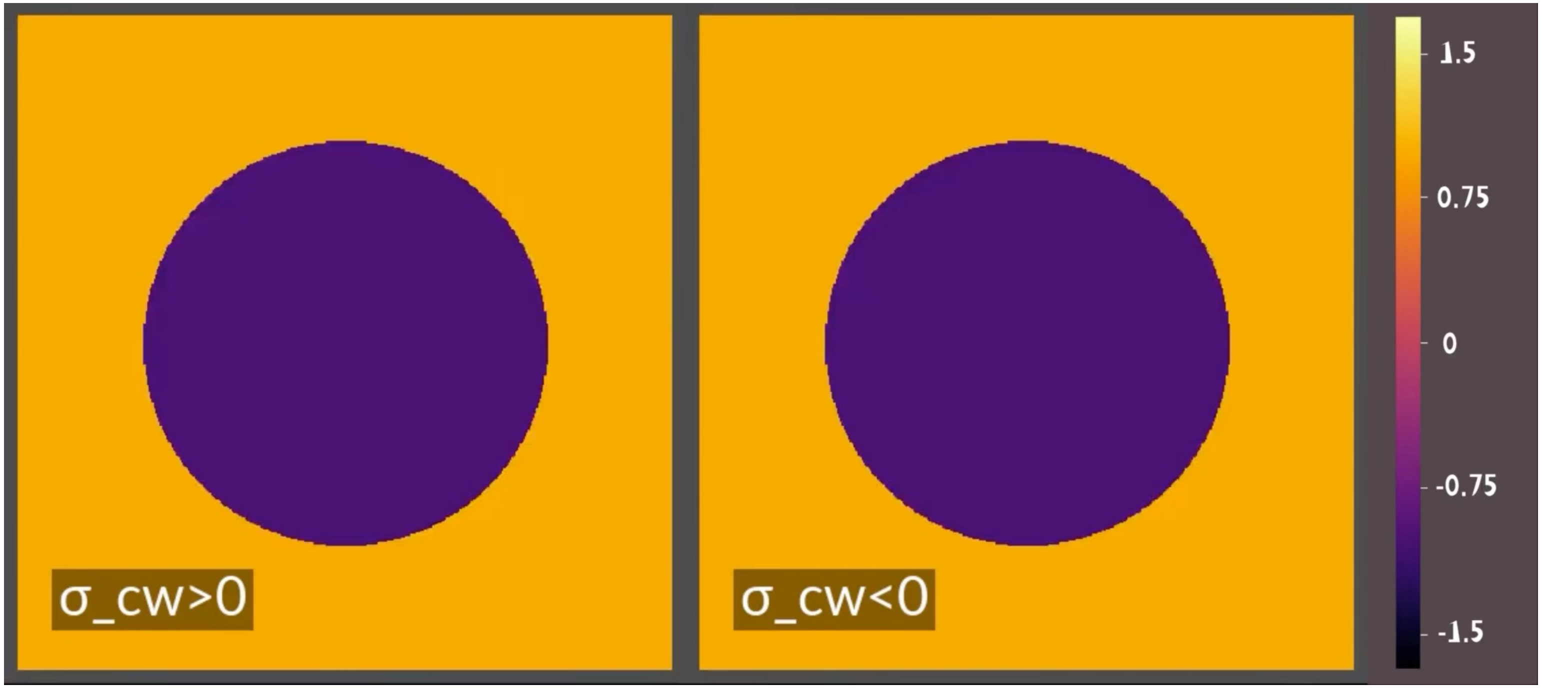
Without DB, these definitions are inequivalent

E. Tjhung et al, PRX 2018

Capillary Interfacial Tension γ_{CW}

Becomes negative at high enough activity ζ

Reverse Ostwald: $\gamma_{CW} > 0 > \gamma_O$ Higher activity: $0 > \gamma_{CW} > \gamma_O$



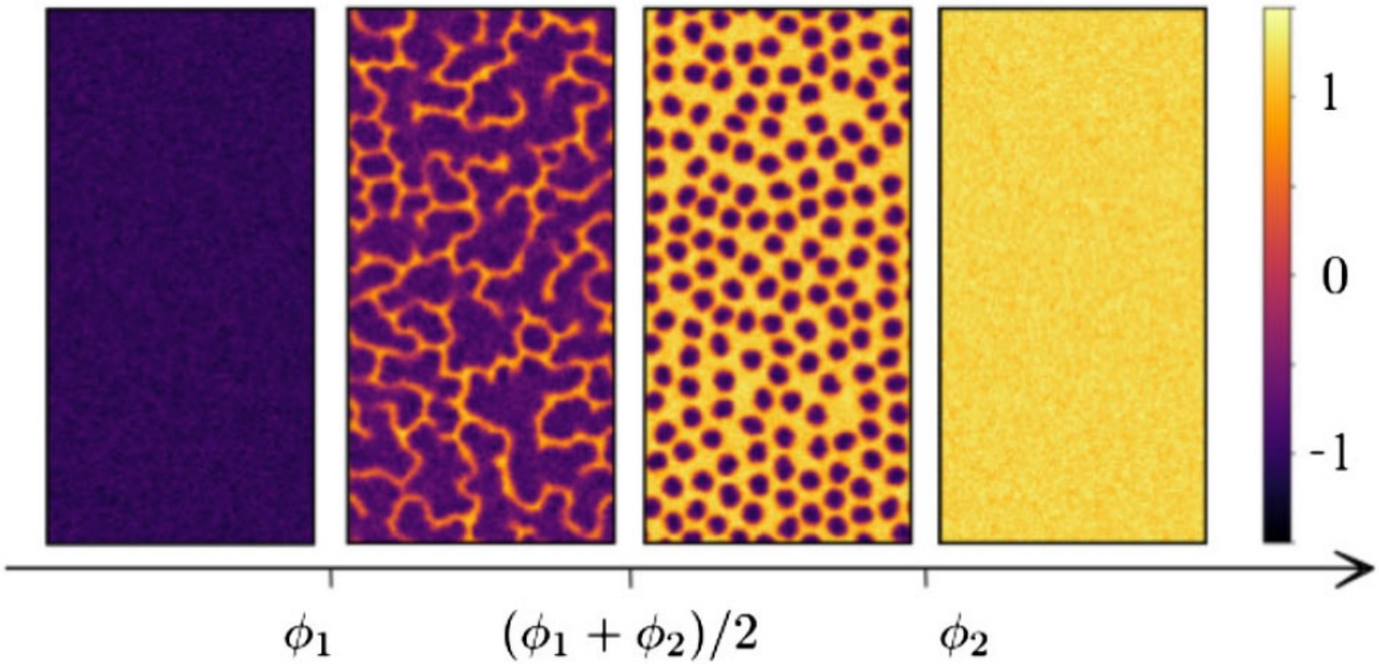
G Fausti et al, PRL 2021

Capillary Interfacial Tension γ_{CW}

Becomes negative at high enough activity ζ

Reverse Ostwald: $\gamma_{CW} > 0 > \gamma_O$ Higher activity: $0 > \gamma_{CW} > \gamma_O$

phase diagram:
now features
'active foam'



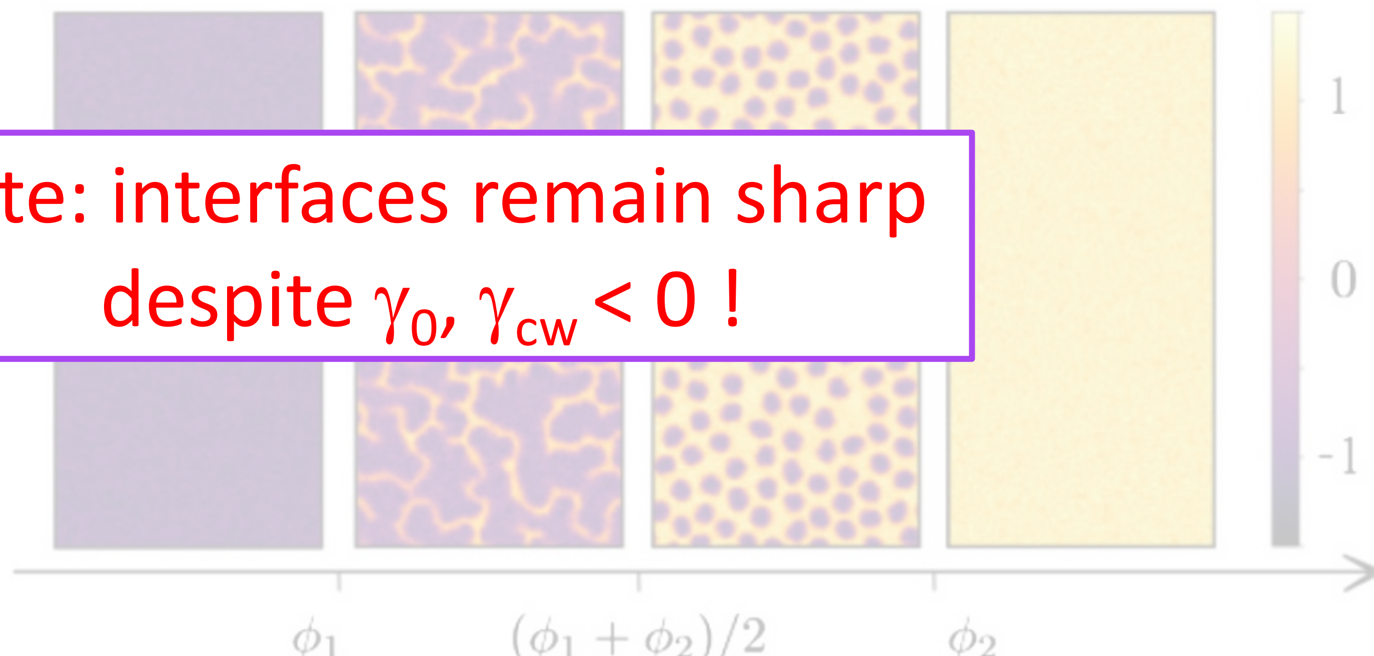
G Fausti et al, PRL 2021

Capillary Interfacial Tension γ_{CW}

Becomes negative at high enough activity ζ

Reverse Ostwald: $\gamma_{CW} > 0 > \gamma_O$ Higher activity: $0 > \gamma_{CW} > \gamma_O$

phase diagram:
now featur
'active foan



Note: interfaces remain sharp despite $\gamma_0, \gamma_{cw} < 0$!

G Fausti et al, PRL 2021

Summary: Model B+

Strengths:

Generically shows anomalous coexistence

Generically admits microphase separation (clusters/bubbles/foam)

Allows explicit calculations of these effects

Allows clean simulations including noise terms

Weakness:

Agnostic of microscopic mechanisms

BUT: ζ, λ terms **are** derivable from microscopics in various cases

E. Tjhung et al, PRX 2018

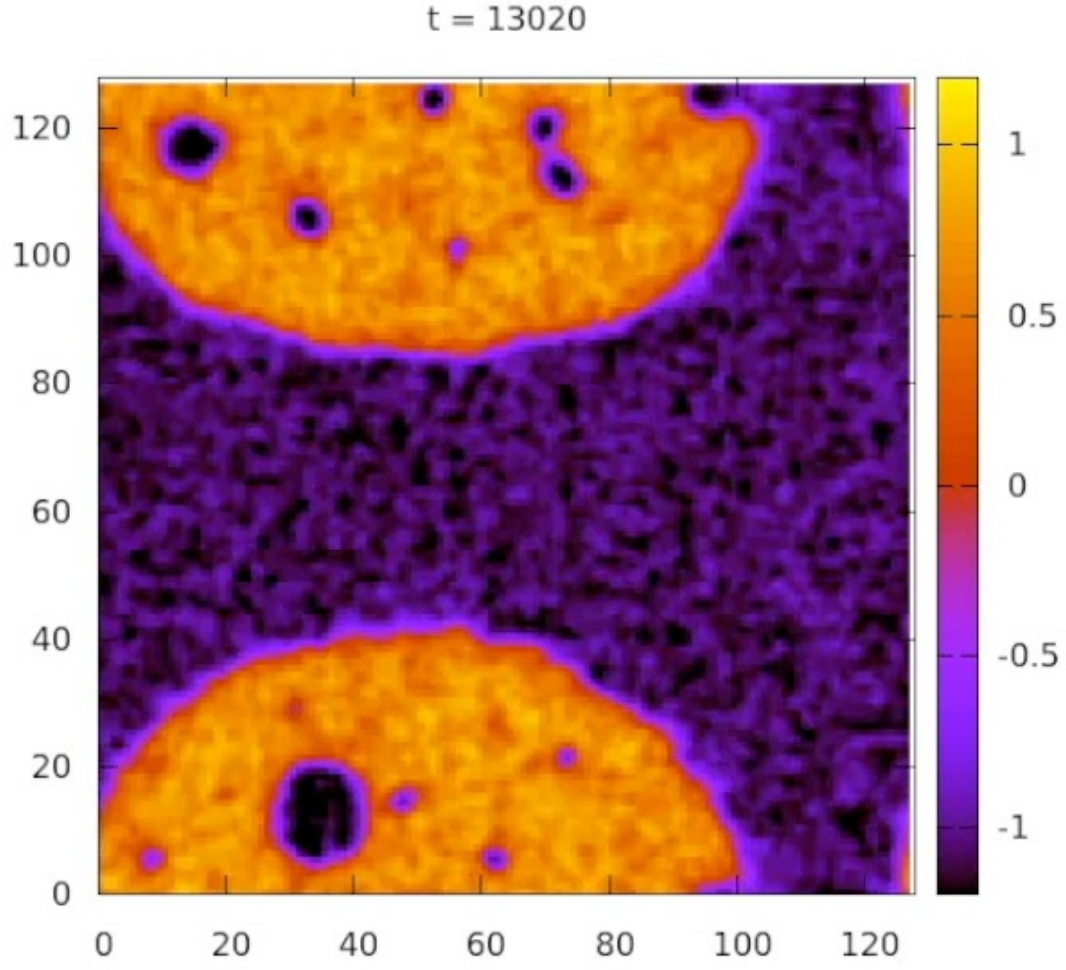
Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics

- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences

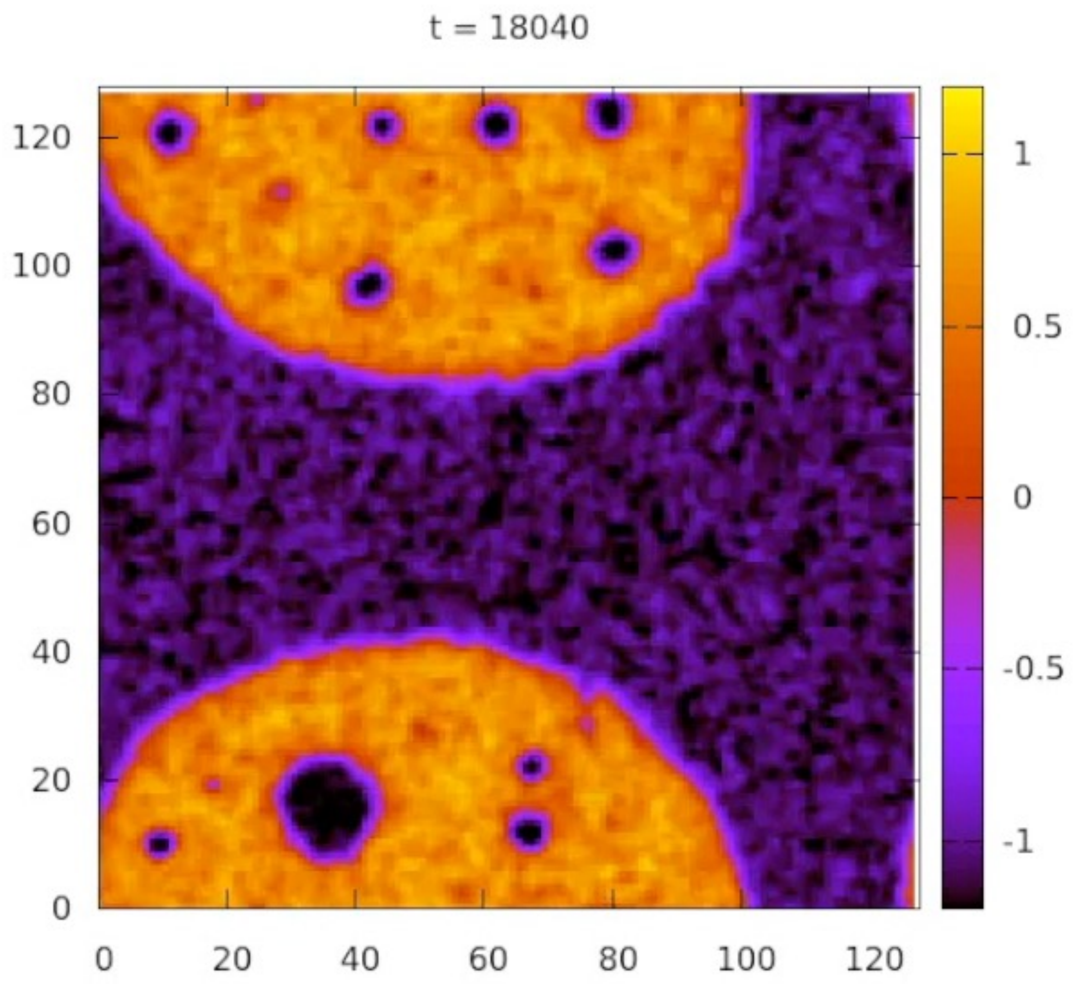
- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production

Active Matter: The Arrow of Life



Active Model B+

Active Matter: The Arrow of Life



Time-Reversed Steady State: **not the same**

Active Model B+

Quantification via Stochastic Thermodynamics

In steady state system attached to heat bath

$$\frac{dS}{dt} = \frac{1}{t_2 - t_1} \log\left(\frac{\text{Prob(forward sequence)}}{\text{Prob(backward sequence)}}\right)$$

dS/dt **directly quantifies** the unlikelihood of reverse processes

Review: U Seifert, Repts Prog Phys 2012

Quantification via Stochastic Thermodynamics

In steady state system attached to heat bath

$$\frac{dS}{dt} = \frac{1}{t_2 - t_1} \log\left(\frac{\text{Prob(forward sequence)}}{\text{Prob(backward sequence)}}\right)$$

dS/dt **directly quantifies** the unlikelihood of reverse processes

- dS/dt depends on scale of observation
- Field theories are coarse-grained $\Rightarrow$
  our dS/dt quantifies "visible" irreversibility only
- = **Informatic** Entropy Production Rate (IEPR)
- IEPR calculable from path integral

IEPR: Active Model B

$$\nabla^{-1}\dot{\phi} = \mathbf{J} = -\nabla(\mu_{\mathsf{E}} + \mu_{\mathsf{NE}}) + (2D)^{1/2}\mathbf{\Lambda}$$

$$\ln P[\phi] = -\frac{1}{4D}\int_0^{\tau} dV\,dt\,|\nabla^{-1}\dot{\phi} + \nabla(\mu_{\mathsf{E}} + \mu_{\mathsf{NE}})|^2$$

$$\ln(P_{\mathsf{f}}/P_{\mathsf{b}}) = \frac{1}{D}\int_0^{\tau} dV\,dt\,(\underline{\nabla^{-1}\dot{\phi}\nabla\mu_{\mathsf{E}}} + \nabla^{-1}\dot{\phi}\nabla\mu_{\mathsf{NE}})$$

- ΔF

steady state: $$\frac{dS}{dt} = \lim_{\tau\to\infty}\frac{[\text{this}]}{\tau} = -\frac{1}{D}\int\langle\dot{\phi}\mu_{\mathsf{NE}}\rangle\,dV$$

C. Nardini et al, PRX 2017

IEPR: Active Model B

$$\frac{dS}{dt} = \int \sigma(\mathbf{r})\, dV$$

steady state **local** entropy production rate density

$$\sigma(\mathbf{r}) = \langle \hat{\sigma}(\mathbf{r}) \rangle$$

$$\hat{\sigma}(\mathbf{r}) = -\lambda \dot{\phi} (\nabla \phi)^2 / D$$

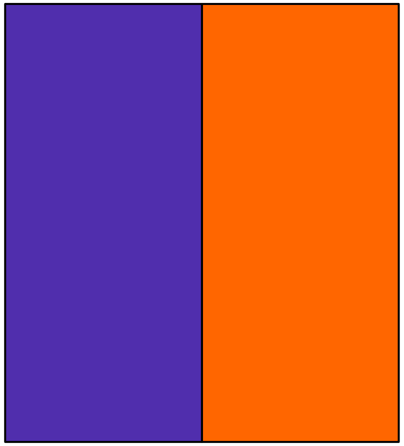
$$\doteq \nabla(a\phi + b\phi^3 - \kappa \nabla^2 \phi + \lambda(\nabla\phi)^2).\nabla(\lambda(\nabla\phi)^2)/D$$

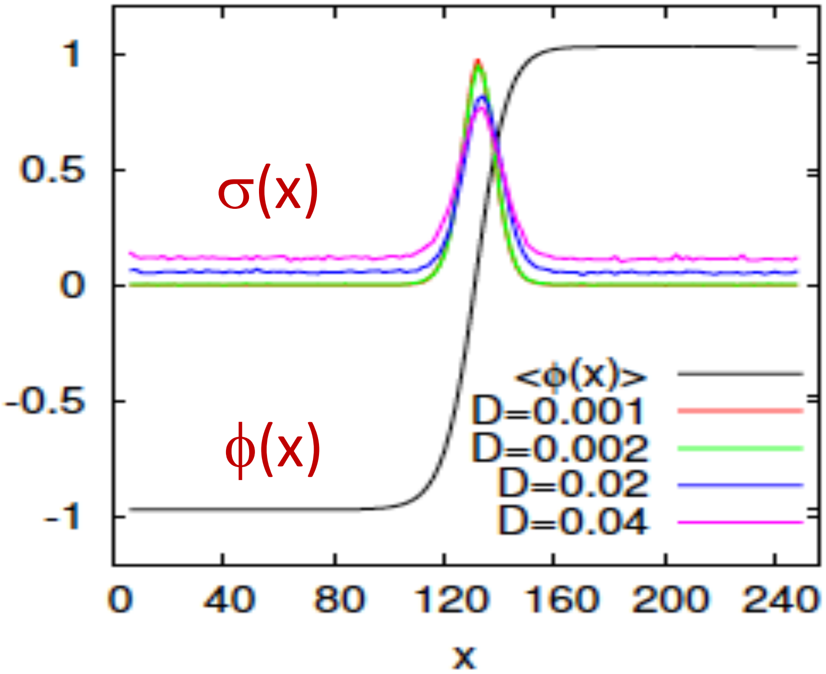
Global IEPR calculable from stationary measure $P[\phi(\mathbf{r})]$

C. Nardini et al, PRX 2017

IEPR as Irreversibility Measure

Active Model B





$\sigma \propto D^1$ bulk phases, $\propto D^0$ at interface

Generic scalings of IEPR density σ for coarse-grained fields:

- D^{-1}: dynamics breaks TRS at deterministic level
- D^{0}: TRS broken by leading-order fluctuations
- $D^{1,2\ldots}$: only broken at higher order

Active Phase Separation

- Phase Separation in Passive Systems
  - Thermodynamics of the Density Field
  - Diffusive Time Evolution: Model B
  - Phase Separation Kinetics

- Active Particles
  - Broken Time Reversal Symmetry
  - Macroscopic Consequences

- Adapting Model B for Active Matter
  - Phase Coexistence
  - Microphase Separation
  - Stead-State Entropy Production