Focused workshop on Rare Isotope Physics @ Yeosu Expo, Nov. 24~26, 2022

Functional Renormalization Group method for Nuclear Matter and Neutron Star

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in collaboration with

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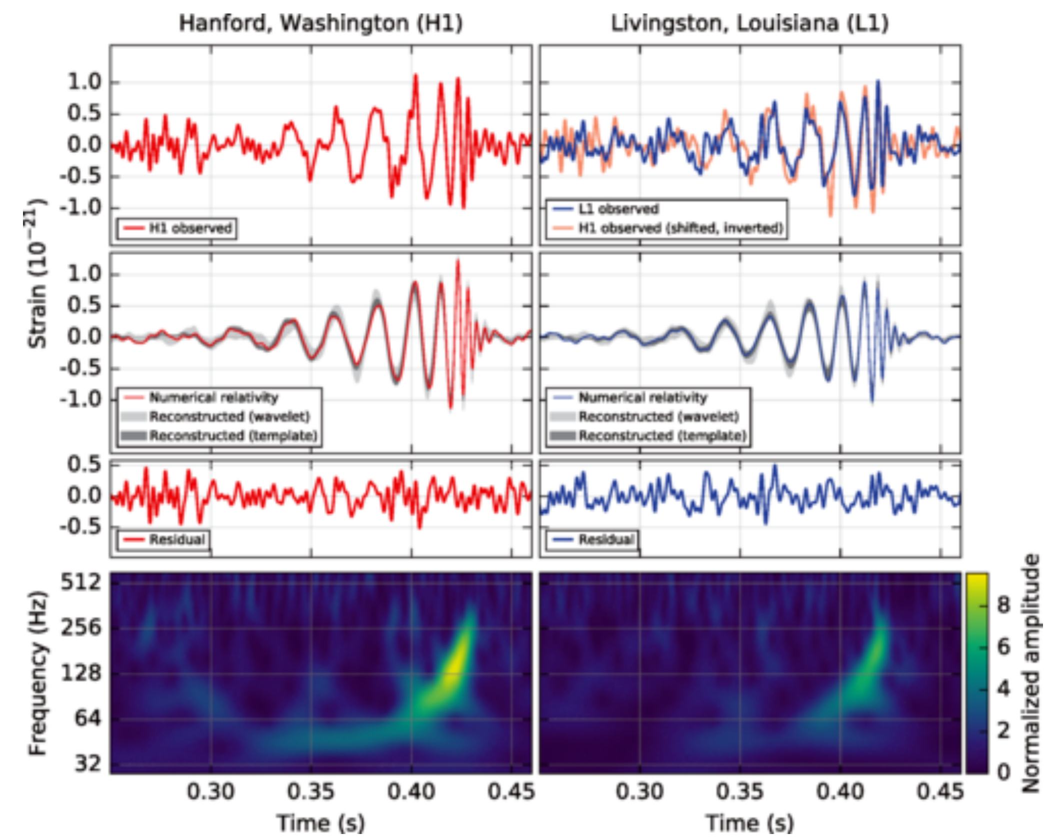
and Sangyong Jeon (McGill. Univ)

My Research is …

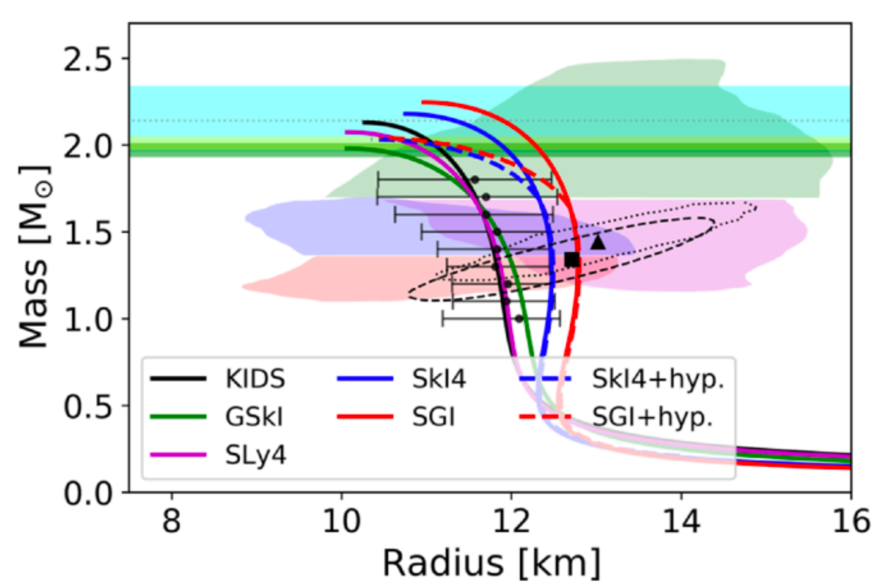
Gravitational Wave Data Analysis

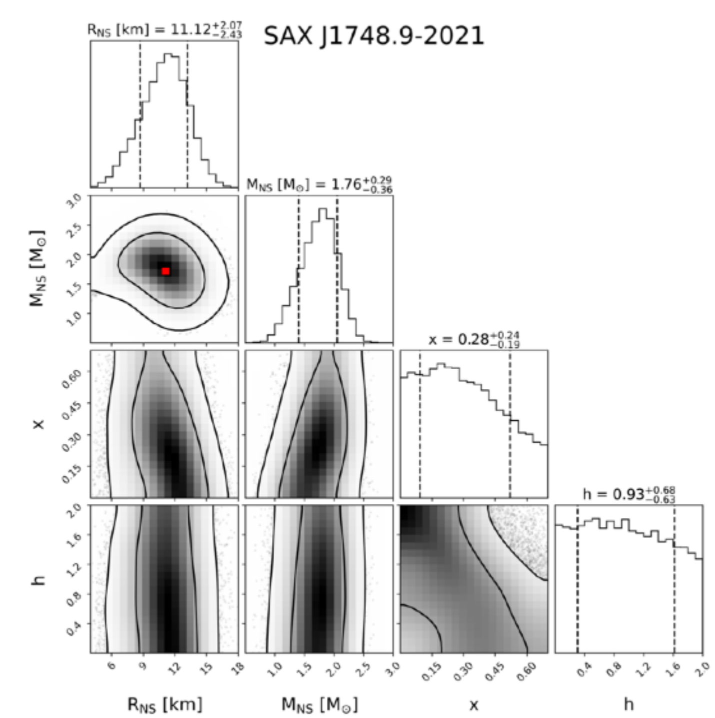
- Detector Characterization + Machine Learning
- Bayesian Inference on Neutron Star properties : NS mass, radius, tidal deformability, etc.



Dense Nuclear Matter & Neutron Star

- Neutron Star Equation of State
- DJBUU + RAON





Polytropic equation of state

K1<K2<K3

M. Kim et al. (JKPS 2021)

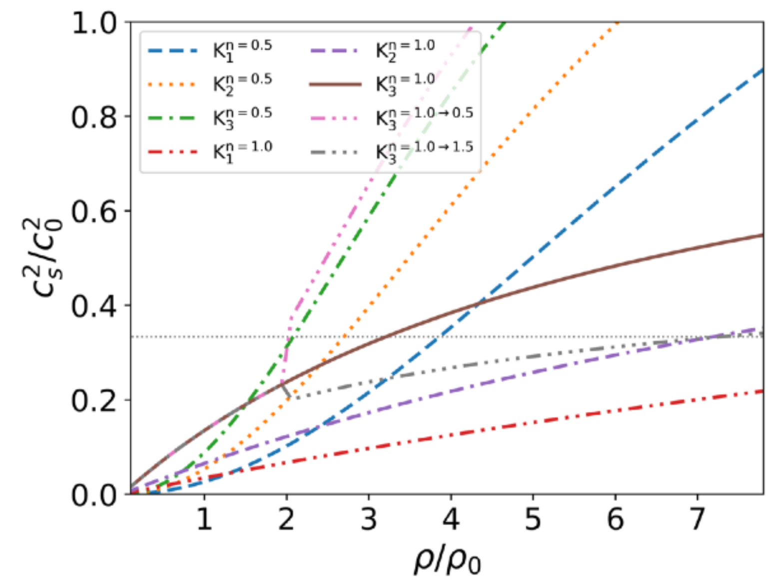
$$p = K\rho^{\Gamma} = K\rho^{1+1/n}$$

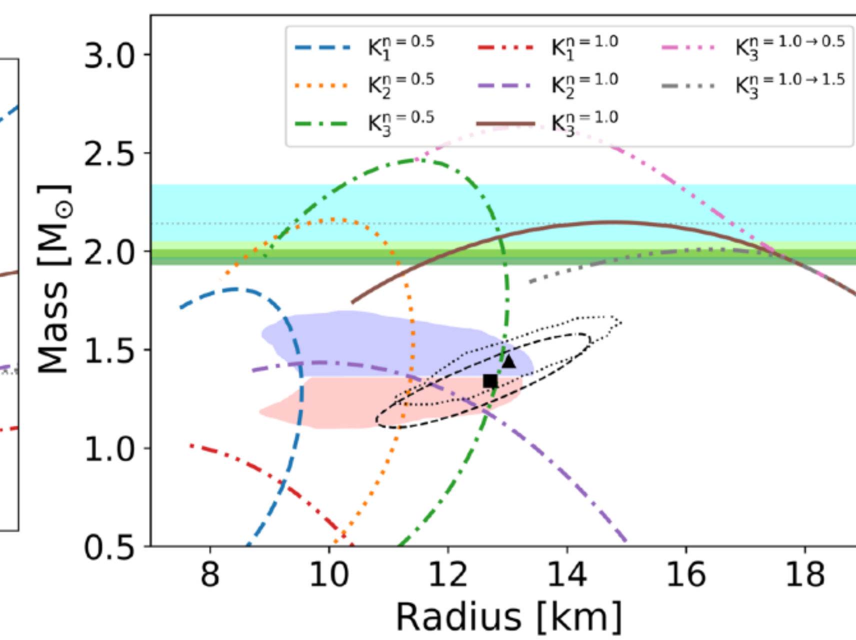
$$\begin{aligned}\epsilon =&(1+a)\rho c_0^2 + \frac{p}{\Gamma - 1} \\ =&(1+a)\rho c_0^2 + \frac{K}{\Gamma-1}\rho^{\Gamma},\end{aligned}$$

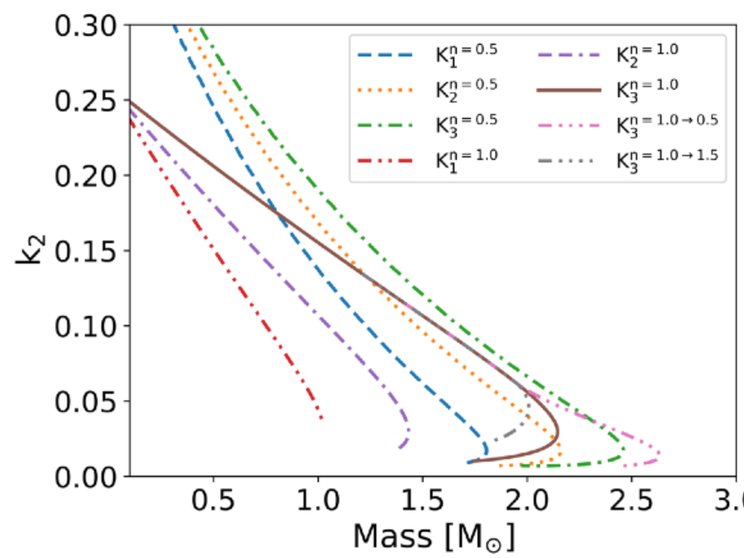
$$\epsilon_2 =(1+a_2)\rho c_0^2 + \frac{K_2}{\Gamma_2 - 1}\rho^{\Gamma_2},$$

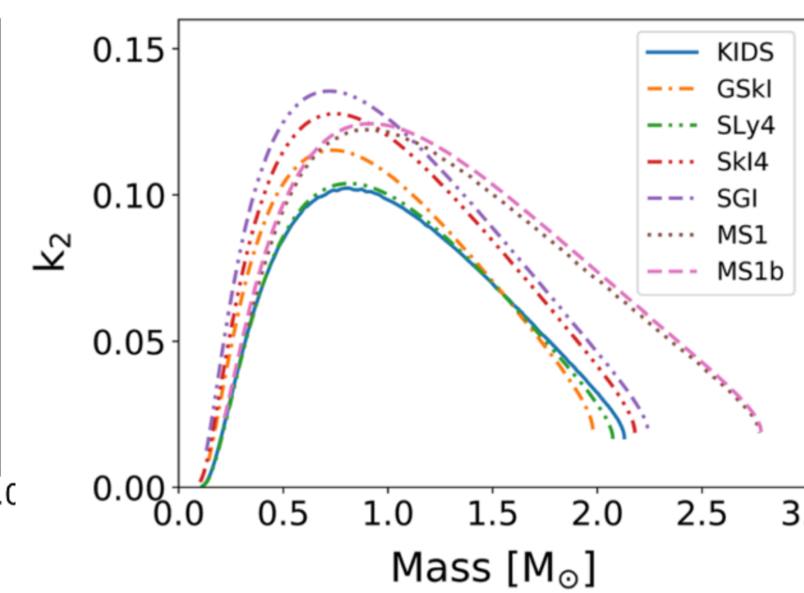
$$a_2 =\left(\frac{1}{\Gamma_1 - 1} - \frac{1}{\Gamma_2 - 1}\right)K_2\rho_t^{\Gamma_2 - 1}$$

$$\frac{c_s^2}{c_0^2} = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \Gamma\frac{p}{\epsilon + p}$$



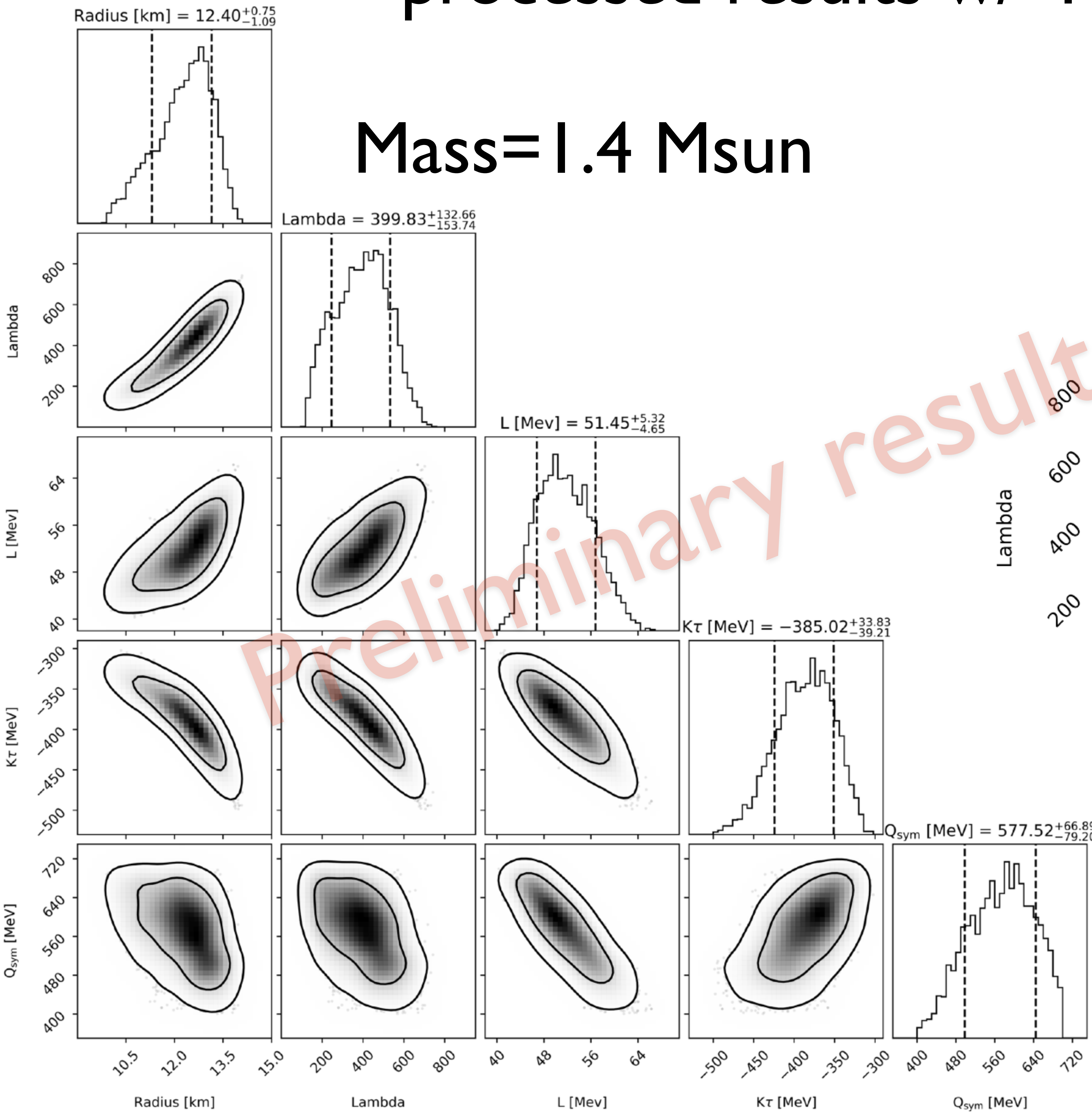


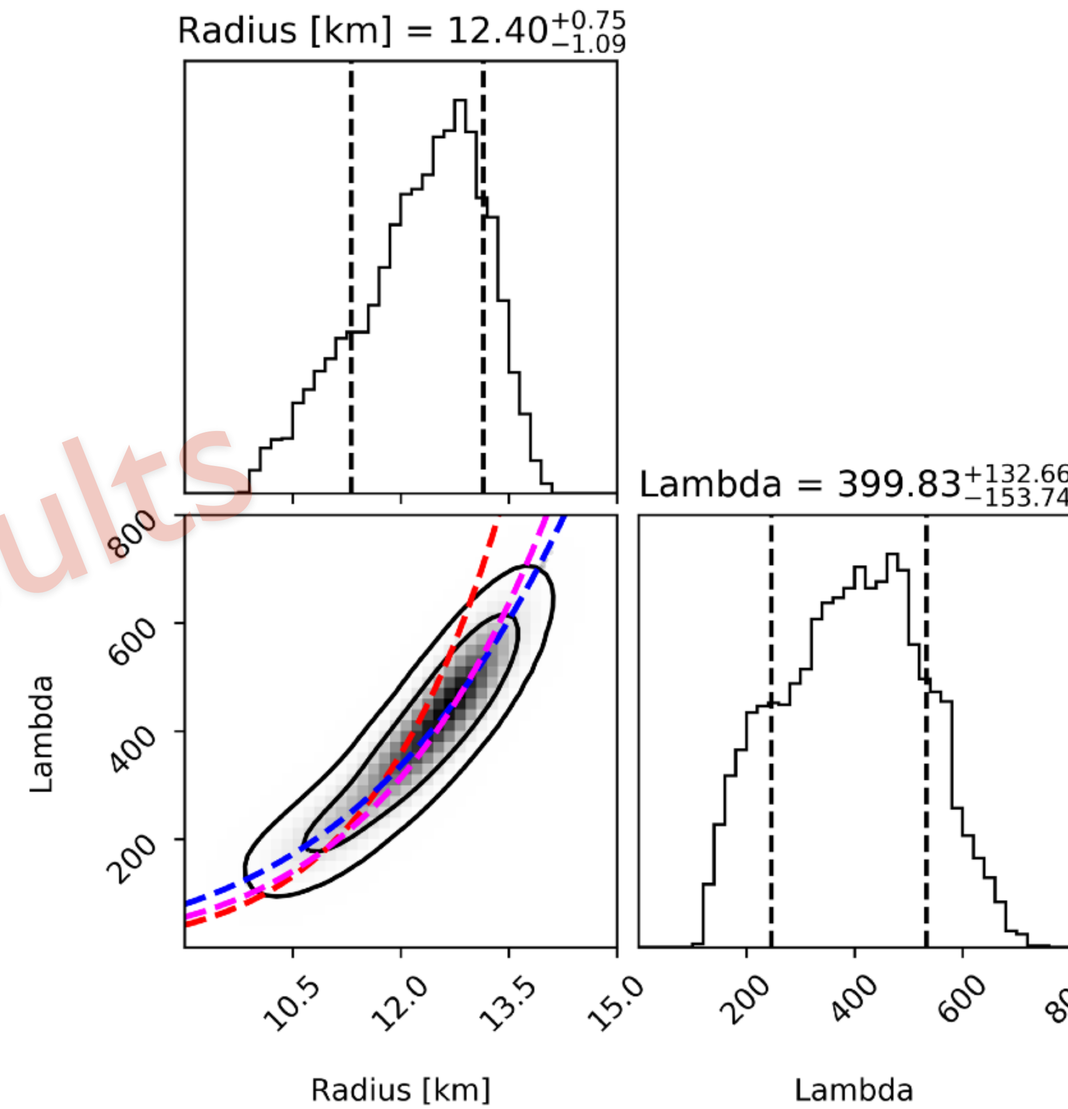




Posterior samples and Post-processed results w/ KIDS

Mass=1.4 Msun





Preliminary results

Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6} (R/km)^{7.5}$

PhysRevLett.120.172703.pdf

Blue line: $\Lambda(1.4M_\odot) = 1.35 * 10^{-3} (R/km)^{5.0}$

Implication of PREX 2 - PhysRevLett.126.172503 (2021)
=> $\Lambda \sim R^{4.8}$

Magenta line: $\Lambda(1.4M_\odot) = 1.05 * 10^{-4} (R/km)^{6.0}$

More reliable theoretical approach is …

Functional Renormalization Group (FRG) method

$$\Gamma_k = \int d^4x \mathcal{L}|_{U\to U_k},$$

$$\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right]$$

Zhang et al., PHYSICAL REVIEW D 96, 114029 (2017)

Example: two flavor quark-meson model w/ omega meson

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu + \mu\gamma_0]\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - U(\sigma, \boldsymbol{\pi}, \omega),\end{aligned}$$

$$\begin{aligned}\partial_k U_k^\phi(T,\mu) = \frac{k^4}{12\pi^2}\bigg\{ & \frac{3[1+2n_\mathrm{B}(E_\pi)]}{E_\pi} + \frac{1+2n_\mathrm{B}(E_\sigma)}{E_\sigma} \\ & - \frac{2\nu_q[1 - n_\mathrm{F}(E_q,\mu_\mathrm{eff}^k) - n_\mathrm{F}(E_q,-\mu_\mathrm{eff}^k)]}{E_q}\bigg\}\end{aligned}$$

2 flavor Quark-Meson model w/ ω meson (1)

Zhang et al., PHYSICAL REVIEW D 96, 114029 (2017)

Flow eq.

$$\partial_k U_k^{\phi}(T,\mu) = \frac{k^4}{12\pi^2}\left\{\frac{3[1+2n_{\mathrm{B}}(E_\pi)]}{E_\pi} + \frac{1+2n_{\mathrm{B}}(E_\sigma)}{E_\sigma} - \frac{2\nu_q[1-n_{\mathrm{F}}(E_q,\mu_{\mathrm{eff}}^k) - n_{\mathrm{F}}(E_q,-\mu_{\mathrm{eff}}^k)]}{E_q}\right\}$$

$$\phi^2 \equiv \sigma^2 + \pi^2.$$

where single-particle energies are

$$E_\pi = \sqrt{k^2 + 2U_k'},$$

$$E_\sigma = \sqrt{k^2 + 2U_k' + 4\phi^2 U_k''},$$

$$E_q = \sqrt{k^2 + g_s^2\phi^2},$$

$$U_k' \equiv \frac{\partial U_k}{\partial \phi^2}$$

$$U_k = U_k^{\phi} + U_k^{\omega}$$

$$U_k^{\omega} = -\frac{1}{2}m_v^2\omega_{0,k}^2$$

$$n_B(E) = \frac{1}{e^{\beta E} - 1}$$

$$n_F(E,\mu) = \frac{1}{e^{\beta(E-\mu)} + 1}$$

2 flavor Quark-Meson model w/ ω meson (2)

$$\partial_k \omega_{0,k} = -\frac{2g_v k^4}{\phi^2 m_v^2 E_q} \frac{\partial}{\partial \mu} (n_F(E_q, \mu_{\text{eff}}^k) + n_F(E_q, -\mu_{\text{eff}}^k))$$

Initial condition w/ Λ=500 MeV,

$$U_\Lambda^\phi = \frac{\lambda}{4} \phi^4$$

g_s = 3.2, λ = 8

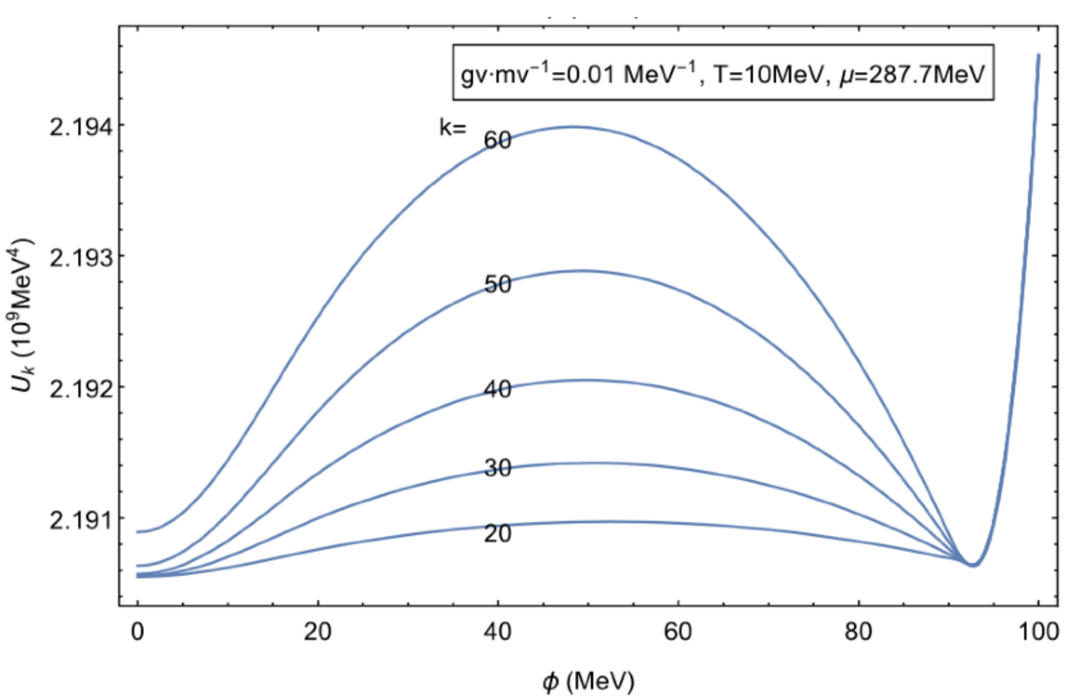
$$\omega_{0,k=\Lambda}(\phi) = 0$$

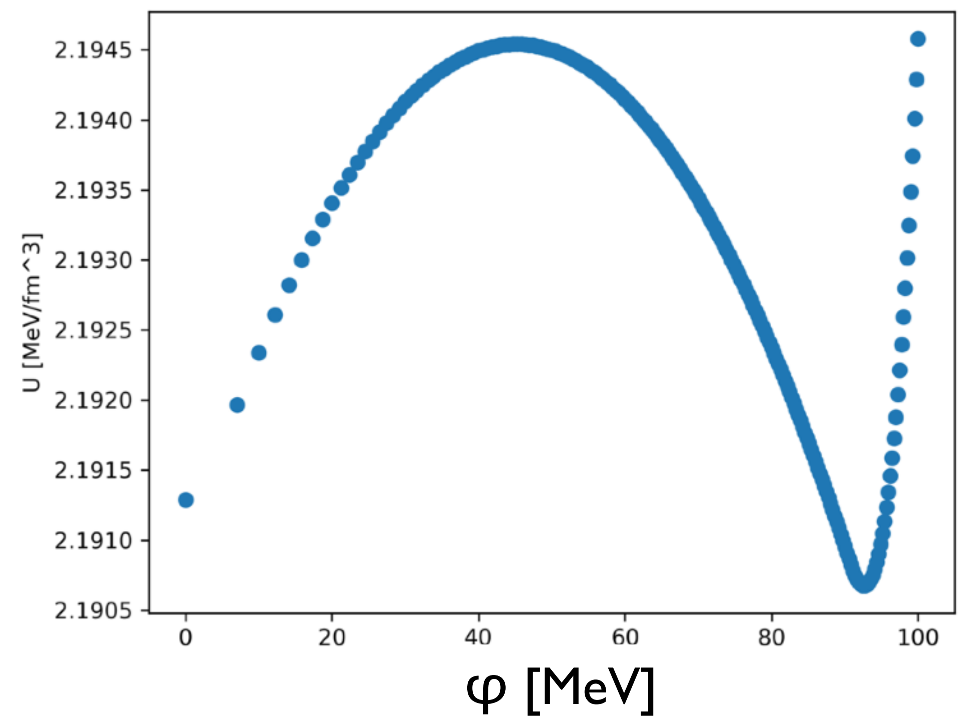
$$\phi = [0, 100] \text{ Mev}$$

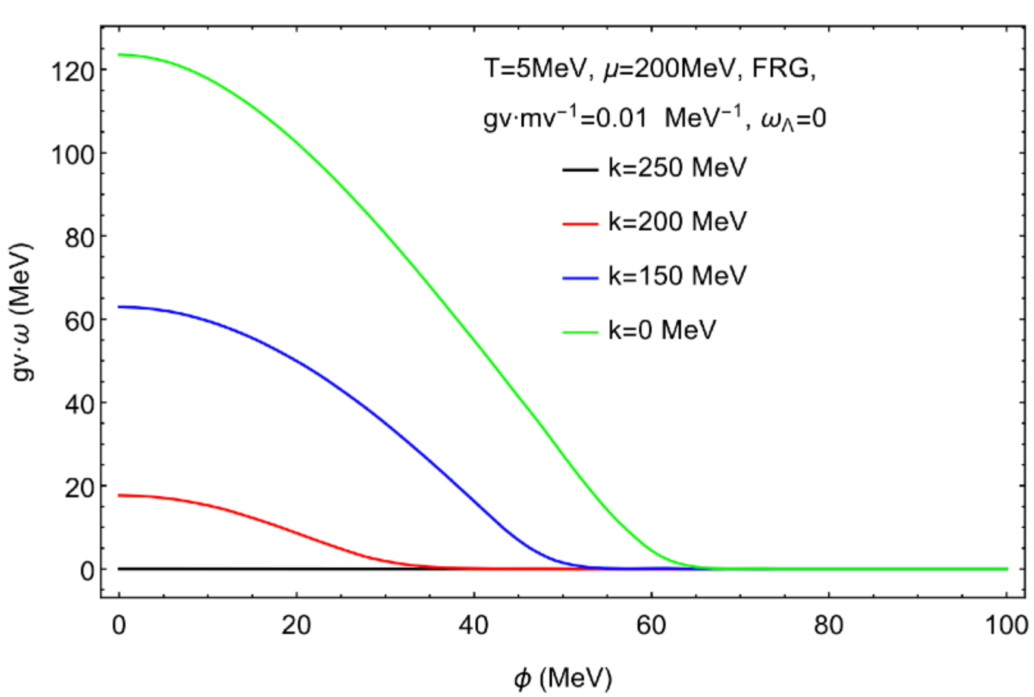
Test Results - Quark-Meson model

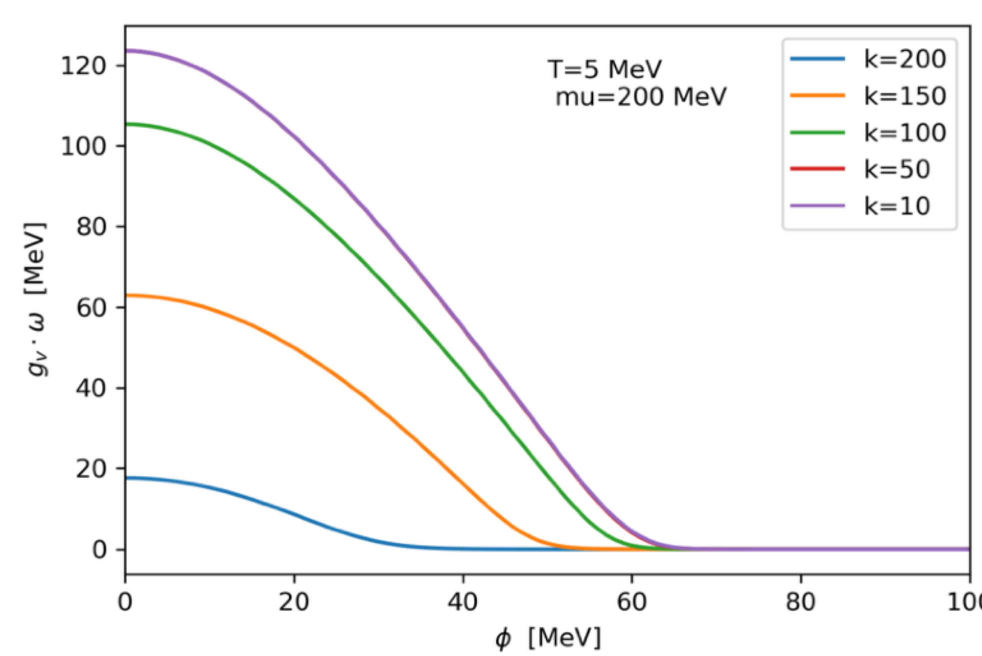
Zhang et al., PHYSICAL REVIEW D 96, 114029 (2017)

Our results









FRG in an Extended Walecka model

$$\mathcal{L} = \bar{\psi}[i \not\partial - M_N + g_\sigma \phi - g_\omega \not\omega - g_\rho \not b]\psi\,,$$
$$+ \frac{1}{2}(\partial_\mu \phi \partial^\phi - m_\sigma^2 \phi^2) - U(\phi)$$
$$+ \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \vec{b} \cdot \vec{b} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} G_{\mu\nu} G^{\mu\nu}$$

$$k \frac{\partial U_{k,\chi}}{\partial k} = \frac{k^5}{12\phi^2}\left[\frac{1 + 2n_B(E_\phi)}{E_\phi} - \Sigma_{i=n,p} 4 \frac{1 - \Sigma_{r=\pm 1}\ n_F(E_N - r\mu_{\mathrm{i,eff}})}{E_N}\right]$$

$$g_\omega \omega_{0,k} = \frac{f_\omega}{3\pi^2} \int_k^\Lambda dp \frac{p^4}{E_N} \Sigma_{r=\pm 1} \frac{\partial}{\partial \mu}[n_F(E_N - r\mu_{p,\mathrm{eff}}) + n_F(E_N - r\mu_{n,\mathrm{eff}})]$$

$$g_\rho \rho_{0,k} = \frac{f_\rho}{3\pi^2} \int_k^\Lambda dp \frac{p^4}{E_N} \Sigma_{r=\pm 1} \frac{\partial}{\partial \mu}[n_F(E_N - r\mu_{p,\mathrm{eff}}) - n_F(E_N - r\mu_{n,\mathrm{eff}})]$$

Parameters for our calculation

Initial condition w/ Λ=1400 MeV,

$$g_\omega \omega_0(\Lambda, \mu, T) = 0$$

$$g_\rho \rho_0(\Lambda, \mu, T) = 0$$

$$U_\Lambda^\phi = \frac{1}{3}\tilde{a}\phi^3 + \frac{1}{4}\tilde{b}\phi^4$$

Parameters to be optimized,

$$f_i \equiv \frac{g_i^2}{m_i^2} \qquad \tilde{a}, \tilde{b}$$

For the reference,

Liu et al., PHYSICAL REVIEW C **65** 045201

TABLE I. Parameter sets.

Parameter	Set I	Set II	NL3
f_σ (fm^2)	10.33	same	15.73
f_ω (fm^2)	5.42	same	10.53
f_ρ (fm^2)	0.95	3.15	1.34
f_δ (fm^2)	0.00	2.50	0.00
A (fm^{-1})	0.033	same	−0.01
B	−0.0048	same	−0.003

$$\rho_0 = 0.16/\mathrm{fm}^3$$

$$E/A = -16\mathrm{MeV}$$

$$M^* = 0.75 M_N$$

$$K_v = 240\mathrm{MeV}$$

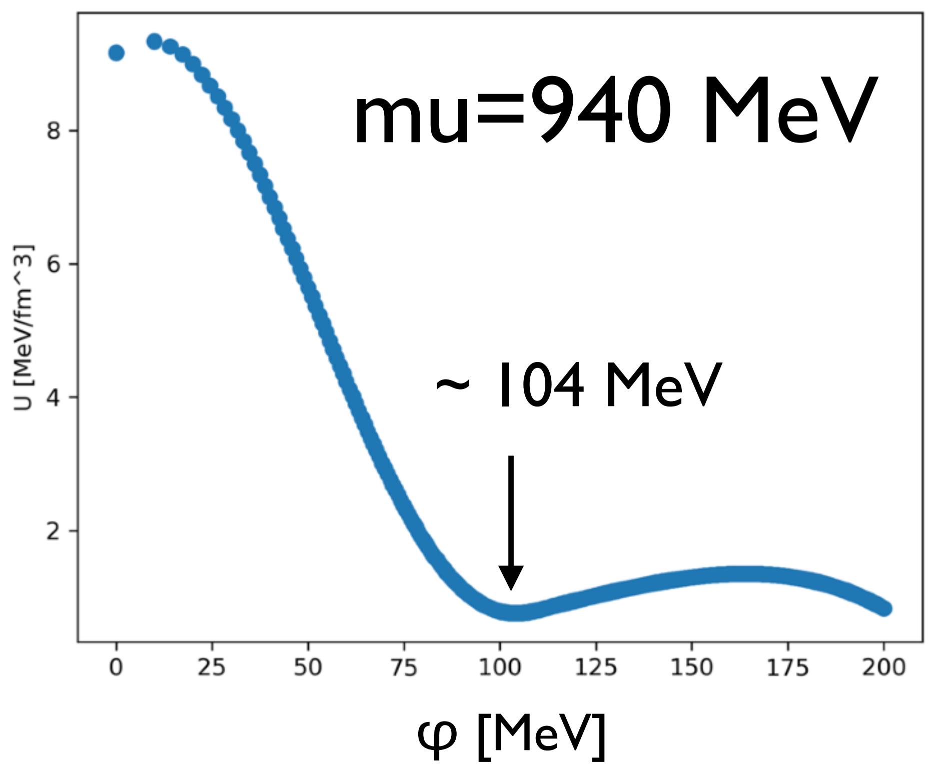
Thermodynamic Properties

$$\Omega(T, \mu_{n,p}) = U(T, \mu_{n,p}, \sigma)$$ **with** $$\sigma = < \phi >$$

$$p = -\Omega(T, \mu_{n,p}), \quad n_i = -\frac{\partial \Omega(T, \mu_{n,p})}{\partial \mu_i},$$ (in our calculation, Δμ=0.1 MeV)

$$s = -\frac{\partial \Omega(T, \mu_{n,p})}{\partial T}, \quad \epsilon = -p + Ts + \sum_{i=n,p} \mu_i n_i .$$

Test Results (1) for SNM (mu=mu_p=mu_n)



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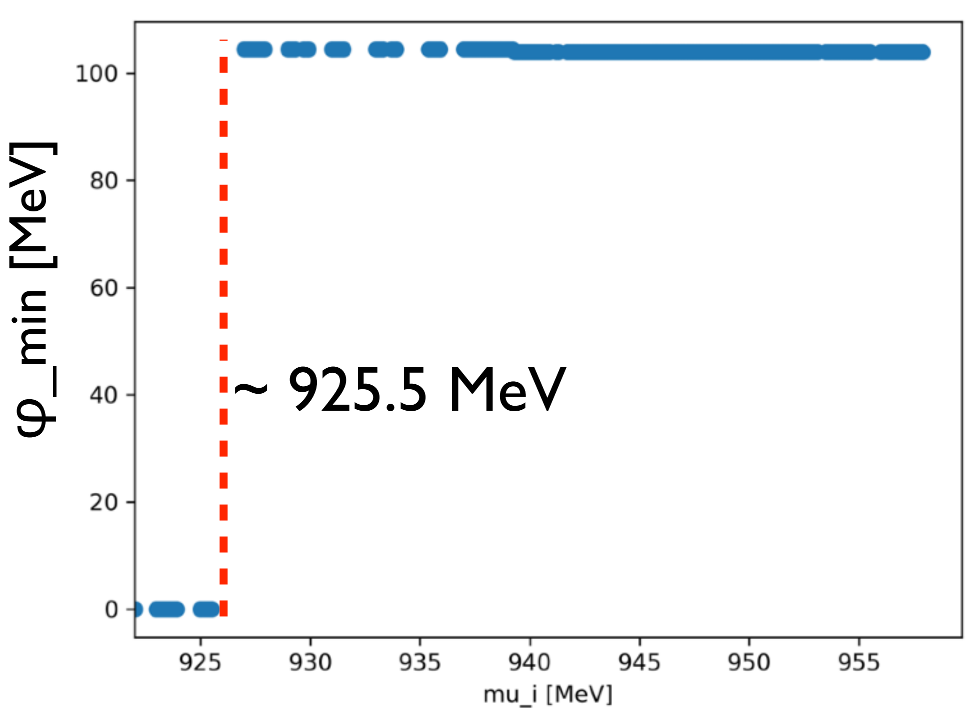
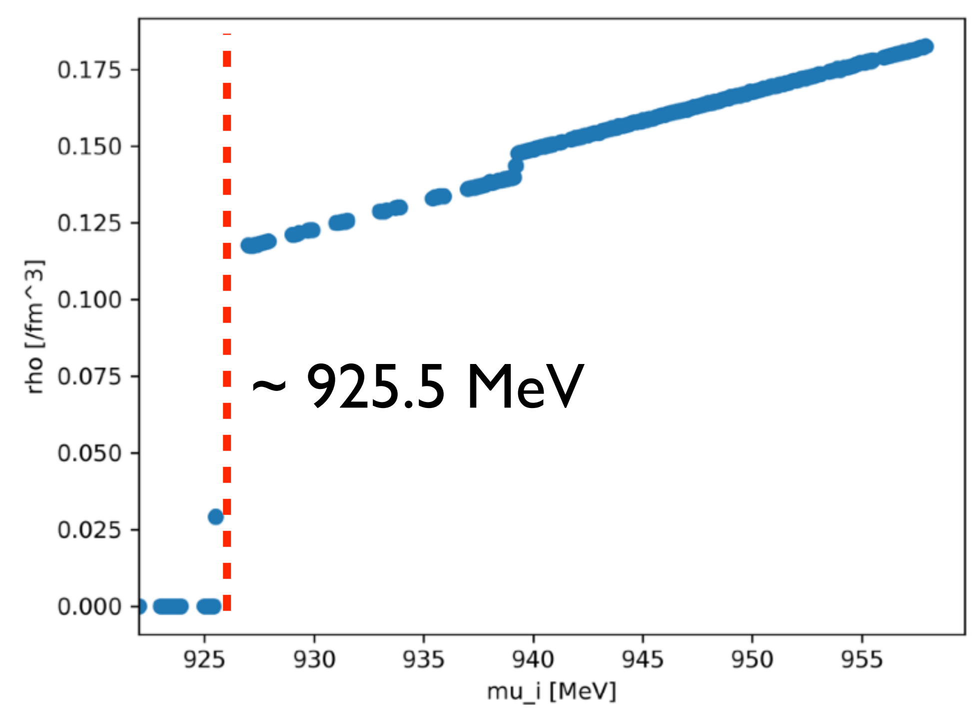
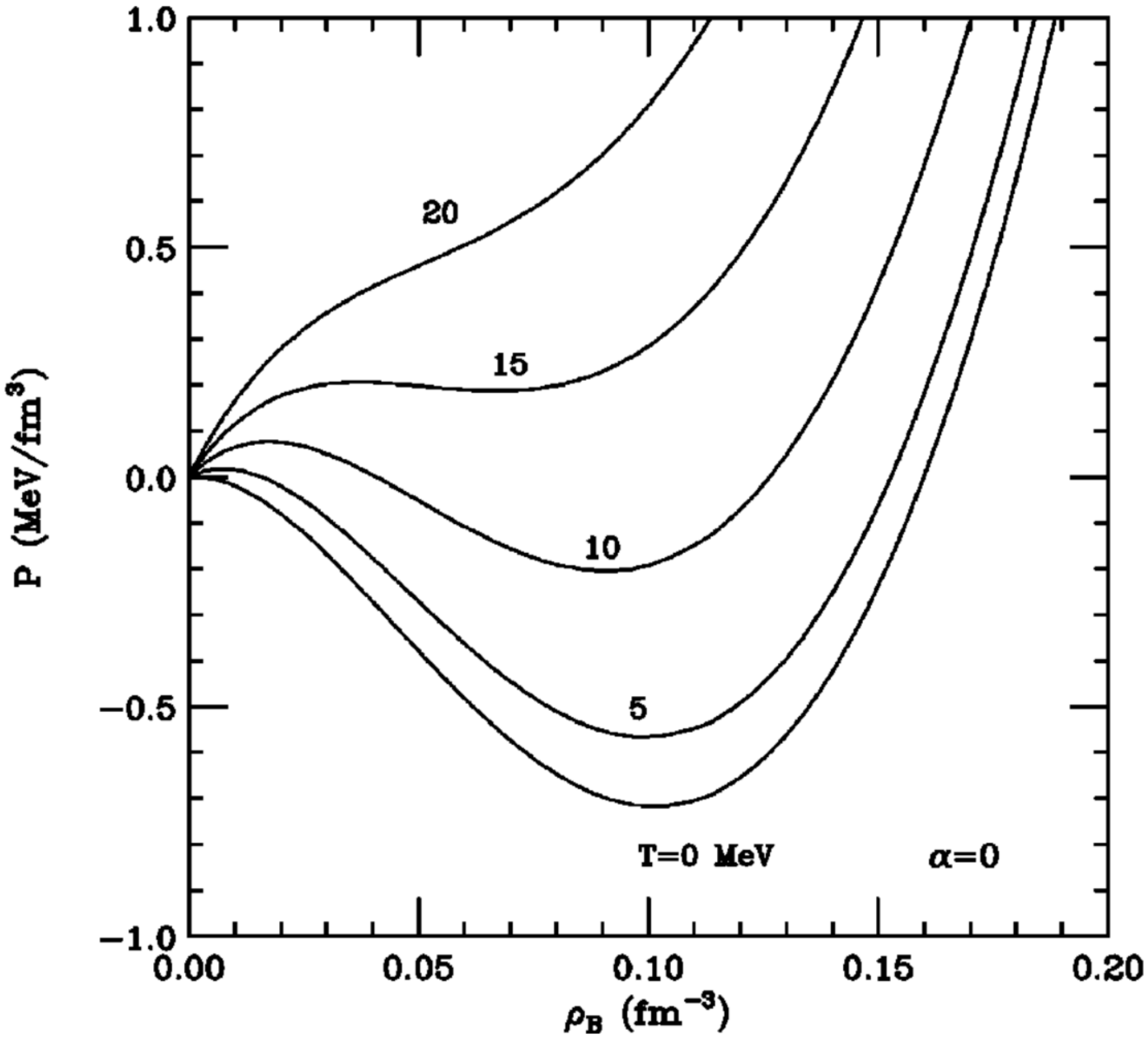
Δμ=0.1 MeV



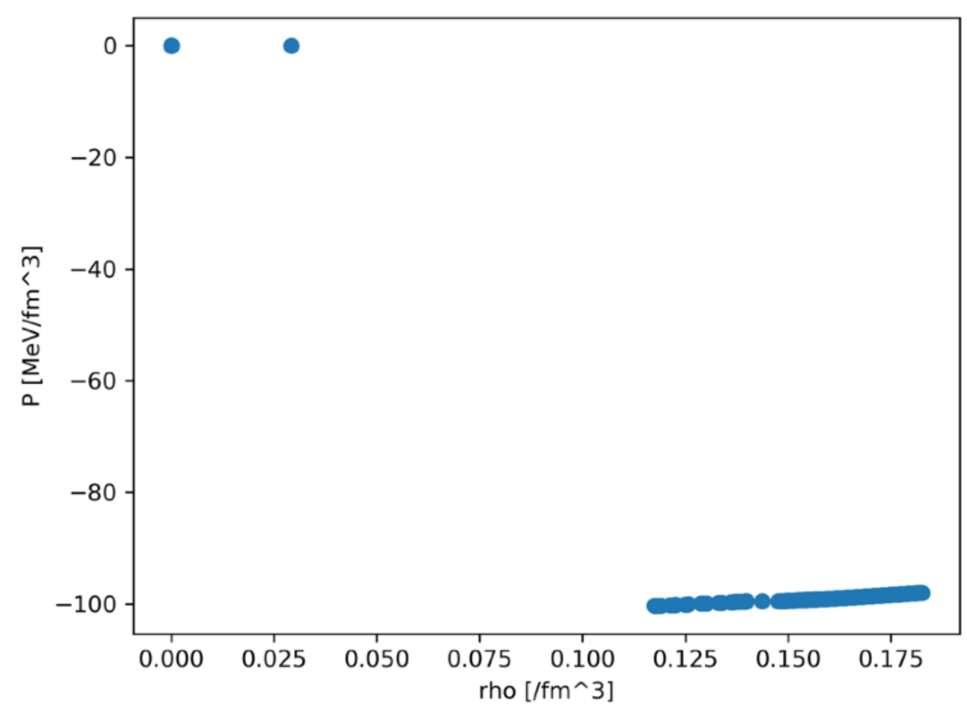


Test Results (2) for SNM

Liu et al., PHYSICAL REVIEW C **65** 045201

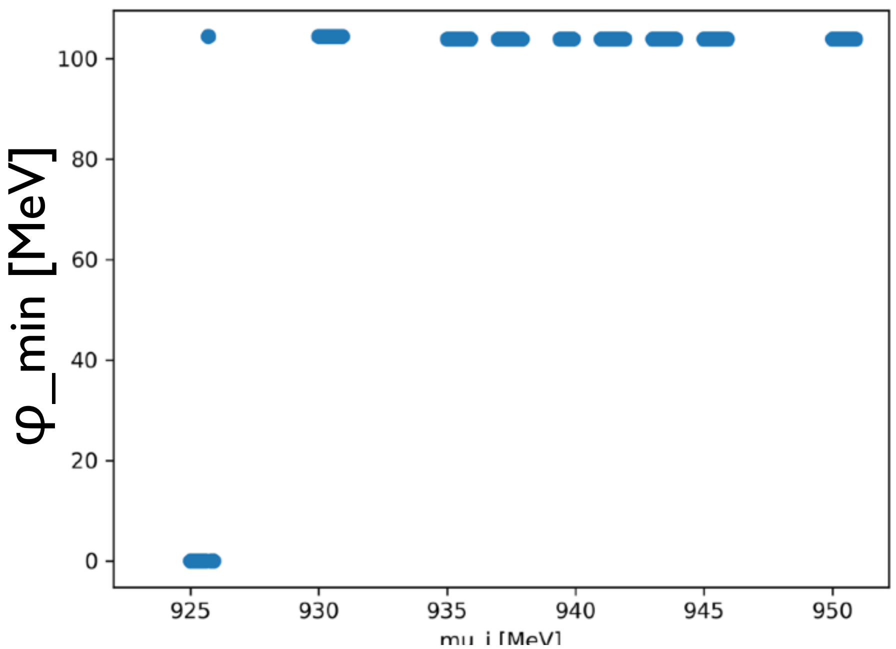


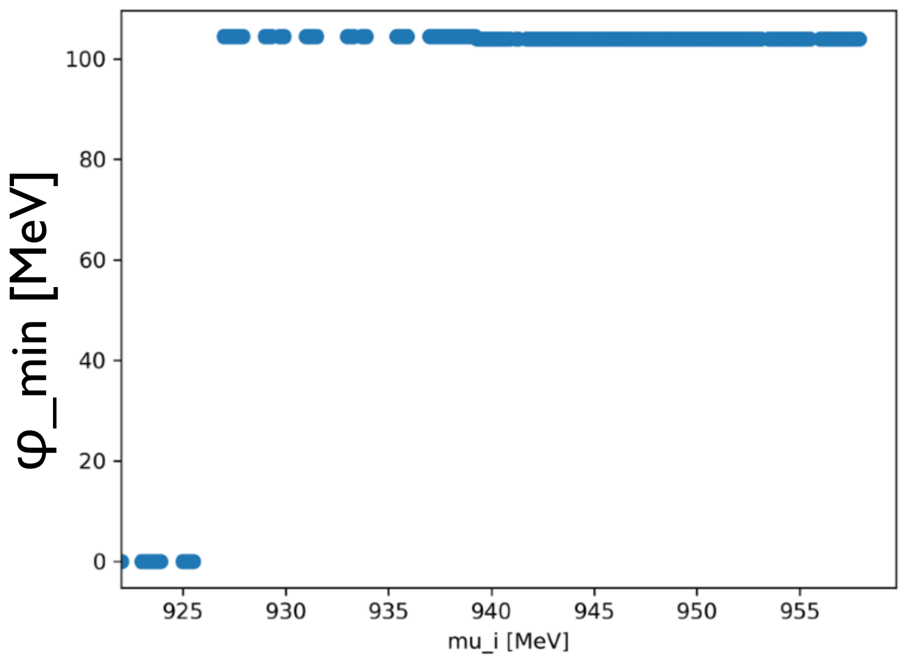
T=0

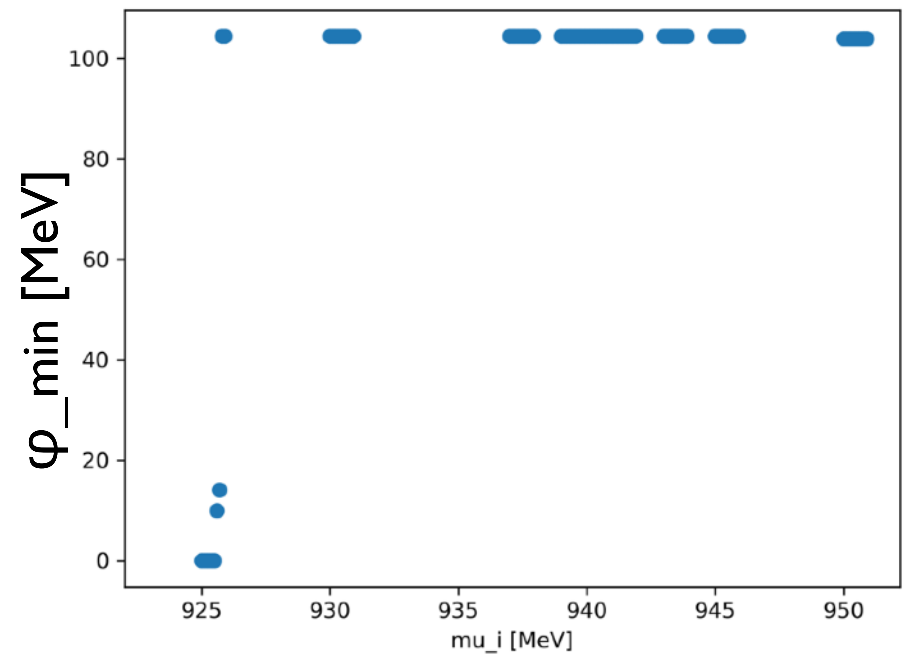


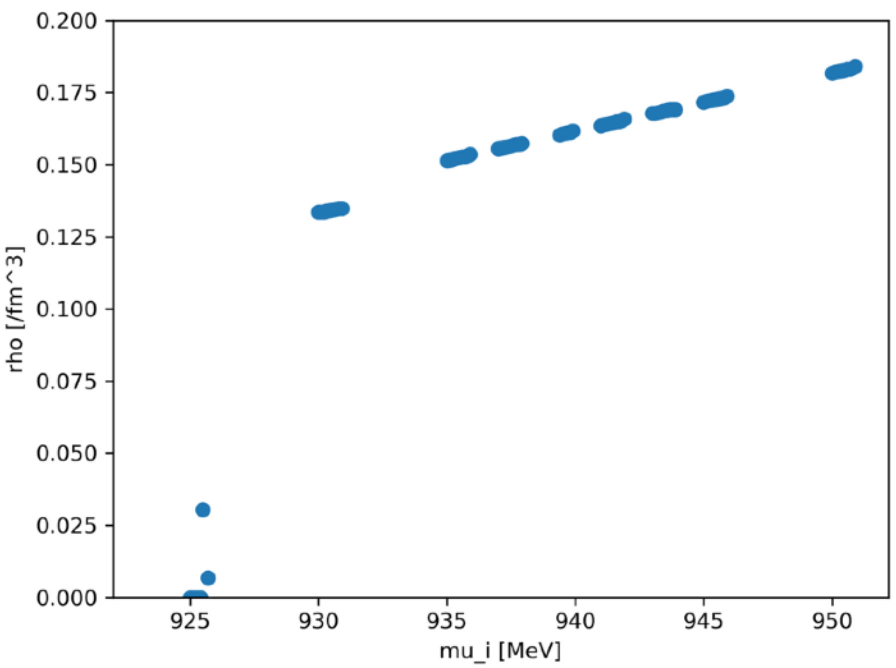
Test Results (3) for SNM

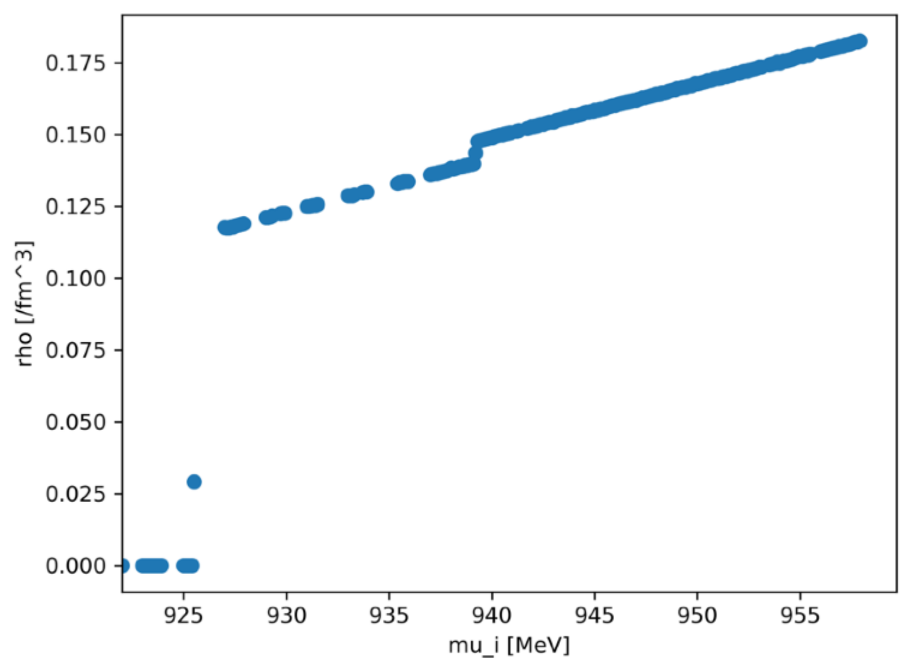
f_w=5.0 fm^2	f_w=5.42 fm^2	f_w=6.0 fm^2

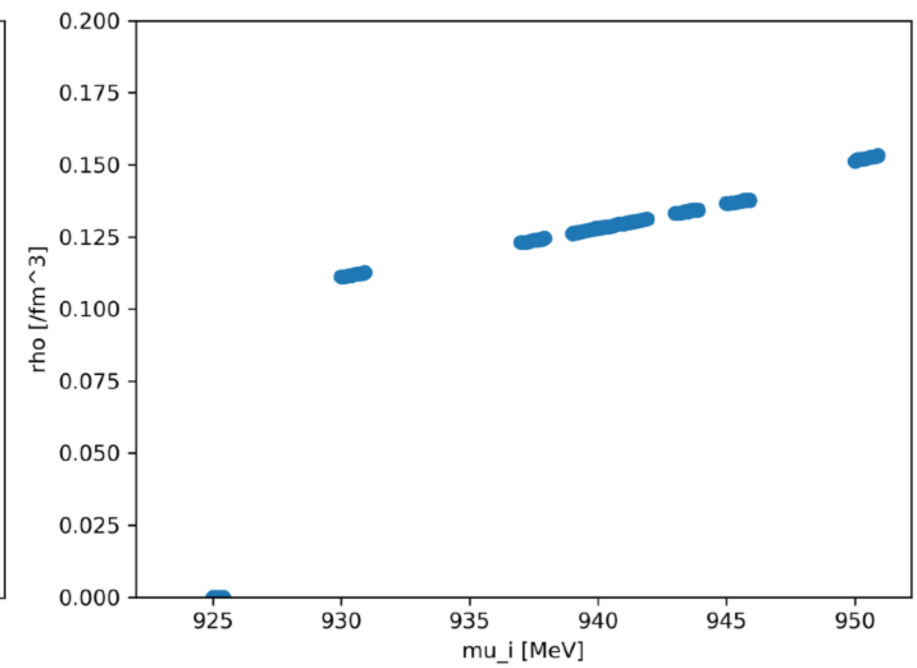






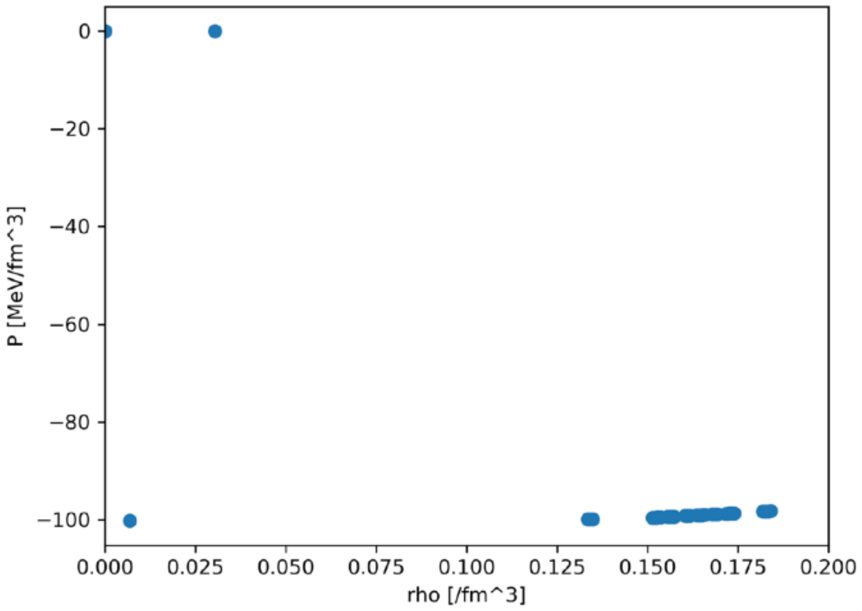




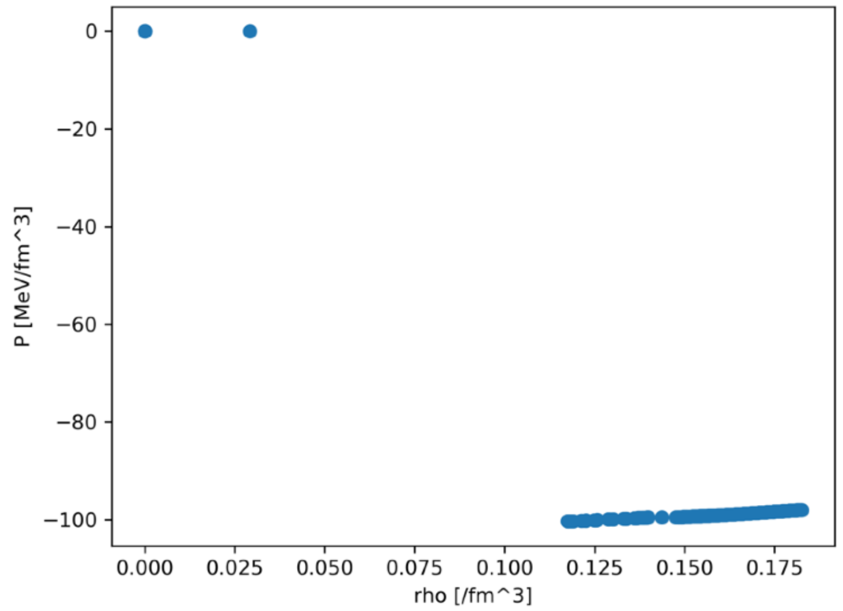


Test Results (4) for SNM

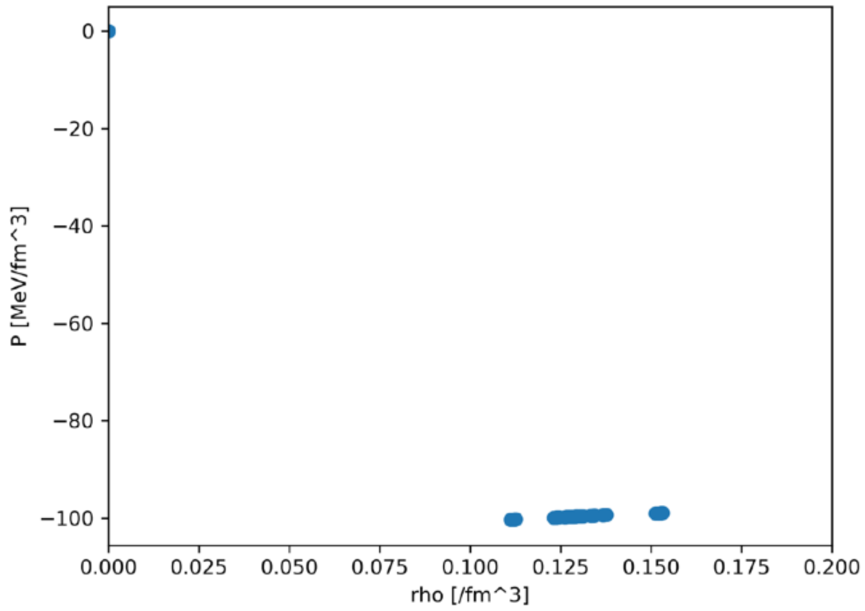
f_w=5.0 fm^2

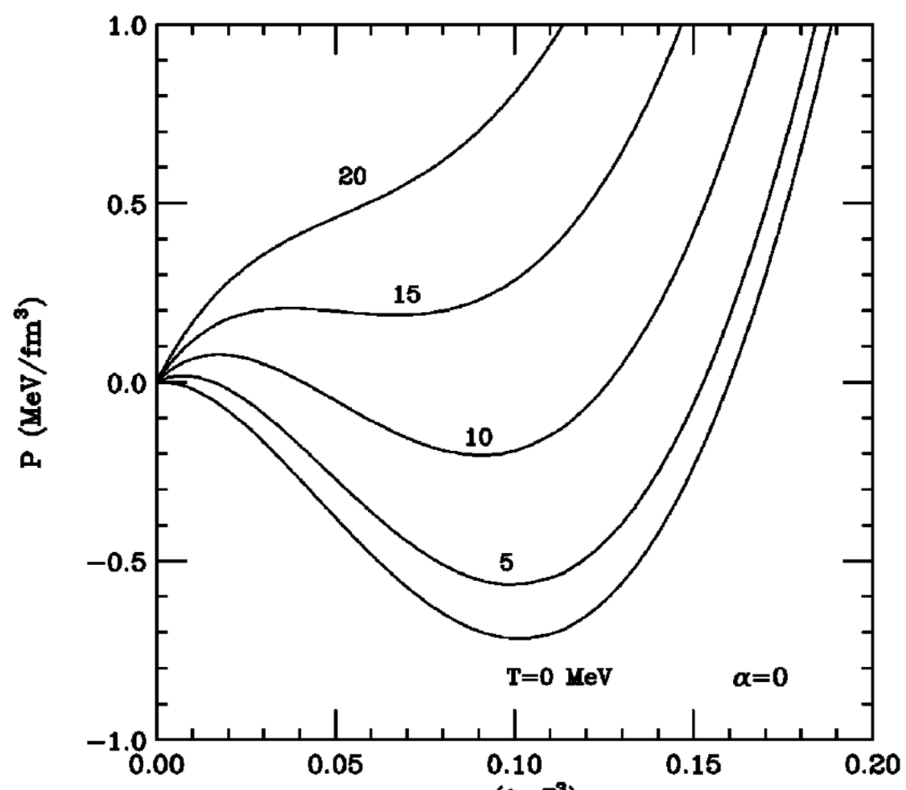


f_w=5.42 fm^2



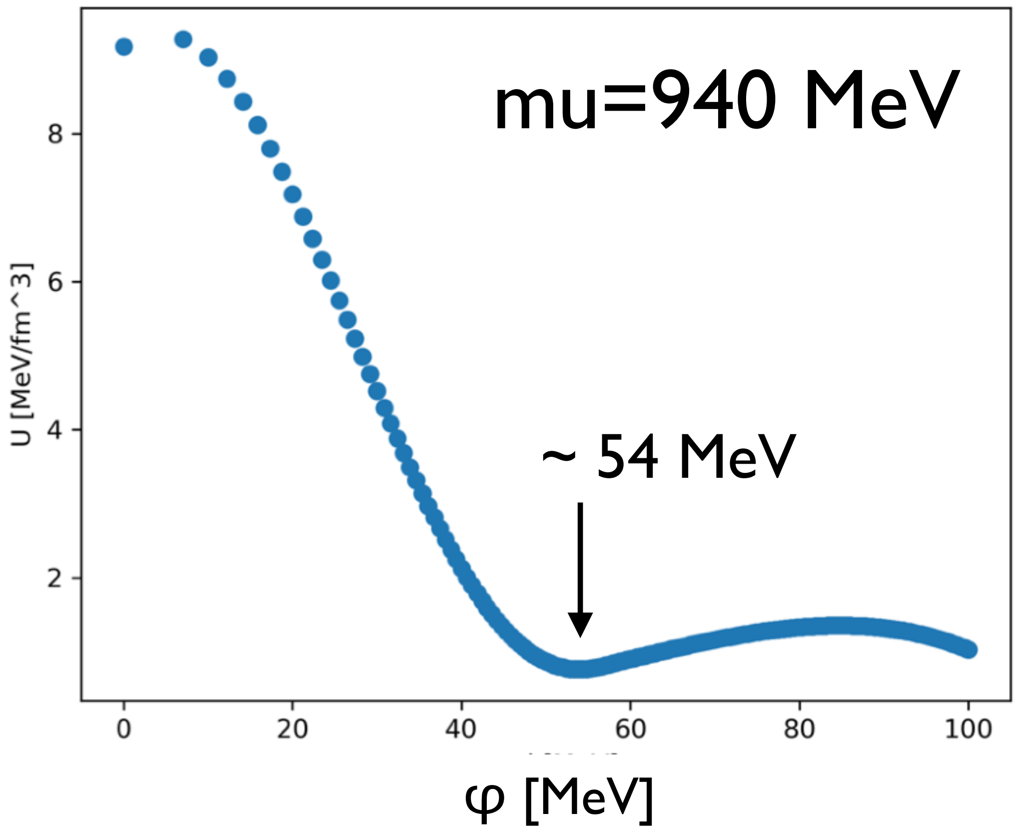
f_w=6.0 fm^2



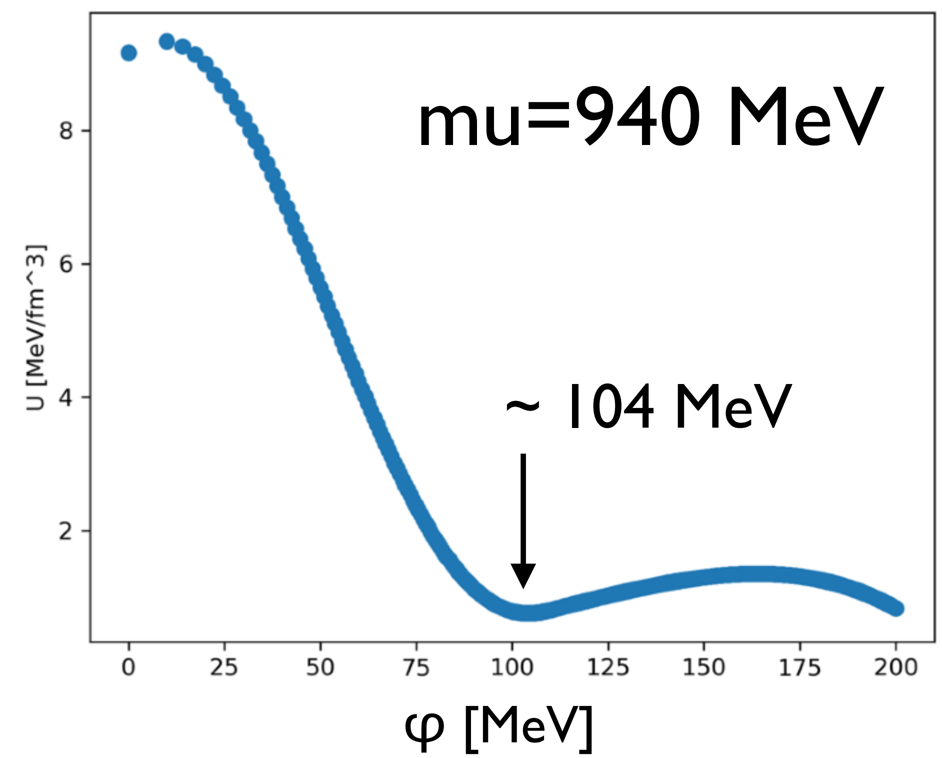


Test Results (5) for SNM

g_s = 16

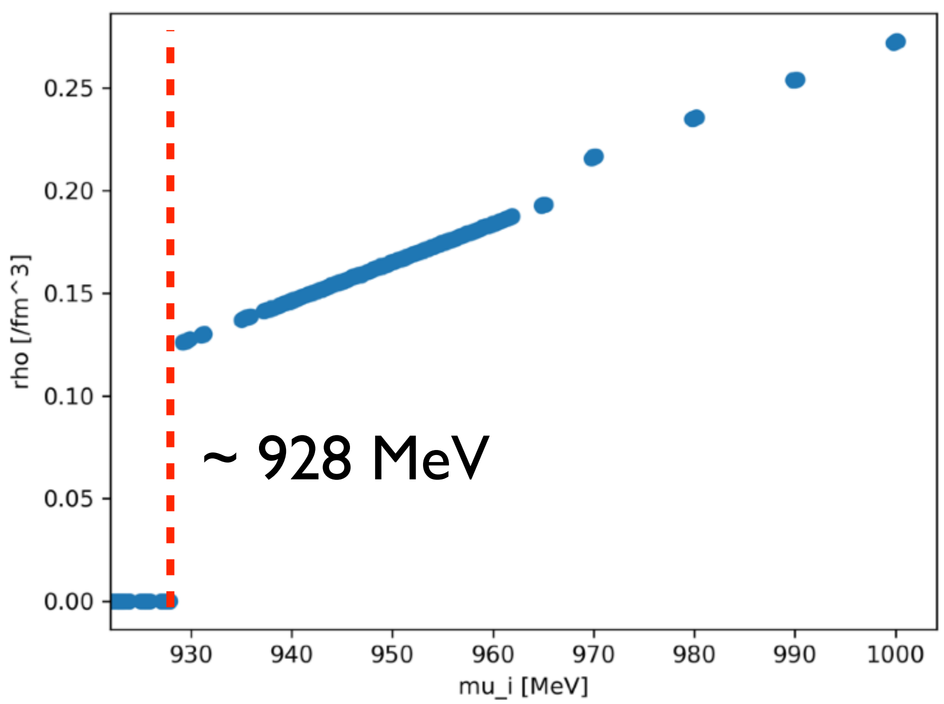


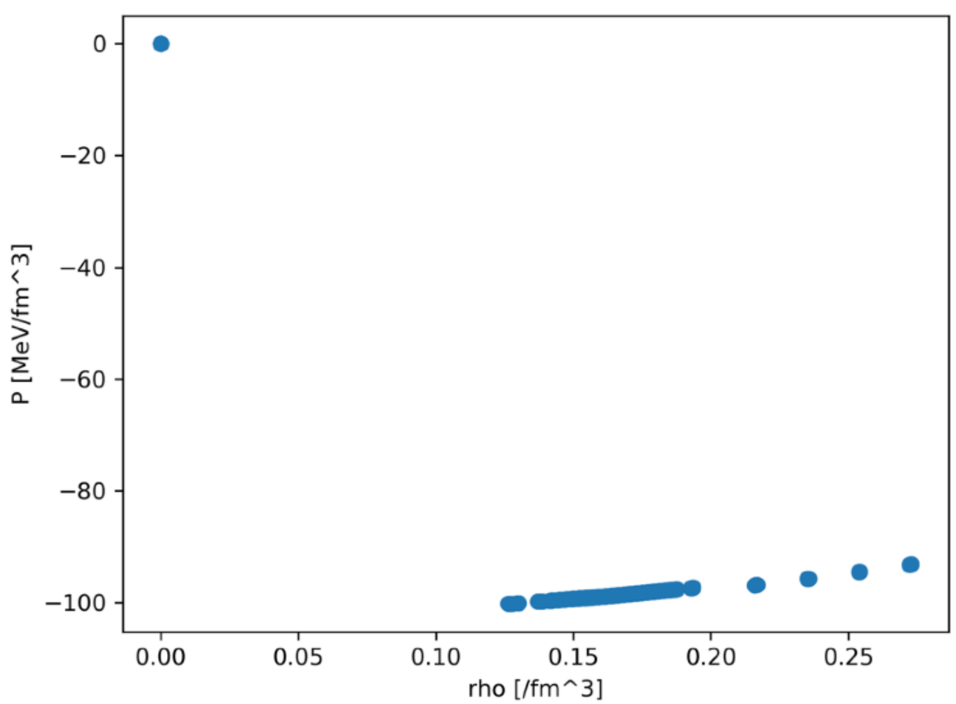
f_w=10.33 fm^2 —> g_s = 8.28



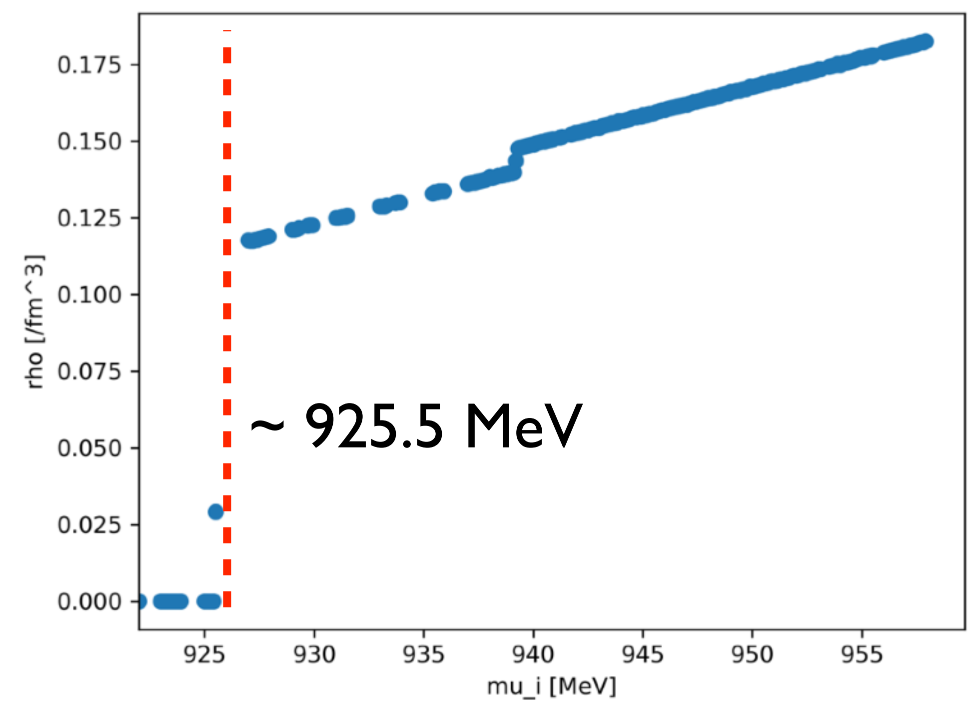
Test Results (6) for SNM

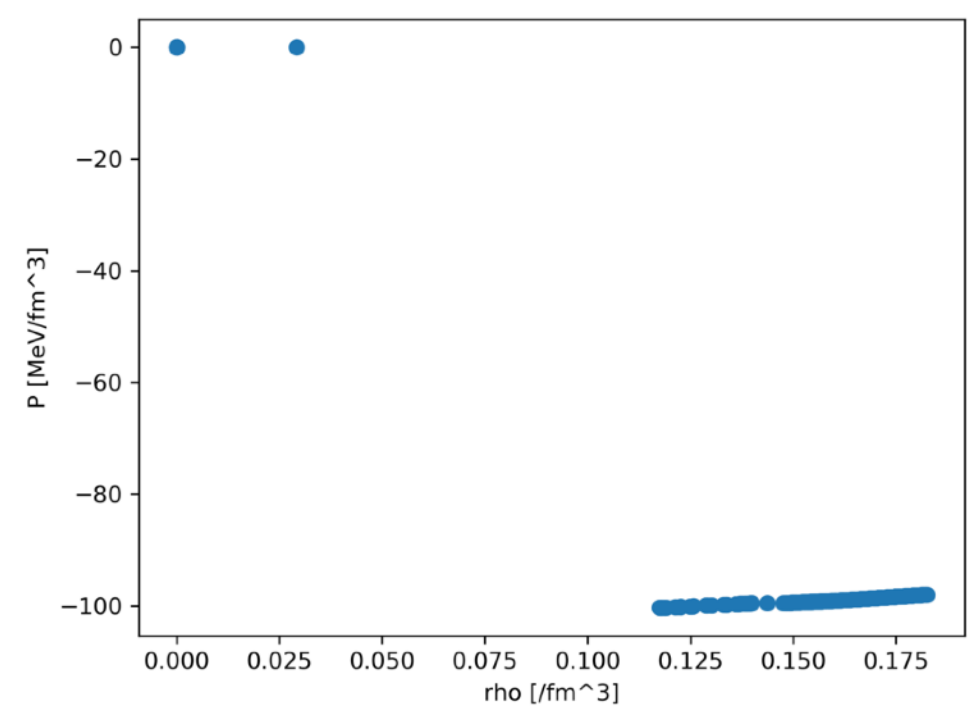
$g\_s = 16$





$f\_w = 10.33\ fm^2 \longrightarrow g\_s = 8.28$





Test Results (7) for SNM

Initial condition

$$U_\Lambda^\phi = \frac{1}{3}\tilde{a}\phi^3 + \frac{1}{4}\tilde{b}\phi^4$$

A—> A/100, B—> B/100

Liu et al., PHYSICAL REVIEW C **65** 045201

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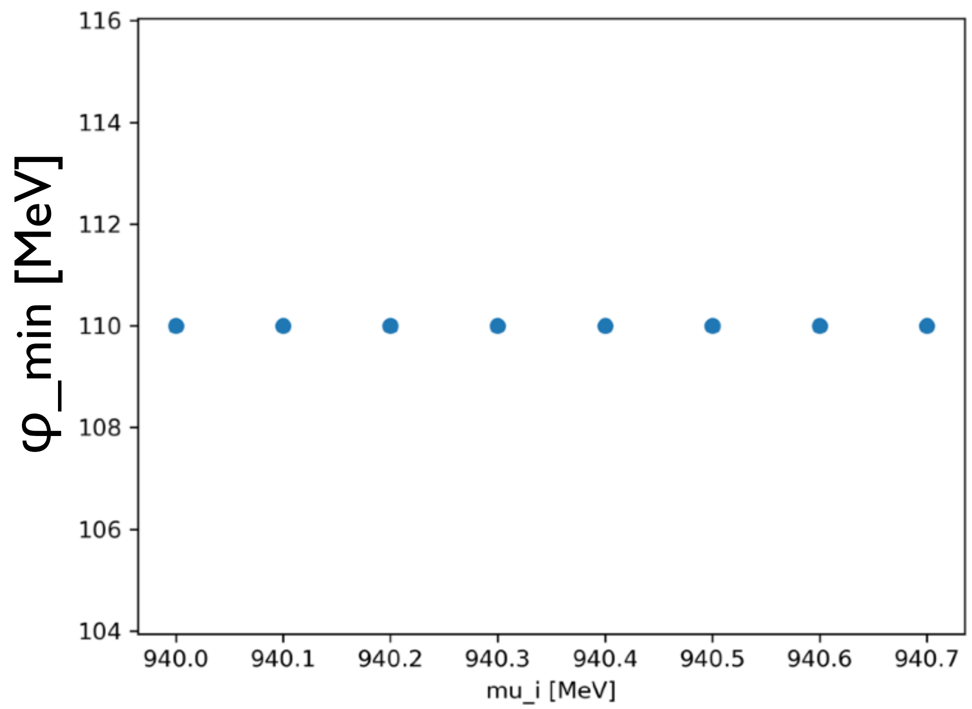

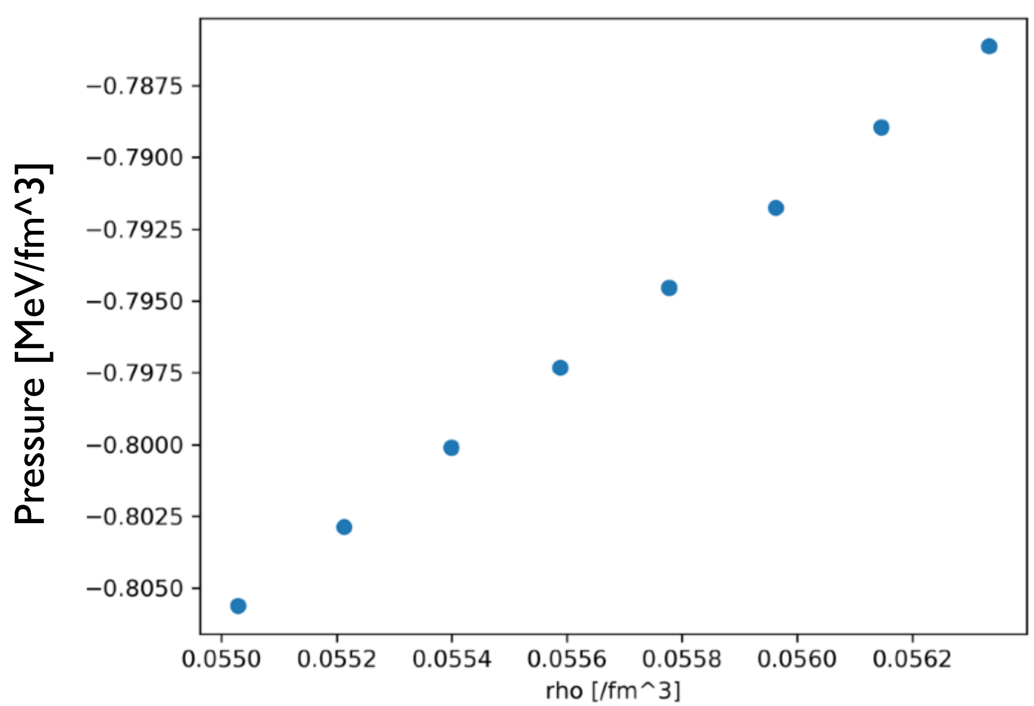

Summary and future plans

1. FRG calculation is still on-going.
2. We need to optimize model parameters.

   - Number of calculation : ~ 10^6,   running time of a single job : 10~60 min.
   - Bayesian optimization and/or deep learning approach

3. Future plans

   - Nuclear matter equation of state —> NS mass, radius, tidal deformability
   - Application to Finite Nuclei
   - Application to Heavy Ion Collision w/ DJBUU

Thank you for your attention.