Refs

Dutta - Faulkner arxiv: 1905.00577

Akers - Rath arxiv: 1911.07852

Hayden - Parrikar - Sorce arxiv: 2107.00009

Nezami - Walter 1608.02595

(1+1) D systems

Zou - Siva - Soejima - Mong - Zaletel arxiv: 2011.11864

Calabrese - Cardy hep-th/0405152

(2+1)D Topological phases

Siva - Zou - Siva - Soejima - Mong - Zaletel

arxiv: 2110.11965

Liu - Sohal - Kudler-Flam - Ryu

arxiv: 2110.11980

Liu - Kusuki - Kudler-Flam - Sohal - Ryu

(forth coming)

(1+1)D Non-equilibrium

Kudler-Flam - Kusuki - Ryu arxiv: 2001.05501

Kudler-Flam - Kusuki - Ryu arxiv: 2008.11266

Multipartite entanglement in many-body systems.

✓ Quantum entanglement in Many-body physics

Many-body physics and Q-entanglement.

- ✓ order parameter
- ✓ RG.
- ✓ numerics.
- ✓ holography

✓ Entanglement and Bell pairs (EPR pairs)

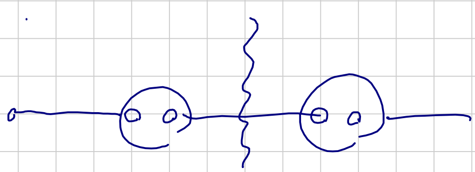
$$\frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

Although this is a two-body state, we can use this as a <u>building block</u> of many-body states

e.g. Haldane state (AKLT state)

 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

1d gapped spin liquid (SPT phase)

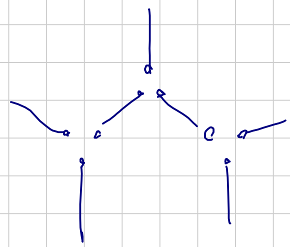


$$S_A = -\mathrm{Tr}\, \rho_A \ln \rho_A$$

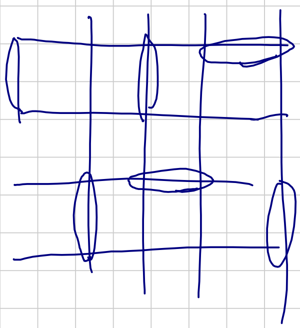
Area law $\sim$ Area.

2d (more complicated)

states in higher dimensions are more complicated.

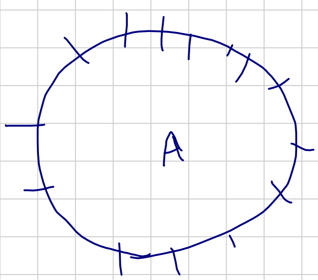


2d AKLT



resonating valence bond

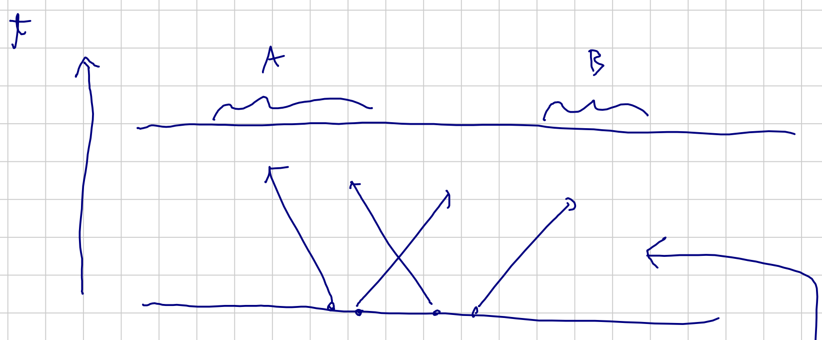
may be PEPS



Area law

Another important example.

e.g. quasi particle picture.



free propagation of Bell pairs

- Multiparty entanglement ?

$$|GHZ\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle\right) \quad \text{"cat"}$$

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\uparrow\uparrow\downarrow\rangle\right)$$

Dür - Vidal - Cirac (00)

"Three qubits can entangled in two inequivalent ways"

equivalent classes under SLOCC

$|W\rangle$: entanglement for any pairs

$|GHZ\rangle$: no " for any pairs.

$$\mathrm{Tr}_3\left(|GHZ\rangle\langle GHZ|\right) = \frac{1}{\sqrt{2}}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

- Is multipartite entanglement important in many-body physics?

  If so, how do we characterize it?

- These lectures: reflected entropy.

Outline

reflected entropy

0d, 1d

2d Topological theories

1d non equilibrium

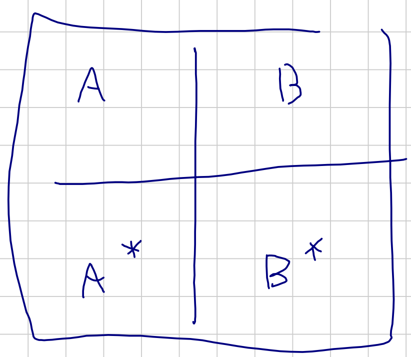
Reflected Entropy $R_{A:B}$

vN entropy of canonical purification

$$\rho_{AB} = \sum_i p_i \, |\psi_i\rangle\langle\psi_i|_{AB} \quad \text{on} \quad \mathcal{H}_A \otimes \mathcal{H}_B$$

$$\downarrow$$

$$|\sqrt{\rho_{AB}}\rangle \equiv \sum_i \sqrt{p_i} \, |\psi_i\rangle_{AB} \, |\psi_i^*\rangle_{A^*B^*}$$



$$R_{A:B} \equiv S_{vN}(\rho_{AA^*})$$

Holographic dual $\quad R_{A:B} = 2E_W / 4G_N$

$R_{A:B} - I_{A:B} > 0 \Rightarrow$ Markov gap h

* note h is symmetric $A \leftrightarrow B$

$$h = S_{AA^*} + S_{AB} - S_A - S_B$$

$$S_{AA^*} = S_{BB^*}$$ since total system is pure

* note h can be interpreted as conditional mutual info btw A^* and B

Akers and Rath (19) noticed that R (or h) can be used to detect W-type tripartite entanglement

$$h_{GHZ} = 0, \qquad h_W = 1.49 \ln 2 - 0.49 \ln 2 > 0$$

Cotext there was holographic duality

One way to phrase this story is as a comparison btw holograpy and tensor network model.

Random tensor network have been used as Toy models of holography. e.g. RT formula.

Akers and Ruth however noted that Some random tensor networks do not reproduce reflected entropy $\simeq$ EW.

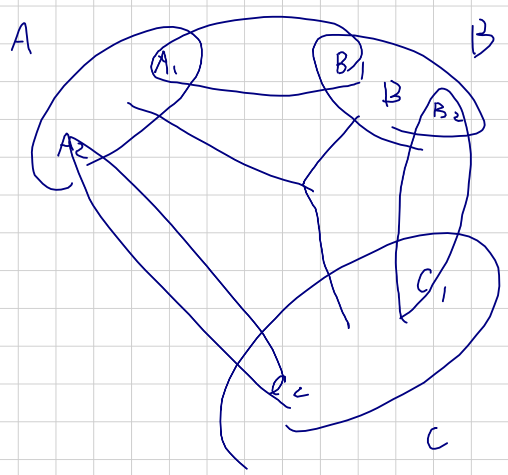
Random stabilizer Tensor network

mostly bipartite

order 1 in $N \to \infty$

$$U_A U_B U_C |\psi\rangle_{ABC}$$

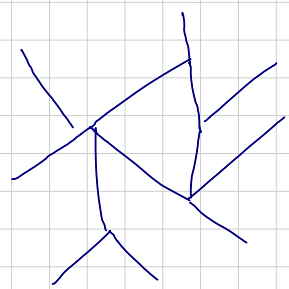
$$= \left(|\Phi\rangle_{A_1B_1}\right)^{c\sim N} \left(|\Phi\rangle_{B_2C_1}\right)^{a\sim N} \left(|\Phi\rangle_{A_2C_2}\right)^{b\sim N} \left(|GHZ\rangle_{A_3B_3C_3}\right)^{g}$$



$$|\Phi\rangle_{AB} = \frac{1}{\sqrt{p}} \sum_{i=0}^{p-1} |i\rangle_A |i\rangle_B$$

$$|GHZ\rangle_{ABC} = \frac{1}{\sqrt{p}} \sum_{i=0}^{p-1} |i\rangle_A |i\rangle_B |i\rangle_C$$

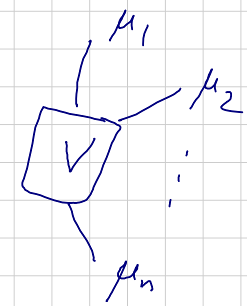
<u>note</u> Random stabilizer state



V: vertices $= \underset{\substack{\uparrow \\ \text{bulk}}}{V_b} \cup \underset{\substack{\uparrow \\ \text{bdry}}}{V_\partial}$

E: edges

Assign a tensor V_x for $x \in V_b$



$$|V\rangle = \sum_{\mu_1 \cdots \mu_n} V_{\mu_1 \cdots \mu_n} |\mu_1\rangle \otimes \cdots \otimes |\mu_n\rangle$$

Assign each edge $|e\rangle \propto \sum_{\mu} |\mu_1\rangle |\mu_2\rangle$

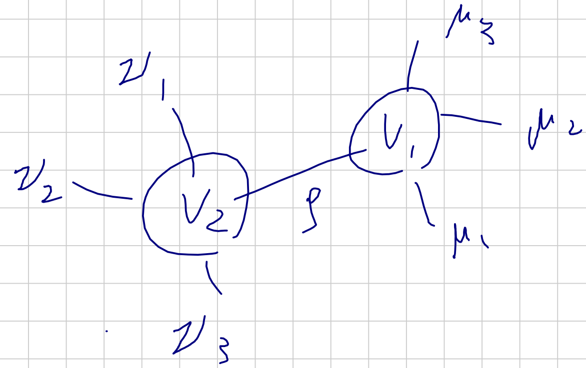
$\Longrightarrow$

$$|\psi\rangle = \left(\bigotimes_{x \in V_b} \langle V_x | \right) \left(\bigotimes_{e \in E} |e\rangle \right)$$

Choose V_x independently, uniformly from the set of stabilizer states e.g.

$$V^1 \quad |\mu_1\rangle \cdots |\mu_3\rangle \quad |\rho\rangle$$

$$V^2 \quad |\nu_1\rangle \cdots |\nu_3\rangle \quad |\rho'\rangle$$



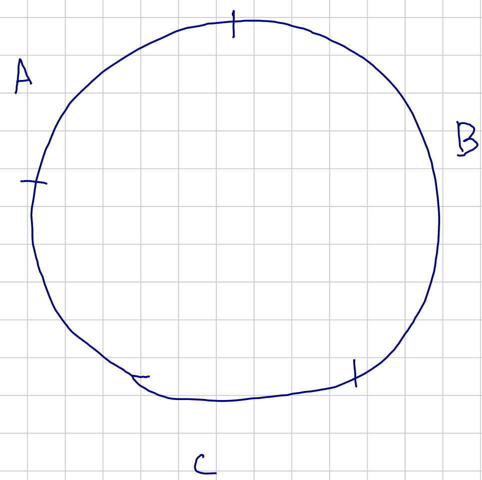
$$\langle e| \propto \sum_{\rho} \langle\rho|\langle\rho|$$

$$\langle V^1|\langle V^2|e\rangle$$

$$= \sum_{\vec{\mu}} \sum_{\vec{\nu}} V^1_{\mu_1\cdots\mu_3\,\rho} \; V^2_{\nu_1\cdots\nu_3\,\rho} \; |\mu_1\cdots\mu_3\rangle |\nu_1\cdots\nu_3\rangle$$

(1+1)d systems

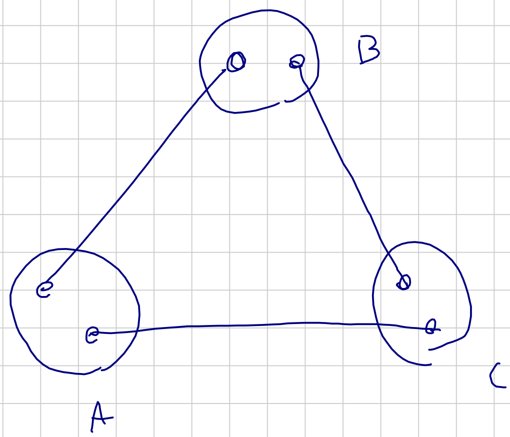
Ref
(Zou - Siva - Soejima - Mong - Zaletel)



$$h_{A:B} = R_{A:B} - I_{A:B}$$

$\boxed{h = 0 \text{ if and only if gapped}}$

Triangle state



$$\mathcal{H}_A = \mathcal{H}_{A_L} \otimes \mathcal{H}_{A_R}$$

$$|ABC\rangle = |A_R B_L\rangle |B_R C_L\rangle |C_R A_L\rangle$$

Sum of triangle states

$$|ABC\rangle = \sum_\mu \sqrt{p_\mu}\, |A_R^\mu B_L^\mu\rangle |B_R^\mu C_L^\mu\rangle |C_R^\mu A_L^\mu\rangle$$

$\boxed{h = 0 \text{ if and only if sum of triangle states}}$

fixed pt MPS (when gapped, $\xi < \infty$)

$\|$ (coarse graining. $\xi \longrightarrow 0$)

N-party version of a triangle state.

state with SSB ($\xi = \infty$)

= N-party version of sum of triangle state.

2d CFT replica + twist operators

$$S_A = -\mathrm{Tr}\,(\rho_A \ln \rho_A)$$

replica trick

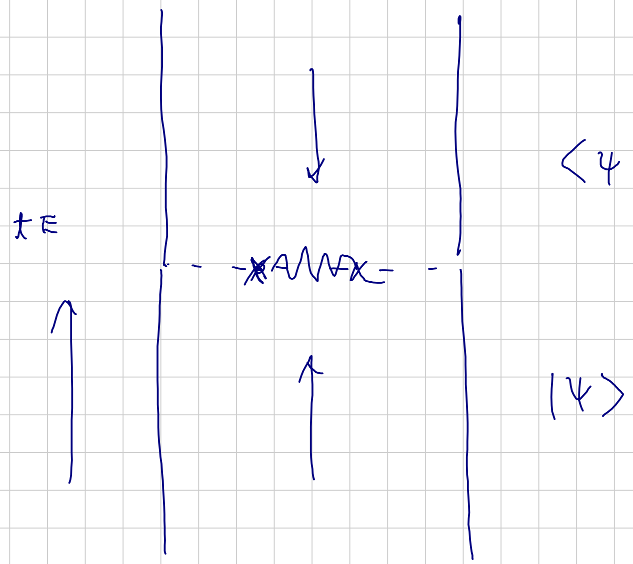
$$\mathrm{Tr}\,\rho_A \ln \rho_A = \lim_{n\to 1} \frac{\partial}{\partial n} e^{n \ln \rho_A}$$

$$= \lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{Tr}\,\rho_A^n$$

$$\left.\mathrm{Tr}\,\rho_A = 1 \right\} \quad = \lim_{n\to 1} \frac{1}{\mathrm{Tr}\,\rho_A^n} \frac{\partial}{\partial n} \mathrm{Tr}_A\,\rho_A^n$$

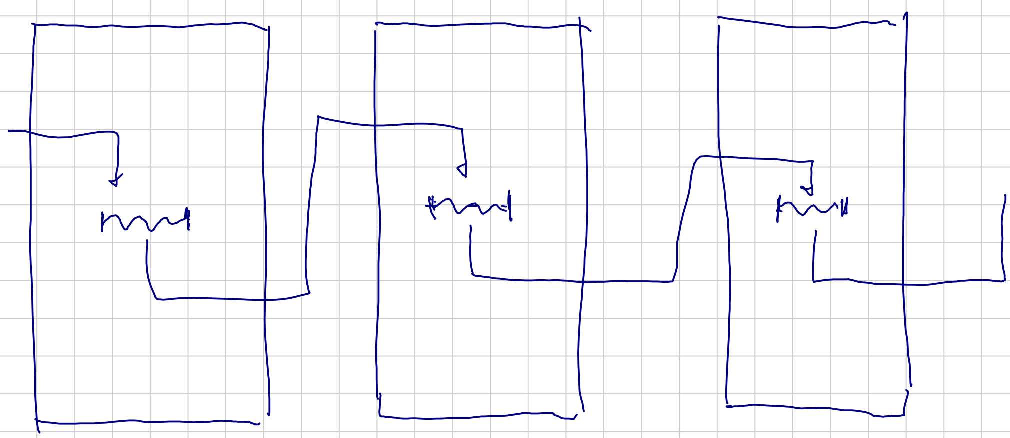
$$= \lim_{n\to 1} \frac{\partial}{\partial n} \ln \mathrm{Tr}\,\rho_A^n$$

$\rho = |\psi\rangle\langle\psi|$:



$= |\psi\rangle\langle\psi| = \rho$

$\mathrm{Tr}\, \rho_A^n$:



$= \mathrm{Tr}\, \rho_A^n$

$\propto \underbrace{\int_{R_n} \mathcal{D}\phi\, e^{-S}}_{Z_n}$

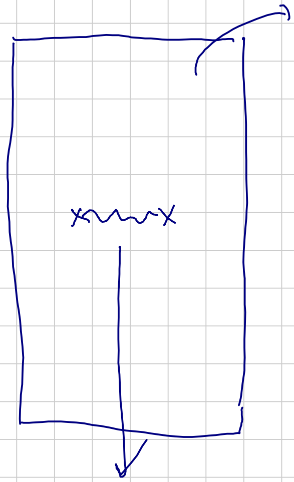
normalization

$\mathrm{Tr}\, \rho_A^n = \dfrac{Z_n}{Z_1}$

Twist operator

$$\mathrm{Tr}\,\rho_A^n \sim \langle 1 \rangle_{R_n}$$

multi component picture



$CFT^{\otimes n}$ $\quad \vec{\phi}(x)$

$$\mathrm{Tr}\,\rho_A^n = \langle \sigma_n(x_1)\,\sigma_{-n}(x_2) \rangle_{\mathbb{C}}$$

$$\vec{\phi}(x) = \begin{pmatrix} 0 & 1 & 0 & & 0 \\ & & \ddots & 1 & \\ & & & \ddots & 1 \\ 1 & & & & \end{pmatrix} \vec{\phi}$$

$$\mathrm{Tr}\,\rho_A^n = \langle \sigma_n(x_1)\,\sigma_{-n}(x_2) \rangle_{\mathbb{C}}$$

$$= \left| \frac{x_1 - x_2}{\varepsilon} \right|^{-4h_n} = R^{\Delta(n)}$$

$$\frac{\partial}{\partial n} \mathrm{Tr}\,\rho_A^n = \frac{\partial}{\partial n} e^{\Delta(n)\ln R} = \frac{\partial \Delta}{\partial n} e^{\Delta(n)\ln R}$$

$$\longrightarrow \left.\frac{\partial \Delta}{\partial n}\right|_{n=1} \ln R \cdot e^{\Delta(1)\ln R}$$

$$\boxed{h_n = \frac{c}{24}\left(n - \frac{1}{n}\right)}$$

$$\frac{\partial}{\partial n} \mathrm{Tr}\, \rho_A^n = -4 \frac{c}{24}\left(1 + \frac{1}{n^2}\right) \ln R \Big|_{n=1} = -\frac{c}{3} \ln\left|\frac{x_1 - x_2}{\varepsilon}\right|$$

$$S_A = \frac{c}{3} \ln\left|\frac{x_1 - x_2}{\varepsilon}\right|$$

h_n ?

compute $\langle T(z)\, \sigma_n(x_1)\, \sigma_{-n}(x_2) \rangle_{\mathbb{C}}^{CFT^{\otimes n}}$ in two ways

$$T(z) = \sum_{i=1}^{n} T_i(z)$$

$$\langle T(z)\, \sigma_n(x_1)\, \sigma_{-n}(x_2) \rangle_{\mathbb{C}} = n \langle T_i(z) \rangle_{R_n}$$

multicomponent theory $\qquad$ original theory

R_n and $\mathbb{C}$ are related by $\quad w = \left(\frac{z - x_1}{z - x_2}\right)^{1/n}$

$$T(z) = \left(\frac{dw}{dz}\right)^2 T(w) + \frac{c}{12}\{w, z\}$$

$$\langle T(z) \rangle_{R_n} = \frac{c}{12}\{w, z\}$$

$$\langle T \sigma \sigma \rangle_{\mathbb{C}} = \sum \left(\frac{h_n}{(z - x_i)^2} + \frac{1}{z - x_i}\partial_{x_i} + \mathrm{reg}\right) \langle \sigma \sigma \rangle$$

adjacent intervals

$$I_{A:B} = S_A + S_B - S_{A\cup B}$$

$$= \frac{c}{3} \ln \frac{|x_2 - x_1|}{\varepsilon} + \frac{c}{3} \ln \frac{|x_3 - x_2|}{\varepsilon} - \frac{c}{3} \ln \frac{|x_3 - x_1|}{\varepsilon}$$

$$= \frac{c}{3} \ln \frac{(x_2 - x_1)(x_3 - x_2)}{(x_3 - x_1)} \frac{\varepsilon}{\varepsilon^2}$$

$$\to \frac{c}{3} \ln \frac{(x_2 - x_1)(x_3 - x_2)}{\varepsilon (x_3 - x_1)}$$

* use only 2-pt functions.

they are normalized

no $O(1)$ term?

reflected entropy

$$|\sqrt{\rho_{AB}}\rangle$$

$$\rightarrow \quad |\psi_m\rangle = \frac{1}{\sqrt{\mathrm{Tr}\,\rho_{A}^{m}}} |\rho_{AB}^{m/2}\rangle \qquad m \in 2\mathbb{Z}^+$$

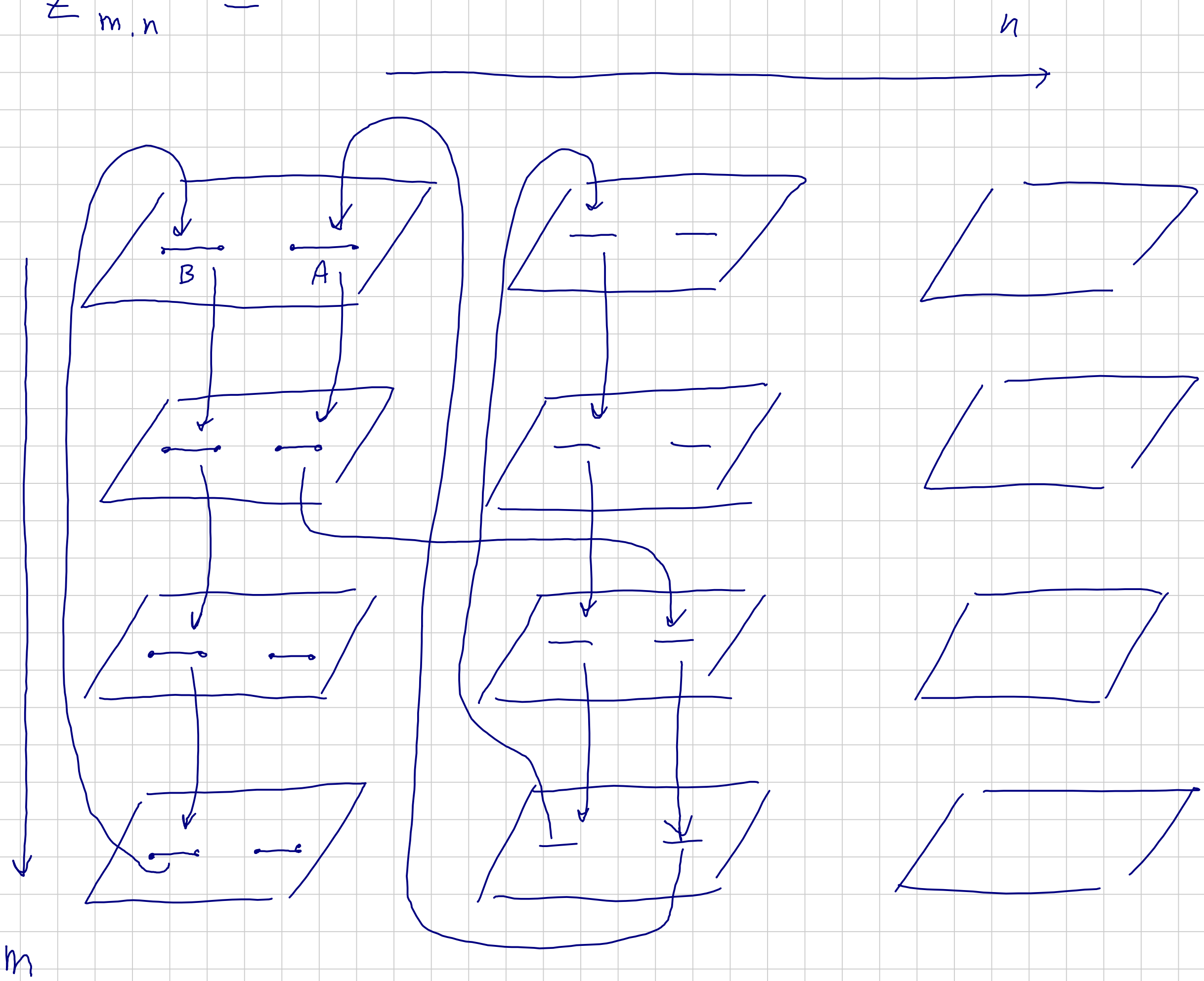
$$\rho_{AA^*}^{(m)} = \mathrm{Tr}_{BB^*} |\psi_m\rangle\langle\psi_m|$$

$$\frac{1}{n-1} \ln \mathrm{Tr} \left(\rho_{AA^*}^{(m)}\right)^n$$

$$Z_{n,m} = \mathrm{Tr}_{AA^*}\left[\mathrm{Tr}_{BB^*} |\rho_{AB}^{m/2}\rangle\langle\rho_{AB}^{m/2}|\right]^n$$

$$R^{(n)} = \frac{1}{n-1} \ln \frac{Z_{n,m}}{(Z_{1,m})^n}$$

$Z_{m,n} =$



$$R_{A:B} \propto \ln \langle \sigma_{g_A}(x_1)\, \sigma_{g_A^{-1}}(y)\, \sigma_{g_B}(y')\, \sigma_{g_B^{-1}}(x_3) \rangle$$

$$h_{g_A} = h_{g_B} = n \frac{c}{24}\left(m - \frac{1}{m}\right)$$

(n copies of σ_m)

$$g_A,\ g_B \in S_{nm}$$

$$y, y' \longrightarrow x_2$$

$$\sigma_{g_A^{-1}}\, \sigma_{g_A^{-1}} \rightarrow \sigma_{g_B\, g_A^{-1}}$$

$\uparrow$ (two copies of σ_n)

$$h_{g_B\, g_A^{-1}} = 2 \frac{c}{24}\left(n - \frac{1}{n}\right)$$

OPE coeff $= C_{n,m}$

$$R_{A:B} = \lim_{n,m\to 1} \frac{1}{1-n}\Big[\ln \left\langle \sigma_{g_A}(x_1)\,\sigma_{g_A^{-1}g_B}(x_2)\,\sigma_{g_B^{-1}}(x_3)\right\rangle$$

$$+ \ln C_{n,m}\Big]$$

$$= \frac{c}{3}\ln\frac{(x_2-x_1)(x_3-x_2)}{\varepsilon\,(x_3-x_1)} + O(1)$$

$$O(1) = \lim_{n,m\to 1}\frac{1}{1-n}\ln C_{n,m} = \frac{-4h_n \ln 2m}{1-n}$$

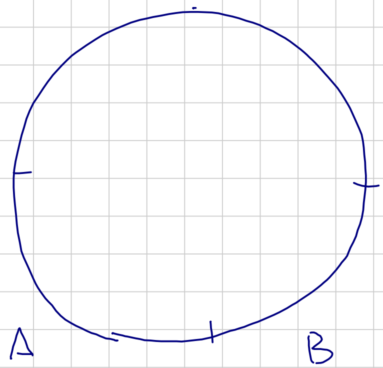
$$C_{n,m} = (2m)^{-4h_n} = \frac{\frac{c}{6}\left(n-\frac{1}{n}\right)}{n-1}\ln 2$$

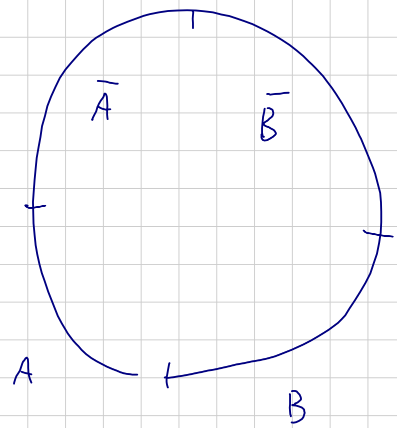
same cut off as $I_{A:B}$. $= \frac{c}{6}\left(1+\frac{1}{n^2}\right)\ln 2$

$$= \frac{c}{3}\ln 2.$$

$C_{n,m}$: Lunin - Mathur (00)

$$|C| = 2\,|A \cup B|$$





$$|\sqrt{\rho_{AB}}\rangle = |GS\rangle$$

$$h = S_{A\bar{A}} + S_{AB} - S_A - S_B$$

$$S = \frac{c}{3} \ln \sin \frac{\pi l}{L}$$

$$\frac{c}{3}\left(\ln \underbrace{\sin \frac{\pi}{2}}_{1} + \ln \underbrace{\sin \frac{\pi}{2}}_{1} - \ln \underbrace{\sin \frac{\pi}{4}}_{1/\sqrt{2}} - \ln \underbrace{\sin \frac{\pi}{4}}_{1/\sqrt{2}} \right)$$

$$= \frac{c}{3} \left(-2 \ln \frac{1}{\sqrt{2}} \right)$$

$$= \frac{c}{3} \left(2 \ln \sqrt{2} \right) = \frac{c}{3} \ln 2.$$

$$h = S_{A\bar{A}} + S_{AB} - S_A - S_B$$

$$\ln \sin \frac{\pi\, 2\ell_A}{L} + \ln \sin \frac{\pi(\ell_A+\ell_B)}{L}$$

$$- \ln \sin \frac{\pi\, \ell_A}{L} - \sin \frac{\pi\, \ell_B}{L}$$

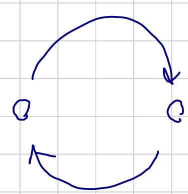
$$= \ln \left[\frac{\sin \frac{\pi\, 2\ell_A}{L} \sin \frac{\pi(\ell_A+\ell_B)}{L}}{\sin \frac{\pi \ell_A}{L} \sin \frac{\pi \ell_B}{L}} \right] \qquad \ell_A + \ell_B = \frac{L}{2}$$

$$= \ln 2$$

(2+1)D Topological states. eg. FQHE

- ✓ fully gapped, all local correl decay exp. fast
- ✓ no local order parameter
- ✓ GS, topology-dependent deg

excited states: anyons



$$|\psi\rangle = e^{i\theta}|\psi\rangle \qquad \theta \neq 0, \pi$$

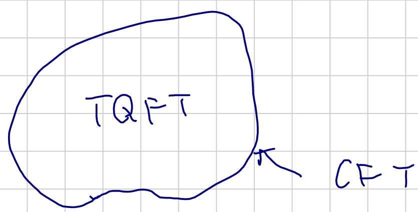
or

$$|\psi\rangle_a = U_{ab}|\psi_b\rangle$$

- ✓ described by TQFT

	SSB	Topological
GS	deg	top. deg
ex.	NG bosons	anyons
effective theories	LG theory	TQFT

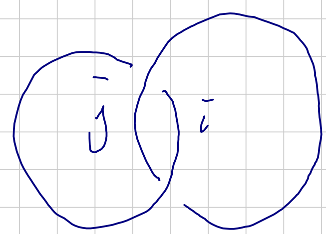
✓ Bulk - bdry correspondence

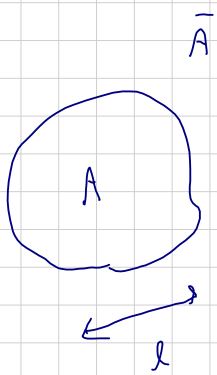


anyons primaries

✓ Topological EE

S_{ij}





$$S_A = \alpha l - S_{top}$$

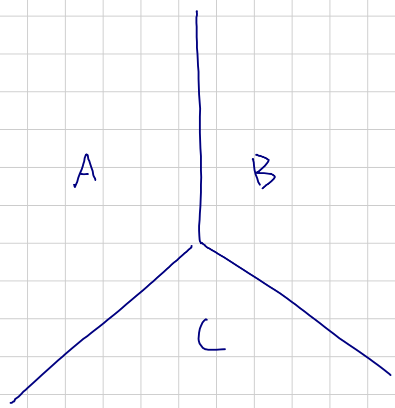
$$S_{0a} = d_a/D$$

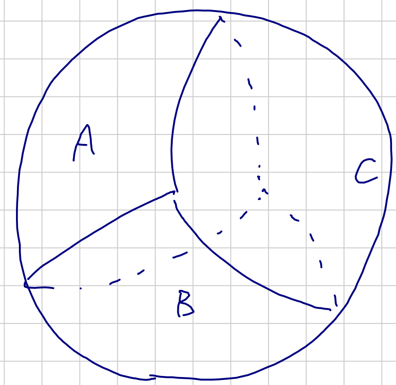
$$S_{00} = 1/D$$

$$S_{top} \approx \ln D$$

$$= -\ln S_{00}$$

$(2+1)D$ TQFT





Result for TQFT

$$h_{A:B} = R_{A:B} - I_{A:B} = \frac{c}{3} \ln 2,$$

More generally.

$$\min_{U} h_{A:B} = c/_3 \ln 2$$

Refs Liu - Sohal - Kudler - Flam - SR (21)

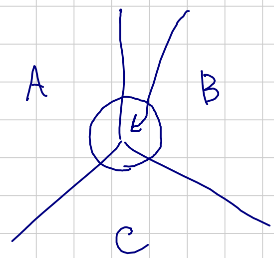
Siva - Zou - Soejima - Mong Zaletel (21)

Conjecture

$$\min_{U} h_{A:B} = \frac{c}{3} \ln 2$$

optimization over U

where $U|\psi\rangle$



c: Total central charge of ungappable degrees of freedom

[= chiral central charge for fully chiral phase]

or

$$h_{A:B} \geq \frac{c}{3} \ln 2$$

Example

ungappable edge

What is the motivation for this conjecture?

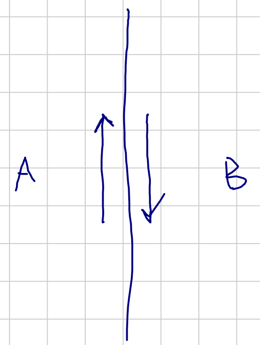
c.f. ungappable edge

e.g. $$K = \begin{bmatrix} k_1 & 0 \\ 0 & -k_2 \end{bmatrix}$$

$(k_1, k_2) = (1, 3)$ $\quad \nu = 2/3$ FQH state

may not be gapped because of anomaly

Calculations

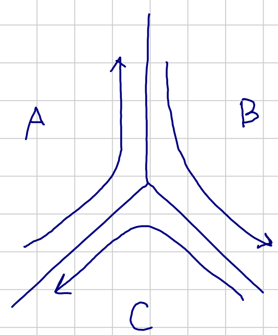


Idea. write $|GS\rangle$ using

degrees of freedom near

entangling bdry.

edge theory.



e.g. bipartition

$$|GS\rangle \sim \sum_{N=0}^{\infty} \sum_{k=1}^{d(N)} |h_i, N, k\rangle \otimes \overline{|h_i, N, k\rangle}$$

maximally entangle state in edge CFT

$= $ Ishibashi state $|I\rangle$

$$|GS\rangle = N e^{-\beta H_0} |I\rangle \qquad \left(J^A(\sigma) - \bar{J}^B(\sigma)\right)|I\rangle = 0$$

$$|GS\rangle = N e^{-\beta H_0} |I\rangle \qquad H_0 = H_L + H_R.$$

$$\rho_A = \mathrm{Tr}_B |GS\rangle\langle GS|$$

$$= N^2 \mathrm{Tr}_B \sum_{NN'} \sum_{k,k'} |h N k\rangle \overline{\underline{|h N k\rangle}} \overbrace{\langle h N' k'|}^{e^{-4\beta E}} \overline{\underline{\langle h N' k'|}}$$

$$= N^2 \sum_{N,k} |h N k\rangle\langle h N k| \, e^{-4\beta E}$$

$$= N^2 e^{-4\beta H_L} \quad (ES) \qquad q = e^{2\pi i \frac{4\beta i}{L}} = e^{-\frac{8\pi\beta}{L}}$$

$$\mathrm{Tr}\, \rho_A^n = N^{2N} \mathrm{Tr}\, e^{-4\beta n H_L} = N^{2N} \chi_i(q^n)$$

$$N^{-2} = \mathrm{Tr}\, \rho_A = \chi_i(q)$$

$$\mathrm{Tr}\, \rho_A^n = \frac{\chi_i(q^n)}{(\chi_i(q))^n} \underset{\beta/L \to 0}{=} \frac{\sum S_{ij} \chi_j(\tilde{q}^{\frac{1}{n}})}{(\sum S_{ij} \chi_j(\tilde{q}))^n}$$

$$\longrightarrow \frac{S_{i0} \chi_0(\tilde{q}^{1/n})}{(S_{i0} \chi_0(\tilde{q}))^n}$$

$$q = e^{2\pi i \frac{4\beta i}{L}} \qquad \tilde{q} = e^{2\pi i\left(-L/4\beta i\right)} = e^{-\frac{\pi L}{4\beta}}$$

$$\chi_i(q) = \mathrm{Tr}\, q^{L_0 - c/24}$$

$$\chi_i(\tilde{q}) \sim \tilde{q}^{\,h_i - c/24}$$

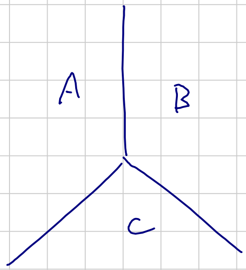
$$\mathrm{Tr}\,\rho_A^n = \frac{S_{i0}\,\tilde{q}^{\frac{1}{n}\left(\overset{=0}{h_i} - c/24\right)}}{\left(S_{i0}\,\tilde{q}^{\left(h_i - c/24\right)}\right)^n}$$

$$= S_{i0}^{1-n}\,\underbrace{\tilde{q}^{\left(\frac{1}{n} - n\right)\left(\quad - \frac{c}{24}\right)}}_{\substack{e^{+\frac{2\pi i L}{4\beta i}\frac{c}{24}\left(\frac{1}{n} - n\right)} \\ e^{+\frac{\pi L c}{48\beta}\left(\frac{1}{n} - n\right)}}}$$

$$\frac{\frac{1}{n} - n}{1-n} \qquad \frac{\frac{1}{n}\left(1-n^2\right)}{1-n} = \frac{1}{n}(1+n)$$

$$\frac{1}{1-n}\,\mathrm{Tr}\,\rho_A^n = \frac{1}{1-n}\left(\ln S_{i0}^{1-n} + \frac{\pi L c}{48\beta}\left(\frac{1}{n} - n\right)\right)$$

$$= \ln S_{i0} + \frac{\pi L c}{48\beta}\,\underbrace{\frac{1+n}{n}}_{2}$$



$$0 \le \sigma \le L/2.$$

$$[\, J^{i}(\sigma) - J^{i+1}(L-\sigma)\,]\, |V\rangle = 0$$

$$[\, \psi^{i}(\sigma) + i\, \psi^{i+1}(L-\sigma)\,]\, |V\rangle = 0$$

Explicit Construction is possible for

free theories.

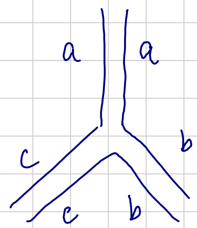
✓ SFT

✓ Liu - Sohal - Kudler - Flam - SR (21)

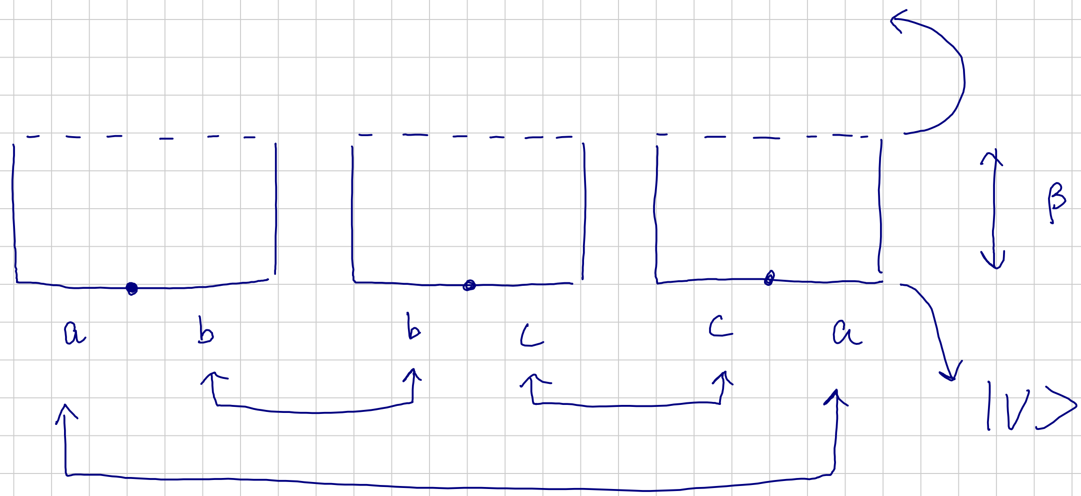
path-integral method

Liu - Kusuki - Kudler - Flam - Sohal - SR

(23 ?)

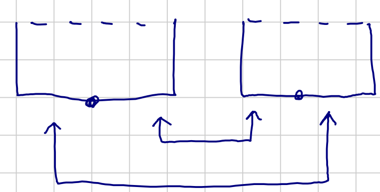


$$|\psi\rangle = e^{-\beta H_0}|V\rangle$$

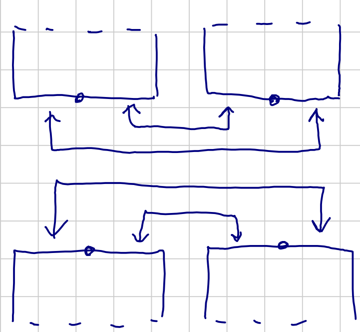


2- copies

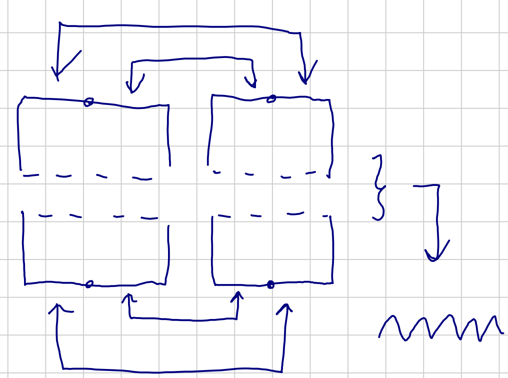
$$|\psi\rangle = e^{-\beta H}|V\rangle$$



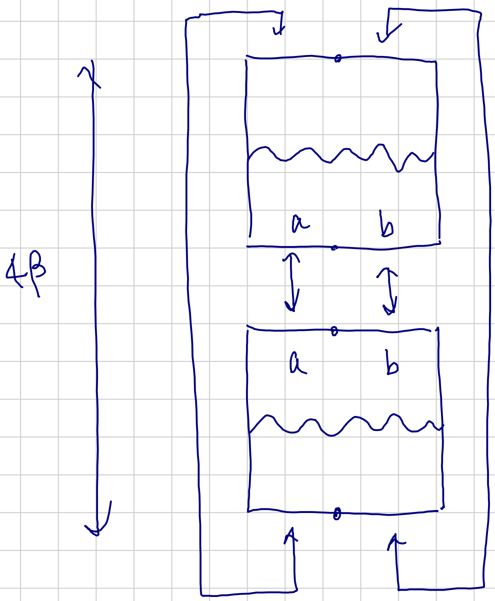
$$\rho = |\psi\rangle\langle\psi|$$



or



Redrawing



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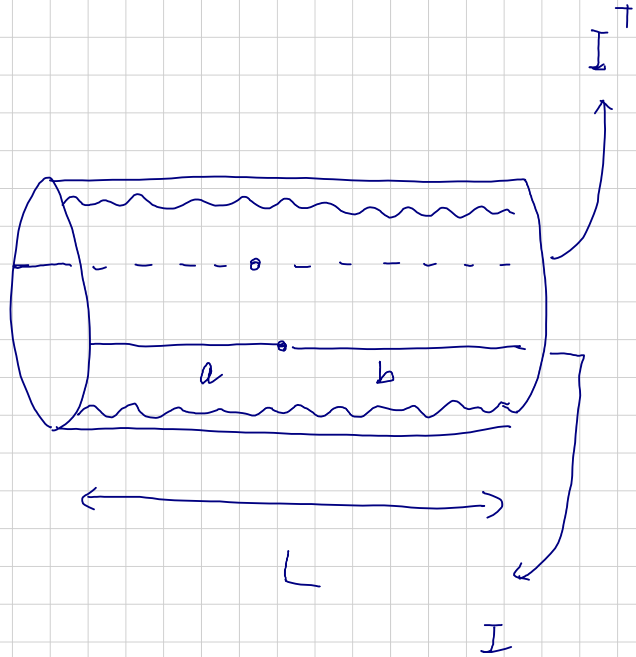
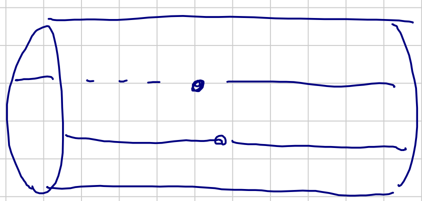
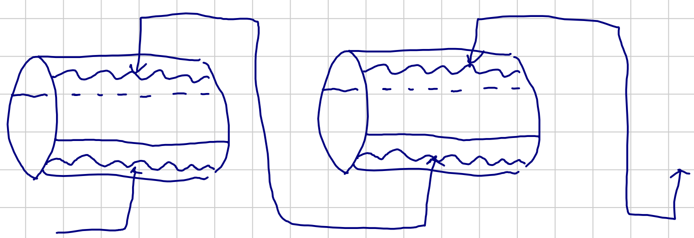


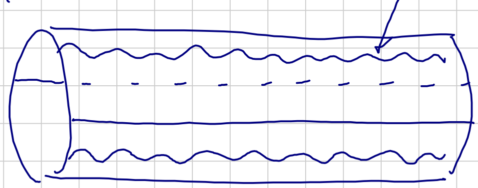
$\mathrm{Tr}\,\rho$

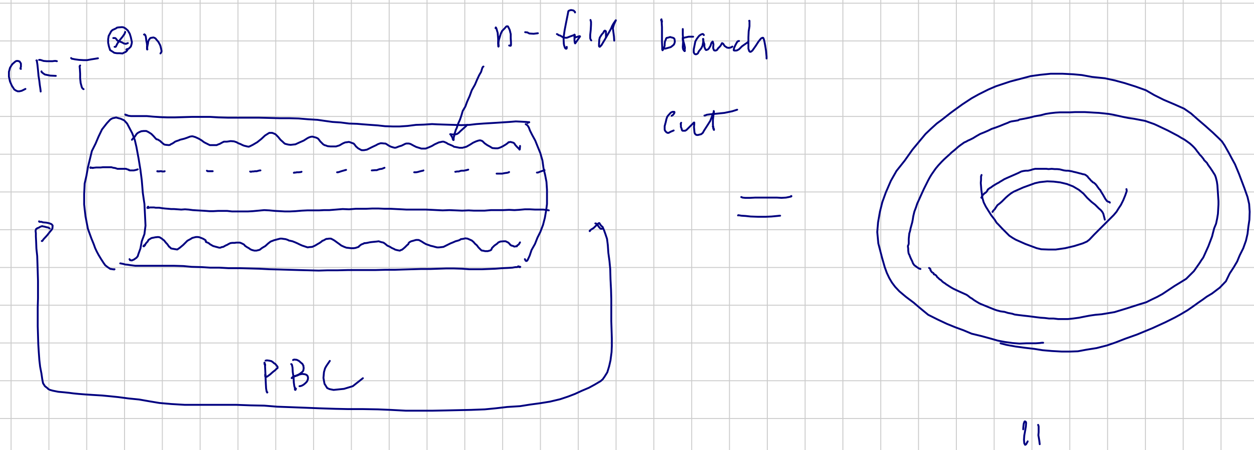


$\mathrm{Tr}\,\rho^n$

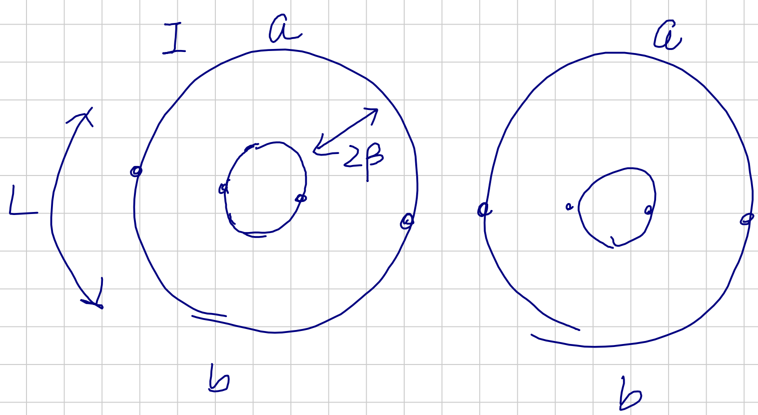


$\mathrm{CFT}^{\otimes n}$



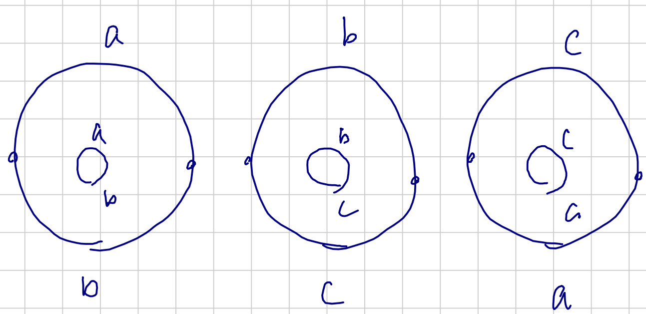


$CFT^{\otimes n}$
n-fold branch cut
PBC
=
||

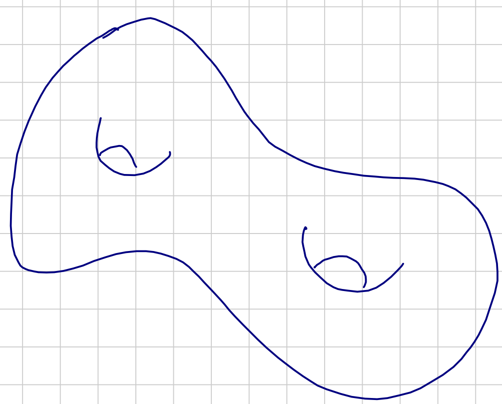


I
a
L
2β
b
a
b

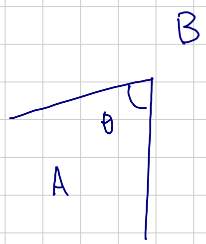
for 3-copies



a
a
b
b
b
b
c
c
c
c
a
a



Results



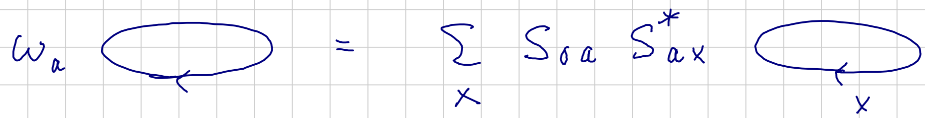
① $$S_A = \frac{\pi c L}{24\beta} + \underbrace{\frac{c}{3} \ln \sin \frac{\theta}{2}}_{\text{corner contrib}} - S_{top}.$$

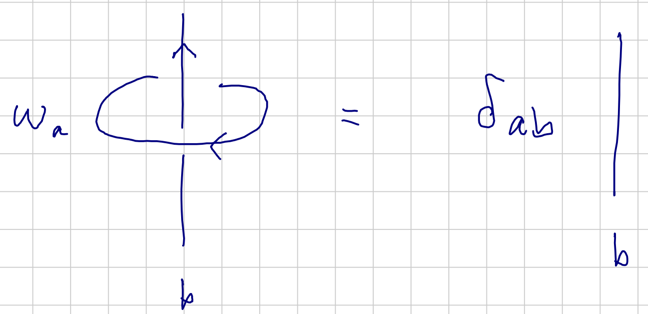
② $$h = \frac{c}{3} \ln 2.$$

non-chiral theories (Levin-Wen string-net)

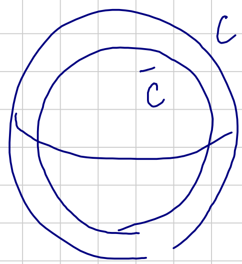
idea use anyon diagram to write GS

ω_a - loop

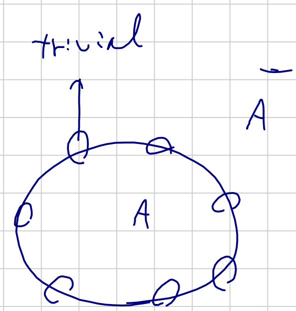




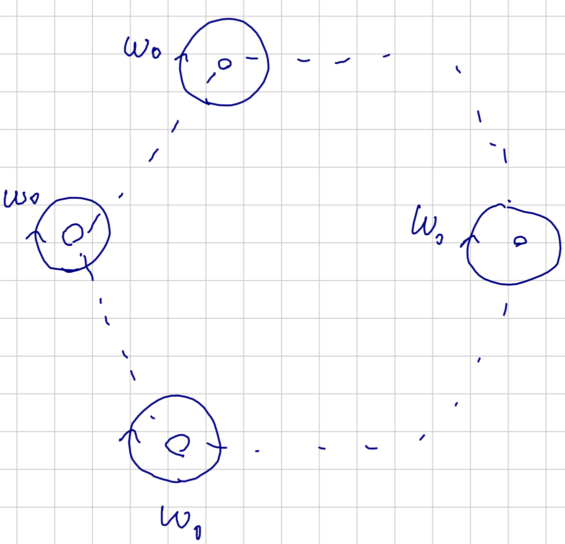
① introduce another sphere with $\bar{C}$



② introduce wormholes



$$|\psi\rangle = d^{n-1}$$



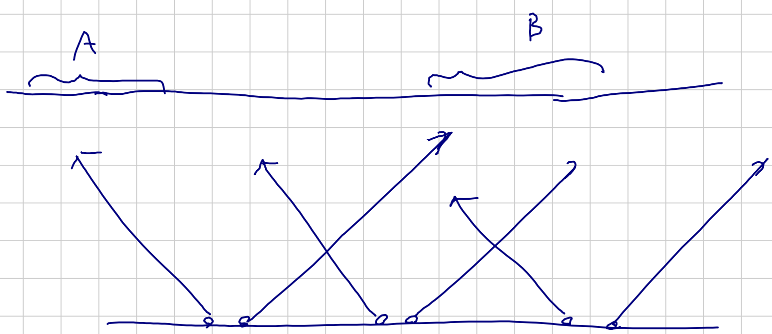
Non-equilibrium

How entanglement is generated ?

propagates ?

e.g. quantum quench

Quasi particle picture



free propagation of Bell pairs

Within Q. particle picture

$$R^{(n)} = I^{(n)}$$

ref. ent mutual info

$$\mathcal{E} = \frac{1}{2} I^{(1/2)}$$

Negativity

$$\Longrightarrow \quad h = R - I = 0$$

Markov gap. detects breakdown of

Q particle picture (and multi partite Ent)

Studied global and local operator quench

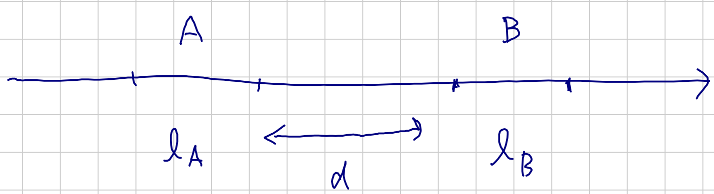
$$|\psi(t)\rangle = e^{-itH} |\psi_{init}\rangle$$

$$|\psi(t)\rangle \propto N \, e^{-itH} O(0) |0\rangle$$

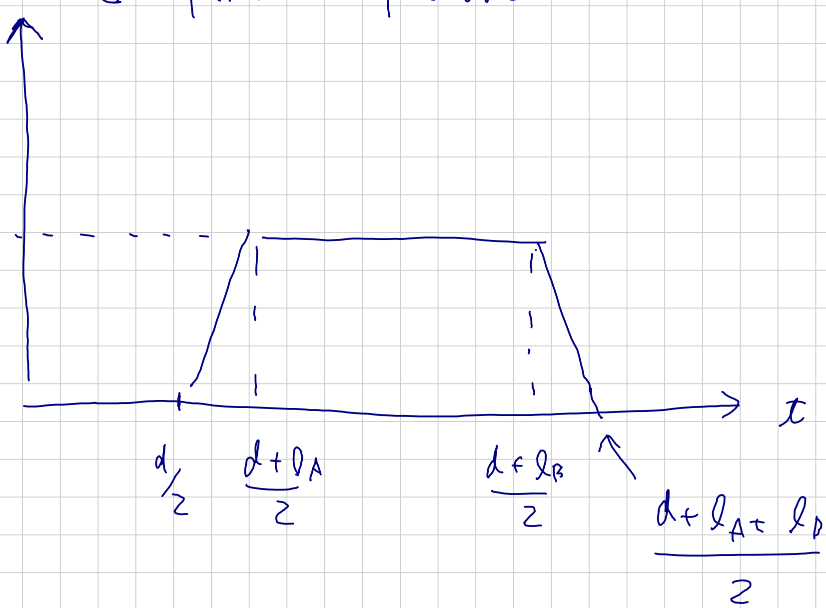
global quench

Assume

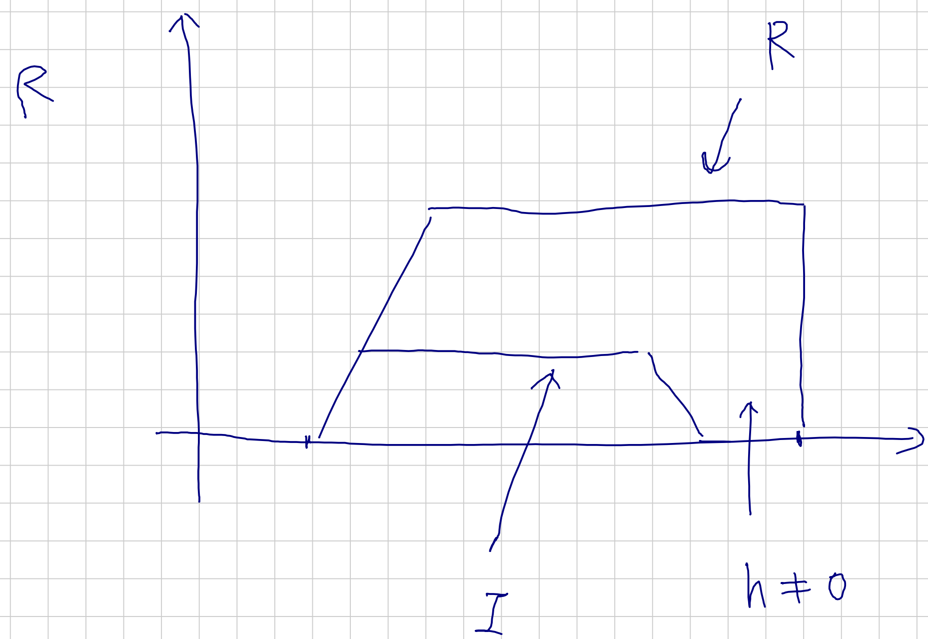
$d < \min(l_A, l_B)$



Q-particle picture



pure CFT



The difference comes from the difference of light-cone singularity.

Also: negativity > MI

Ent cost $\gg$ distillable Ent

significant bound Ent

How many Bell pair (ebit) is necessary to prepare the state