NUMERICAL IMPROVEMENTS ON S-WAVE PENETRATION FOR GAMOW FACTOR

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I. Motivation

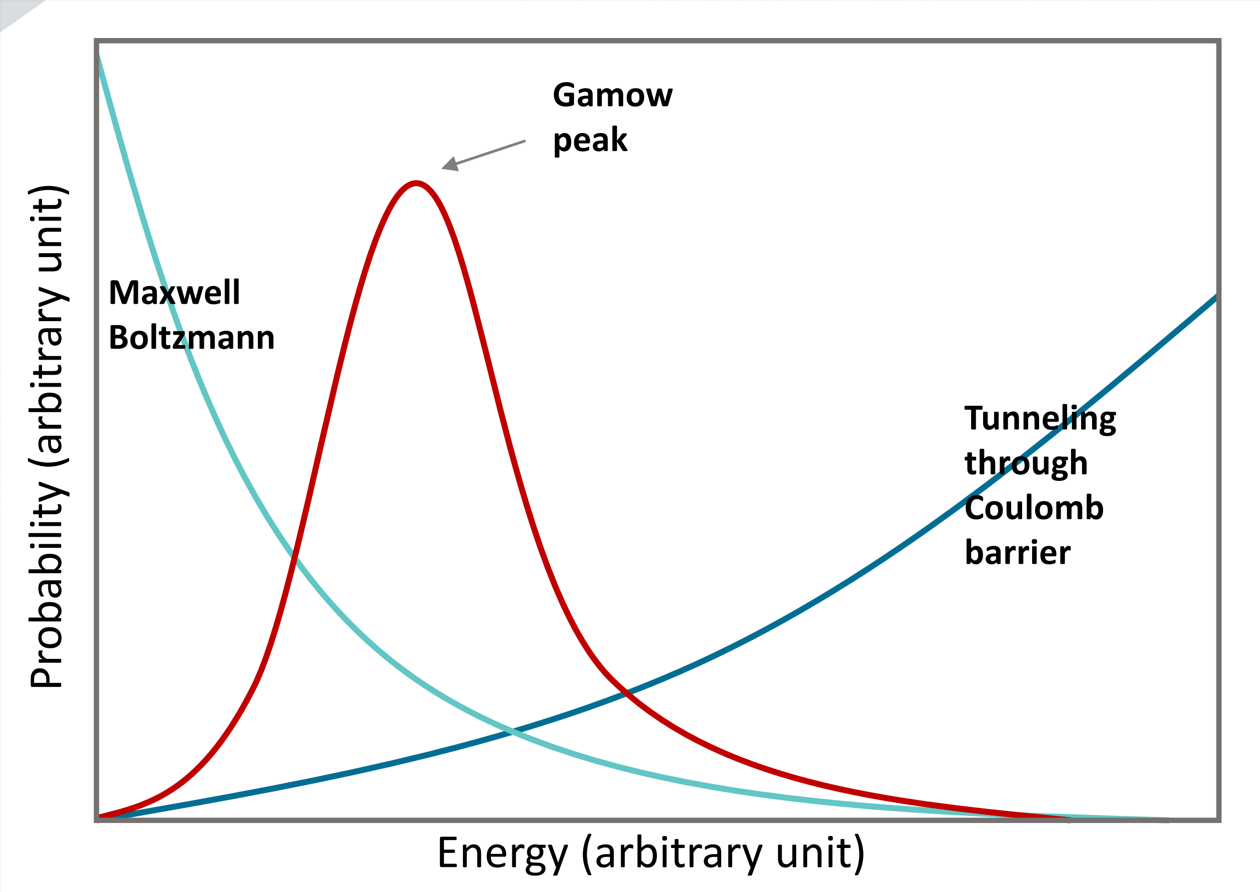
- Thermonuclear reaction rate

$$\langle\sigma v\rangle \propto \int E e^{-\frac{E}{T}} \sigma(E) dE$$
$$= \int e^{-\frac{E}{T}} e^{-2\pi\eta} S(E) dE$$

$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)$

$e^{-2\pi\eta}$: Gamow factor

$S(E)$: astrophysical S-factor



- Gamow factor

$$e^{-2\pi\eta} = \exp\left(-2\sqrt{2\mu}\int_{R_0}^{R_c}\sqrt{\frac{Z_1 Z_2 e^2}{r} - E}\,dr\right)$$

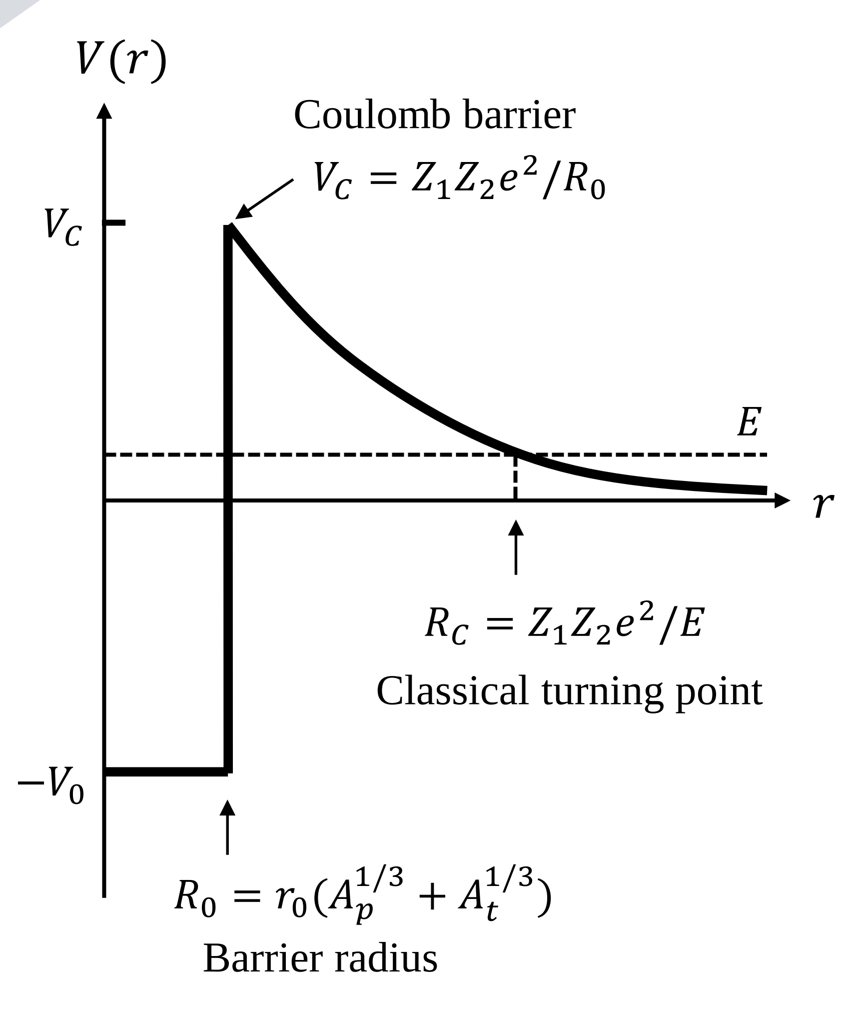
➢ The Gamow factor comes from the assumption that the energy is much smaller than the Coulomb barrier.

I. Motivation

- Coulomb barrier height

$$V_c = \frac{Z_1 Z_2 e^2}{R_0} \qquad R_0 = R_p + R_t \approx r_0(A_p^{1/3} + A_t^{1/3})$$

Reaction	Coloumb barrier (MeV)	A_1	A_2	Z_1	Z_2
$d+d$	0.4762	2	2	1	1
$d+p$	0.5309	2	1	1	1
$t+d$	0.4440	3	2	1	1
$t+\alpha$	0.7921	3	4	1	2
^{3}He + d	0.8881	3	2	2	1
$^3\mathrm{He}+\alpha$	1.5843	3	4	2	2
^{7}Li + p	1.2358	7	1	3	1



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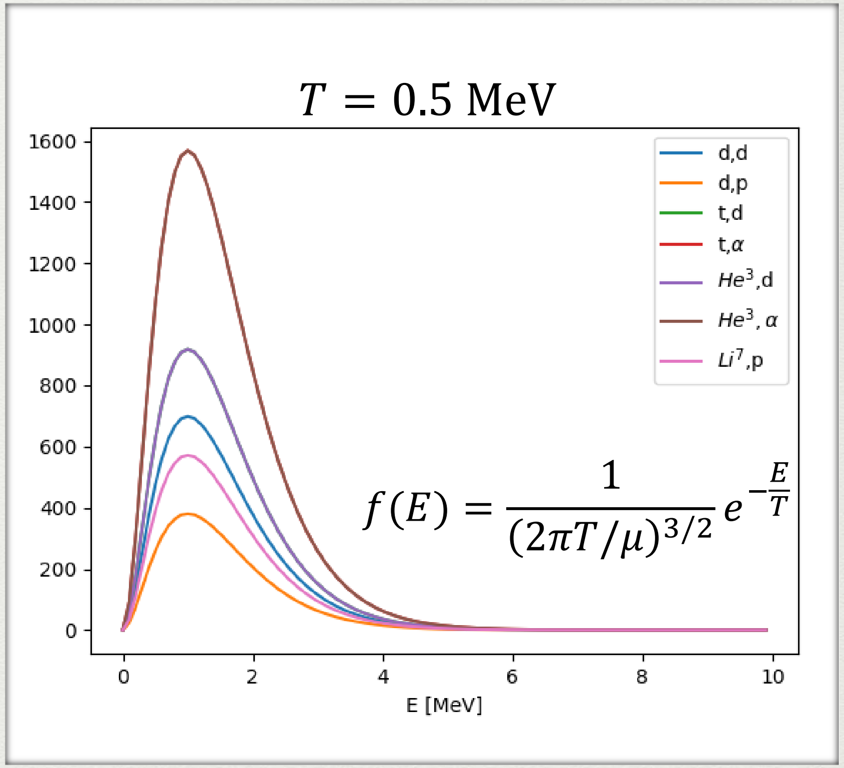


II. Formalism

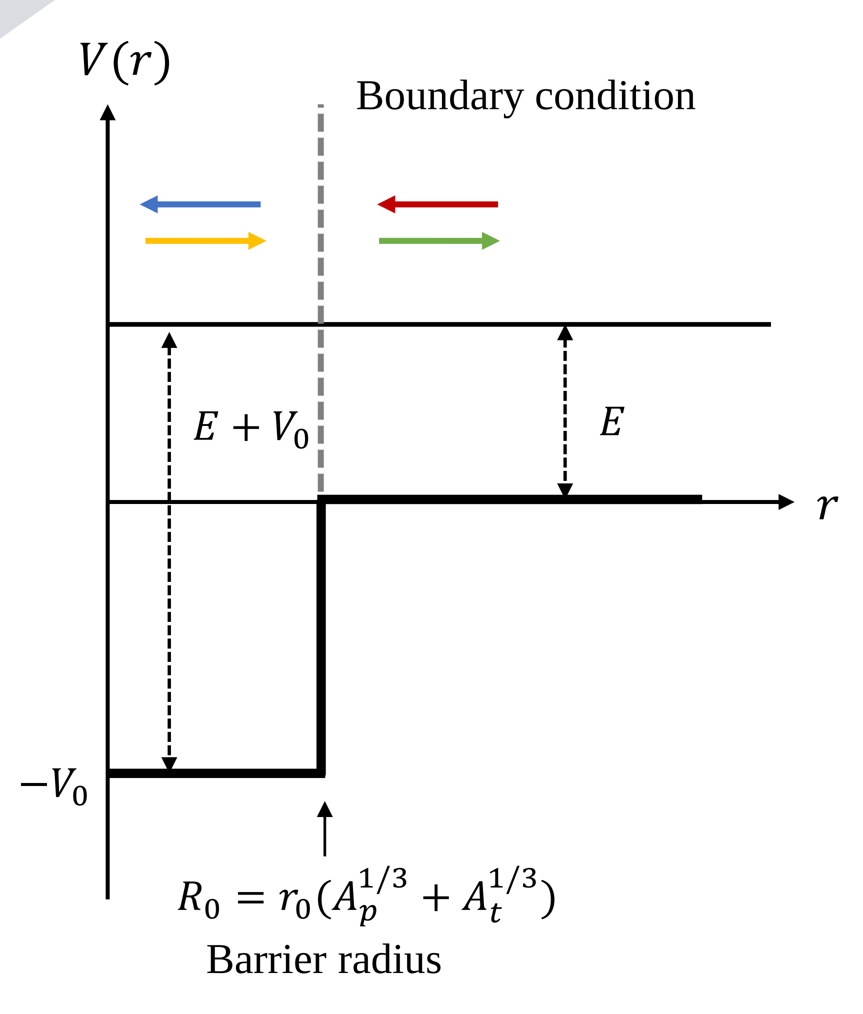
- Square-well potential

Radial equation

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}[E - V(r)]u = 0 \qquad \boxed{(l = 0)}$$

$$\frac{d^2u}{dr^2} + \hat{k}^2 u = 0 \rightarrow u(r) = \alpha e^{i\hat{k}r} + \beta e^{-i\hat{k}r}$$

- $r < R_0$ $\qquad u_{in} = A' e^{iKr} + B' e^{-iKr} \qquad K^2 \equiv \frac{2m}{\hbar^2}(E + V_0)$
- $r > R_0$ $\qquad u_{out} = C' e^{ikr} + D' e^{-ikr} \qquad k^2 \equiv \frac{2m}{\hbar^2}E$



II. Formalism

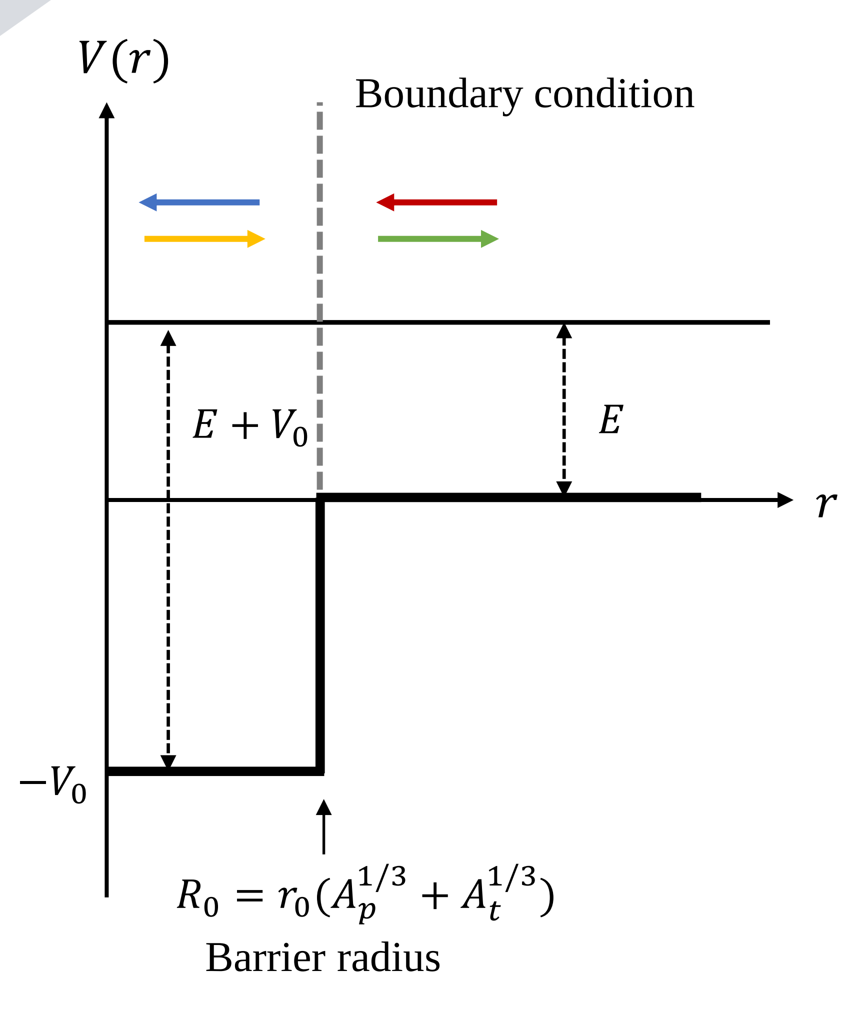
- Square-well potential

Matching condition

$$(u_{in})_{R_0} = (u_{out})_{R_0} \quad \rightarrow \quad A'e^{iKR_0} + B'e^{-iKR_0} = C'e^{ikR_0} + D'e^{ikR_0}$$

$$\left(\frac{du_{in}}{dx}\right)_{R_0} = \left(\frac{du_{out}}{dx}\right)_{R_0} \quad \rightarrow \quad \frac{K}{k}\left(A'e^{iKR_0} - B'e^{-iKR_0}\right) = \left(C'e^{ikR_0} - D'e^{ikR_0}\right)$$

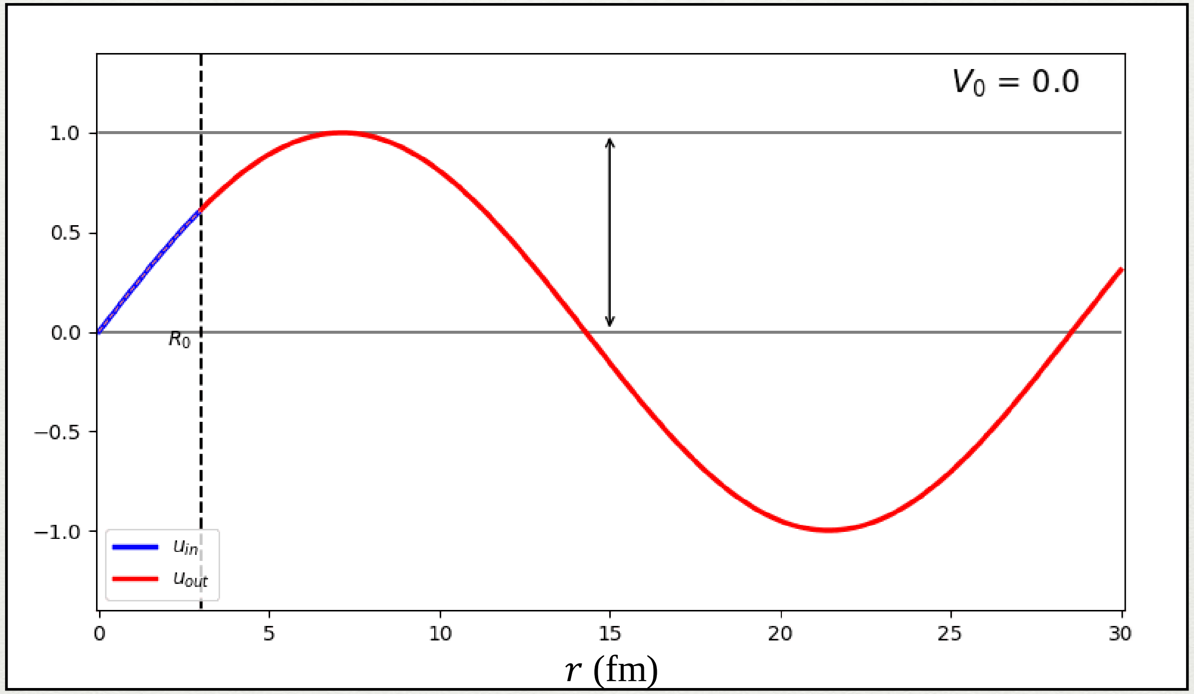
$$\boxed{\hat{T} = \frac{K|B'|^2}{k|D'|^2} = 4\frac{kK}{(K+k)^2} = 4\frac{\frac{2m}{\hbar^2}\sqrt{(E+V_0)E}}{\left[\sqrt{\frac{2m}{\hbar^2}(E+V_0)} + \sqrt{\frac{2m}{\hbar^2}E}\right]^2}}$$



II. Formalism

- Square-well potential

  - ➢ The transition probability is not always equal to 1 even when only attractive potential are present.
  - ➢ The transition probability of the potential well depends on the potential depth.



$$R_0 = 3\text{ fm},\qquad E = 1\text{ MeV}$$

II. Formalism

- Square-Barrier potential

Wave function

$$u_{\mathrm{I}} = Ae^{iKx} + Be^{-iKx}$$

$$u_{\mathrm{II}} = Ce^{-\kappa x} + De^{\kappa x}$$

$$u_{\mathrm{III}} = Fe^{ikx} + Ge^{-ikx}$$

$$K^2 = \left(\frac{2m}{\hbar^2}\right)(E + V_0)$$

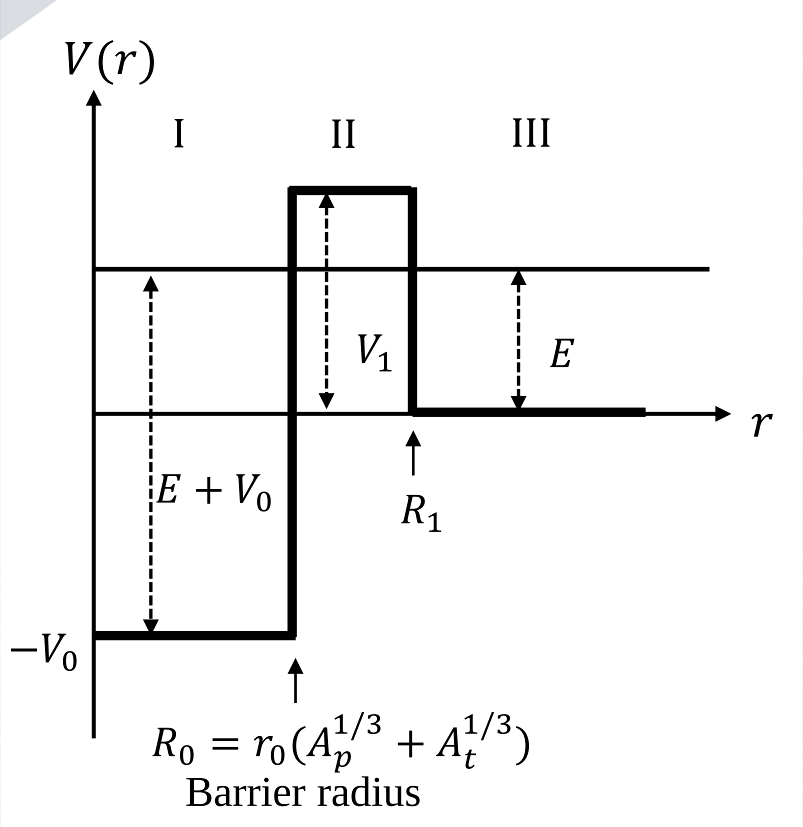
$$i^2\kappa^2 = i^2\left(\frac{2m}{\hbar^2}\right)(V_1 - E)$$

$$k^2 = \left(\frac{2m}{\hbar^2}\right)E$$

Matching condition

$$(u_{\mathrm{I}})_{R_0} = (u_{\mathrm{II}})_{R_0} \qquad (u_{\mathrm{II}})_{R_1} = (u_{\mathrm{III}})_{R_1}$$

$$\left(\frac{du_{\mathrm{I}}}{dx}\right)_{R_0} = \left(\frac{du_{\mathrm{II}}}{dx}\right)_{R_0} \qquad \left(\frac{du_{\mathrm{II}}}{dx}\right)_{R_1} = \left(\frac{du_{\mathrm{III}}}{dx}\right)_{R_1}$$



$$\boxed{\hat{T} = \frac{K}{k}\frac{|B|^2}{|G|^2} = \frac{4\sqrt{E(E+V_0)}}{\left\{\left[2E + V_0 + 2\sqrt{E(E+V_0)} + \left[E + V_0 + V_1 + \frac{E(E+V_0)}{V_1 - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_1 - E)}(\mathrm{R}_1 - \mathrm{R}_0)\right]\right]\right\}}}$$

II. Formalism

- Square-Barrier potential

$$\hat{T} = \frac{K}{k}\frac{|B|^2}{|G|^2} = \frac{4\sqrt{E(E+V_0)}}{\left\{\left[2E + V_0 + 2\sqrt{E(E+V_0)} + \left[E + V_0 + V_1 + \frac{E(E+V_0)}{V_1 - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_1 - E)}(\mathrm{R}_1 - \mathrm{R}_0)\right]\right]\right\}}$$

Assumption : The energy E is much smaller than the potential barrier V_1.

$$\frac{\sqrt{2m(V_1 - E)}}{\hbar}(R_1 - R_0) \gg 1$$

$$\Downarrow$$

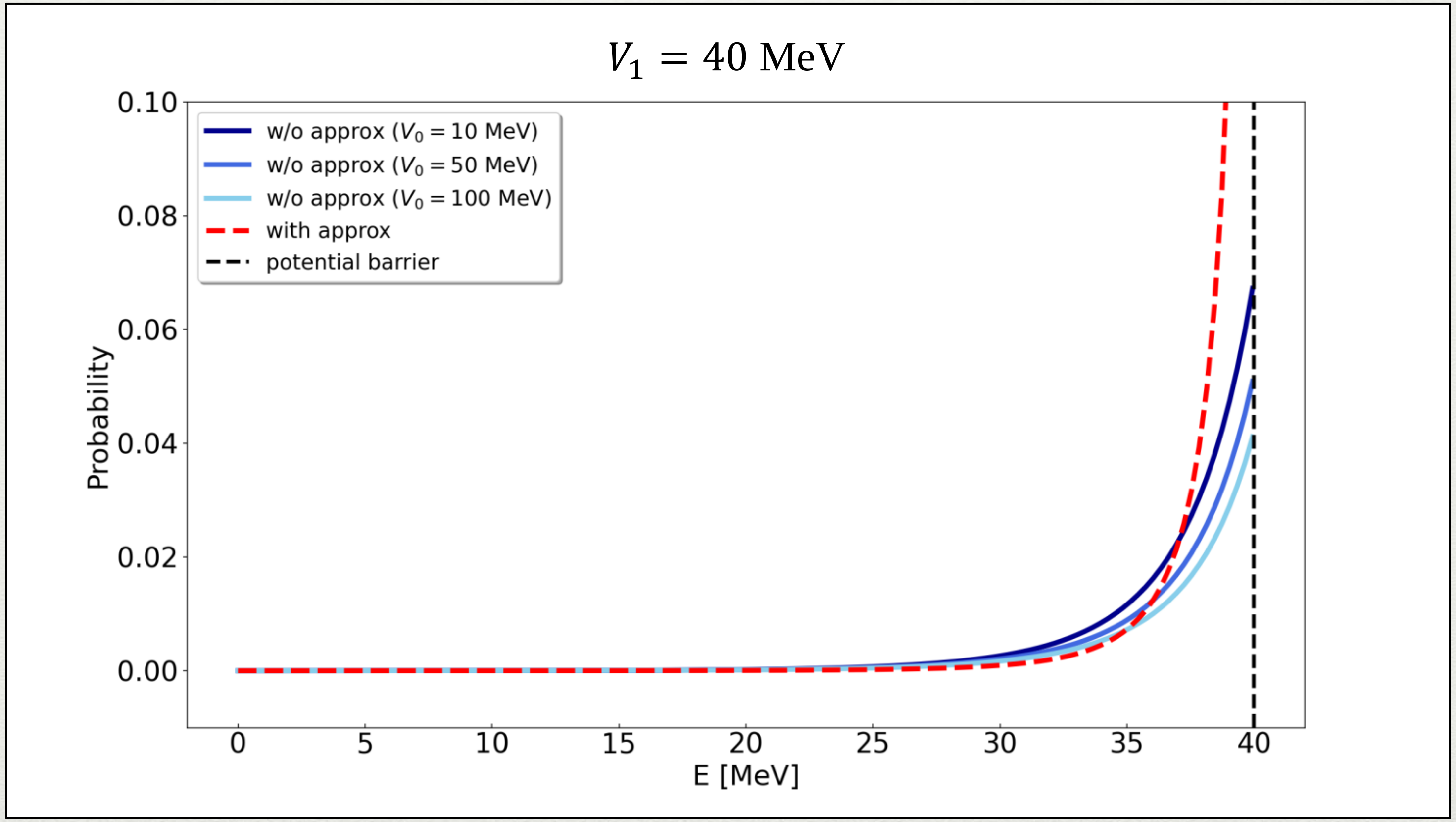
$$\hat{T} \approx e^{-\left(\frac{2}{\hbar}\right)\sqrt{2m(V_1 - E)}(\mathrm{R}_1 - \mathrm{R}_0)}$$

II. Formalism

- Square-Barrier potential

$$\hat{T} = \frac{4\sqrt{E(E+V_0)}}{\left\{\left[2E + V_0 + 2\sqrt{E(E+V_0)} + \left[E + V_0 + V_1 + \frac{E(E+V_0)}{V_1 - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_1 - E)}\Delta\right]\right]\right\}}$$

$$\hat{T} \approx e^{-\left(\frac{2}{\hbar}\right)\sqrt{2m(V_1 - E)}(\mathrm{R}_1 - \mathrm{R}_0)}$$



II. Formalism

- Coulomb potential

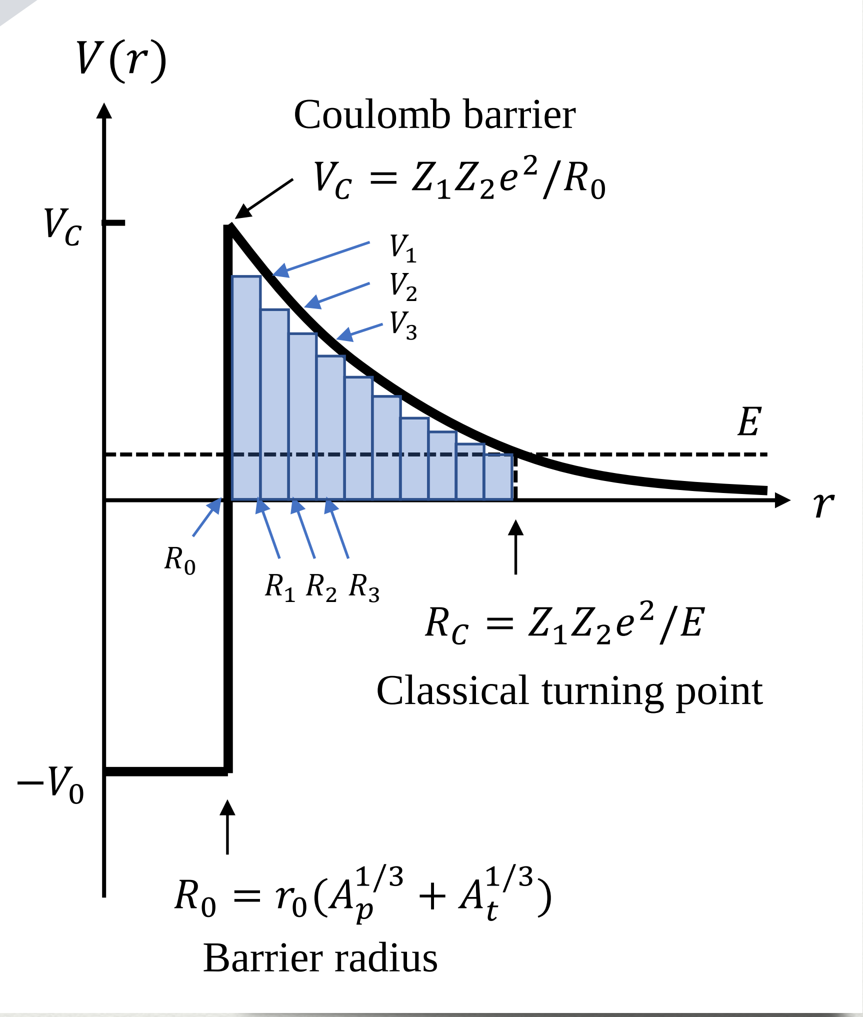
$$\hat{T} = \hat{T}_1 \cdot \hat{T}_2 \cdot \ldots \cdot \hat{T}_n \approx \exp\left[-\frac{2}{\hbar}\sum_i \sqrt{2m(V_i - E)}(R_{i+1} - R_i)\right]$$

$$\text{when } N \gg 1 \;\rightarrow\; \exp\left[-\frac{2}{\hbar}\int_{R_0}^{R_C} \sqrt{2m[V(r) - E]}\, dr\right]$$

$$\hat{T} \approx \exp\left(-\frac{2}{\hbar}\sqrt{2m}\int_{R_0}^{R_C} \sqrt{\frac{Z_0 Z_1 e^2}{r} - E}\, dr\right)$$

$$\approx \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m}{2E}} Z_0 Z_1 e^2\right) \equiv e^{-2\pi\eta}$$

η : Sommerfeld parameter



II. Formalism

- Coulomb potential

$$\hat{T} = \hat{T}_1 \hat{T}_2 \hat{T}_3 \cdots \hat{T}_n$$

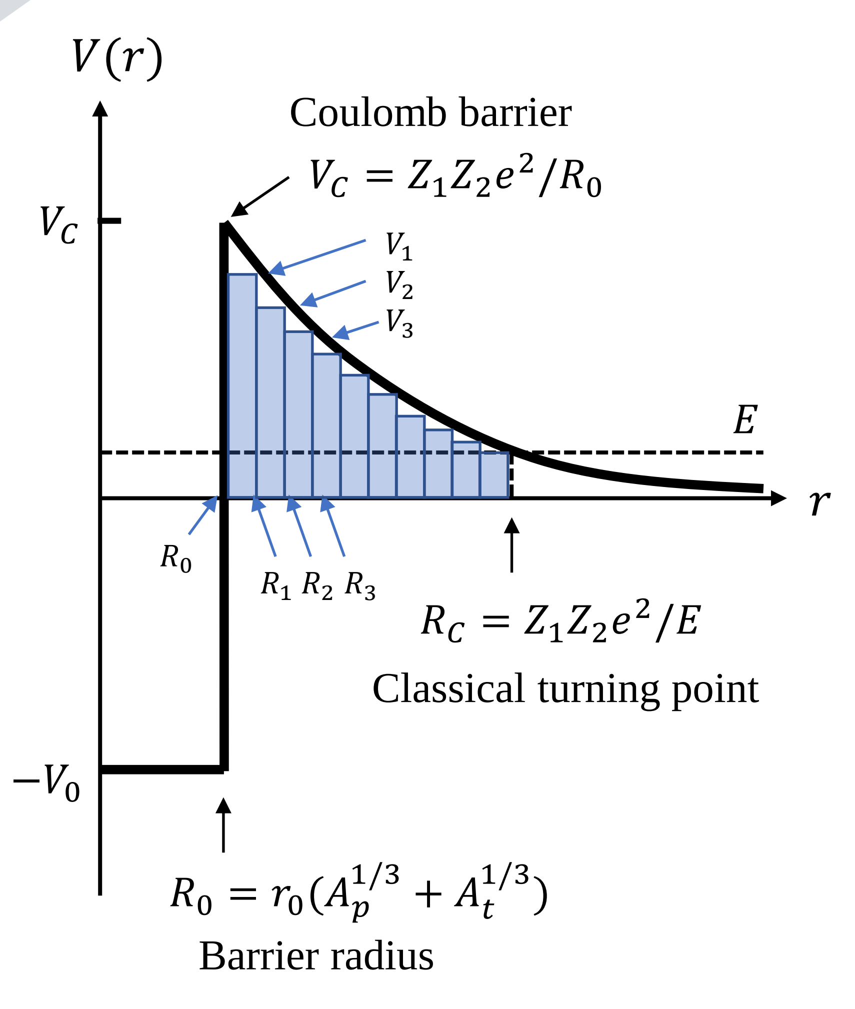
$$= \frac{4\sqrt{E(E+V_0)}}{\left\{\left[2E + V_0 + 2\sqrt{E(E+V_0)} + \left[E + V_0 + V_{C1} + \frac{E(E+V_0)}{V_1 - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_{C1}-E)}(R_1 - R_0)\right]\right]\right\}}$$

$$\times \frac{4E}{\left\{\left[4E + \left[E + V_{C2} + \frac{E^2}{V_{C2} - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_{C2}-E)}(R_2 - R_1)\right]\right]\right\}}$$

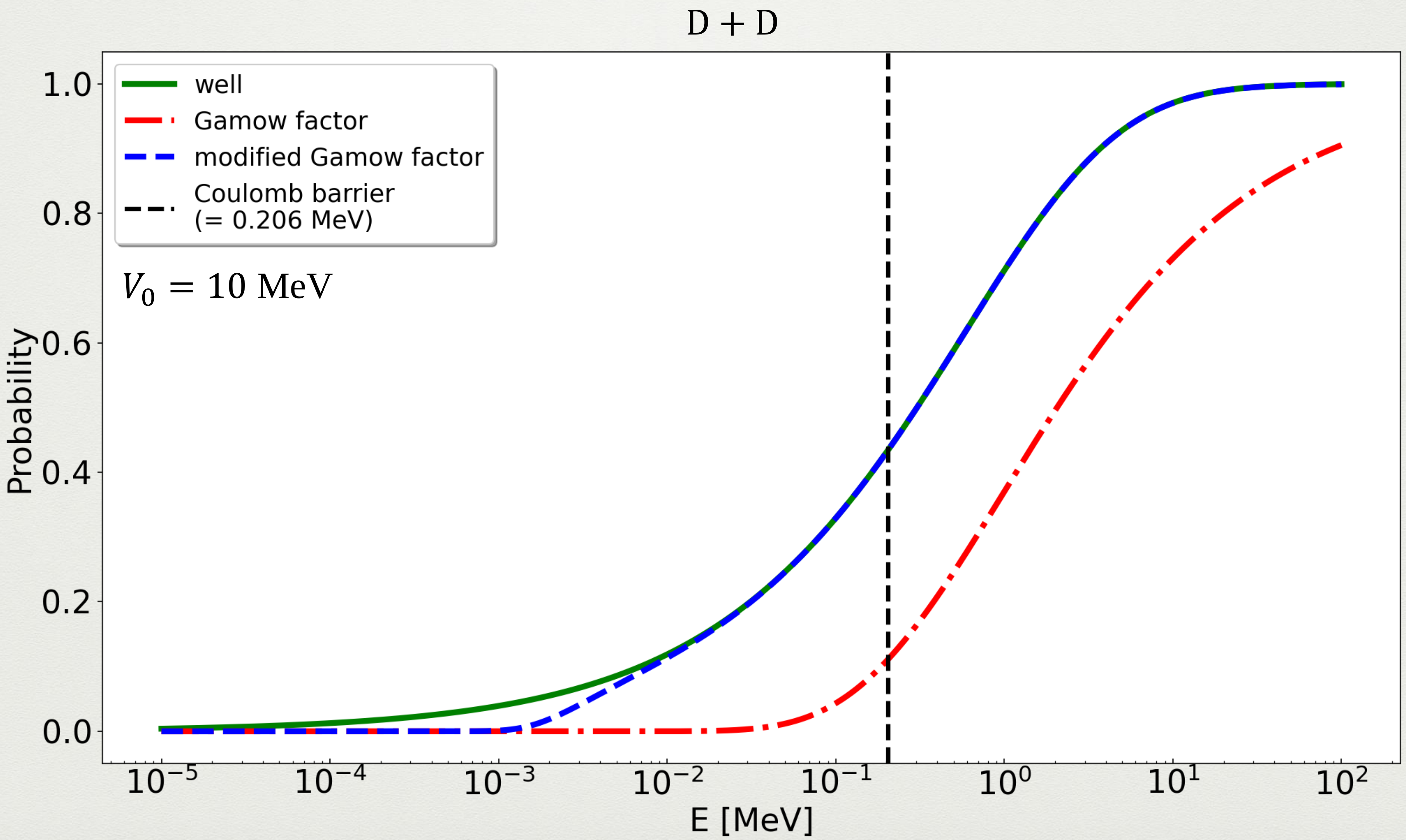
$$\vdots$$

$$\times \frac{4E}{\left\{\left[4E + \left[E + V_{C_n} + \frac{E^2}{V_{C_n} - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_{C_n}-E)}(R_n - R_{n-1})\right]\right]\right\}}$$

$$= T_1 \prod_{i=2}^{n} \frac{4E}{\left\{\left[4E + \left[E + V_{C_i} + \frac{E^2}{V_{C_i} - E}\right]\sinh^2\left[\sqrt{\frac{2m}{\hbar^2}(V_{C_i}-E)}(R_i - R_{i-1})\right]\right]\right\}}$$



III. Result : Gamow factor



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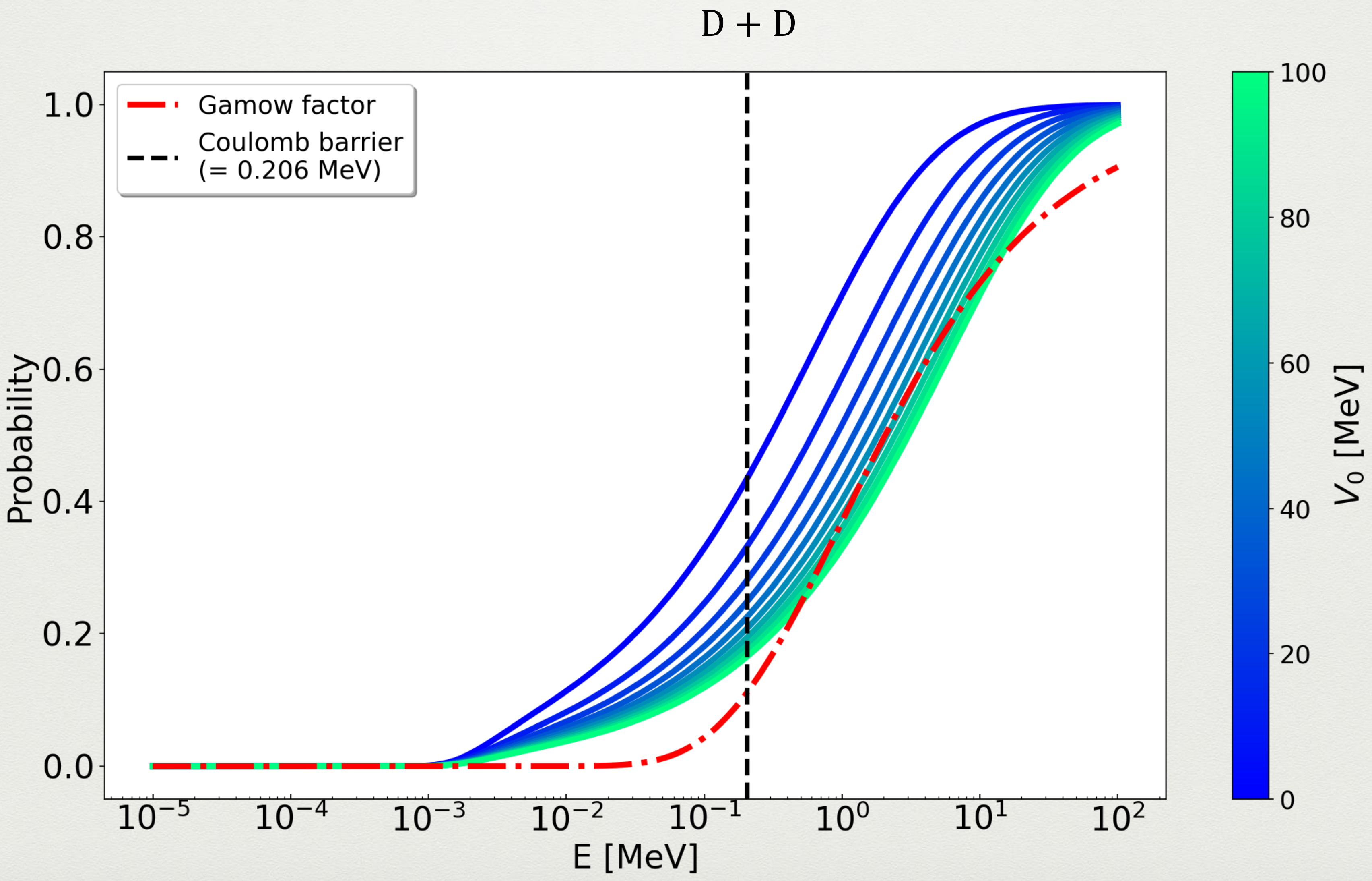

III. Result : Gamow factor

Astrophysical S-factor for deep sub-barrier fusion reactions of light nuclei

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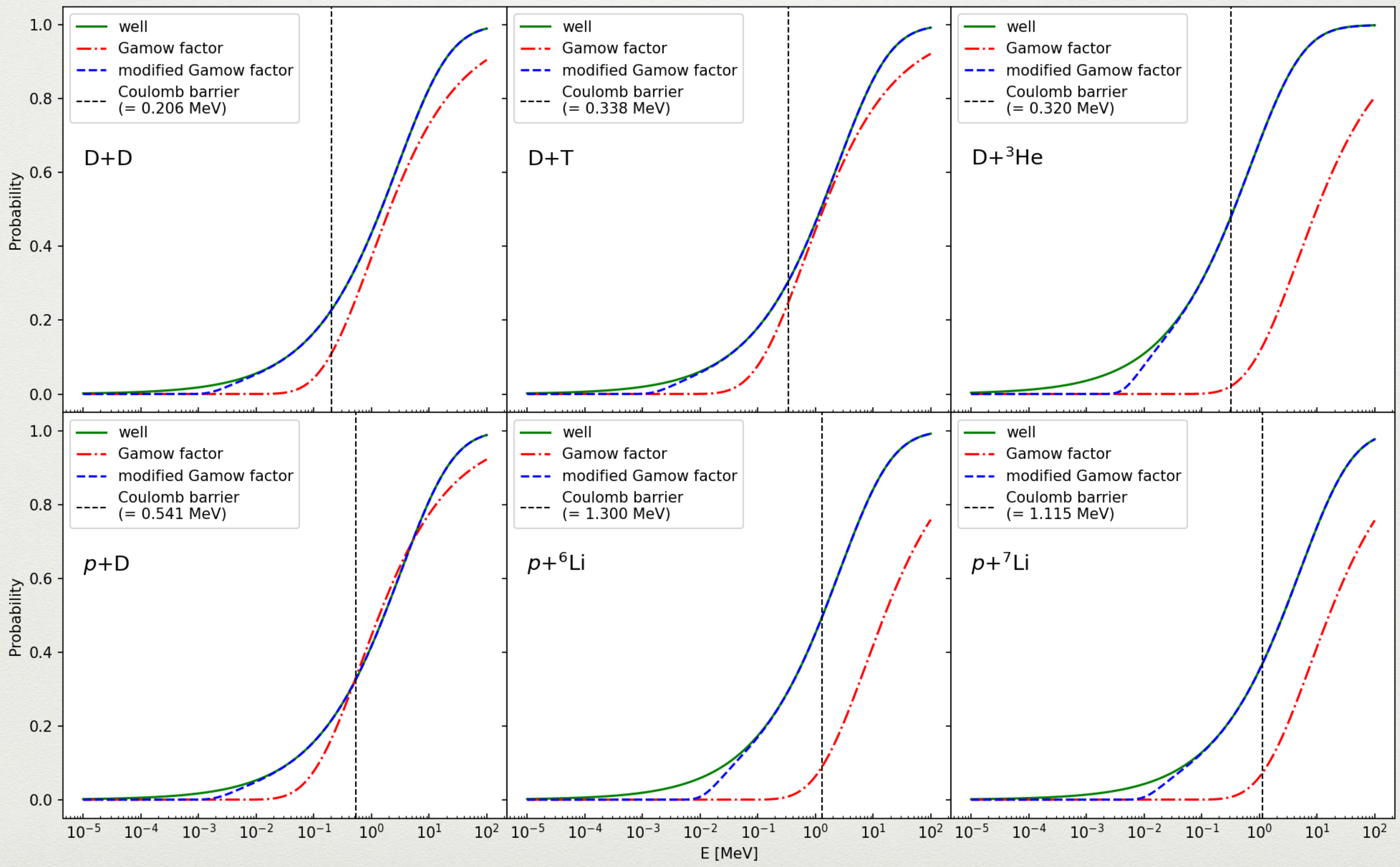
Table 1
Values of potential parameters for fusion reactions.

Reactions	V_r [MeV]	V_i [keV]	r_0 (fm)
D+D	−48.52	−263.27	2.778
D+T	−40.69	−109.18	1.887
D+^{3}He	−11.859	−259.02	3.331
p+D	−55.0	−0.0235	1.177
p+^{6}Li	−44.25	−7500	1.180
p+^{7}Li	−85.0	−395.0	1.330

III. Result : Gamow factor



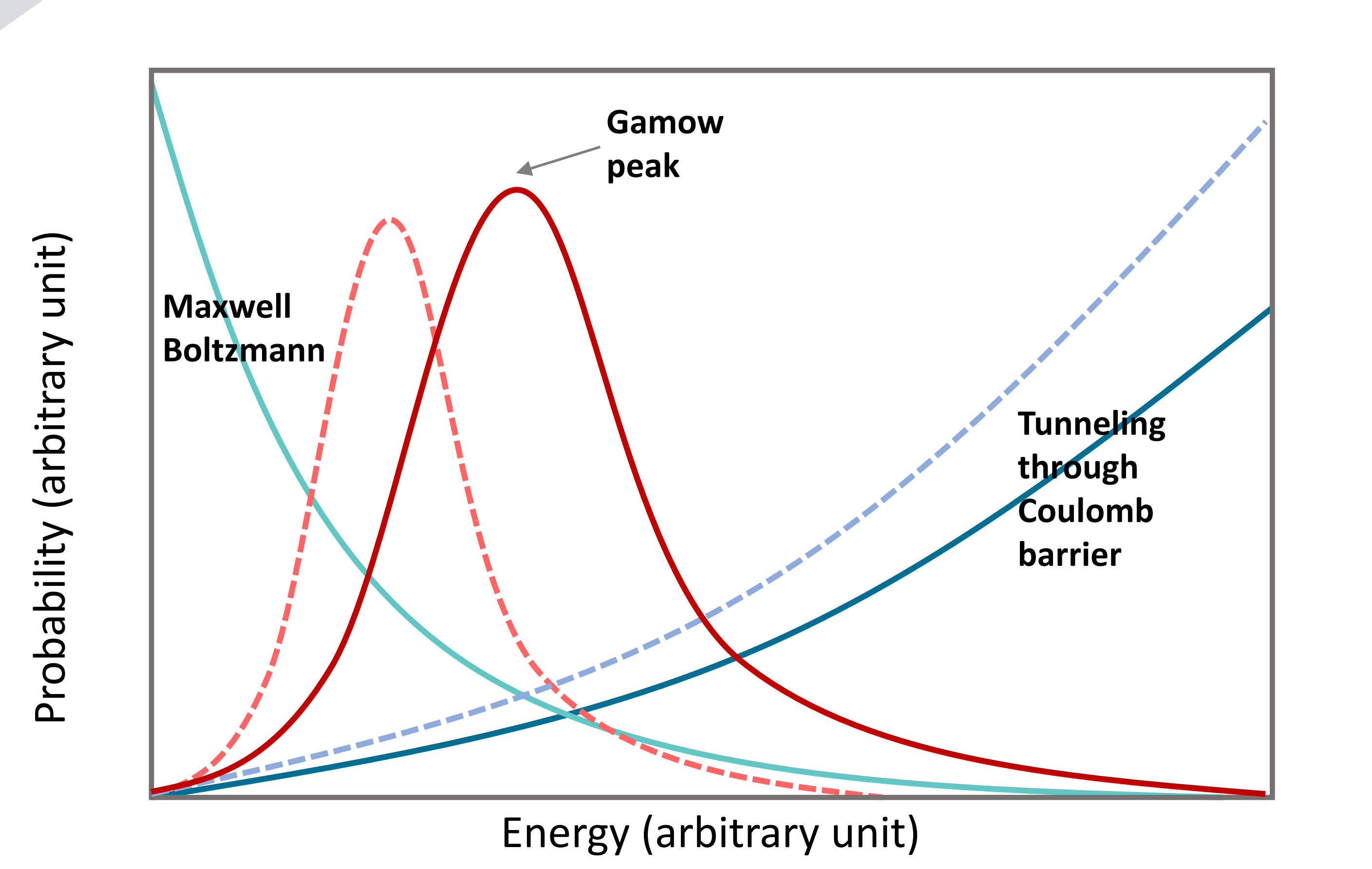
III. Result : Gamow energy

- **Gamow peak**

$$\langle \sigma v \rangle \propto \int E e^{-\frac{E}{T}} \sigma(E) dE = \int e^{-\frac{E}{T}} e^{-2\pi\eta} S(E) dE$$

$e^{-2\pi\eta}$: Gamow factor

$S(E)$: astrophysical S-factor



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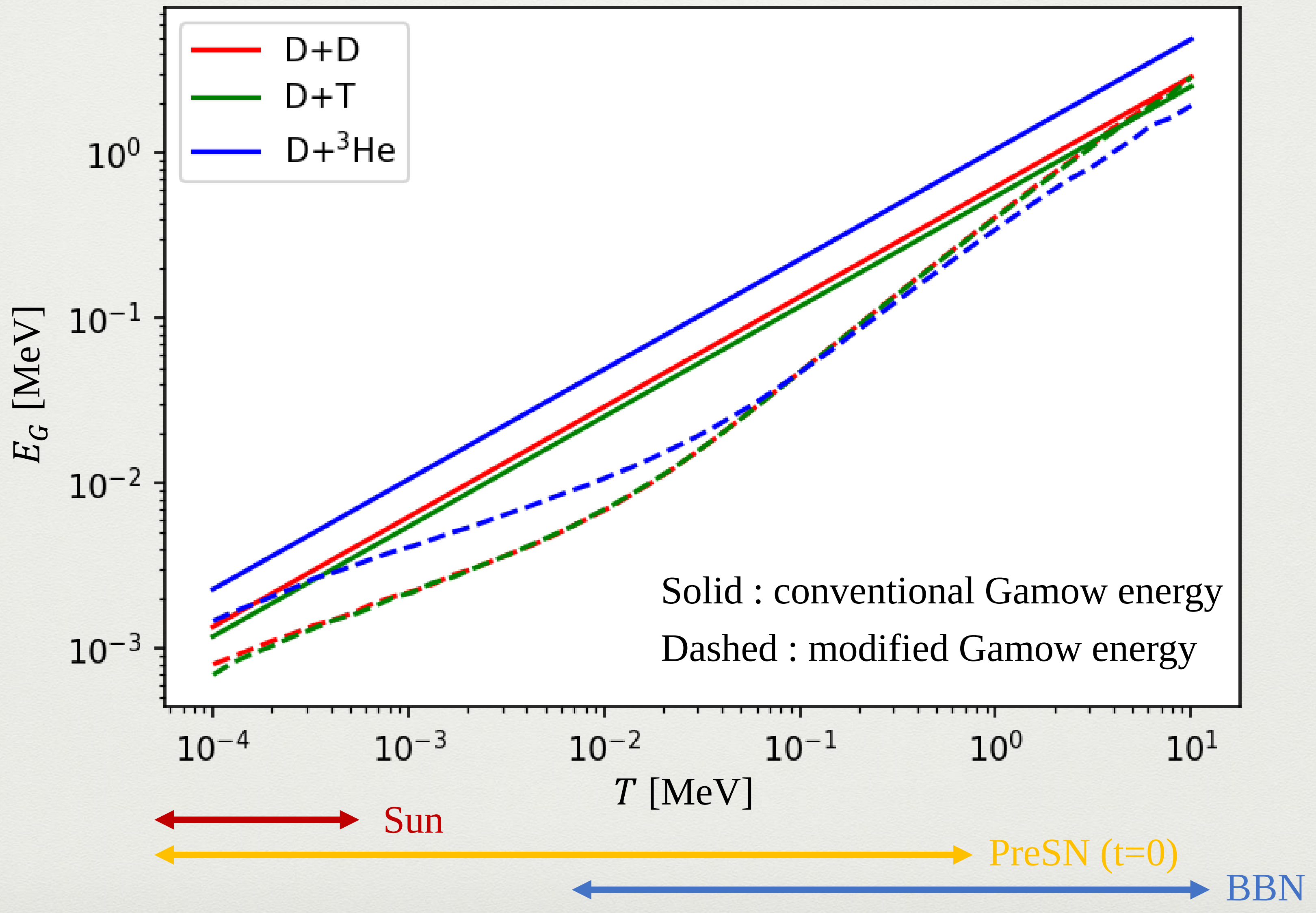

IV. Summary

- We found that the Gamow factor has two problems.

  1. The assumption used in the Gamow factor could be wrong.

  2. The Gamow factor is independent of the potential depth.

- We modified the Gamow factor using numerical calculation without the assumption.

- The modified Gamow factor depends on the potential depth, and it tends to be greater than the Gamow factor.

- We modified Gamow energy using our modified Gamow factor model.

THANK YOU FOR YOUR ATTENTION!