Alpha decay half-lives in the ground states of 74 ≤ Z ≤ 90 even-even isotopes

82 ≤ Z ≤ 90 :courtesy of Seonghyun Kim (SSU)

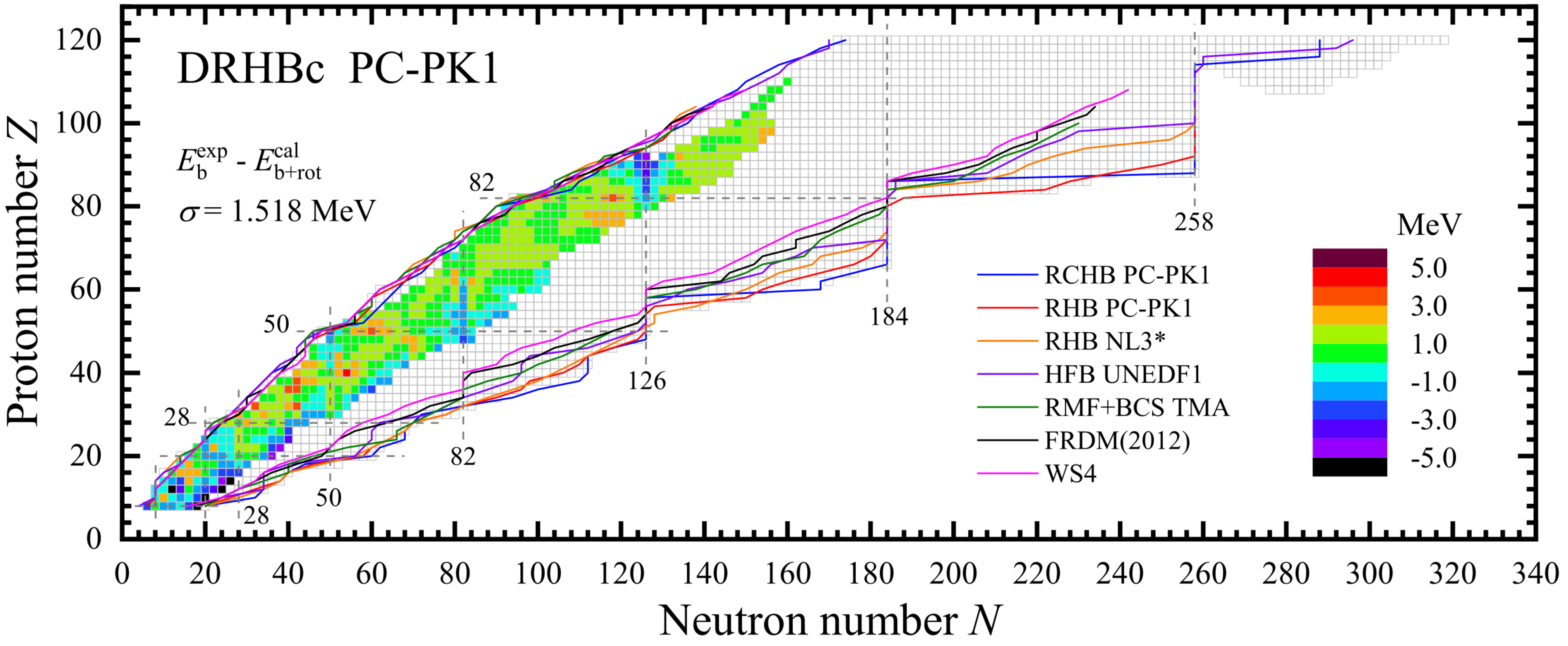
Yong-Beom Choi (UNIST)

Contents

- Introduction to DRHBc
- Alpha decay with WKB approximation
- Half-life comparison with experimental data
  - Empirical formulae
  - Two-parameter Fermi form
  - DRHBc
- Summary

DRHBc mass table project

Deformed **R**elativistic **H**artree-**B**ogoliubov theory in **c**ontinuum

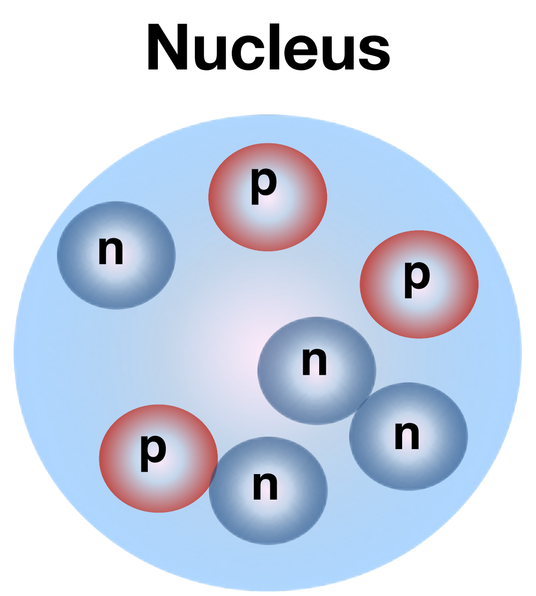


K. Zhang and DRHBc Mass Table Collaboration, Atomic Data and Nuclear Data Tables **144** 101488 (2022).

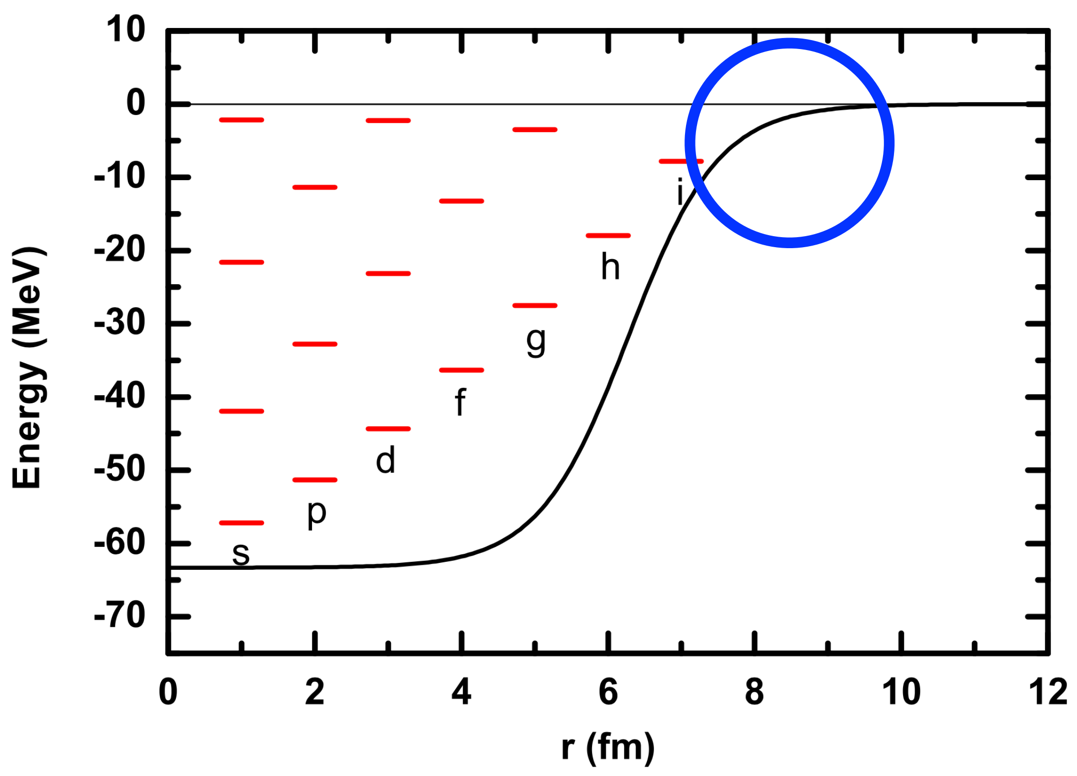
- DRHBc mass table project for ground state of nuclei with axial symmetry has begun in 2018 (Mass table for even-even nuclei is published in 2022).

Introduction to DRHBc with PC-PK1

Deformed **R**elativistic **H**artree-**B**ogoliubov theory in **c**ontinuum

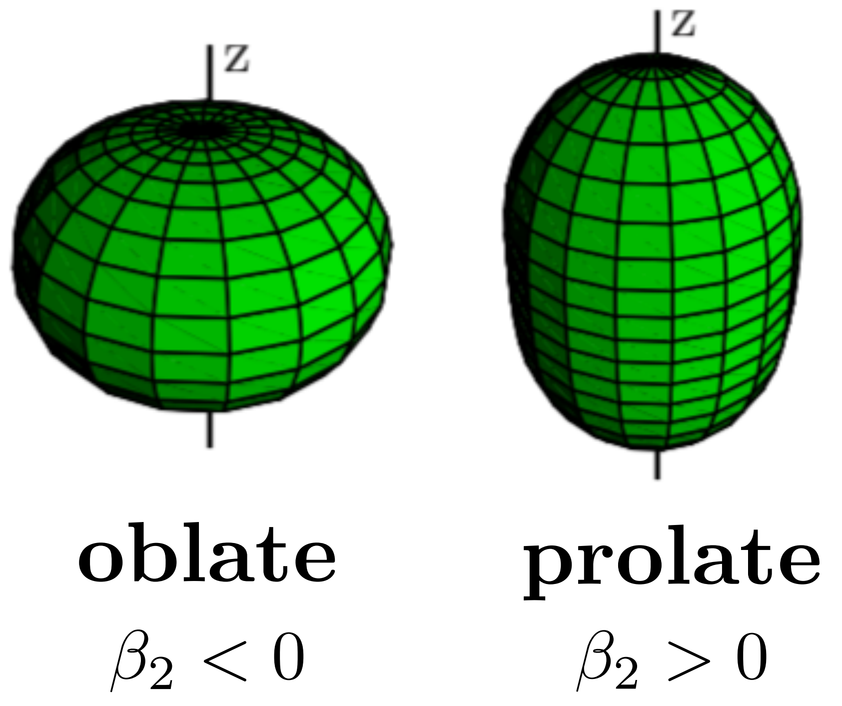


- Contact interactions between nucleons using PC-PK1 are adopted to Lagrangian.

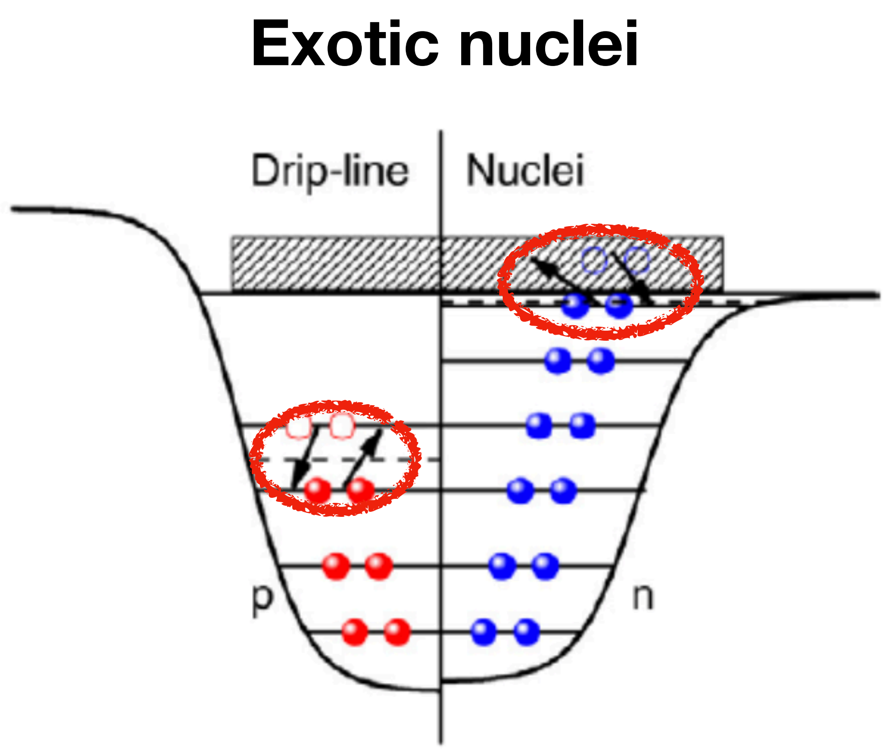


H. Liang *et al,* Phys. Rev. C **87**, 014334 (2013).

- Dirac Woods-Saxon basis is adopted; continuum is considered.



- Axial deformation degrees of freedom are included.



J. Meng *et al,* Prog. Part. Nucl. Phys. **57**, 470 (2006).

- Pairing potential is consistently included and solved together with mean field using HB equation.

Neutron drip line shift

Element (*Z*)	Neutron number No pairing	Neutron number RCHB	Element (*Z*)	Neutron number No pairing	Neutron number RCHB	Element (*Z*)	Neutron number No pairing	Neutron number RCHB
O (8)	20	20	Pd (46)	112	118	Po (84)	184	184
Ne (10)	20	32	Cd (48)	112	126	Rn (86)	184	184
Mg (12)	34	34	Sn (50)	126	126	Ra (88)	184	258
Si (14)	34	38	Te (52)	126	126	Th (90)	184	258
S (16)	40	40	Xe (54)	126	126	U (92)	234	258
Ar (18)	40	44	Ba (56)	126	126	Pu (94)	258	258
Ca (20)	40	60	Ce (58)	126	126	Cm (96)	258	258
Ti (22)	54	62	Nd (60)	126	168	Cf (98)	258	258
Cr (24)	62	68	Sm (62)	126	168	Fm (100)	258	258
Fe (26)	62	68	Gd (64)	142	176	No (102)	258	258
Ni (28)	70	70	Dy (66)	168	184	Rf (104)	258	258
Zn (30)	70	72	Er (68)	184	184	Sg (106)	258	258
Ge (32)	70	82	Yb (70)	184	184	Hs (108)	258	258
Se (34)	82	94	Hf (72)	184	184	Ds (110)	258	258
Kr (36)	82	100	W (74)	184	184	Cn (112)	258	258
Sr (38)	102	110	Os (76)	184	184	Fl (114)	258	258
Zr (40)	112	112	Pt (78)	184	184	Lv (116)	258	288
Mo (42)	112	112	Hg (80)	184	184	Og (118)	258	288
Ru (44)	112	112	Pb (82)	184	184	$Z = 120$	288	288

X. W. Xia *et al*, At. Data Nucl. Data Tables **121-122**, 1 (2018).

Element (Z)	neutron number (N) RCHB	neutron number (N) DRHBc
O (8)	20	20
Ne (10)	32	28
Mg (12)	34	34
Si (14)	38	38
S (16)	40	40
Ar (18)	44	52
Ca (20)	60	60

E. J. In *et al*, Int. J. Mod. Phys. E **30**, 2150009 (2021)

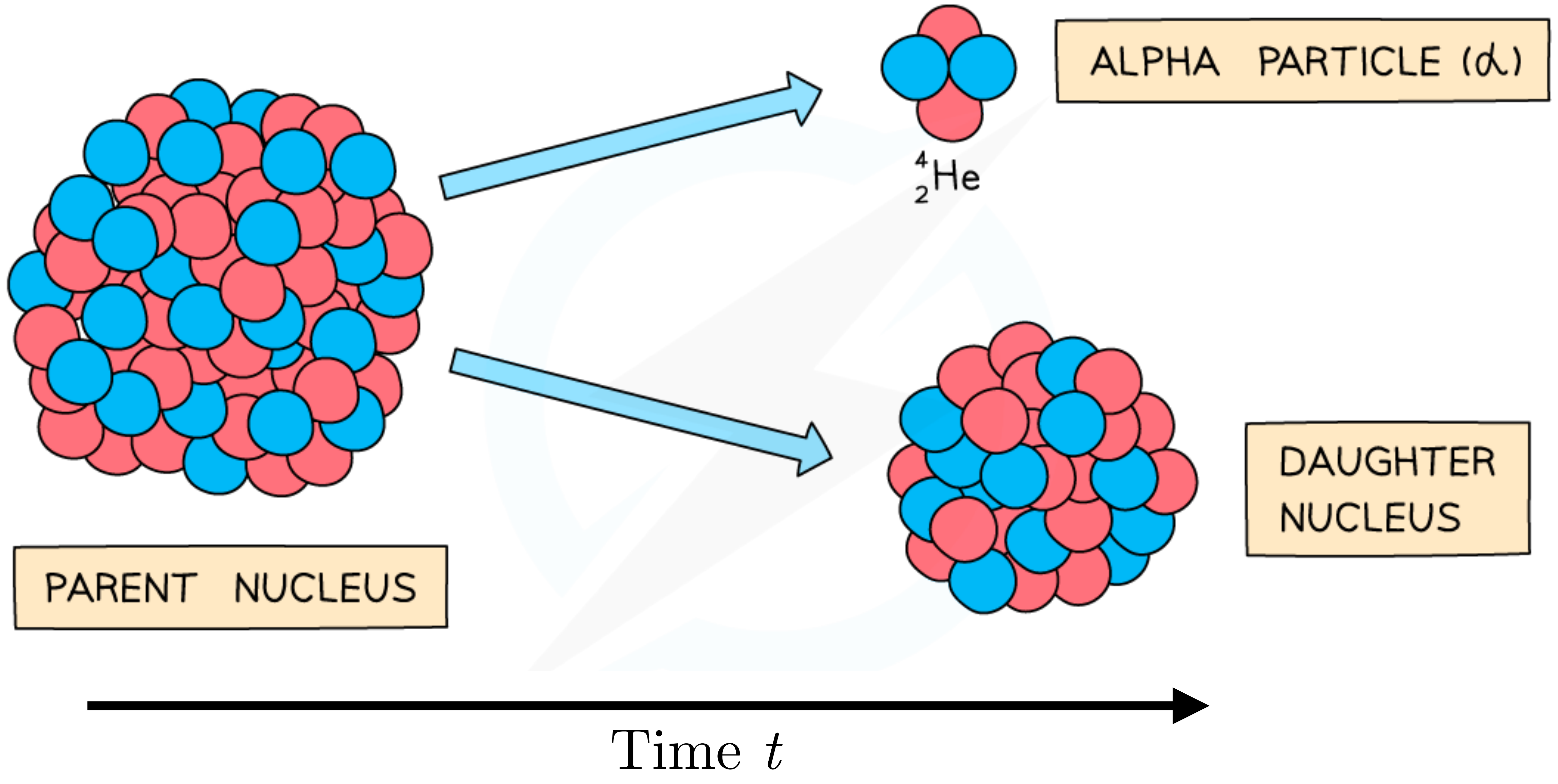
(RCHB: continuum+pairing potential)

(DRHBc: continuum+pairing potential+deformation)

- In the present of the continuum, including pairing potential and deformation could change the neutron drip line.

X. Y. Qu *et al*, Sci. Chin. Phys. Mech. Astron. **56**, 2031 (2013).

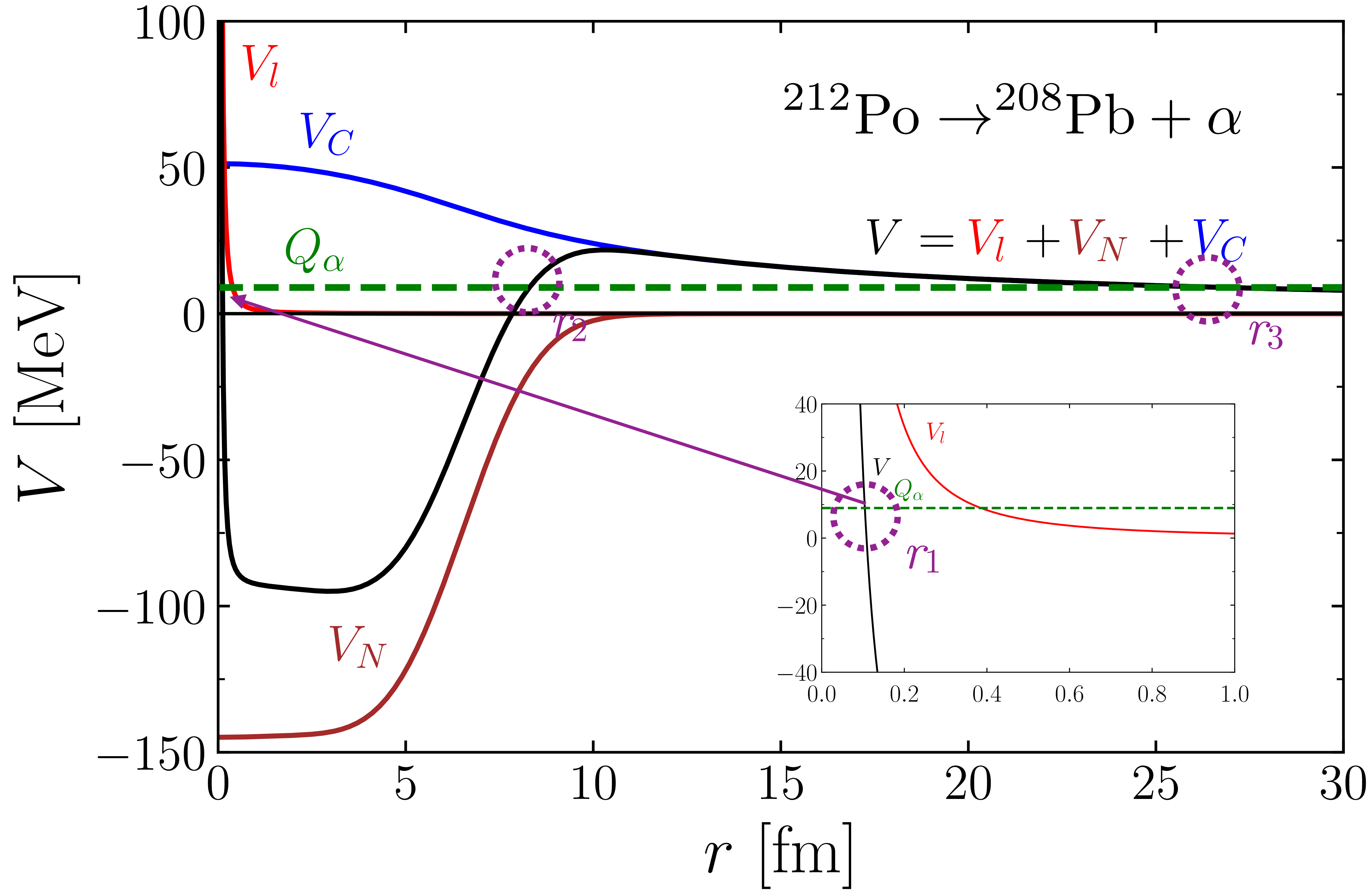
Research motivation

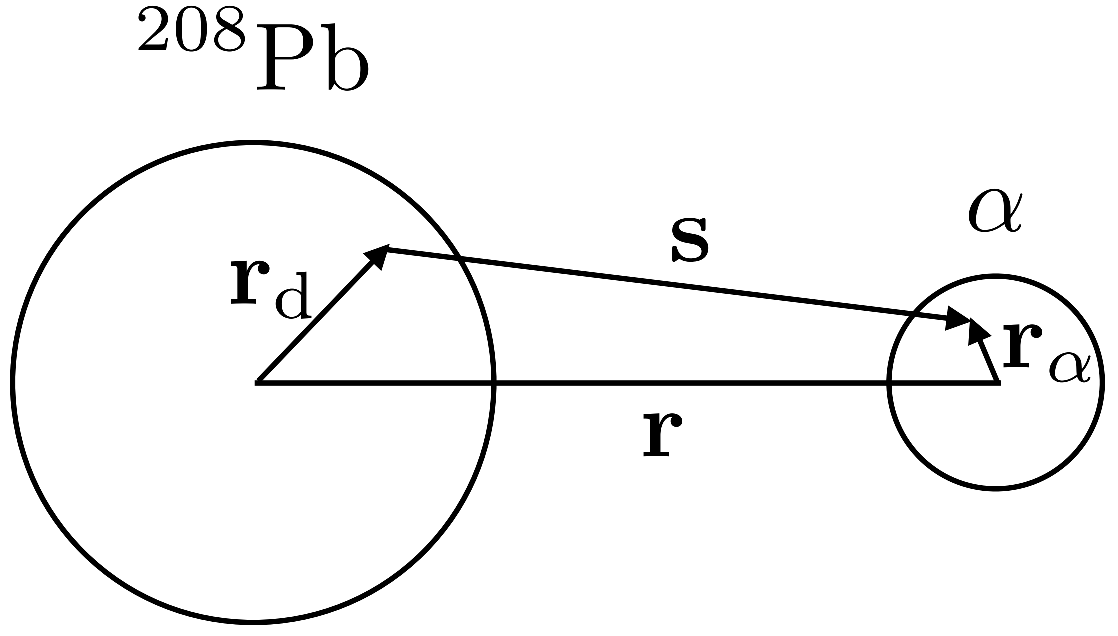


- The alpha decay process could be studied using DRHBc theory.
- Alpha decay mainly occurs in neutron-deficient nuclei.

Alpha decay with WKB approximation

Potential between ^{208}Pb and alpha particle





$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r)|}$$

$$\Gamma = P_\alpha N(k)\frac{\hbar^2}{4\mu}\exp[-2\int_{r_2}^{r_3} k(r')dr']$$

$$T_{1/2} = \hbar \ln 2/\Gamma$$

S. A. Gurvitz *et al*, Phys. Rev. Lett. **59**, 262 (1987)

C. Xu *et al*, Phys. Rev. C **74**, 014304 (2006)

- We need to know the wave number $k(r)$ consisting of V_l, V_N, and V_C and the points r_1, r_2, and r_3 ($V = Q_\alpha$).

Alpha decay with WKB approximation

$$T_{1/2} = \hbar \ln 2/\Gamma$$

$$\Gamma = P_\alpha N \frac{\hbar^2}{4\mu} \exp[-2\int_{r_2}^{r_3} k(r')dr']$$

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r)|}$$

Q_α from AME2020

normalization factor (for bound-stated wave function)

$$N \simeq \left[\int_{r_1}^{r_2} \frac{dr'}{2k(r')}\right]^{-1}$$

Phys. Rev. Lett. **59**, 262 (1987)

Preformation factor (of alpha particle)
to fit experimental data

$$P_\alpha = 1$$

$$V_l(r) = \frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2} \simeq \frac{\hbar^2}{2\mu}\frac{(l+1/2)^2}{r^2}$$ (l=0 for even-even nuclei)

R. E. Lange, Phys. Rev. 51, 669 (1937)

$$V_C(r) = \int d\mathbf{r}_d d\mathbf{r}_\alpha \rho_d(\mathbf{r}_d)\rho_\alpha(\mathbf{r}_\alpha)\frac{e^2}{r}$$

$$V_N(\mathbf{r}) = \lambda \int d\mathbf{r}_d d\mathbf{r}_\alpha \rho_d(\mathbf{r}_d)\rho_\alpha(\mathbf{r}_\alpha) v(\mathbf{s} = \mathbf{r} - \mathbf{r}_d + \mathbf{r}_\alpha)$$

M3Y effective nucleon-nucleon interaction

$$v(\mathbf{s}) = 7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s} - 276(1 - 0.005Q_\alpha/A_\alpha)$$

G. F. Bertsch *et al*, Nucl. Phys. A **284**, 399 (1977)
G. R. Satchler *et al*, Phys. Rept. **55**, 183 (1979)
A. M. Kobos, Nucl. Phys. A **384**, 65 (1982)

λ is determined self-consistently using
Bohr-Sommerfeld quantization condition (extension of Bohr model):

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2}(Q - V(r))} = (2n+1)\frac{\pi}{2} = (G - l + 1)\frac{\pi}{2}$$

$$\rho_{\alpha,\ \mathrm{p\ or\ n}}(r) = \frac{0.4299}{2}\exp(-0.7024r^2)$$

G. R. Satchler *et al*, Phys. Rept. **55**, 183 (1979), A. M. Kobos, Nucl. Phys. A **384**, 65 (1982)

Comparison method

1. Empirical formulae

ZZCW (Phys. Rev. C **95**, 014311 (2017))

$$\log_{10}(T_{1/2}/\mathrm{s}) = (a_1\mu_A^x + b_1)[(Z_c Z_d)^y/\sqrt{Q_c} - 7] + (a_2\mu_A^x + b_2)$$

ADF (Nucl. Phys. A **970**, 259 (2018))

$$\log_{10}(T_{1/2}/\mathrm{s}) = aZ_\alpha Z_\mathrm{d}\sqrt{\mu_A/Q_\alpha} + b\sqrt{\mu_A Z_\alpha Z_\mathrm{d}} + c + d$$

UDL (Phys. Rev. Lett. **103**, 072501 (2009))

$$\log_{10}(T_{1/2}/\mathrm{s}) = aZ_\mathrm{c}Z_\mathrm{d}\sqrt{\mu_A/Q_\mathrm{c}} + b\sqrt{\mu_A Z_\mathrm{d} Z_\mathrm{c}(A_\mathrm{d}^{1/3} + A_\mathrm{c}^{1/3})} + c$$

SLH (Nucl. Phys. A **259**, 259 (2018))

$$\log_{10}(T_{1/2}/\mathrm{s}) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + dI + eI^2$$

UNIV (Phys. Rev. C **83**, 014601 (2011))

$$\log_{10}(T_{1/2}/s) = -0.22873\sqrt{\mu_A Z_\mathrm{d} Z_\mathrm{c} R_b}\arccos(\sqrt{r} - \sqrt{r(1-r)}) + 0.598(A_\mathrm{c} - 1) + c_{ee}$$

2. Two-parameter Fermi form

$$\rho_\mathrm{d}(r,\theta) = \frac{\rho_0}{1 + \exp[\frac{r - R(\theta)}{a}]}$$

ρ_0 : normalization factor

$$R(\theta) = R_0(1 + \sum_{l=2,4,6} \beta_l Y_{l0}(\theta))$$

β_l from FRDM2012

$$R_0 = 1.07A_\mathrm{d}^{1/3}$$

$$a = 0.54\ [\mathrm{fm}]$$

3. DRHBc

$$\rho_\mathrm{d}(r,\theta) = \rho_{\mathrm{d},l=0}(r)$$

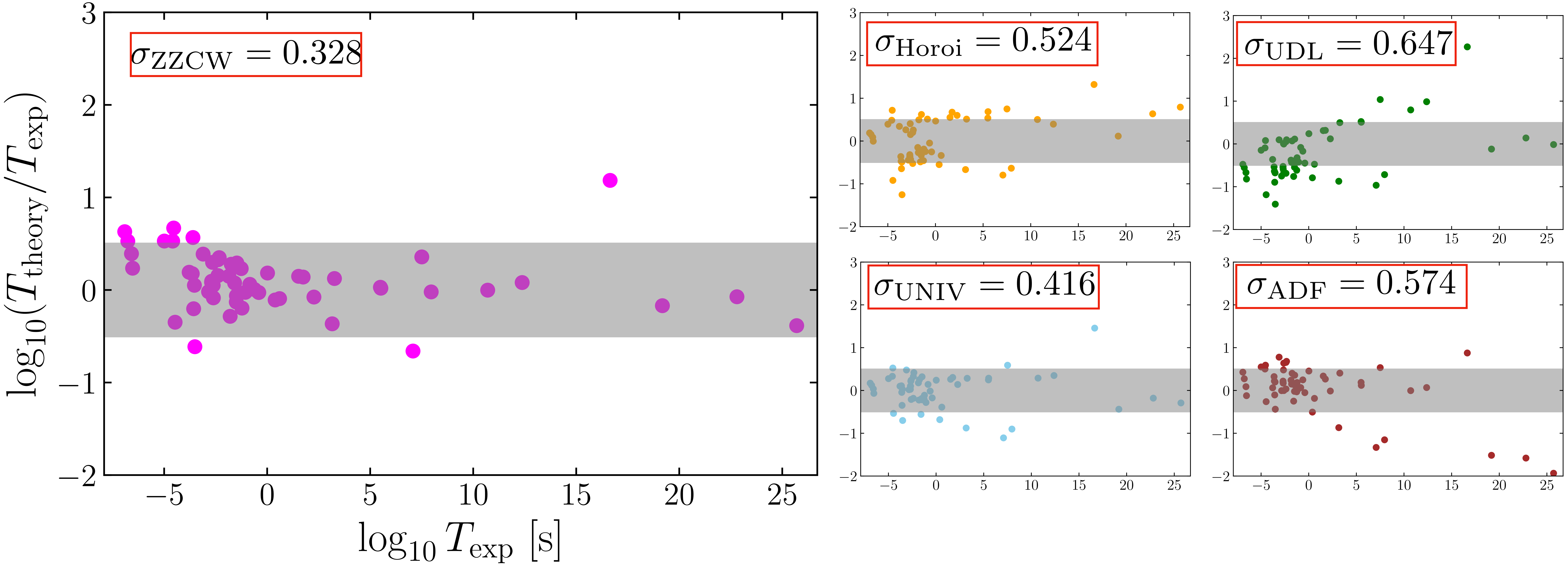
Only spherical component is calculated.

$$\rho_\mathrm{d}(r,\theta) = \sum_{l=0,2,4,\ldots} \rho_{\mathrm{d},l}(r)P_l(\cos\theta)$$

Standard deviations to compare: $\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\log_{10} T_{\mathrm{Theory},i} - \log_{10} T_{\mathrm{NUBASE2020},i})^2}$

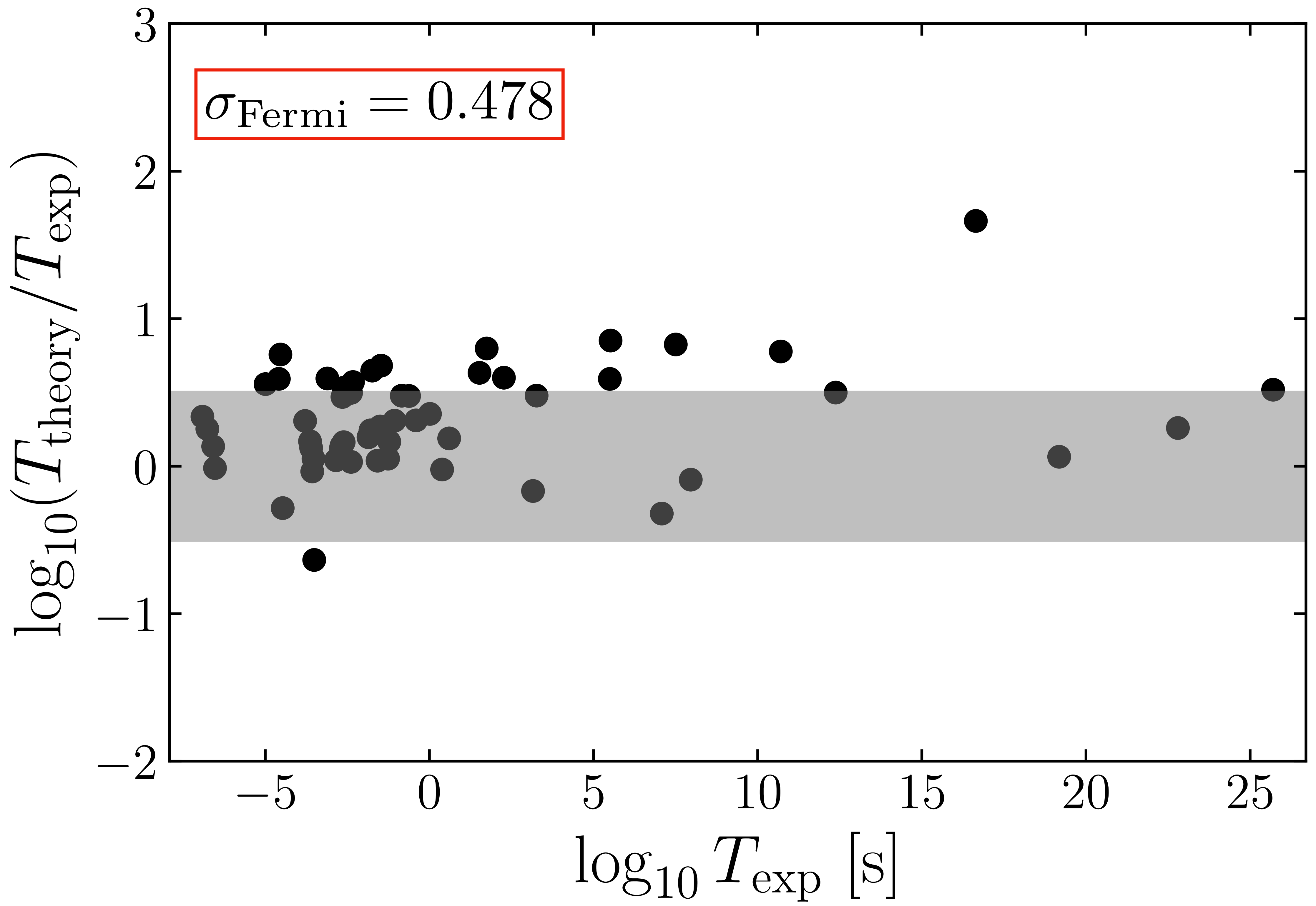
only alpha decay ~ 100%

Result - Empirical formulae



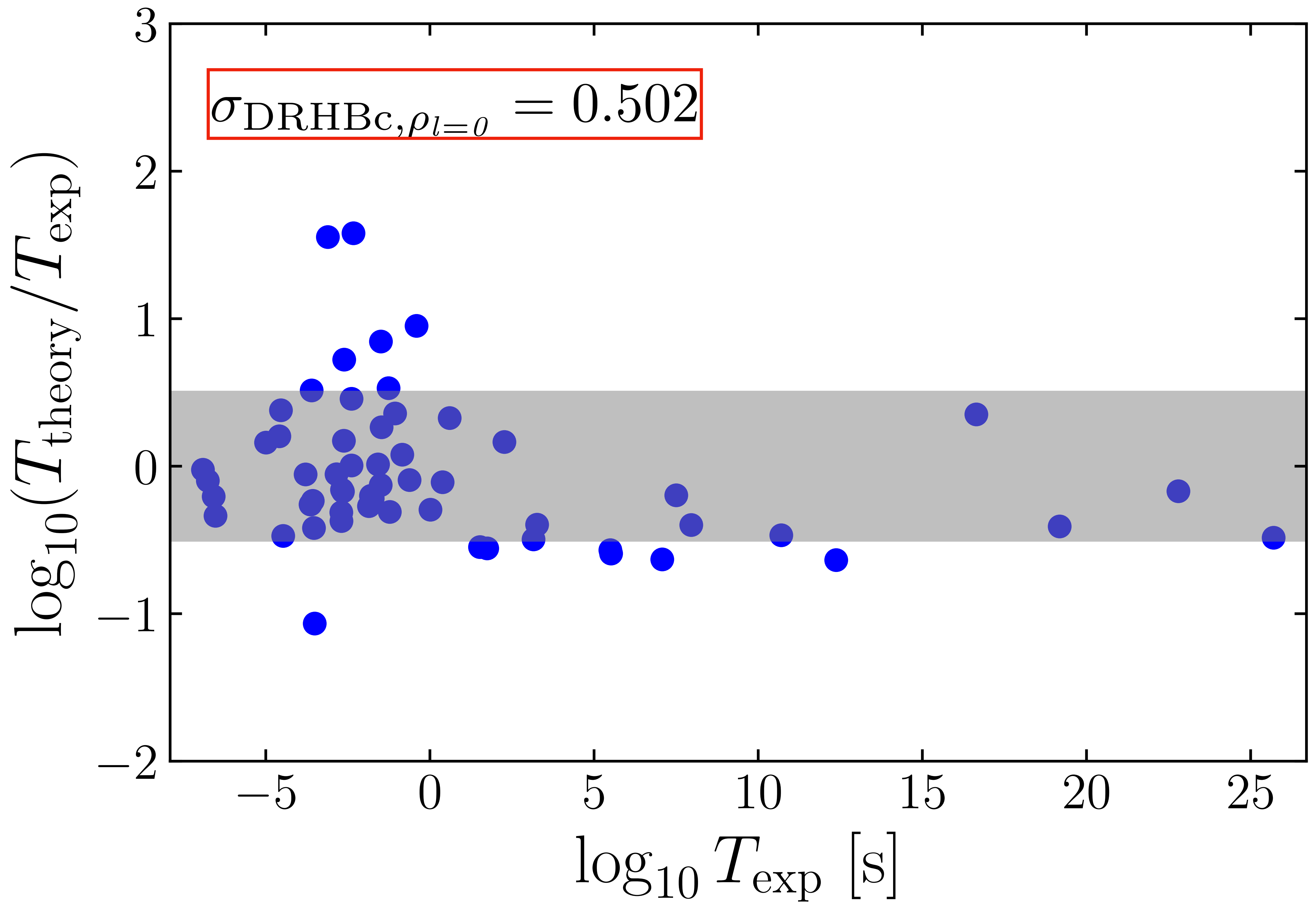
- With the exception of ZZCW, the predictions are biased.
- The results with ZZCW shows a weakness near proton drip line.

Result - Two-parameter Fermi form



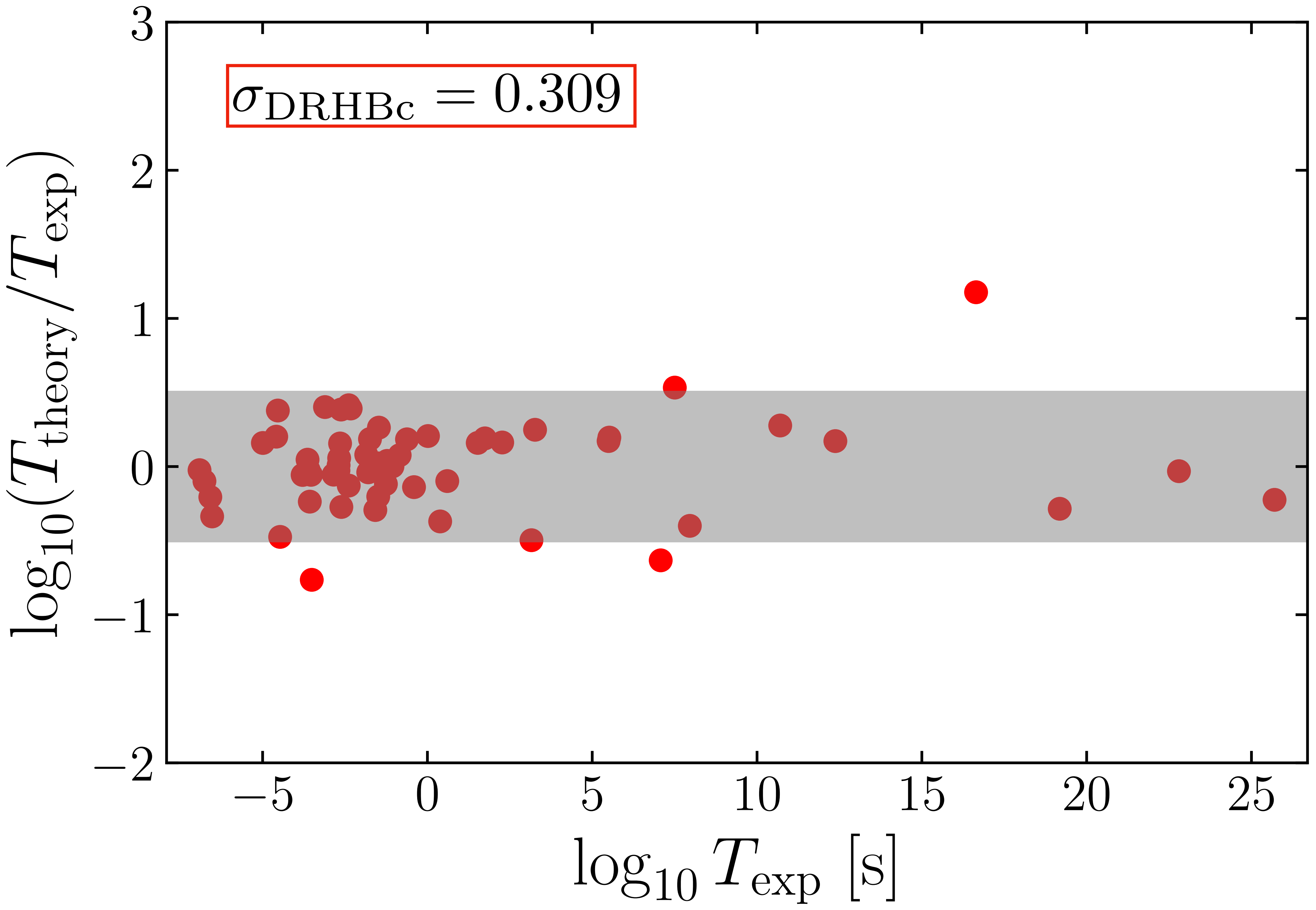
- The results with Two-parameter Fermi form predict experimental data as well as the results with empirical formulae.
- Systematically, Half-lives are greater (decay slowly) than experimental data.

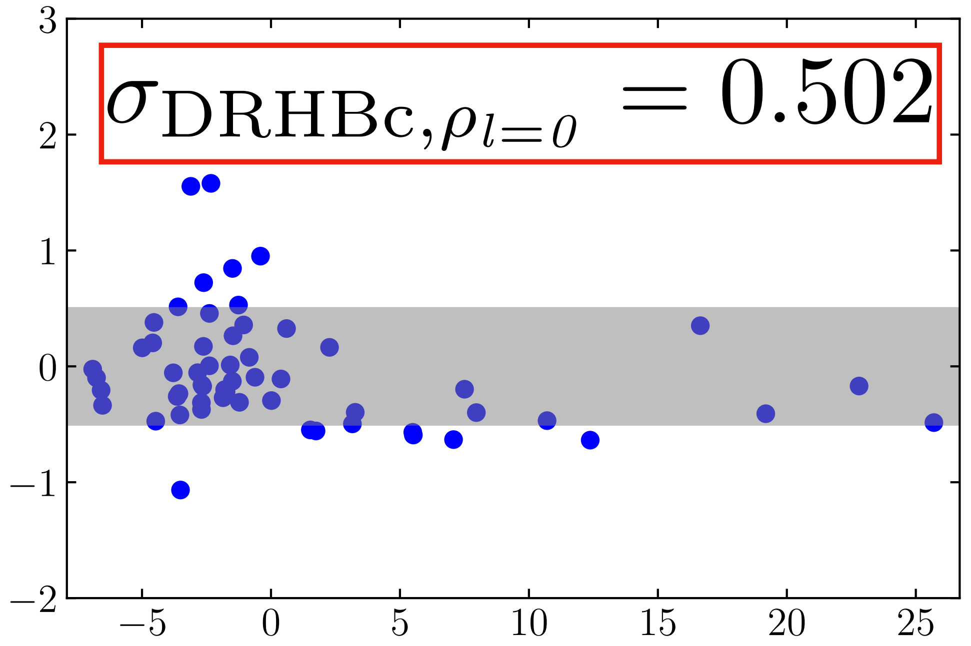
Result - DRHBc (spherical component)



- The results with DRHBc ($\rho_{l=0}$) also predict experimental data as well as the results with empirical formulae.
- Systematically, Half-lives are greater (decay slowly) or less than experimental data.

Result - DRHBc





$\sigma_{\mathrm{DRHBc}} = 0.309$

$\sigma_{\mathrm{Fermi}} = 0.478$

$\sigma_{\mathrm{DRHBc},\rho_{l=0}} = 0.502$

Empirical formulae

$\sigma_{\mathrm{ZZCW}} = 0.328$

$\sigma_{\mathrm{UNIV}} = 0.416$

$\sigma_{\mathrm{Horoi}} = 0.524$

$\sigma_{\mathrm{ADF}} = 0.574$

$\sigma_{\mathrm{UDL}} = 0.647$

- The results using DRHBc best predict the half-lives.
- It shows consistency with experimental data from close proton drip line to stable region.

Summary

- Calculated alpha decay half-lives were compared with experimental data.
- Alpha decay half-lives with WKB approximation predict the experimental data as well as Alpha decay half-lives with empirical formulae.
- Alpha decay half-lives with DRHBc best predict the experimental data and predict consistently not only near proton drip line but also stable region.
- The density distributions of neutron-deficient and stable nuclei predicted by DRHBc are reliable.

Thank you for your attentions.