Proton induced fission process with a Langevin approach

Ik Jae Shin

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- Fission
- Langevin method
- Results : p + ^{238}U
  - empirical formula
  - Langevin approach

◆ **Collaborators**

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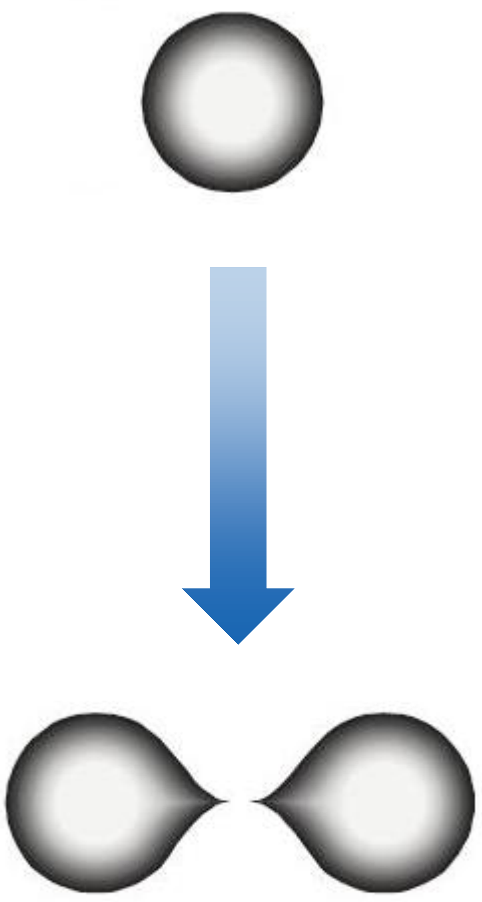
Shinya Takagi, Yoshihiro Aritomo (Kindai Univ.)
Youngman Kim (IBS)

Nuclear fission

❖ Nuclear fission

- Nuclear fission is a kind of reaction by which a nucleus is divided into two (or more) smaller nuclei.

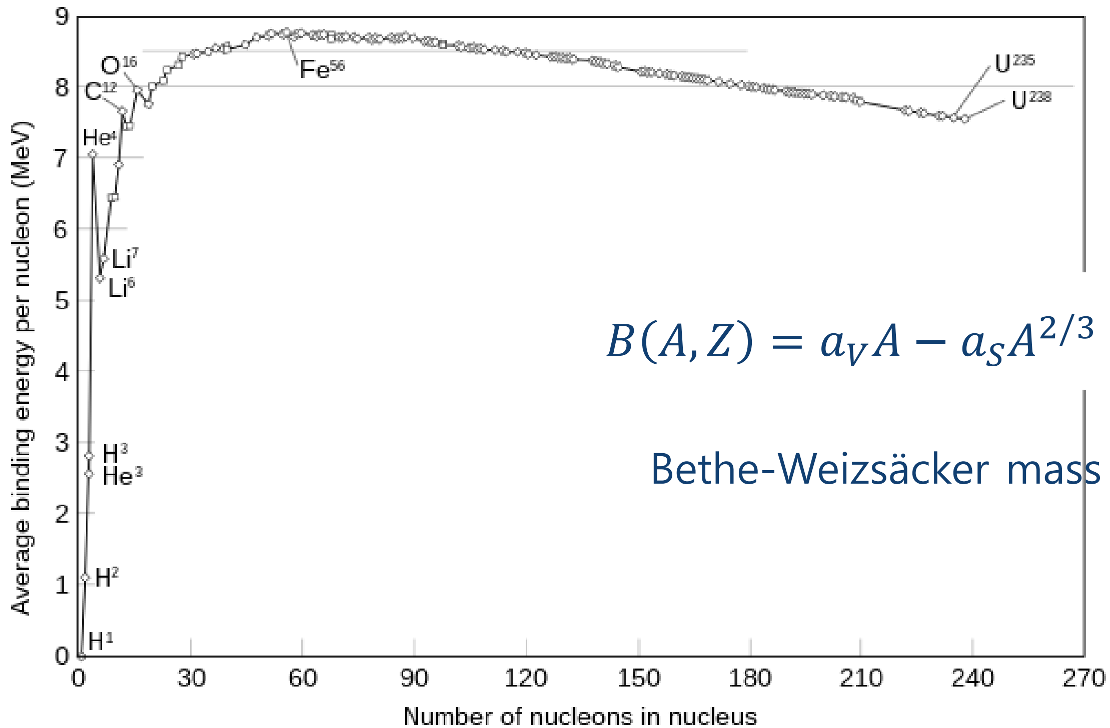
- In 1939, by O. Hahn and F. Starβman, the fission of ^{235}U was first observed.



[https://en.wikipedia.org/wiki/Nuclear_fission]

Nuclear fission

❖ Nuclear fission



$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - \cdots$$

Bethe-Weizsäcker mass formula

[https://en.wikipedia.org/wiki/Nuclear_binding_energy]

Nuclear fission

❖ Nuclear fission

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - \cdots \approx E_V - E_S - E_C$$

If we assume a symmetric fission, i.e. $A_1 = A_2 = A/2$, then

$$Q = 2 \times B(A/2, Z/2) - B(A,Z)$$
$$= E_S^0\left(1 - 2^{1/3}\right) + E_C^0\left(1 - 2^{-2/3}\right) > 0\,.$$

$$\therefore \quad \frac{E_C^0}{E_S^0} > 0.7 \quad , \quad \text{or equivalently} \quad \frac{Z^2}{A} > 0.7 \times \frac{a_S}{a_C}$$

Nuclear fission

❖ Nuclear fission

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - \cdots \approx E_V - E_S - E_C$$

If we assume a symmetric fission, i.e. $A_1 = A_2 = A/2$, then

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$$= E_S^0\left(1 - 2^{1/3}\right) + E_C^0\left(1 - 2^{-2/3}\right) > 0\,.$$

$$\therefore \quad \frac{E_C^0}{E_S^0} > 0.7$$, or equivalently $$\frac{Z^2}{A} > 0.7 \times \frac{a_S}{a_C} \approx 18$$, $$A \geq 70$$

This is not natural..

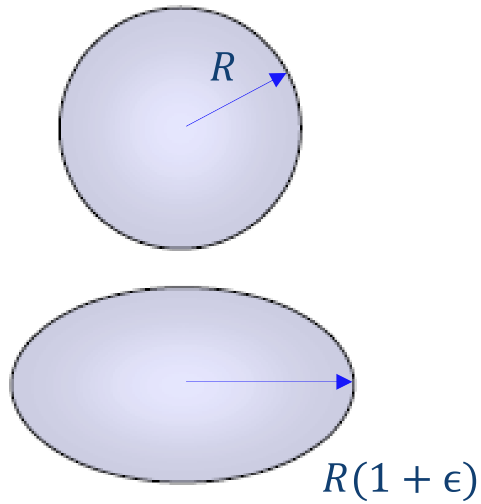
Nuclear fission

❖ Nuclear fission

If we consider a change derived from the ellipsoidal deformation,

$$E_S = E_S^0\left(1 + \frac{2}{5}\epsilon^2\right)$$

$$E_C = E_C^0\left(1 - \frac{1}{5}\epsilon^2\right)$$



$$Q = B - B^0 = -E_S^0 \times \frac{2}{5}\epsilon^2 + E_C^0 \times \frac{1}{5}\epsilon^2 > 0$$

$$\therefore \quad \frac{E_C^0}{2E_S^0} > 1 \quad , \quad \text{or equivalently} \quad \frac{Z^2}{A} > \frac{2a_S}{a_C}$$

Nuclear fission

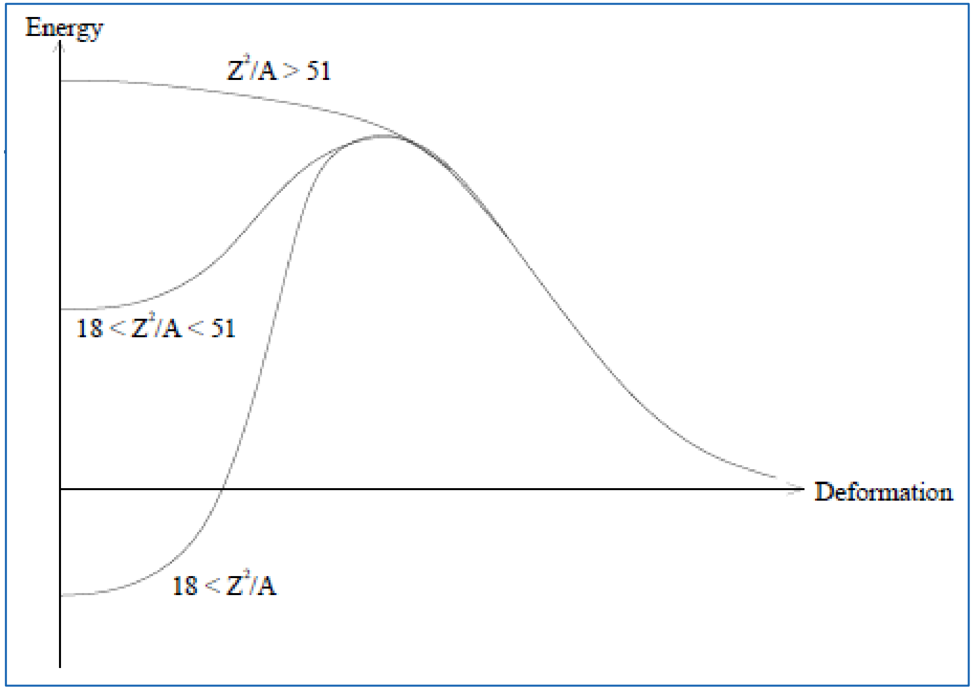
❖ Nuclear fission

If we consider a change derived

$$E_S = E_S^0\left(1 + \frac{2}{5}\epsilon^2\right)$$

$$E_C = E_C^0\left(1 - \frac{1}{5}\epsilon^2\right)$$

$$Q = B - B^0 = -E_S^0 \times \frac{2}{5}\epsilon^2 + E_C^0 \times$$



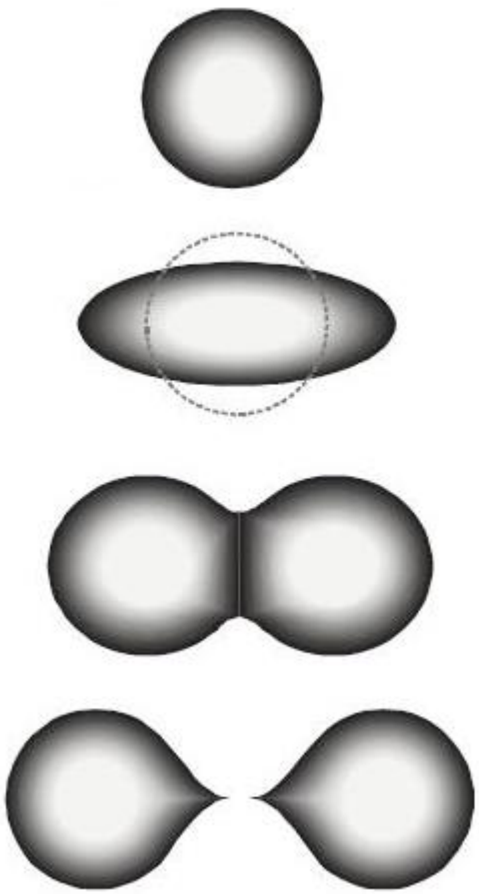
$\therefore \quad \dfrac{E_C^0}{2E_S^0} > 1$, or equivalently $\dfrac{Z^2}{A} > \dfrac{2a_S}{a_C} \approx 51$

Nuclear fission

❖ Theoretical description of fission

▪. Charged LDM (liquid drop model)

Coulomb repulsion (disrupting)
vs.
Surface tension (stabilizing)



[https://en.wikipedia.org/wiki/Nuclear_fission]

Nuclear fission

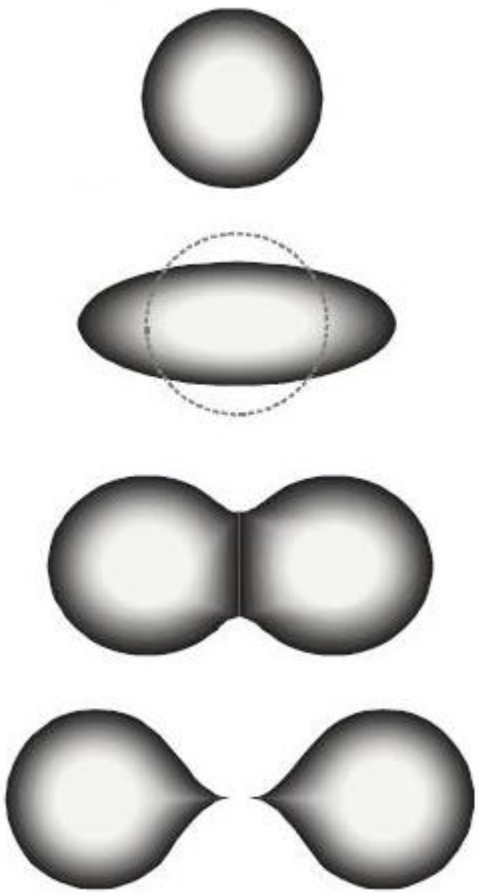
❖ Theoretical description of fission

- Charged LDM (liquid drop model)

Coulomb repulsion (disrupting) vs. Surface tension (stabilizing)

However, LDM can only explain the symmetric fission..

Nature, e.g. in case of actinides, prefers mass-asymmetric splitting.



[https://en.wikipedia.org/wiki/Nuclear_fission]

Nuclear fission

❖ Theoretical description of fission

▪. Charged LDM (liquid drop model)

Coulomb repulsion (disrupting)
vs.
Surface tension (stabilizing)

- To consider the microscopic effect

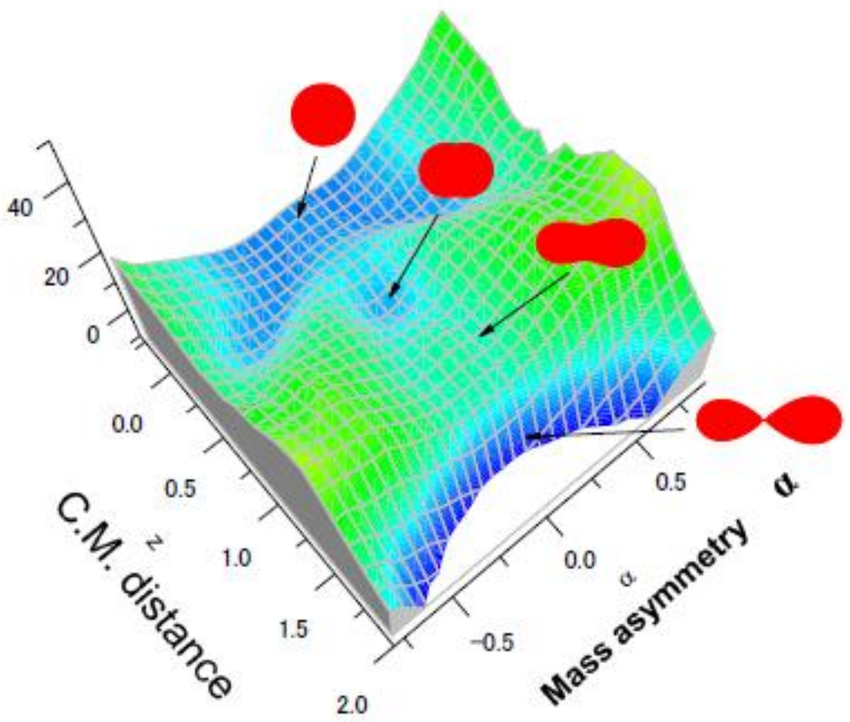
⇒ LDM + shell correction (macroscopic-microscopic potential)

+ mass-asymmetric degree of freedom

Langevin method

❖ Dynamical calculation

Time-evolution of nuclear shape in fission process



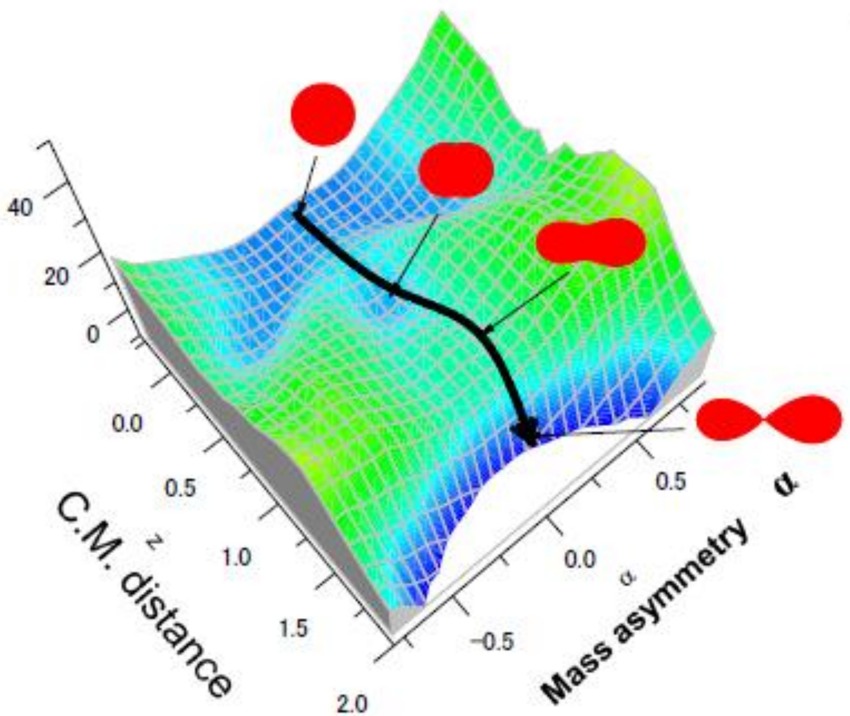
- Potential energy surface

[From Y.Aritomo's talk. 16th ASRC Workshop (2014)]

Langevin method

❖ Dynamical calculation

Time-evolution of nuclear shape in fission process

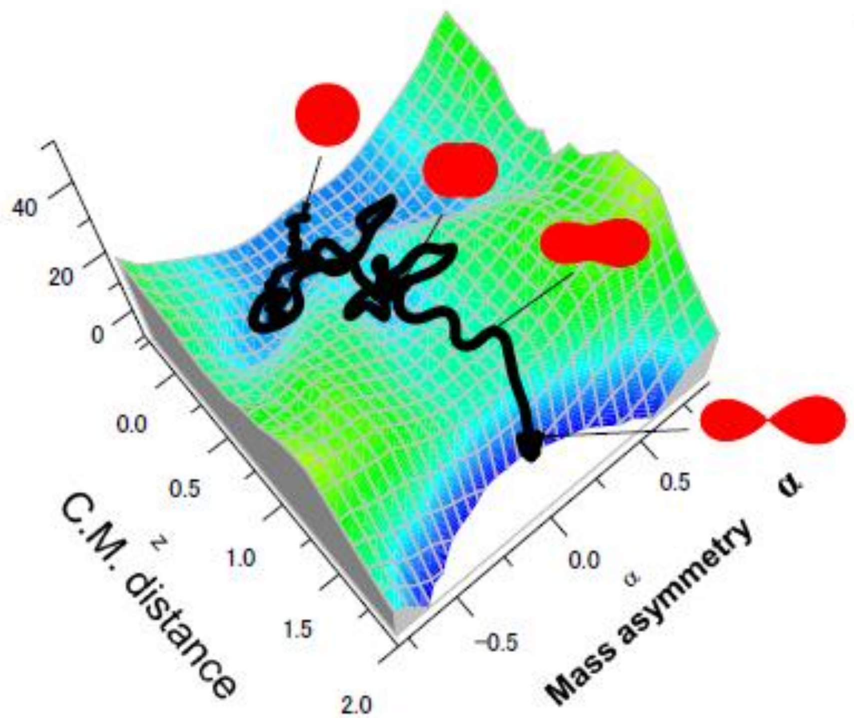


- Potential energy surface
- Trajectory described by equation of motion

Langevin method

❖ Dynamical calculation

Time-evolution of nuclear shape in fission process

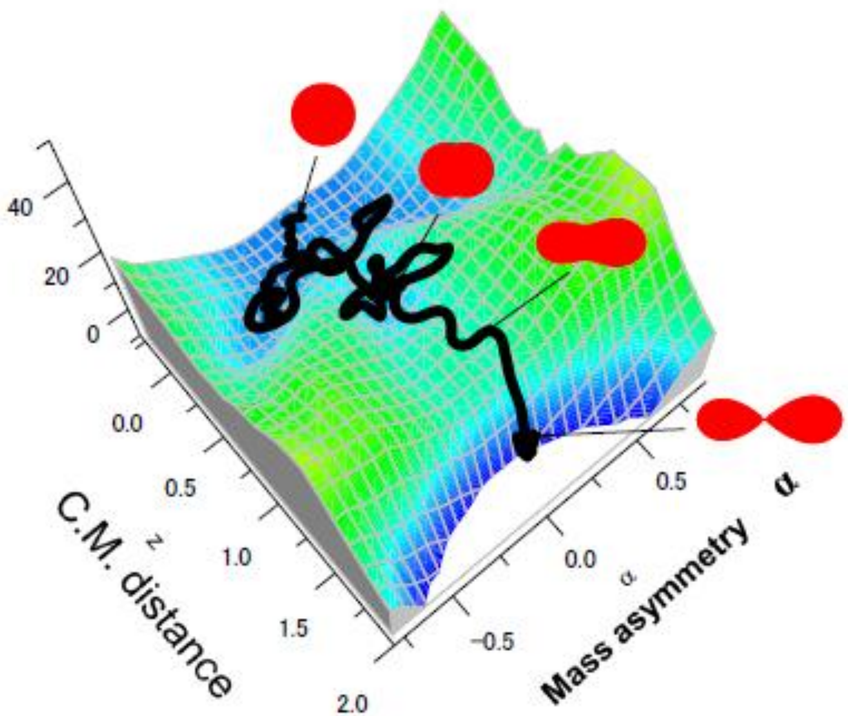


- Potential energy surface
- Trajectory described by equation of motion
- Dissipation effects !

Langevin method

❖ Dynamical calculation

Time-evolution of nuclear shape in fission process



- Potential energy surface
- Trajectory described by equation of motion
- Dissipation effects !

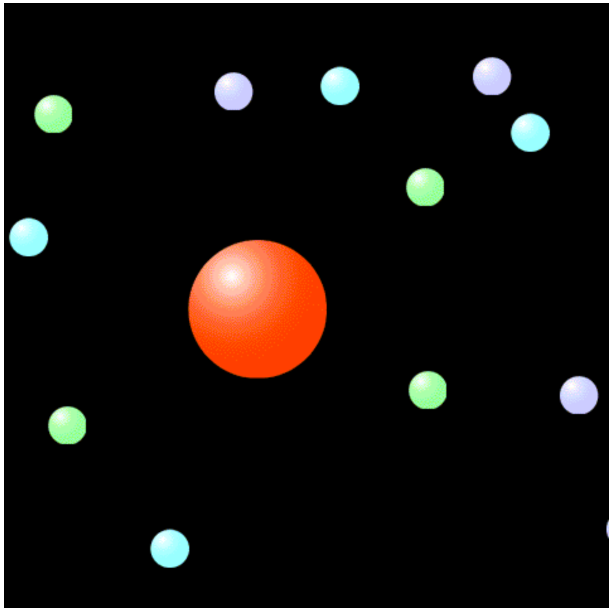
Langevin equation

Langevin method

❖ Langevin equation

- Brownian motion

  : the random motion of particles surrounded by a medium



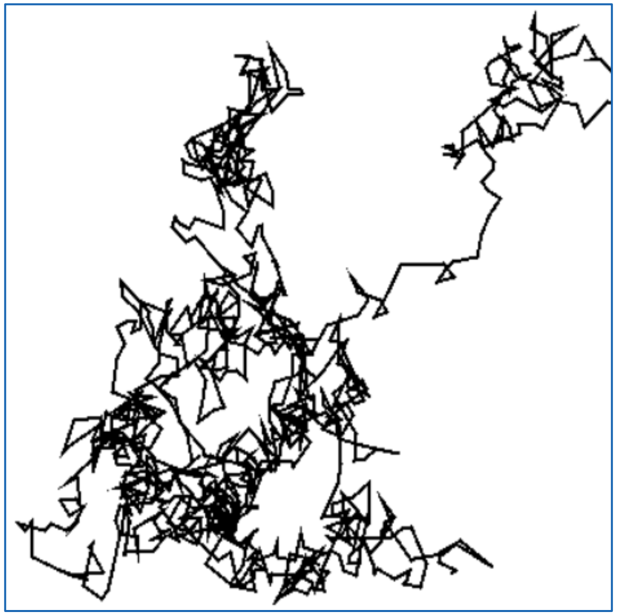
[From S.Chiba's domestic talk. (2019)]

[https://en.wikipedia.org/wiki/Brownian_motion]

Langevin method

❖ Langevin equation

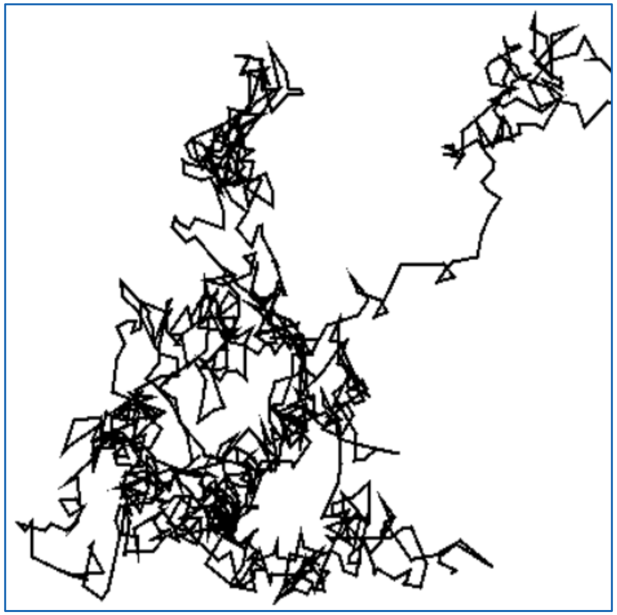
- Brownian motion
  : the random motion of particles surrounded by a medium



[From S.Chiba's domestic talk. (2019)]

Langevin method

❖ Langevin equation



- Brownian motion
  : the random motion of particles surrounded by a medium

- Langevin equations describe the time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a "heat bath".

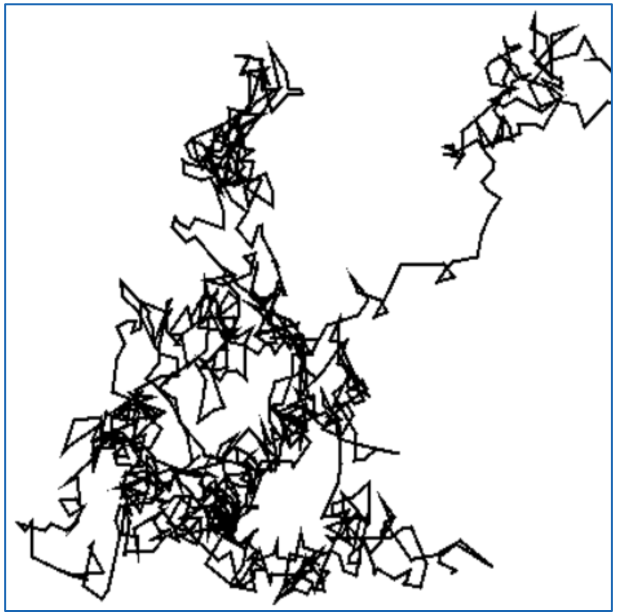
$$\frac{dp}{dt} = f$$

Langevin method

❖ Langevin equation

- Brownian motion

  : the random motion of particles surrounded by a medium



- Langevin equations describe the time evolution of the collective variables like the evolution of Brownian particle that interacts stochastically with a “heat bath”.

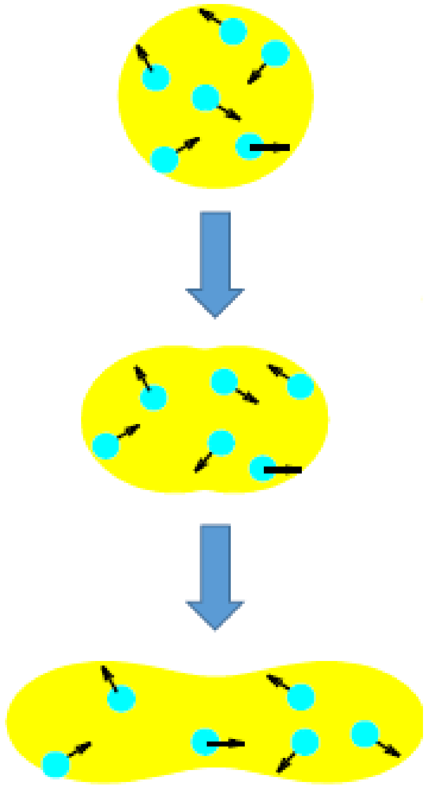
$$\frac{dp}{dt} = f - \gamma \frac{p}{m} + R(t) \qquad \text{where} \quad \langle R(t) \rangle = 0$$

Langevin method

❖ Langevin equation

- Nuclear shape evolution is driven by random kicks of nucleons in thermal equilibrium.

- The average of these impulse constitutes the associated friction force, while the residual fluctuations provide the diffusive properties for shape evolution.



Langevin method

❖ Nuclear shape and Potential energy

Two-center shell model parametrization

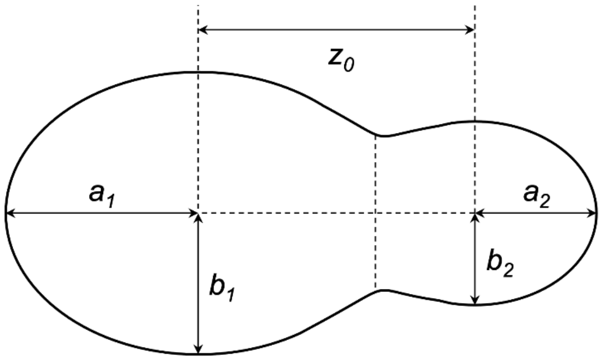
$$\delta_i = \frac{3(a_i - b_i)}{2a_i + b_i}$$ [deformation]

$$z = \frac{z_0}{BR}$$ [rescaled distance]

where $B = \sqrt{B_1 B_2}$ with $B_i = \frac{3 + \delta_i}{3 - 2\delta_i}$

R : radius of compound nucleus

$$\alpha = \frac{A_1 - A_2}{A_1 + A_2}$$ [mass asymmetry]



Langevin method

❖ Nuclear shape and Potential energy

Two-center shell model parametrization

Here, we assume : $\delta_1 = \delta_2 = \delta$

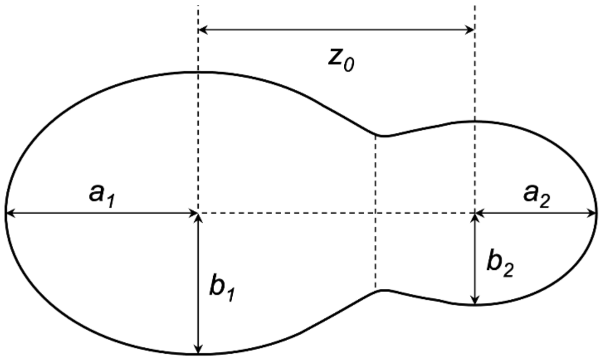
$$\delta = \frac{3(a-b)}{2a+b}$$ [deformation]

$$z = \frac{z_0}{BR}$$ [rescaled distance]

where $B = \dfrac{3+\delta}{3-2\delta}$

R : radius of compound nucleus

$$\alpha = \frac{A_1 - A_2}{A_1 + A_2}$$ [mass asymmetry]



Langevin method

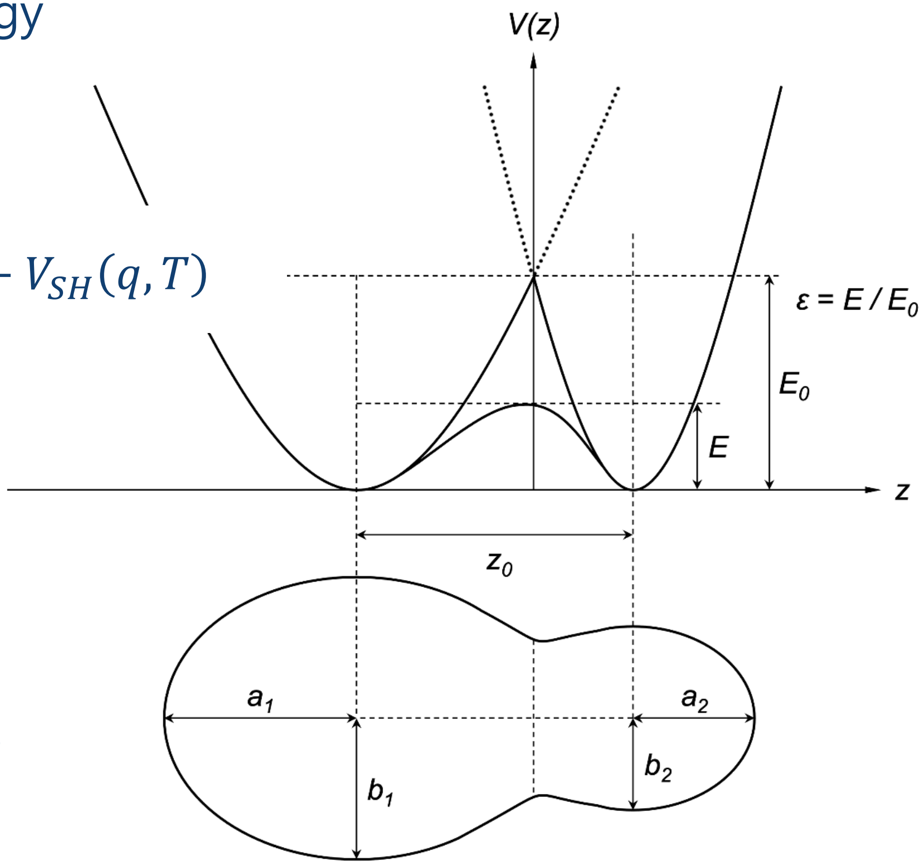
❖ Nuclear shape and Potential energy

$$V(q,l,T) = V_{LDM}(q) + \frac{\hbar^2 l(l+1)}{2I(q)} + V_{SH}(q,T)$$

$$V_{LDM}(q) = E_{surf}(q) + E_{Coul}(q)$$

$I(q)$: moment of inertia of rigid body

$$V_{SH}(q,T) = E^0_{shell}(q)\exp(-aT^2/E_d)$$



Langevin equation

❖ (Multi-dimensional) Langevin equation

$$\frac{dq_i}{dt} = (m^{-1})_{ij}\, p_j$$

friction

$$\frac{dp_i}{dt} = -\frac{\partial V}{\partial q_i} - \frac{1}{2}\frac{\partial (m^{-1})_{jk}}{\partial q_i} p_j p_k \boxed{- \gamma_{ij}(m^{-1})_{jk} p_k} \boxed{+ g_{ij} R_j(t)}$$

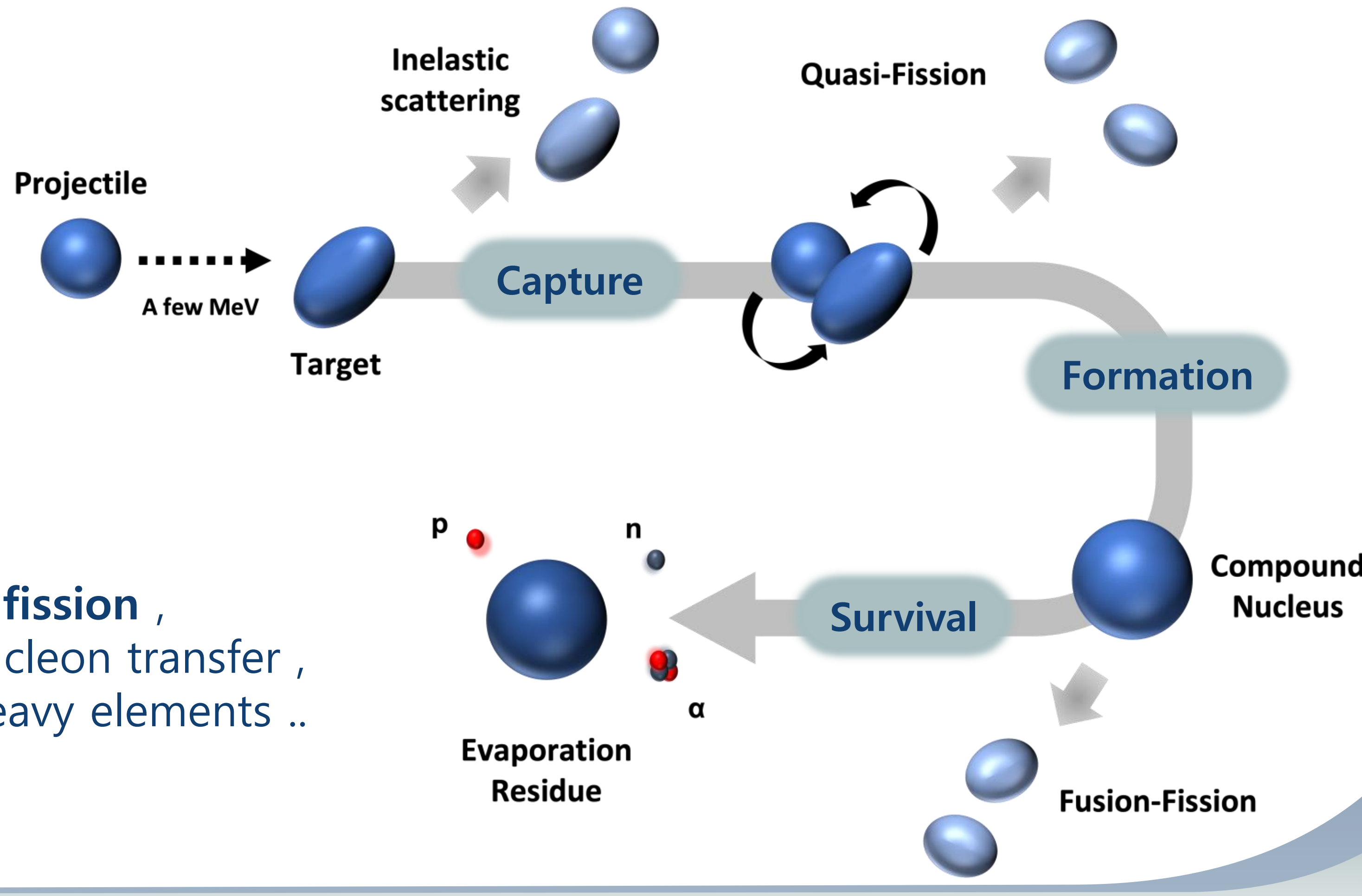
random force

q_i : deformation coordinate $\{z, \delta, \alpha\}$
p_i : conjugate momentum
m_{ij} : inertia mass tensor
γ_{ij} : friction tensor

Here, the transport coefficients m_{ij} and γ_{ij} are obtained microscopically.

Langevin method

❖ Langevin equation



fusion / **fission** ,
multi-nucleon transfer ,
super-heavy elements ..

Result : $p+^{238}$U (preliminary)

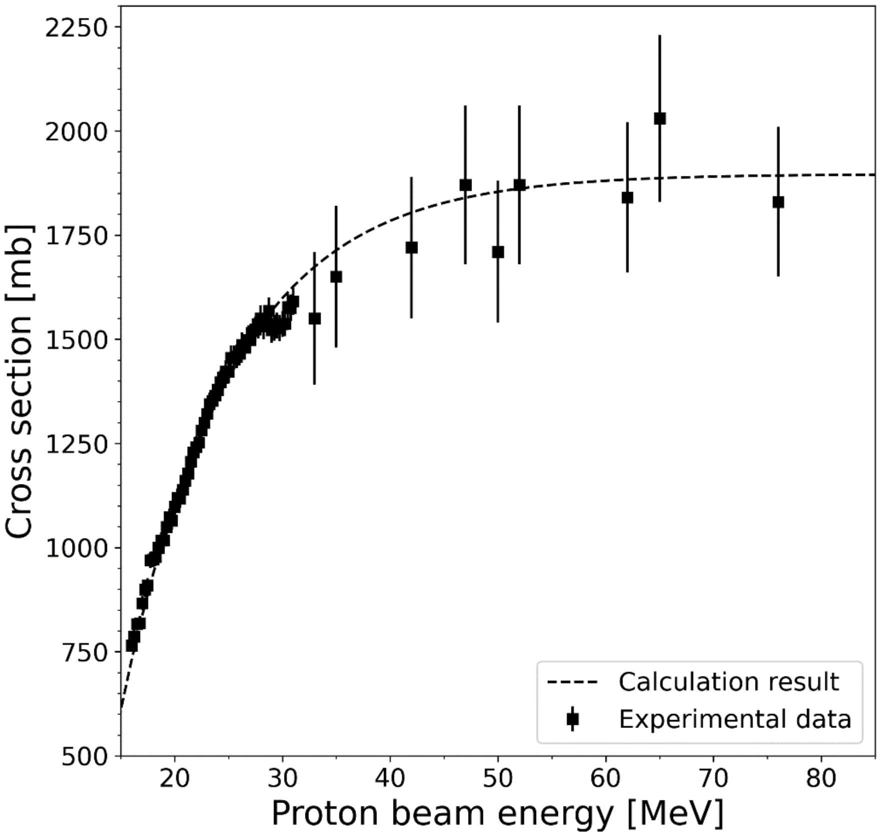
❖ Empirical formula

Total fission cross section :

$$\sigma_f(E_p) = p_1\{1 - \exp[-p_2(E_p - p_3)]\}$$

p_1 : the saturation cross section
p_2 : the increasing rate of c.s. with energy
p_3 : the apparent threshold energy

[F.A.Khan, *et al.*, Phys.Rev.C94, 054605 (2016)]



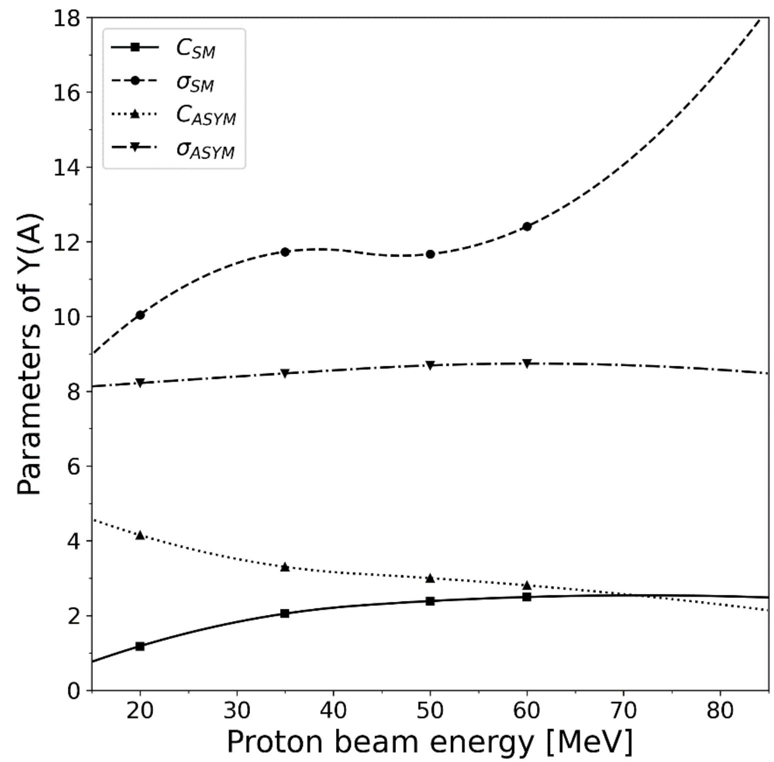
Result : $p+^{238}U$ (preliminary)

❖ Empirical formula

Mass yield distribution :

$$Y(A) = Y_{SM}(A) + Y^1_{ASYM}(A) + Y^2_{ASYM}(A)$$

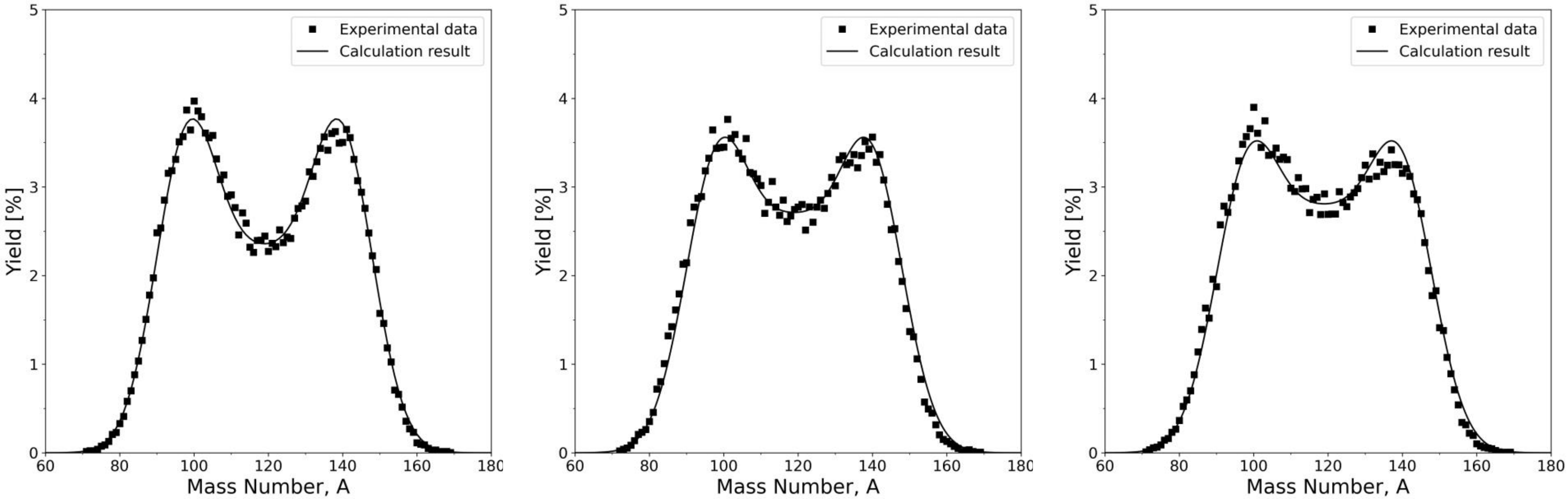
$$= C_{SM}\exp\left[-\frac{(A-A_{SM})^2}{2\sigma^2_{SM}}\right] + C_{ASYM}\exp\left[-\frac{\{A-(A_{SM}-D_{ASYM})\}^2}{2\sigma^2_{ASYM}}\right] + C_{ASYM}\exp\left[-\frac{\{A-(A_{SM}+D_{ASYM})\}^2}{2\sigma^2_{ASYM}}\right]$$



[C.-H.Song, *et al.*, submitted to JKPS]

Result : p+^{238}U (preliminary)

- Empirical formula

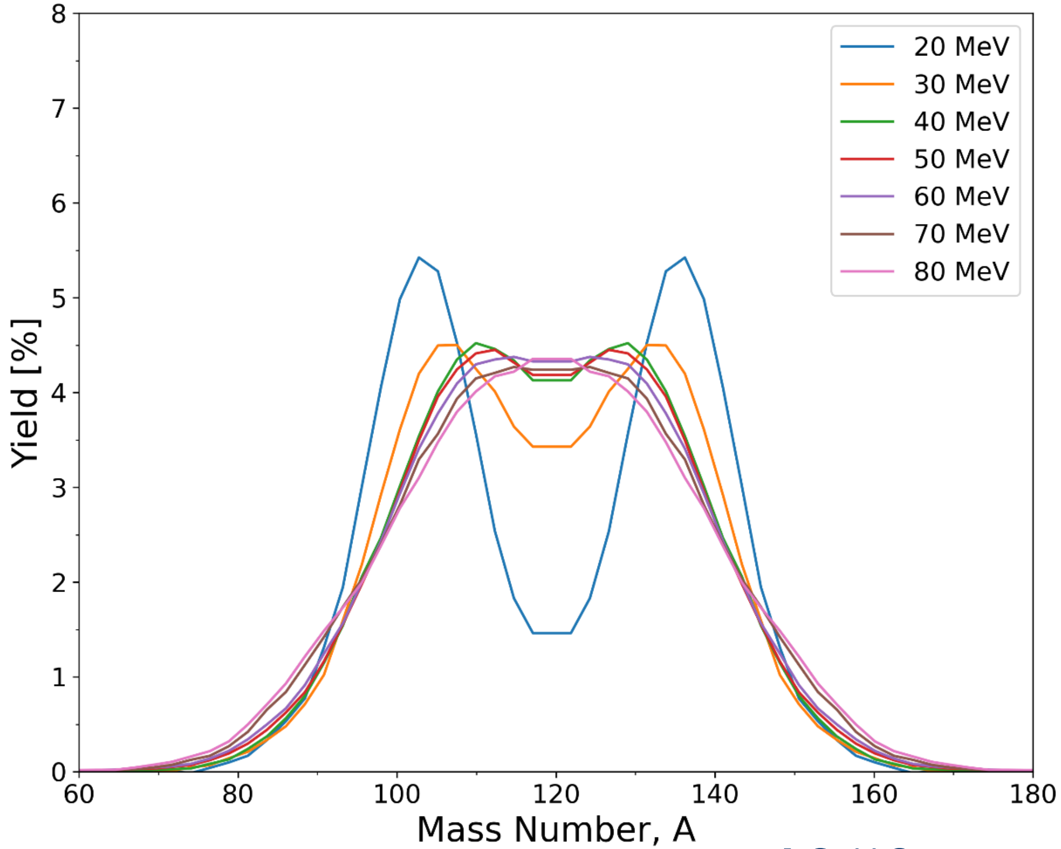


35 MeV 50 MeV 60 MeV

[C.-H.Song, *et al.*, submitted to JKPS]

Result : $p+^{238}U$ (preliminary)

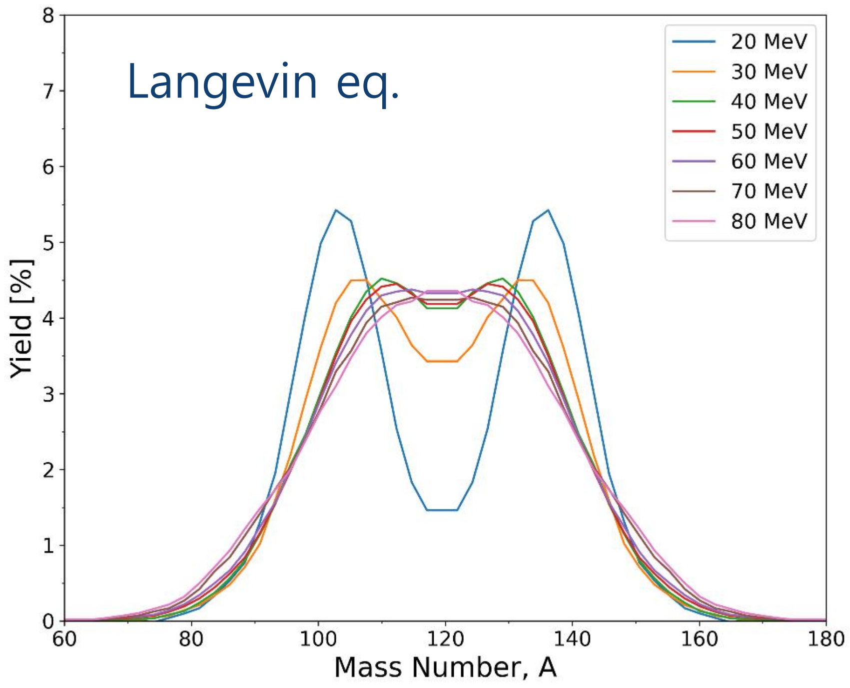
❖ Langevin equation

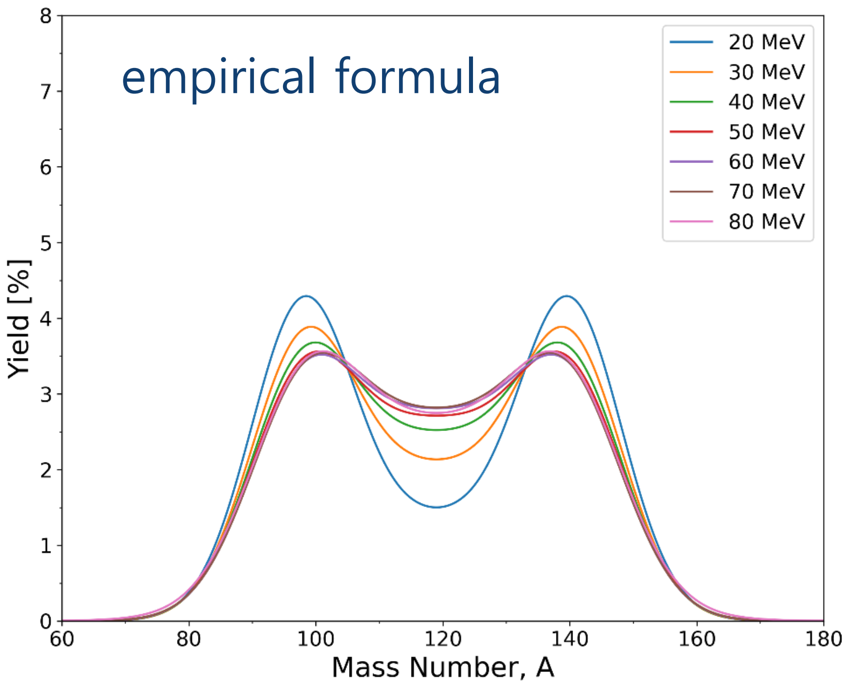


[C.-H.Song, *et al.*, submitted to JKPS]

Result : $p+^{238}U$ (preliminary)

❖ Comparison





[C.-H.Song, *et al.*, submitted to JKPS]

Summary and outlook

- ✓ A dynamical model based on Langevin equations has been applied to study the fission dynamics.

- ✓ We are calculating the p + ^{238}U collision for ISOL beam at the RAON experiment and find the fission cross section with what fission fragments are revealed.

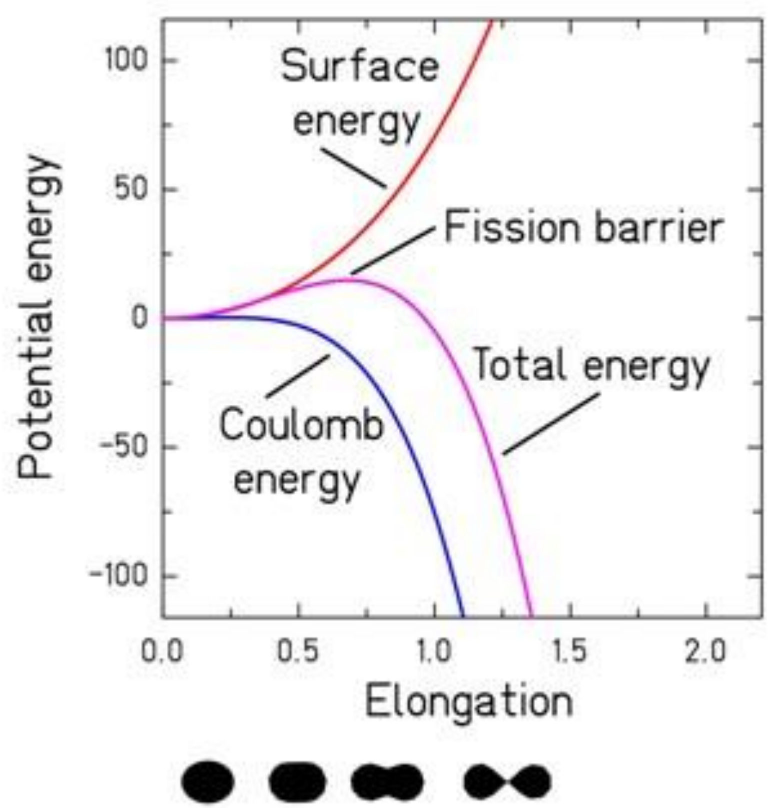
- ✓ Further study
  - To improve the model to reduce the difference between the experiment and theoretical calculation
  - To increase the number of variables: e.g. $\delta_1 \neq \delta_2$


THANK YOU !

Backup

Nuclear fission

❖ Nuclear fission



- When the shape of nuclei is deformed, till some point, the surface tension dominates over Coulomb repulsion.
- So the fission occurs if there is an enough energy to overcome the fission barrier.

Nuclear fission

❖ Nuclear fission

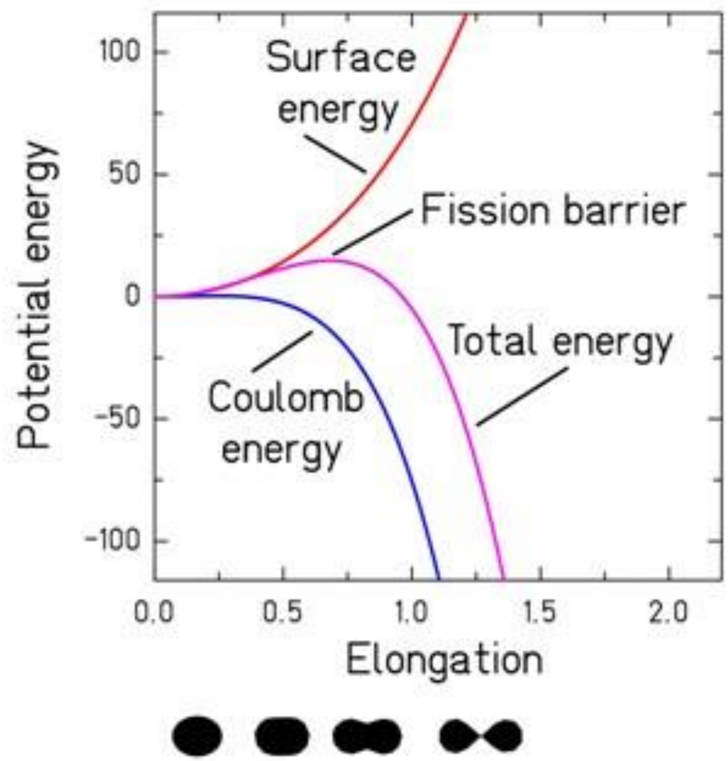
If we consider a change derived from the ellipsoidal deformation,

$$E_S = E_S^0\left(1 + \frac{2}{5}\epsilon^2\right)$$

$$E_C = E_C^0\left(1 - \frac{1}{5}\epsilon^2\right)$$

$$Q = B - B^0 = -E_S^0 \times \frac{2}{5}\epsilon^2 + E_C^0 \times \frac{1}{5}\epsilon^2 > 0$$

$$\therefore \quad \frac{E_C^0}{2E_S^0} > 1 \quad , \quad \text{or equivalently} \quad \frac{Z^2}{A} > \frac{2a_S}{a_C}$$



The types of dissipations

The dissipation of collective energy into internal

The two-body dissipation
(short mean free path)
(Davies et al. 1976)

originates from individual two-body collisions of particles, like in ordinary fluids.

It is currently accepted that one-body mechanism dominates in the dissipation of collective energy. Due to Pauli blocking principe two-body interactions are very unprobable.

The one-body dissipations
(long mean free path)
(Blocki et al. 1978)

originates from collisions of independent particles with moving time-dependent potential well (‘container’ with fixed volume). Two limiting cases: compact shapes (wall formula), necked-in shapes (wall-and-window formula).

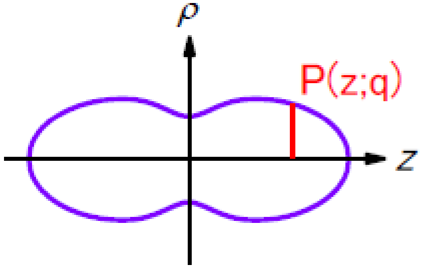
Langevin equation

Transport coefficients (inertia mass)

Inertia Mass (Hydrodynamical mass)

Total kinetic energy of system

$$T = \frac{1}{2}\rho_m \int v^2 d^3r = \frac{1}{2}\sum m_{ij}(q)\dot{q}_i\dot{q}_j$$



Werner-Wheeler approximation

$\nabla \cdot \vec{v} = 0$	Incompressible fluid
$\vec{v} = \dot{\rho}\vec{e}_\rho + \dot{z}\vec{e}_z$	Axially symmetric shape

$$\dot{z} = \sum A_i(z;q)\dot{q}_i$$

$$\dot{\rho} = \frac{\rho}{P}\sum B_i(z;q)\dot{q}_i$$

$$P = P(z;q)$$

For an incompressible fluid the total (convective) time derivative of any fluid volume must vanish

$$m_{ij} = \pi\rho_m \int_{z_{\min}}^{z_{\max}} P^2\left(A_i A_j + \frac{1}{8}P^2 A_i' A_j'\right)dz$$

$$A_i(z;q) = \frac{1}{P^2(z;q)}\frac{\partial}{\partial q_i}\int_z^{z_{\max}} P^2(z';q)dz'$$

$$A_i(z;q) = -\frac{1}{P^2(z;q)}\frac{\partial}{\partial q_i}\int_{z_{\min}}^{z} P^2(z';q)dz'$$

$$B_i(z;q) = -\frac{1}{2}P\frac{\partial A_i}{\partial z}$$

K.T.R. Davies, A.J. Sierk, R. Nix, PRC 13 (1976) 2385

Langevin equation

Transport coefficients (friction tensor)

Friction (One body friction)

Rayleigh dissipation function

Incompressible fluid
constant two-body viscosity coefficient

$$F = \frac{1}{2}\frac{dE}{dt} = \frac{1}{2}\sum \gamma_{ij}(q)\dot{q}_i\dot{q}_j \quad \longleftrightarrow \quad F = \frac{1}{2}\mu\int \Phi(r)\,d^3r = \frac{1}{2}\sum \eta_{ij}(q)\dot{q}_i\dot{q}_j$$

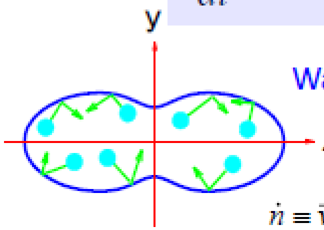
Loss of energy to particles inside the mean filed at the rate

$$\frac{dE}{dt} = \rho_S \bar{v}\int \dot{n}^2 dS$$

$\rho_S = \rho_S(q, z)$ mass density of nucleus

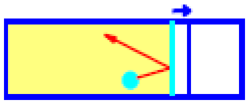
$\bar{v}$ average nucleon speed

$\dot{n}$ relative normal velocity of the wall



Wall formula

$$\dot{n} \equiv \vec{v}\cdot\hat{n} = \frac{\partial \rho_S}{\partial t}\left[1+\left(\frac{\partial \rho_S}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$



$$= \sum_i \dot{q}_i \rho_S \frac{\partial \rho_S}{\partial q_i}\left[\rho_S^2 + \left(\rho_S \frac{\partial \rho_S}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$

$$\gamma_{ij} = \frac{\pi\rho\bar{v}}{2}\int_{z_{\min}}^{z_{\max}} dz \frac{\partial \rho_S^2}{\partial q_i}\frac{\partial \rho_S^2}{\partial q_j}\left[\rho_S^2 + \frac{1}{4}\left(\frac{\partial \rho_S^2}{\partial z}\right)^2\right]^{-\frac{1}{2}}$$

One body friction (Wall formula)

A.J. Sierk, R. Nix, PRC 21 (1980) 982

Nuclear fission

❖ Theoretical description of fission

WRONG???

- Charged LDM (liquid drop model)

Coulomb repulsion (disrupting) vs. Surface tension (stabilizing)

- statistical model

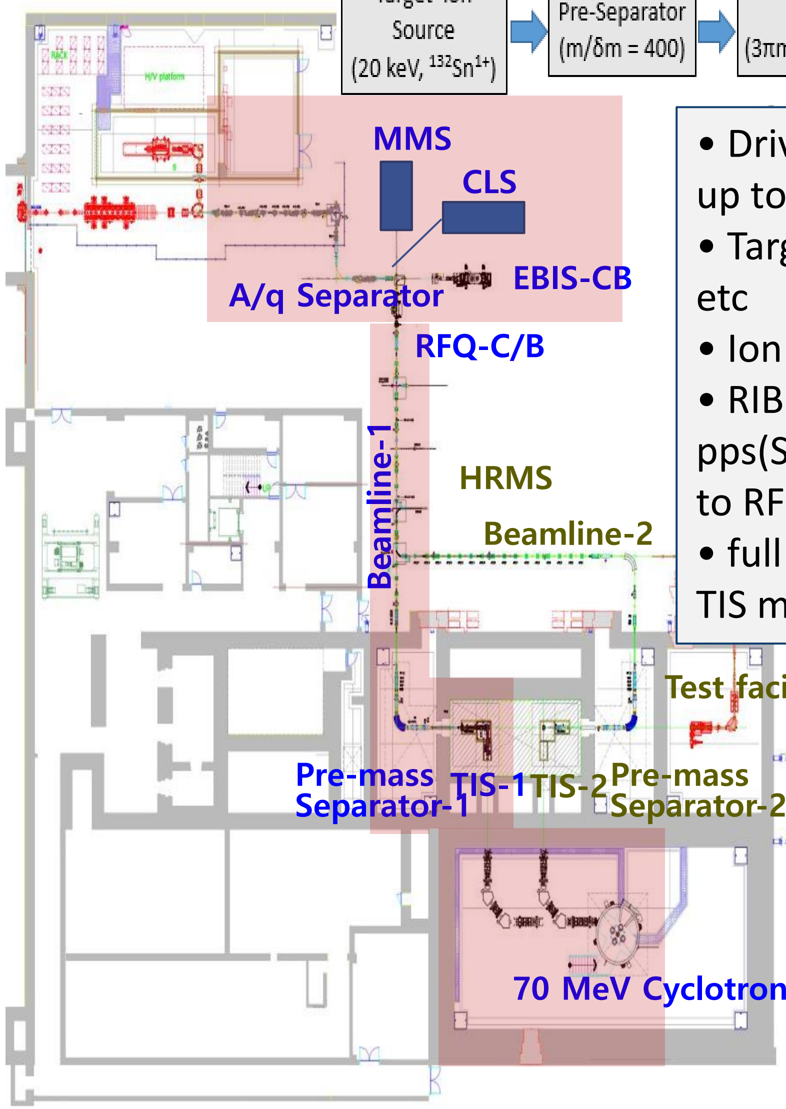
  ✓ including level density
  at the ground state and the saddle point

* micro-macroscopic model using shell correction
* Kramers' theory is used to invoke energy dissipation mechanisms in statistical model calculations

3. Sys. Install.ISOL system

[From T.Shin, Feb. 18 (2022) A3 Joint m

Target ion Source (20 keV, $^{132}Sn^{1+}$) → Pre-Separator (m/δm = 400) → RFQ Cooler (3πmm mrad, < 5ev) → EBIS (10 keV/u, $^{132}Sn^{33+}$) → Post linac (Charge state n+)



- Driver beam : proton 35<K<70 MeV, up to 70 kW
- Target : UCx, MgO, BN, CaO, BeO, SiC, etc
- Ion Source : Surface, RILIS, Plasma
- RIB : 6< A < 250, 10<K< 80 keV, 10^8 pps(Sn), >90% purity @Exp.
- incident to RFQ of Post accelerator 10 keV/u
- full remote maintenance system with TIS mod

ISOL beam line including sub-systems is commissioned in 2021

Try to study proton induced ^{238}U fission