

2212.09571
2304.13042

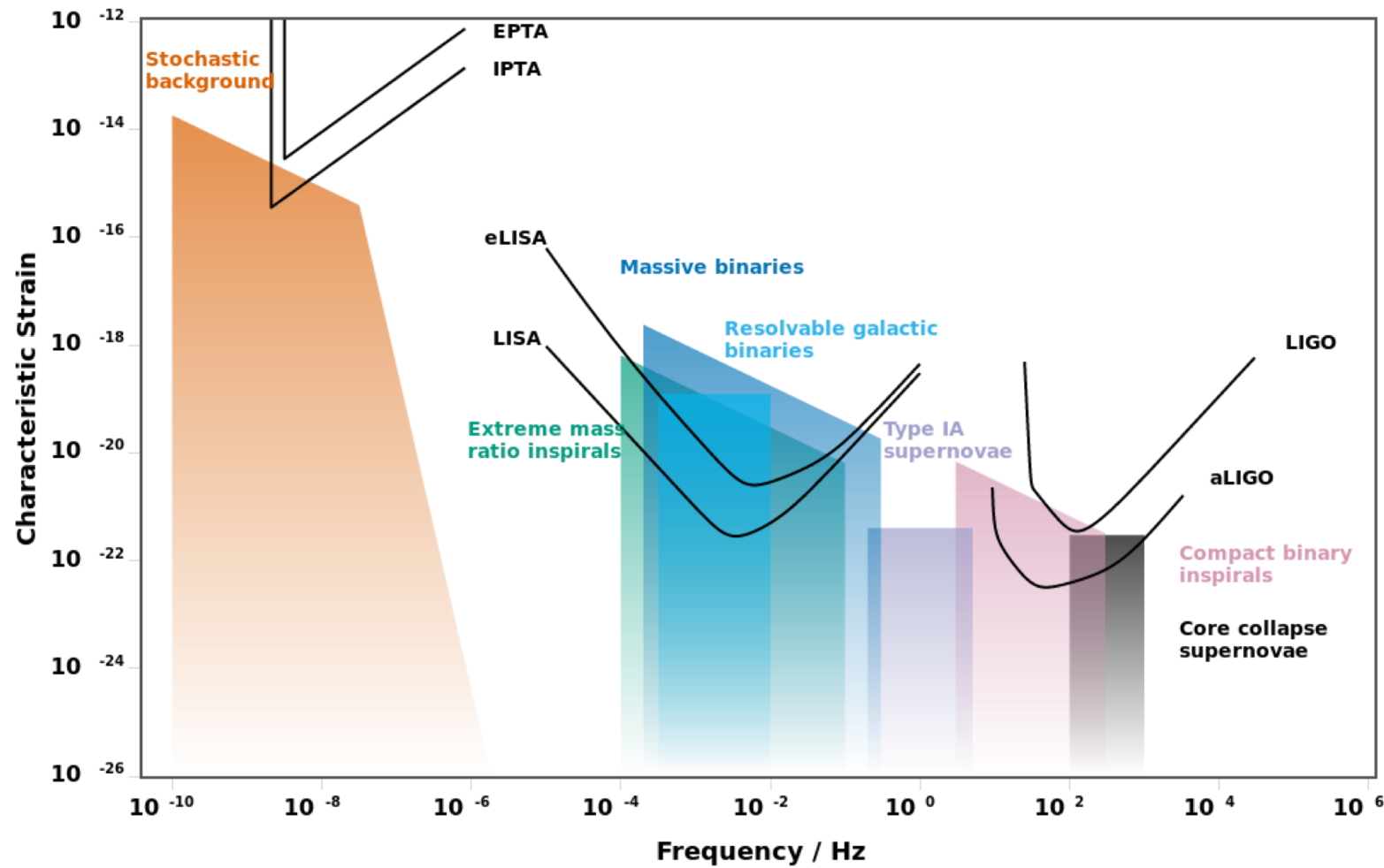
Searching for sub-nHz gravitational waves with pulsar parameter drift

William DeRocco

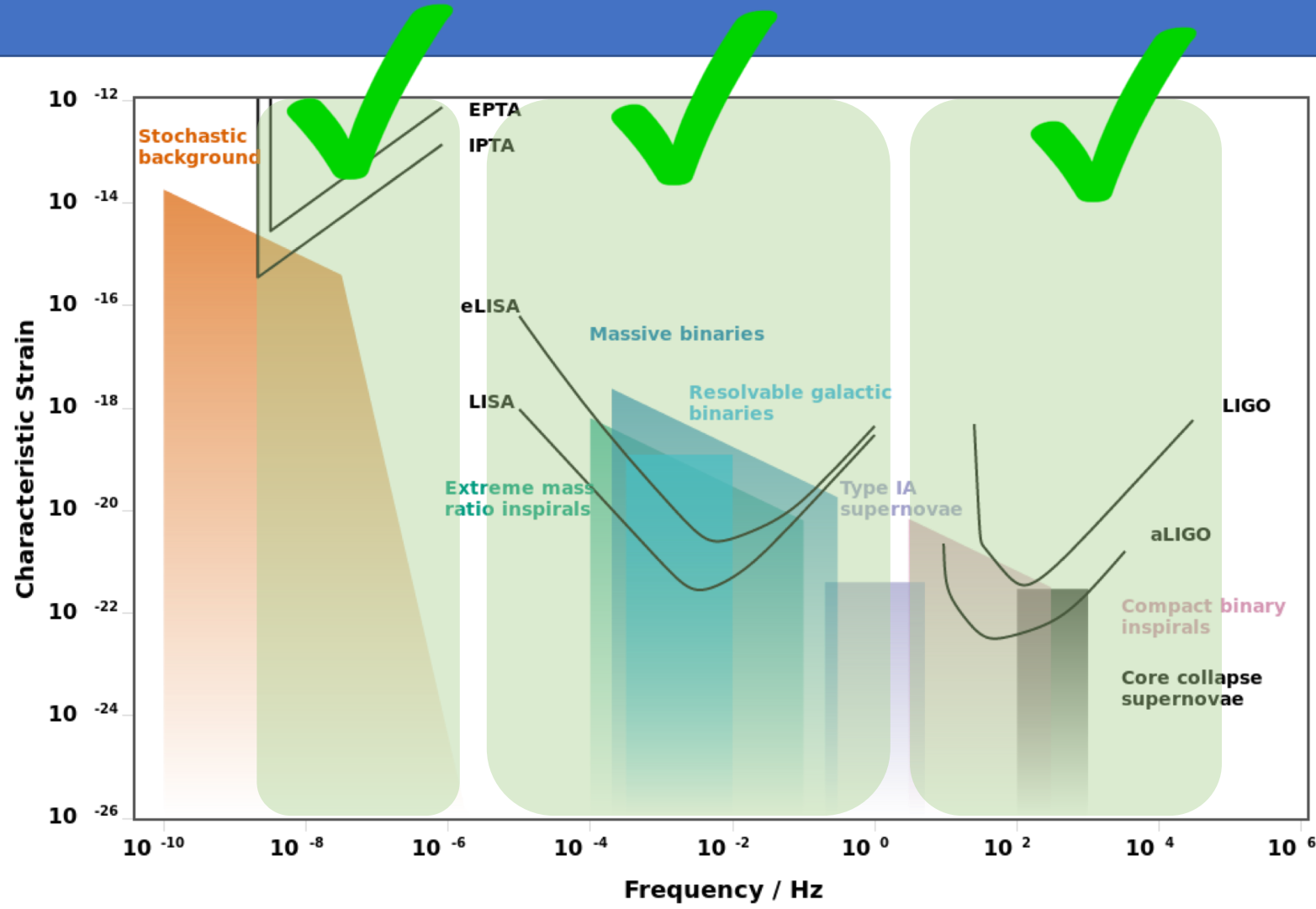
(with Jeff Dror)

University of California, Santa Cruz

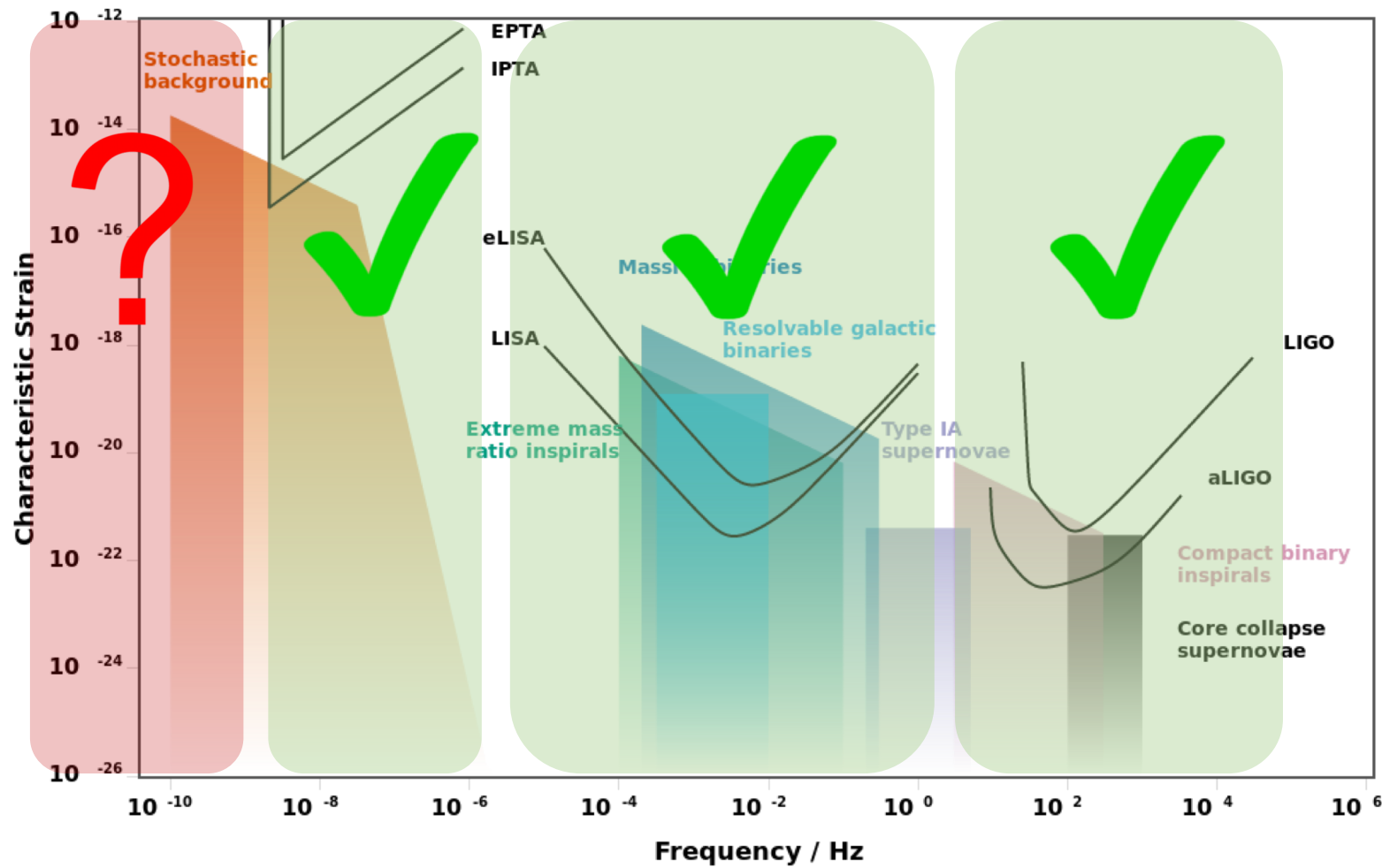
Motivation



Motivation



Motivation



Outline

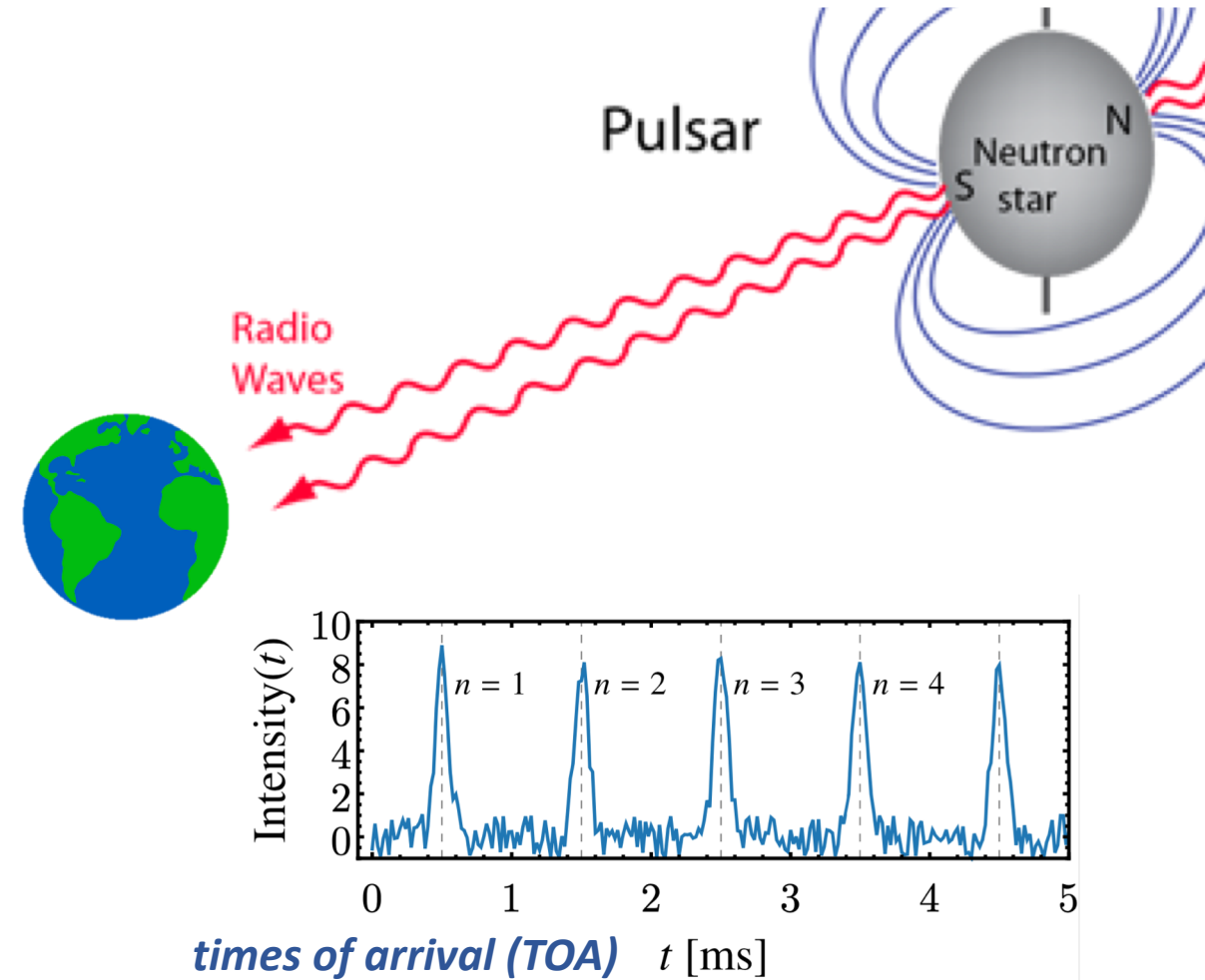
- **Part I:** PTA analysis
- **Part II:** Extracting sub-period signal
- **Part III:** Results

Outline

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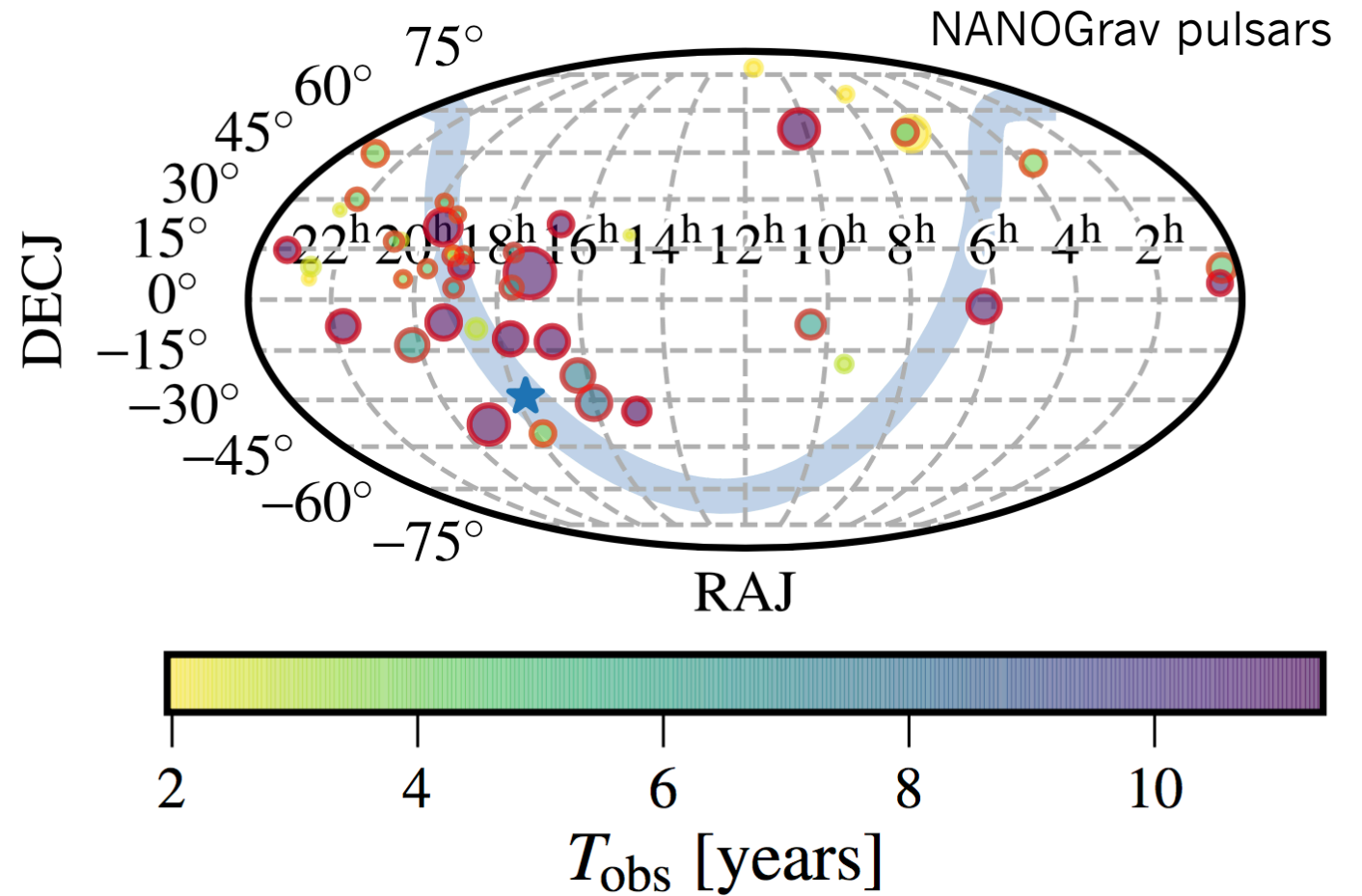
Pulsars

- Rapidly-rotating neutron star with directional beam of radiation
- Pulse arrival = very precise clock
 - Periods $\sim 10^{-3}$ sec
 - Measured to $\sim 10^{-7}$ sec
- Sensitive to variations in path length (GW signal)



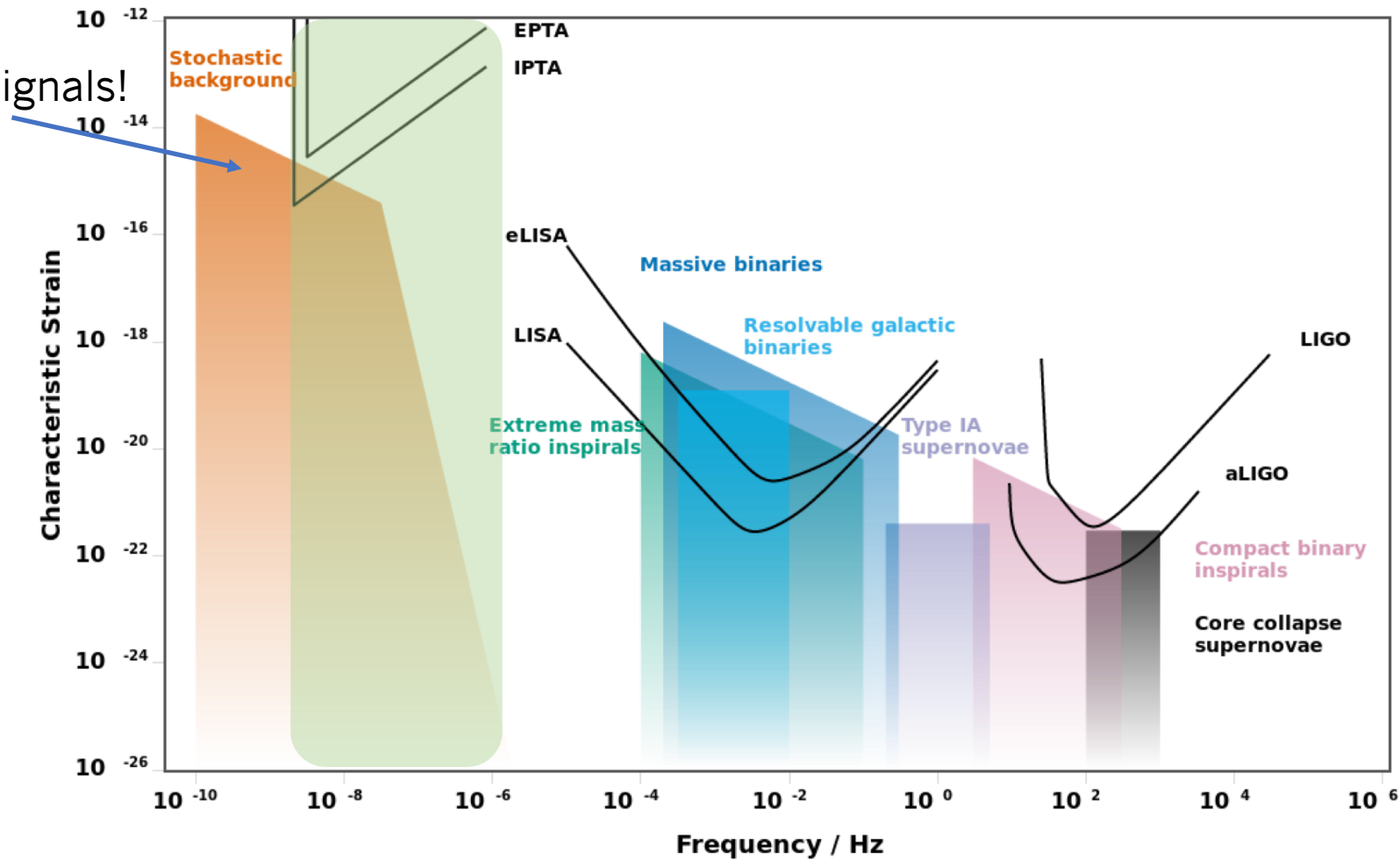
Pulsar timing array

- GWs are quadrupolar
- Variations in arrival times are ***correlated across sky***
- Correlations help discriminate GWs from other sources of variation



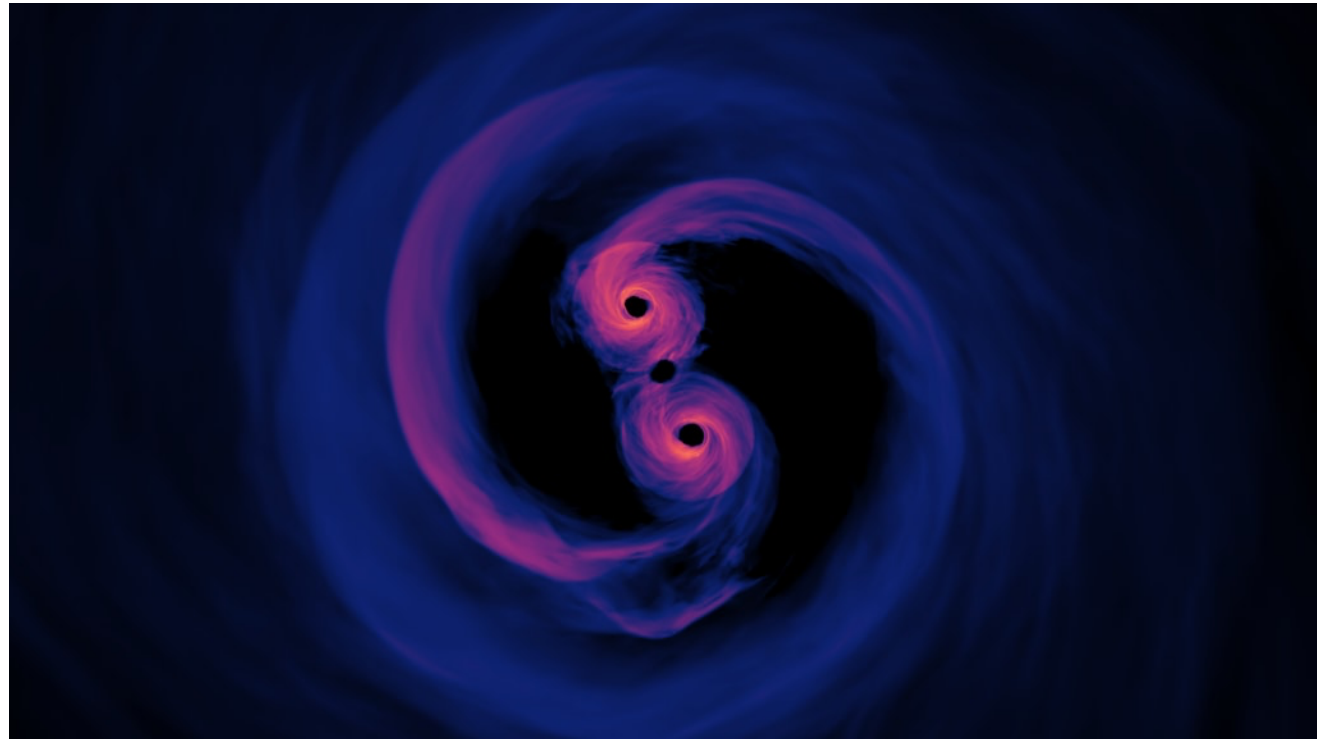
PTA band

astrophysical signals!

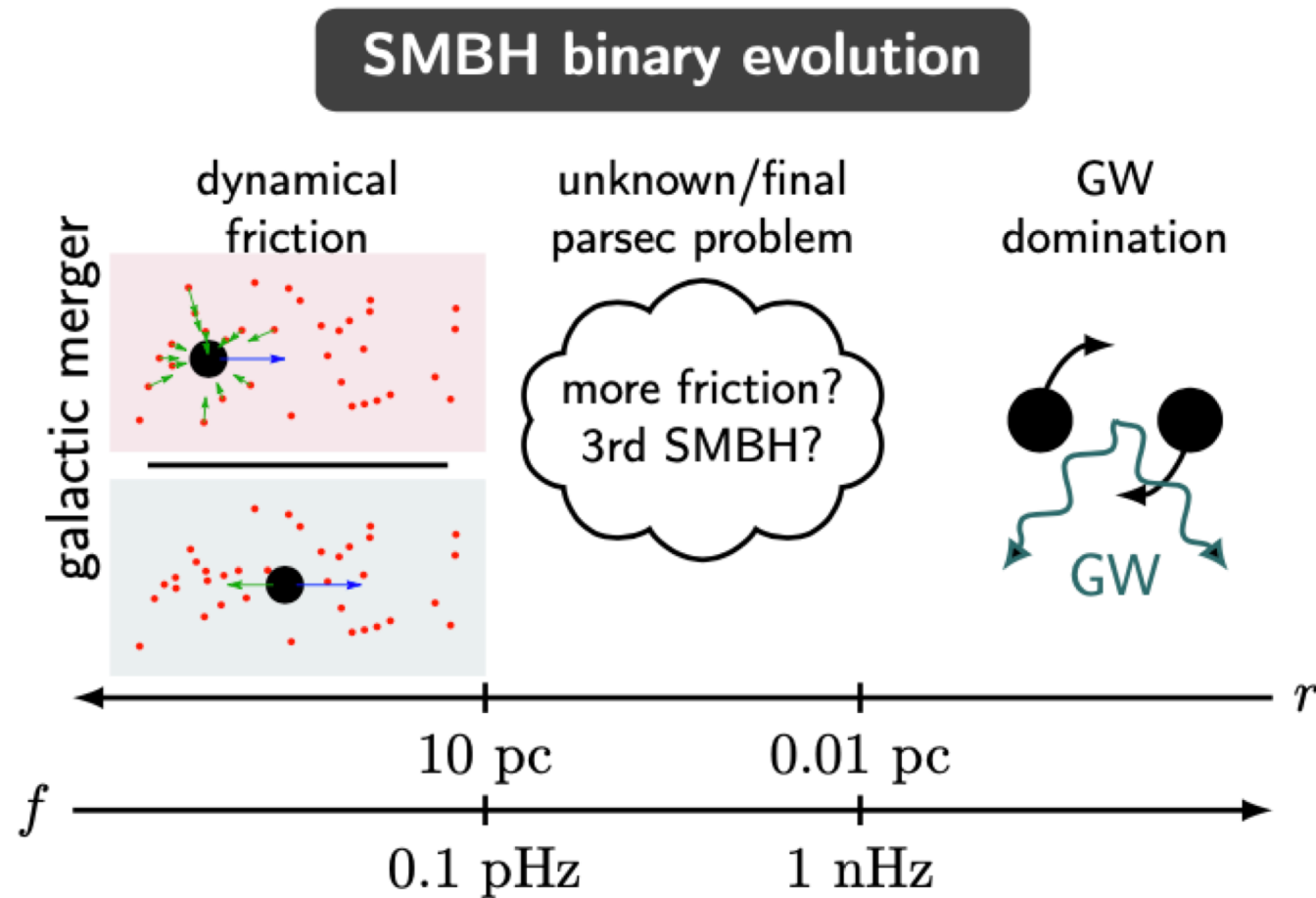


SMBH binaries

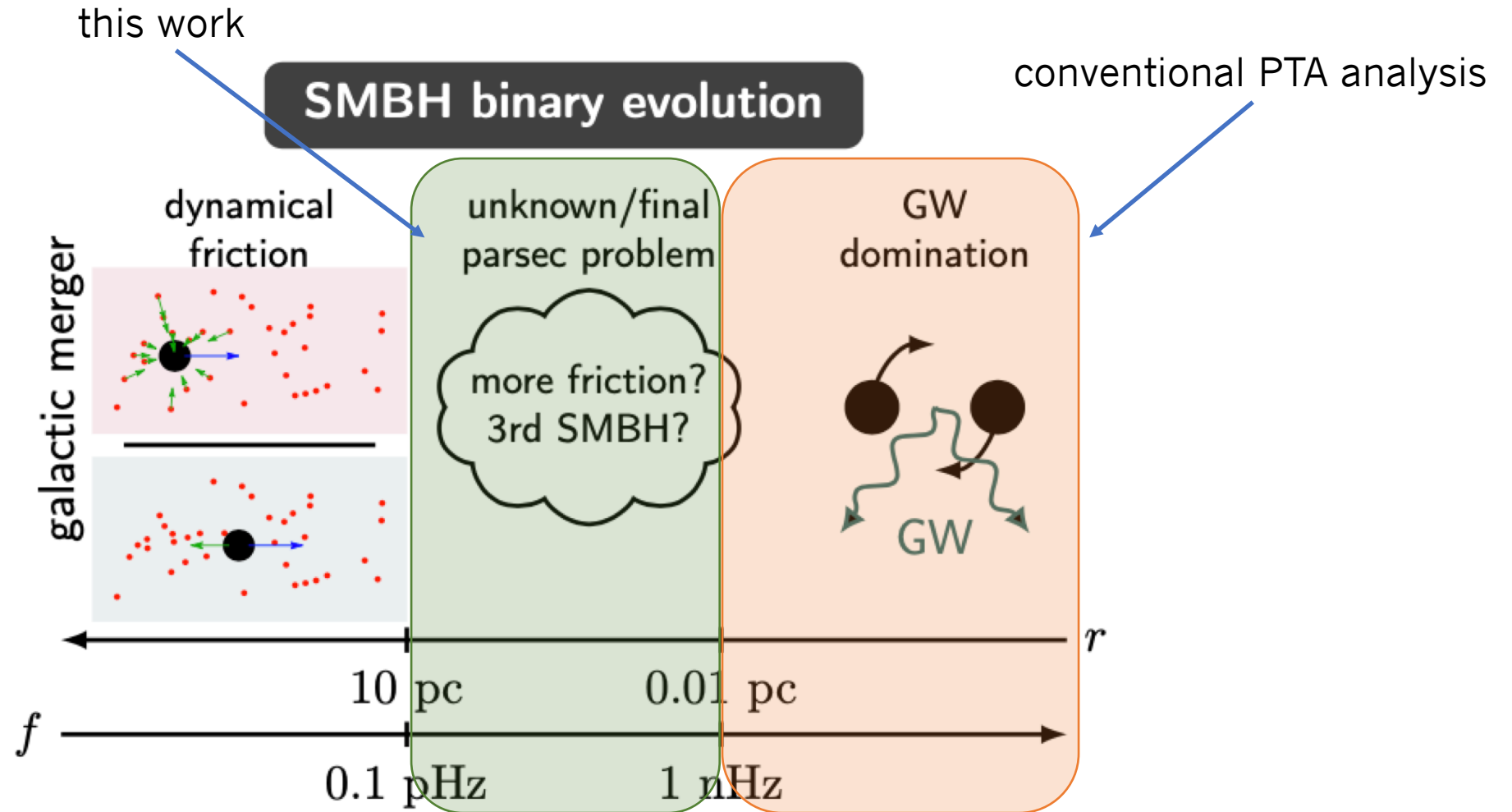
- $10^6 - 10^9 M_{\odot}$ black holes in the center of galaxies
- Merging galaxies produce stochastic background of gravitational waves
- Expected discovery soon! (NANOGrav 2023 DR?)



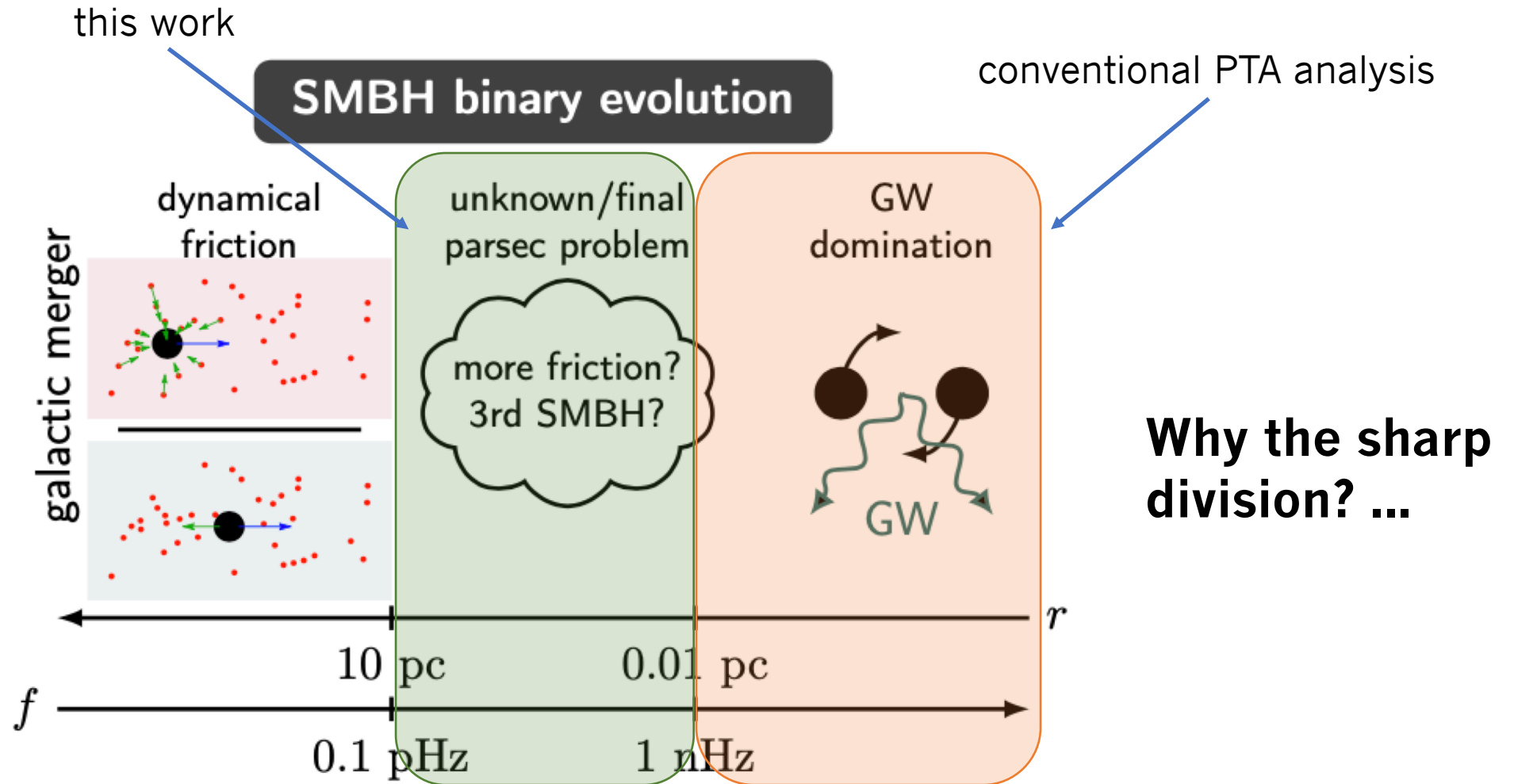
SMBH evolution



SMBH evolution

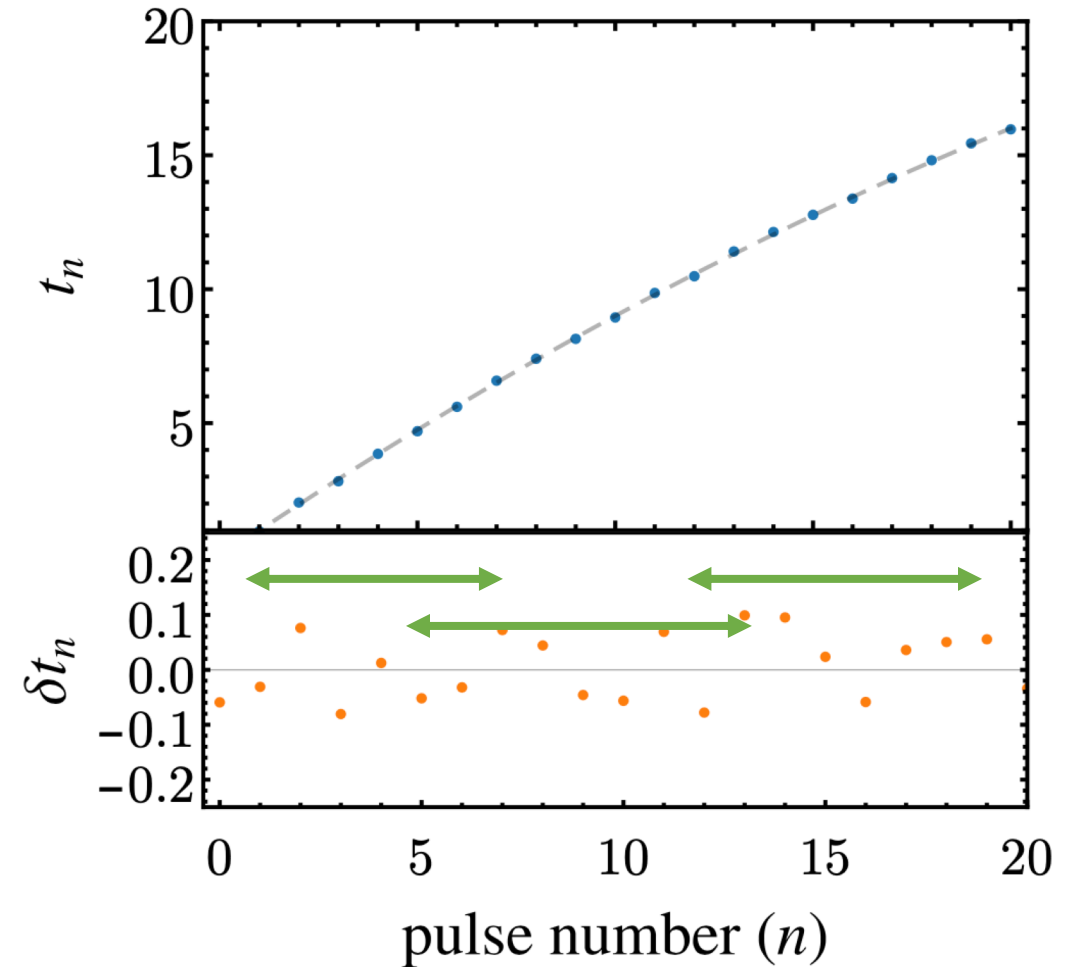


SMBH evolution



PTA Recipe

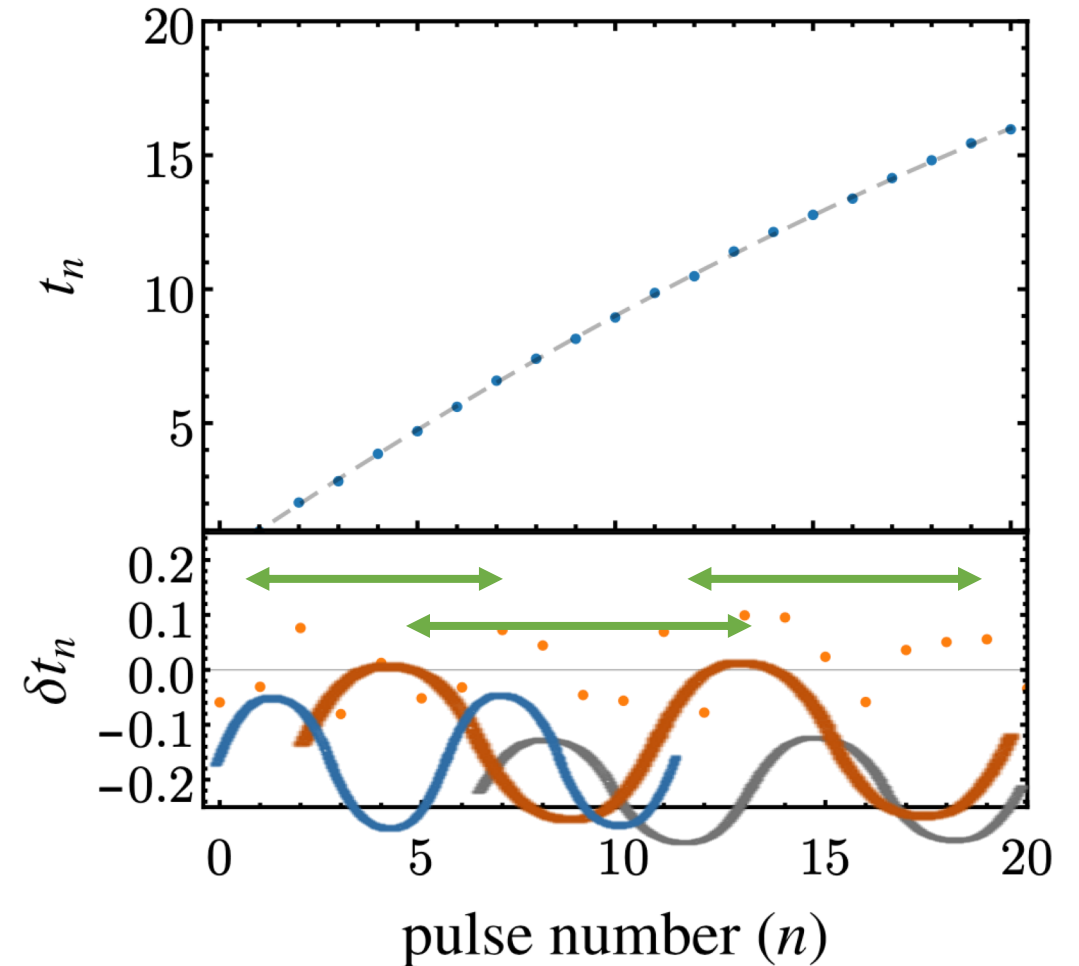
- **Step 1:** Pulses $\rightarrow$ **TOAs**
- **Step 2:** Fit **model** to TOAs
- **Step 3:** Subtract model from TOAs to produce **residuals**
- **Step 4:** Test for **correlations** in residuals



PTA Recipe

- **Step 1:** Pulses $\rightarrow$ TOAs
- **Step 2:** Fit **model** to TOAs
- **Step 3:** Subtract model from TOAs to produce **residuals**
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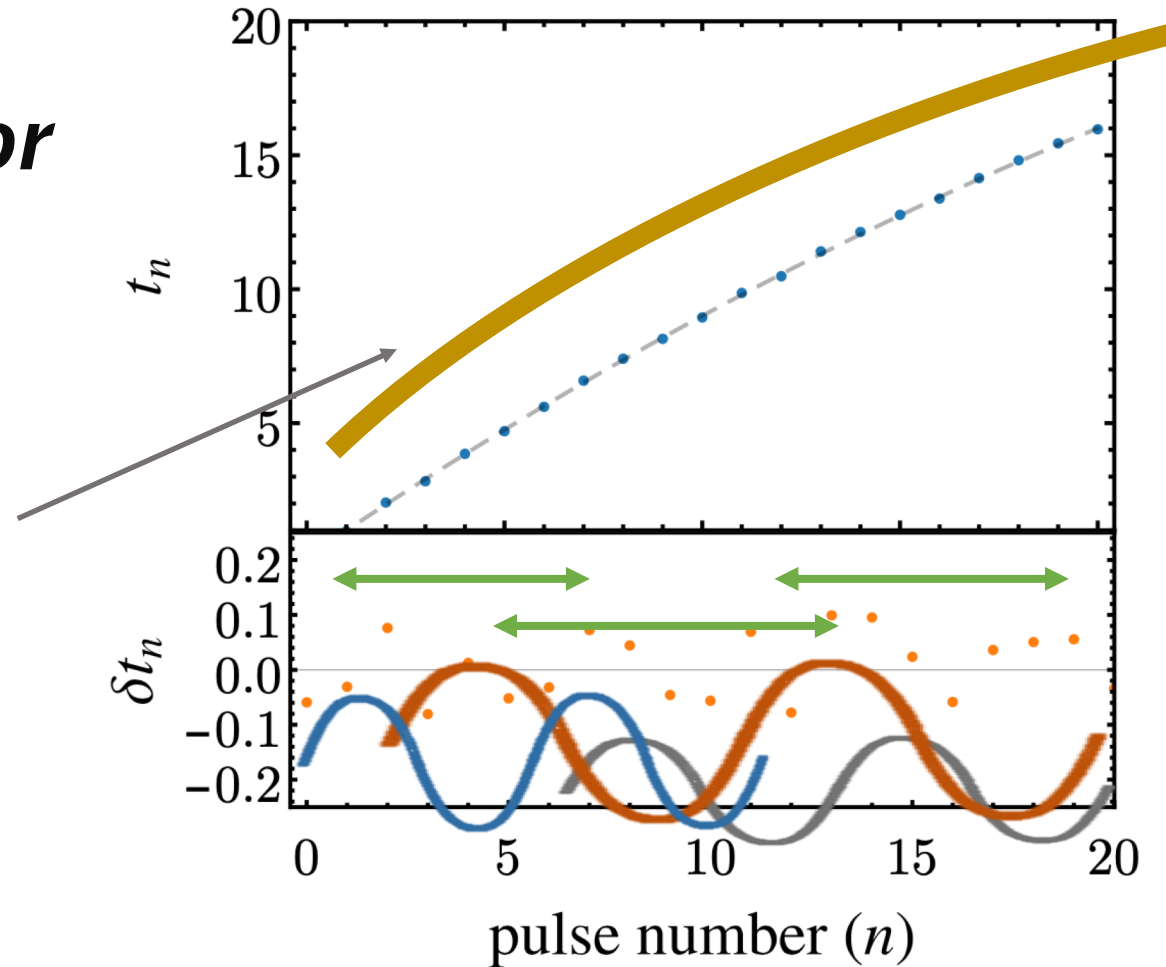
***Correlations only observable
for $f > 1/T$!***



PTA Recipe

Correlations only observable for $f > 1/T$!

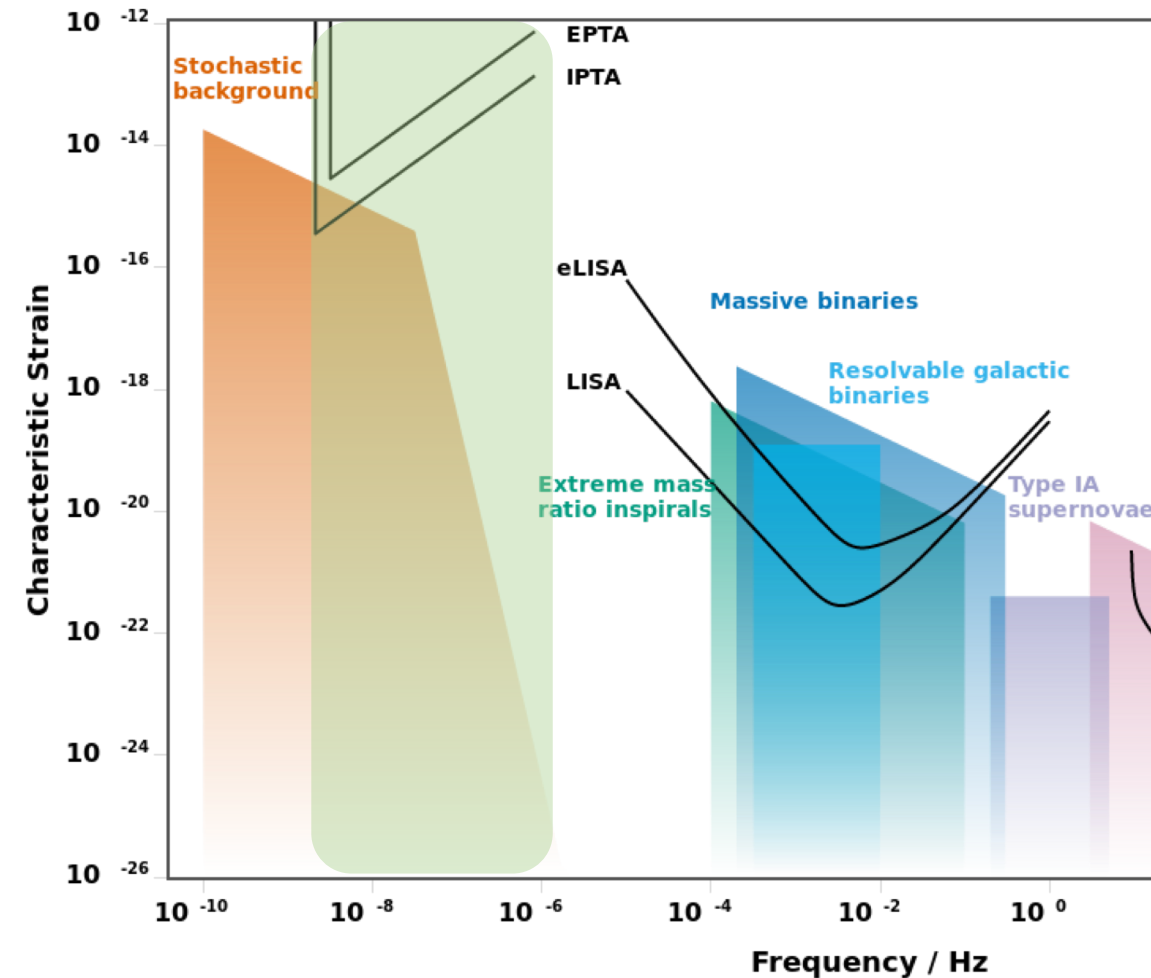
- For $f < 1/T$, signal is “fit away”
- No sensitivity to ultralow-frequency gravitational waves



PTA Recipe

Correlations only observable for $f > 1/T$!

- For $f < 1/T$, signal is “fit away”
- No sensitivity to ultralow-frequency gravitational waves

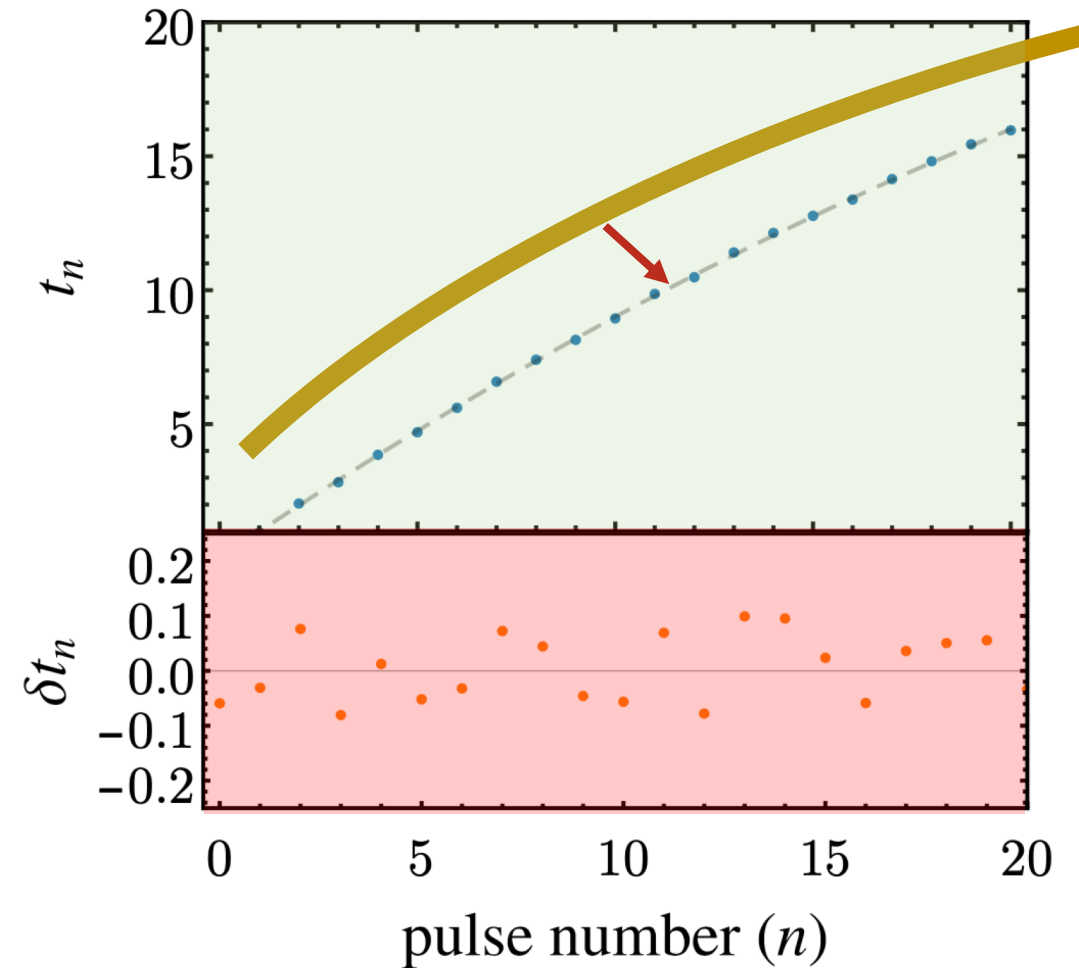


Outline

- **Part I:** PTA analysis
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Fit parameters

Key Insight: “Fitting away” just means low-frequency information is stored in parameter fits!

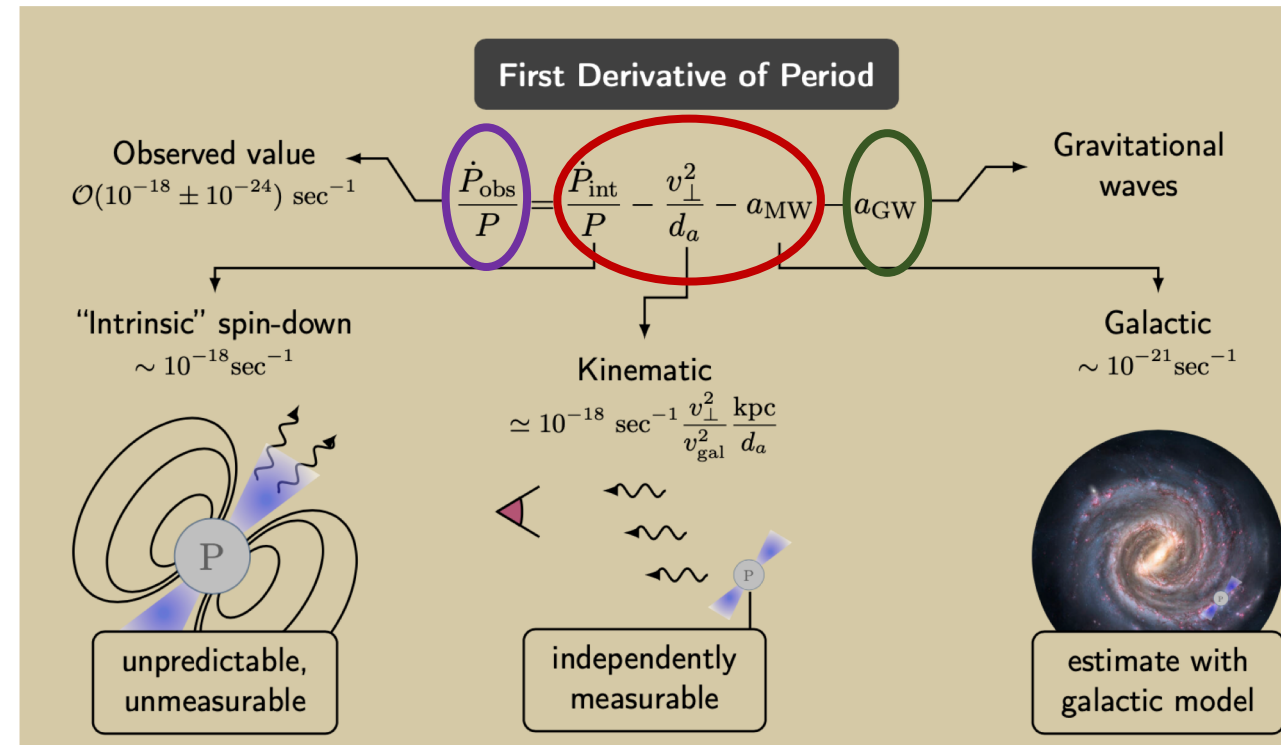


Fit parameters

Key Insight: “Fitting away” just means low-frequency information is stored in parameter fits!

- Example: $\dot{P}$
 - Contains information about GWs!
 - Also pulsar spin down, etc.

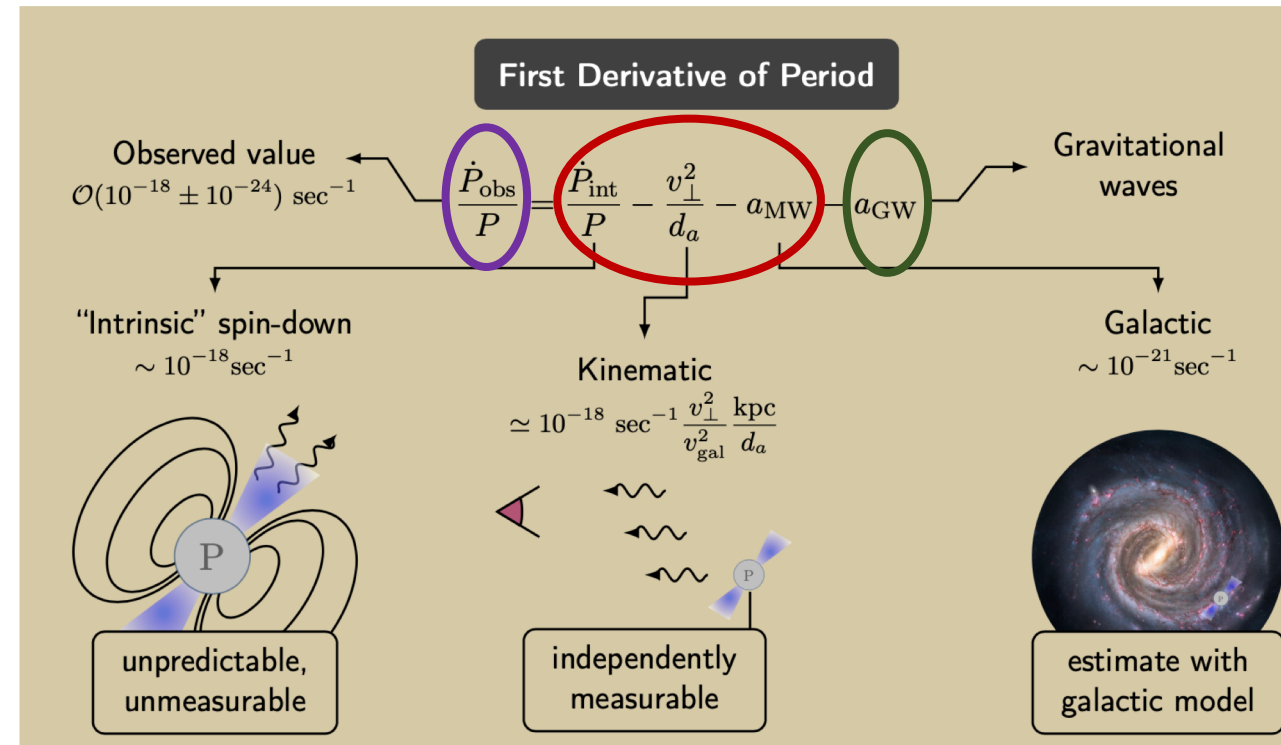
$$\dot{P}_{obs} = \dot{P}_{other} + \dot{P}_{GW}$$



Fit parameters

Key Insight: “Fitting away” just means low-frequency information is stored in parameter fits!

- Example: $\dot{P}$
 - Contains information about GWs!
 - Also pulsar spin down, etc.
- $$\dot{P}_{obs} = \dot{P}_{other} + \dot{P}_{GW}$$
- Intrinsic contributions to $\dot{P}$ unpredictable $\Rightarrow$ GW signal cannot be extracted

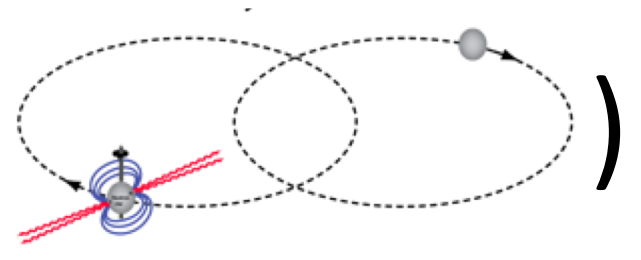


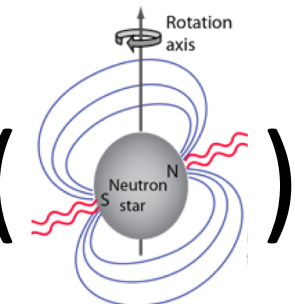
Sensitive parameters

- Non-GW contributions to some parameters are known and can be subtracted away!

- **Best parameters:**

- $\dot{P}_b$: change in *binary* period
- $\ddot{P}$: second derivative of *spin* period

$$\frac{d}{dt} \left(\text{Diagram of binary system} \right)$$


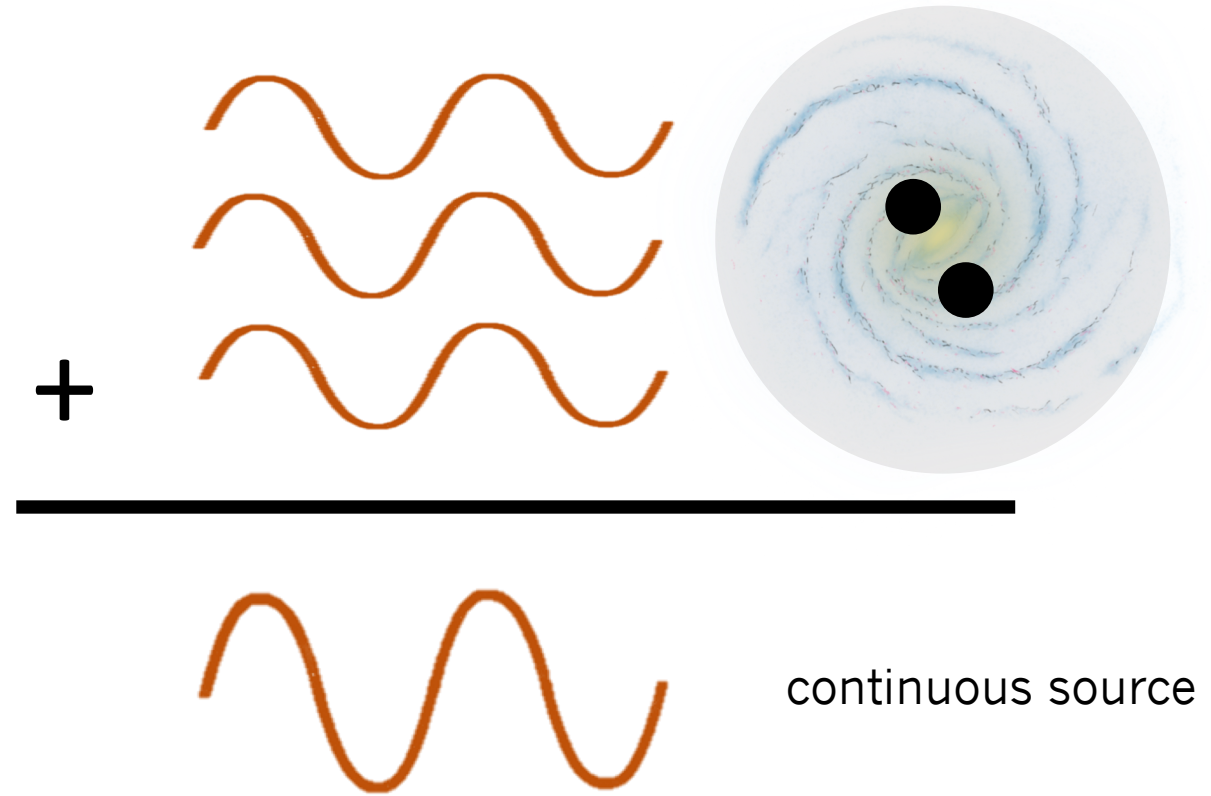
$$\frac{d^2}{dt^2} \left(\text{Diagram of a neutron star} \right)$$


Outline

- **Part I:** PTA analysis
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Continuous SMBH source

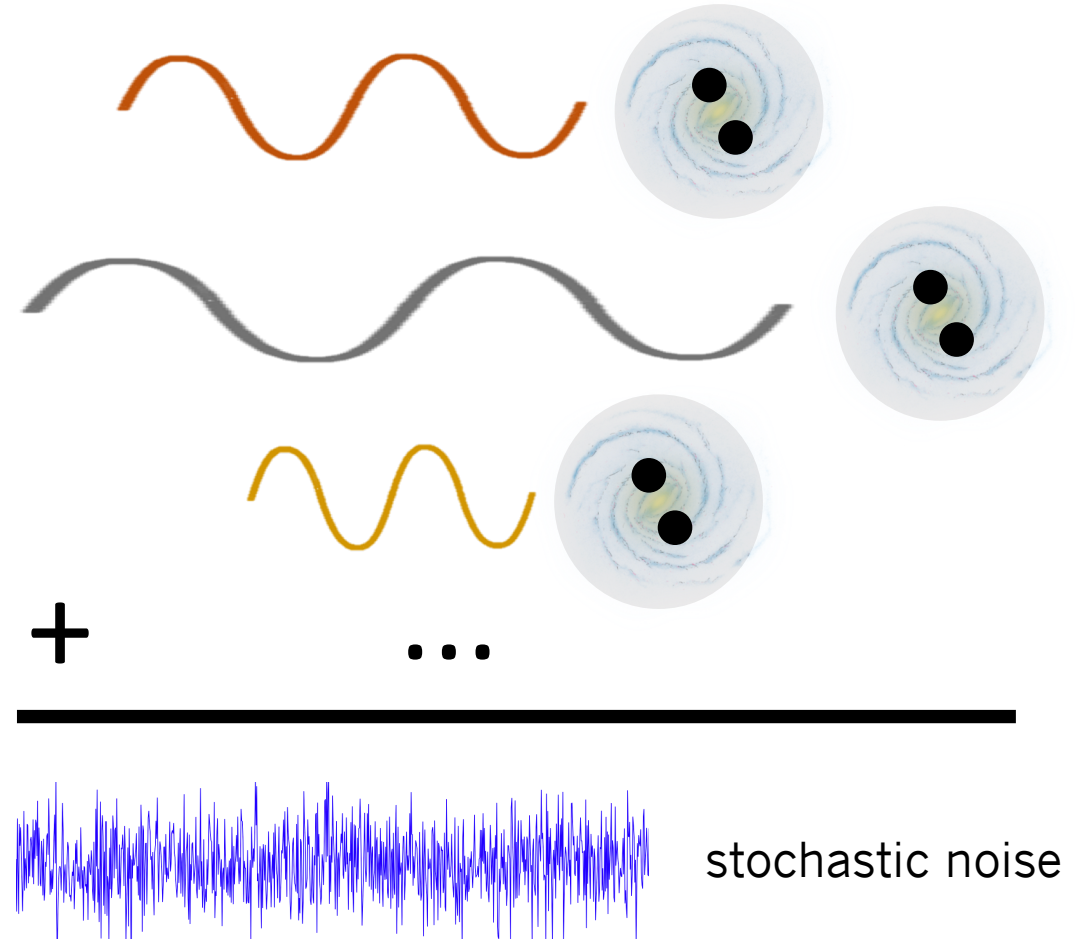
- Measured in terms of *strain* h
 - Relative change in path length
 - $h \sim v_{LOS}$
- Approximately sinusoidal at ultralow frequencies
- SMBH merger = single coherent source



continuous source

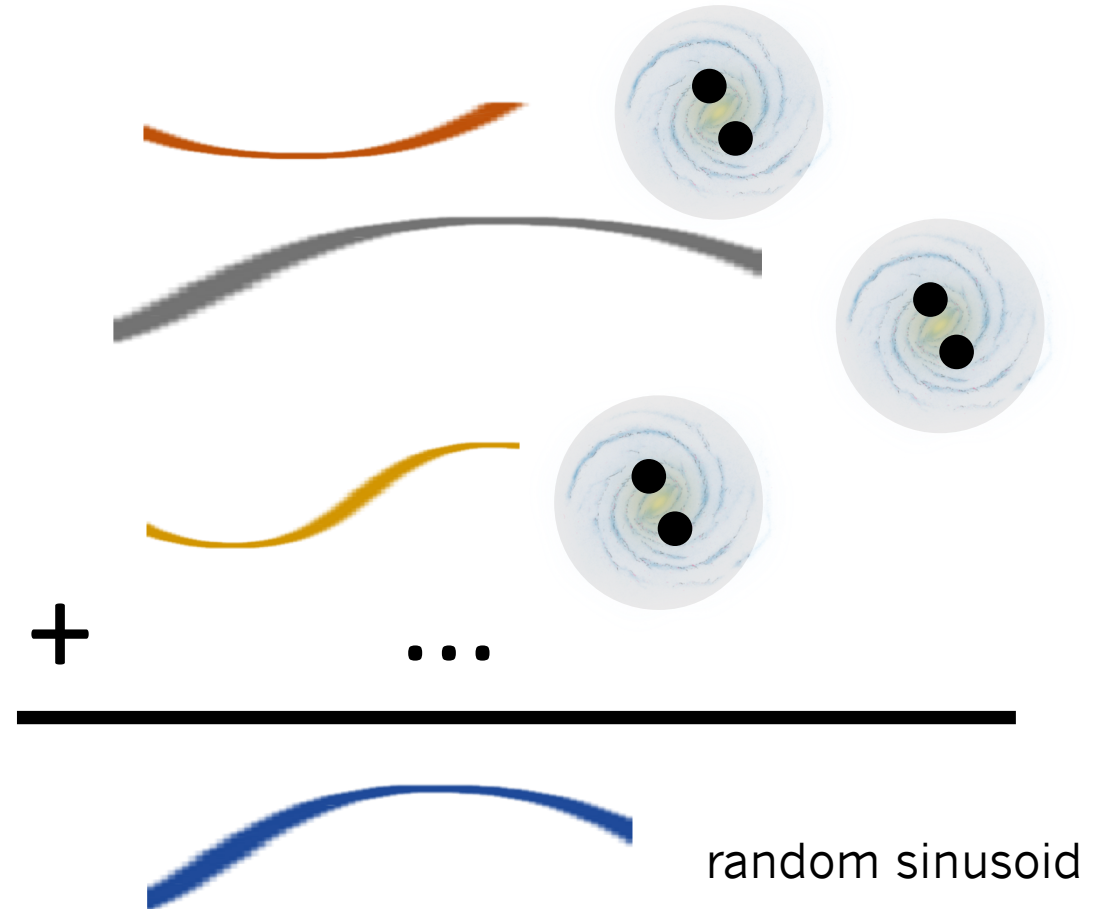
Stochastic signal

- **High frequencies ($f > 1/T$):**
incoherent sum of sources $\Rightarrow$
stochastic noise



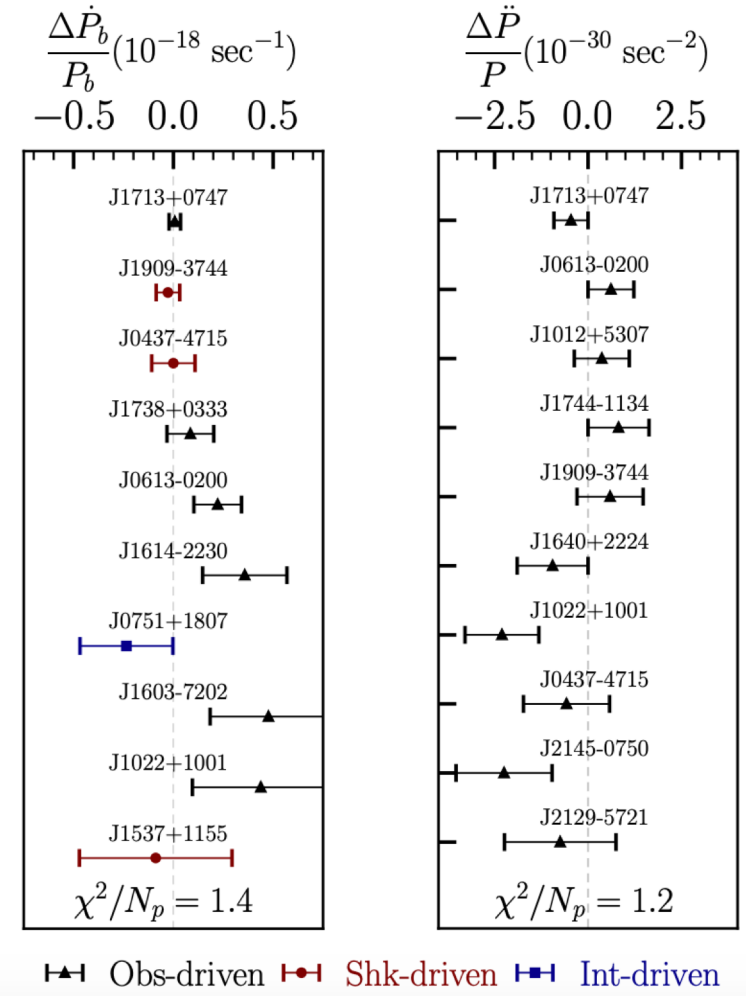
Stochastic signal

- **High frequencies ($f > 1/T$):**
incoherent sum of sources $\Rightarrow$ stochastic noise
- **Ultralow frequencies ($f < 1/T$):**
incoherent sum $\Rightarrow$ sinusoid with random phase and amplitude



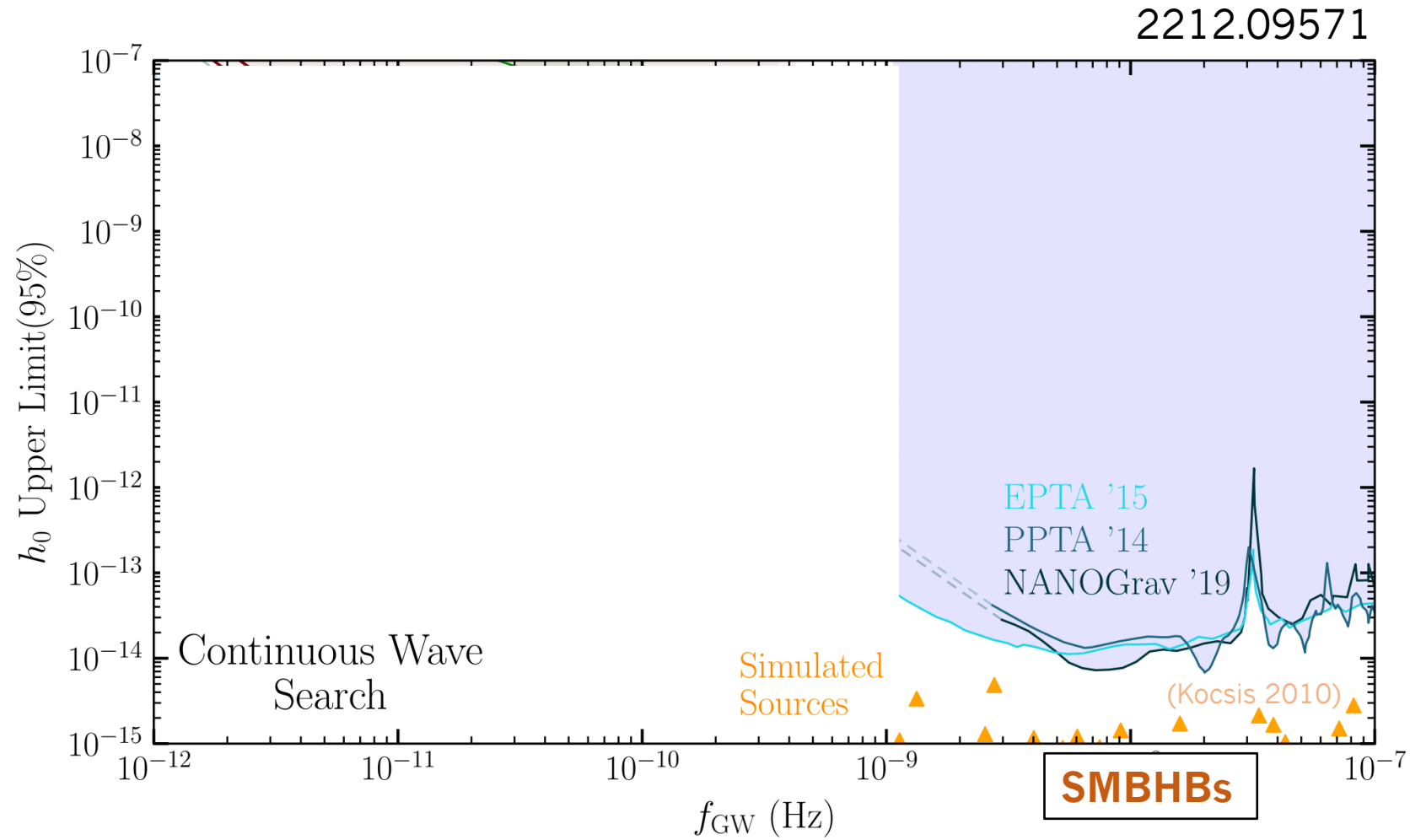
Data

- $\ddot{P}$: Liu et al. (2018)
 - jerk $\sim 10^{-31} \text{ s}^{-2}$
- $\dot{P}_b$: Chakrabarti et al. (2020)
 - acceleration $\sim 10^{-19} \text{ s}^{-1}$
- Pulsars are well-modeled on individual basis (clean environments, dispersion measure, etc.)



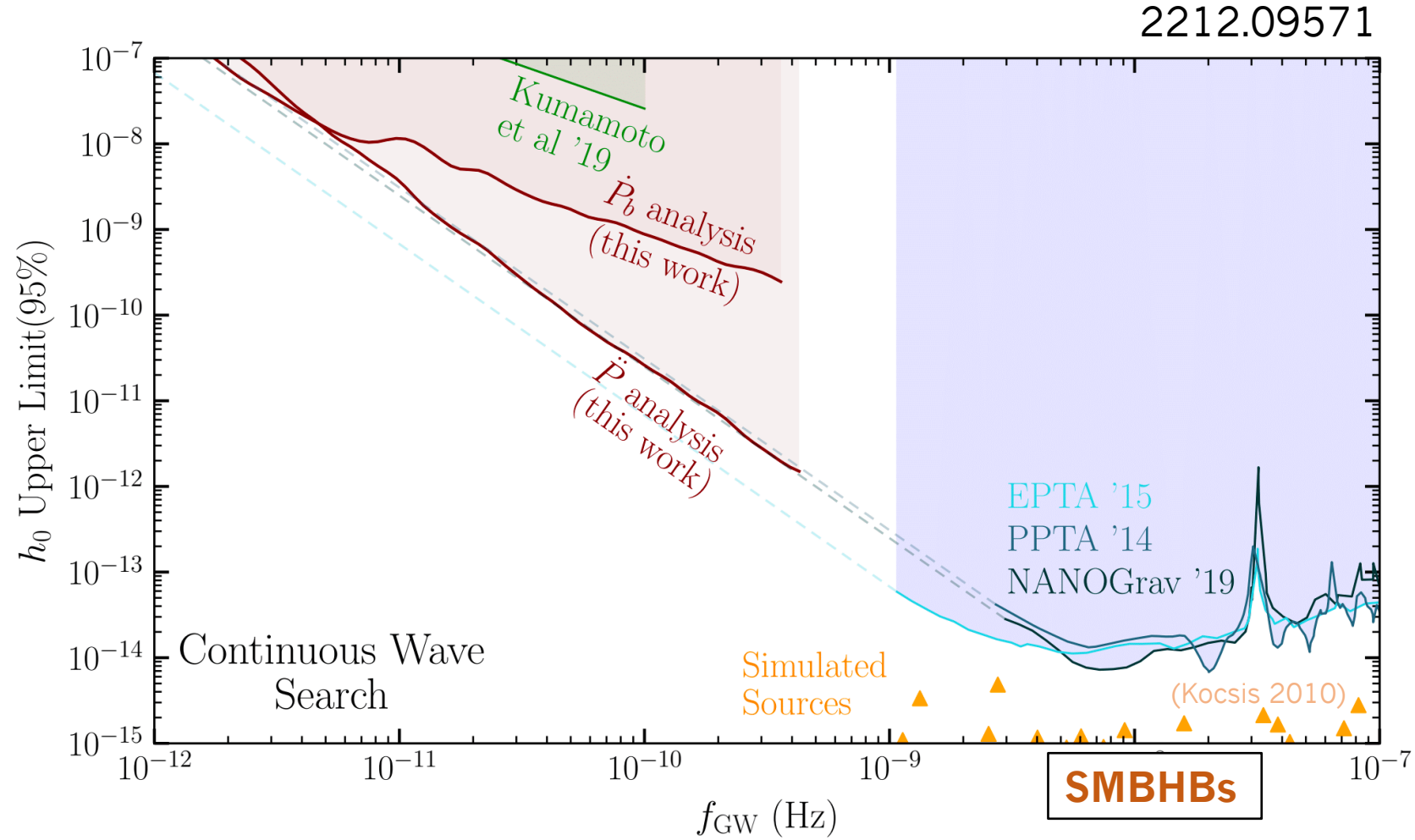
Single SMBHB

- Target: SMBH binaries



Single SMBHB

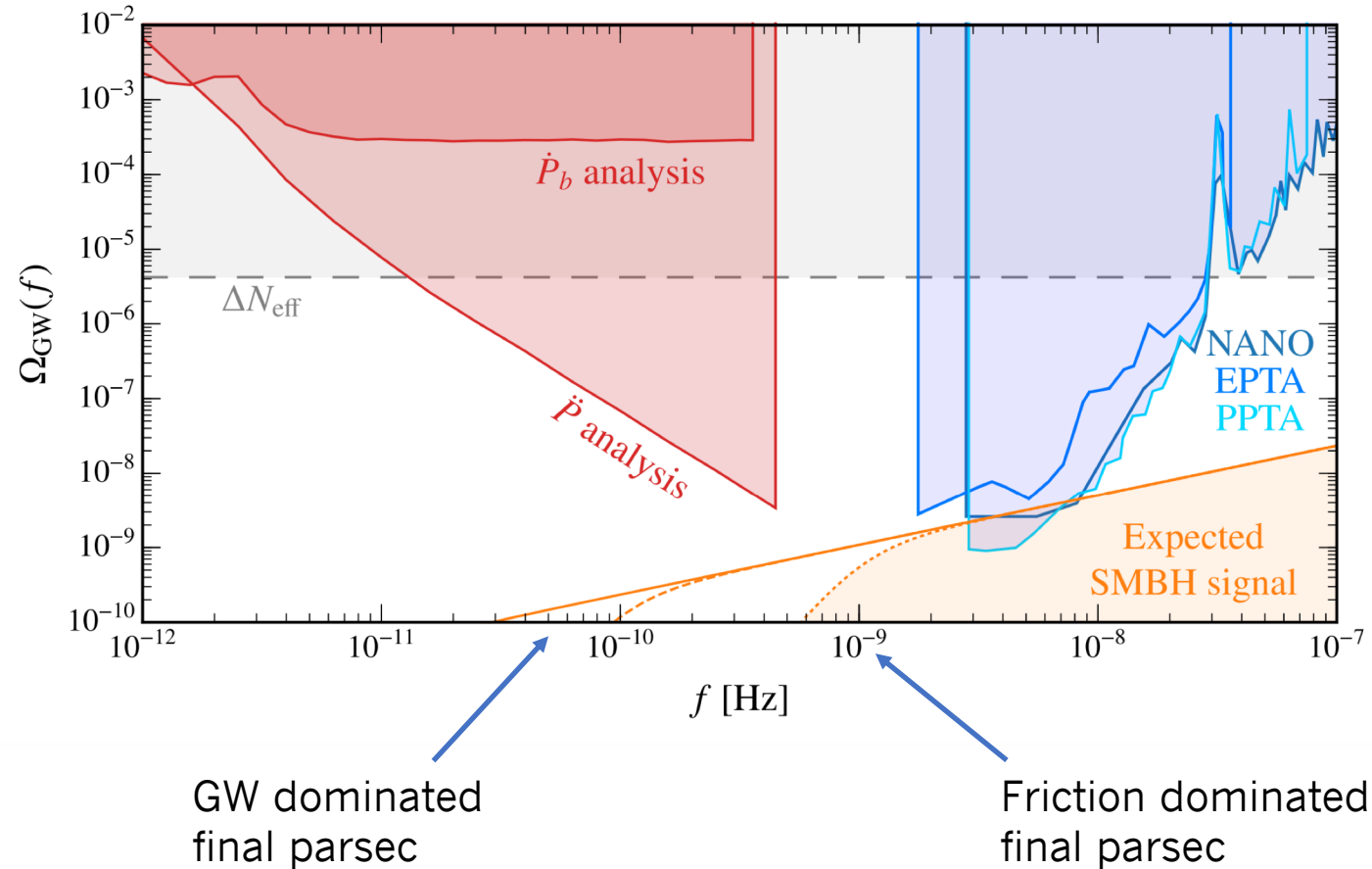
- Target: SMBH binaries
- No statistical preference towards signal ($p > 0.2 \ \forall f_{GW}$)



Results: stochastic source

2303.13042

- Plotted in terms of fractional energy density of the universe
- Technique is sensitive to final parsec problem!



More to come!

- Our data were exclusively EPTA and PPTA...
- Currently working with NANOGrav on in-house analysis of their data
- Observation of ultralow-frequency signal in next data release?



Outline

- **Part I:** PTA analysis
 - **Takeaway I:** Standard PTA analysis “fits away” ultralow-frequency GWs.
- **Part II:** Extracting sub-period signal
 - **Takeaway II:** Low-frequency behavior is stored in parameter estimates.
- **Part III:** Results
 - **Takeaway III:** Certain parameters offer sensitivity to gravitational waves at sub-nanohertz frequencies.

Thank you!



Backup

Results: stochastic source

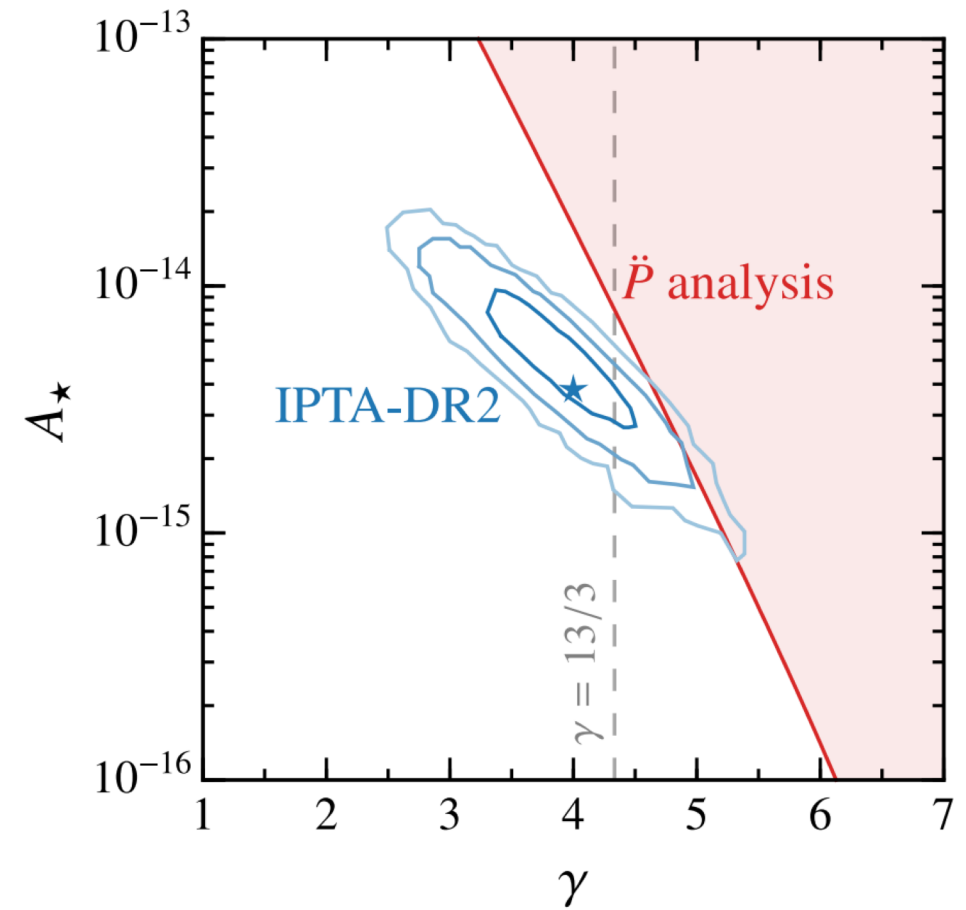
2303.13042

- Power-law constraints

- Scaling:

$$h_c(f) = A_* \left(\frac{f}{\text{yr}^{-1}} \right)^\gamma$$

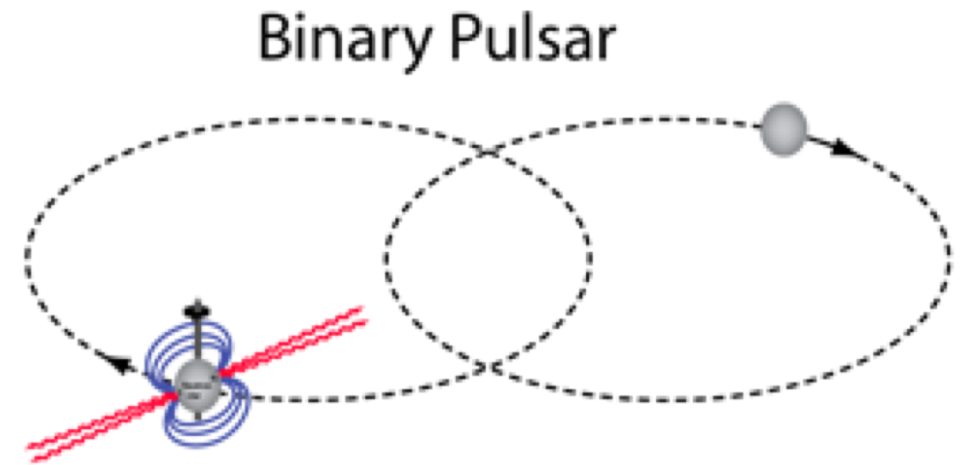
- Adjoins current best fit!



Binary spin-down ($\dot{P}_b$)

$$\frac{\dot{P}_{b,\text{obs}}}{P_b} = \frac{\dot{P}_{b,\text{GR}}}{P_b} + \frac{\dot{P}_{b,\text{Shk}}}{P_b} + a_{\text{MW}} + a_{\text{GW}}$$

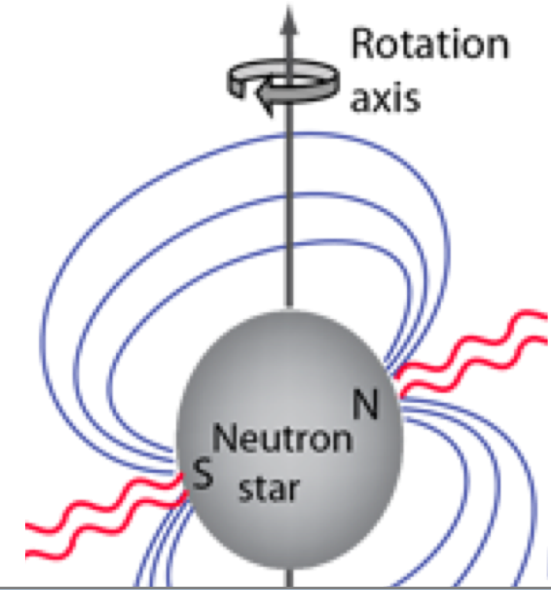
- Change in binary period
- Induces apparent line-of-sight **acceleration**
- Three relevant non-GW contributions



Second derivative of pulsar spin-down ($\ddot{P}$)

$$\frac{\ddot{P}_{\text{obs}}}{P} = j_{\text{GW}}$$

- Change in pulsar period
- Induces apparent line-of-sight **jerk**
- All non-GW contributions negligible →



Spin-down: 10^{-35} s^{-2}

Shklovskii: 10^{-32} s^{-2}

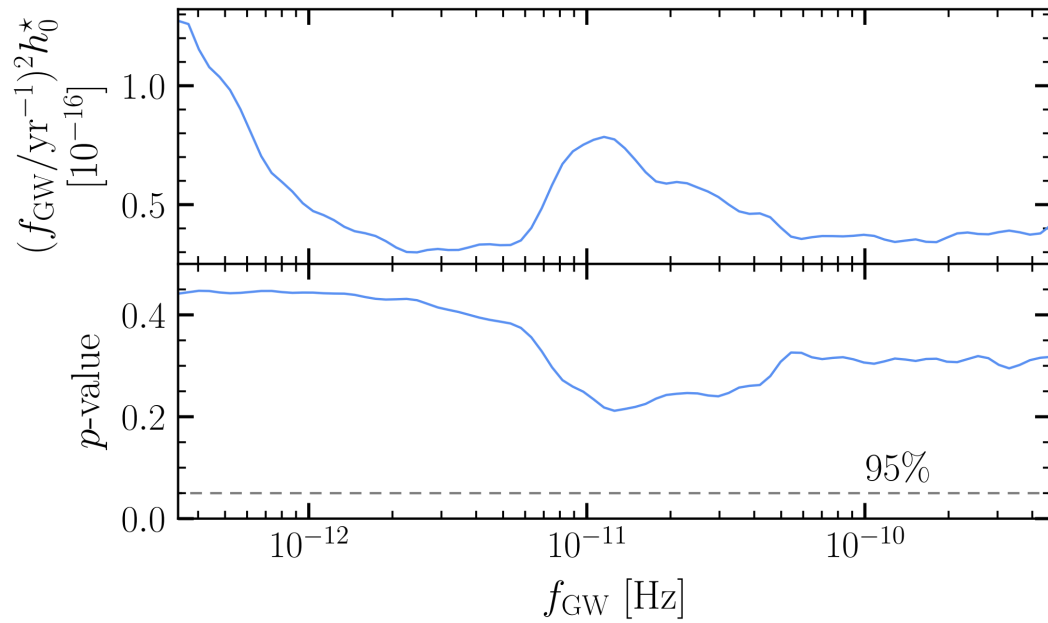
Galactic: 10^{-33} s^{-2}

Observational uncertainty: $>10^{-30} \text{ s}^{-2}$

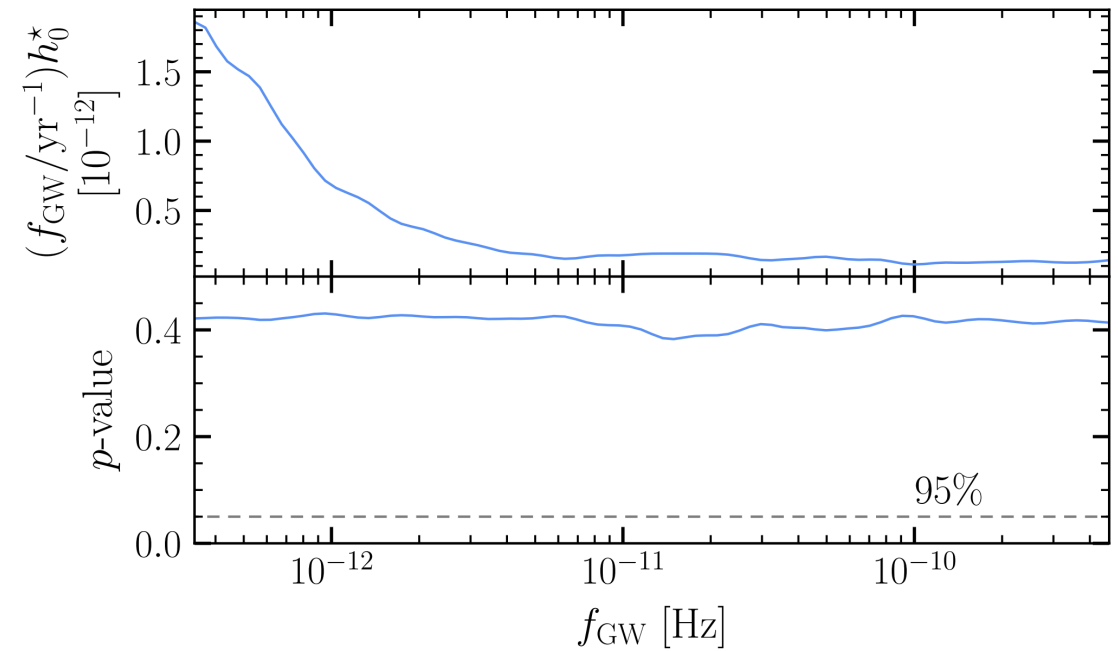
- Target: SMBH binaries
- No statistical preference towards signal ($p > 0.2 \quad \forall f_{GW}$)
- Follows analytic expectation for frequency scalings



Best fits



Jerk



Acceleration

Estimating $\dot{P}_{b,GR}$ independently

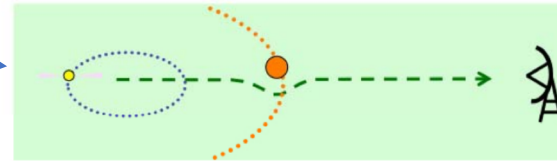
- Shape of Doppler shift curve gives i, ϕ, P_b
- Shapiro delay gives m_2
- Precession + time dilation give (AKA Einstein delay) gives e, m_1
- System is overconstrained $\rightarrow \dot{P}_{b,GR}(m_1, m_2, e)$ from radiation equation is independent of fit value for $\dot{P}_b$

GR Effects Summary



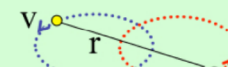
Precession

$$\dot{\omega} = 3 \left(\frac{P_b}{2\pi} \right)^{5/3} \frac{1}{1 - e^2} \left[\frac{G}{c^3} (m_1 + m_2) \right]^{2/3}$$



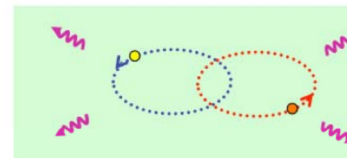
Shapiro Delay

$$\Delta t = 2 \frac{G}{c^3} m_2 \ln \left[1 + \sin i \sin(\varphi - \varphi_0) \right]$$



Grav Redshift/Time Dilation

$$\gamma = \frac{G^{2/3}}{c^2} \left(\frac{P_b}{2\pi} \right)^{1/3} e \frac{m_2(m_1 + 2m_2)}{(m_1 + m_2)^{5/3}}$$



Gravitational Radiation

$$\dot{P}_b = \left(\frac{192\pi}{5} \right) \frac{G^{5/3}}{c^5} \left(\frac{P_b}{2\pi} \right)^{5/3} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{1}{(1 - e^2)^{5/2}} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}$$

Swiped from David Nice

https://users.astro.ufl.edu/~eiken/AST7939_files/Lecture_6_Pulsar_Timing_Binaries.pdf

Scaling

- Line-of-sight velocity induced by GW:

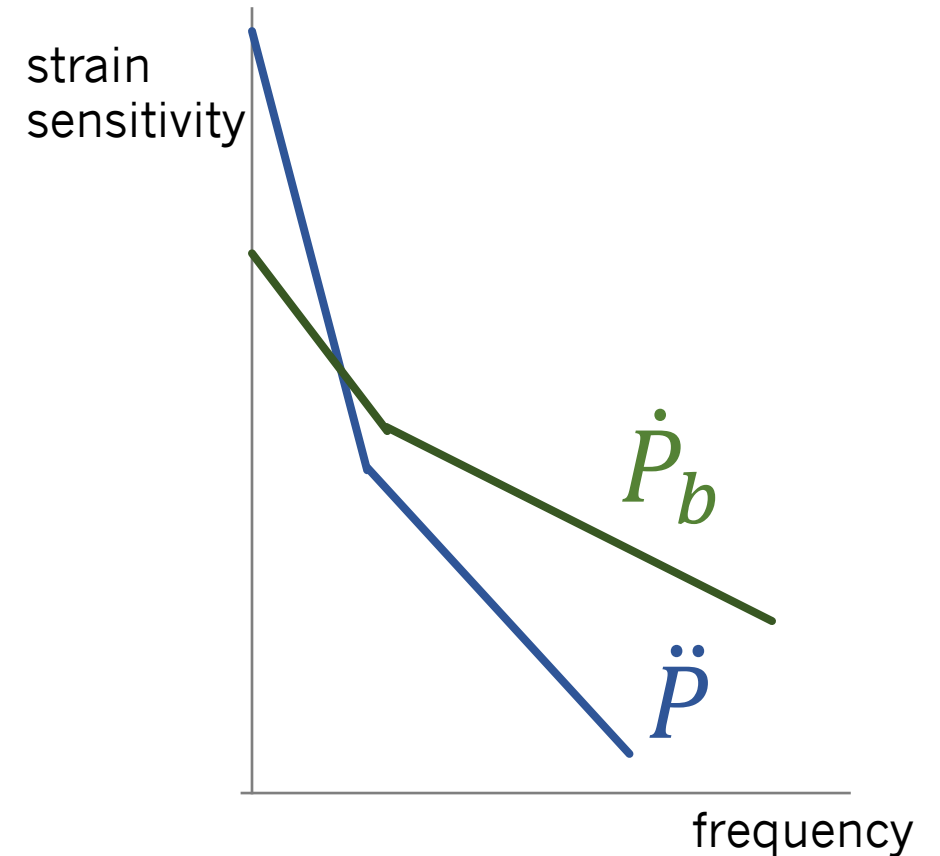
$$\dot{x}_{LOS} \sim h_0 (\cos 2\pi f t_{\text{em}} - \cos 2\pi f t_{\text{obs}})$$

- Wavelength < distance to pulsar:

- Acceleration:** $\dot{P}_b \propto a_{LOS} = \ddot{x}_{LOS} \sim h_0 f$
- Jerk:** $\ddot{P} \propto j_{LOS} = \dddot{x}_{LOS} \sim h_0 f^2$

- Wavelength > distance to pulsar:

- Additional f from expansion of cosine
- Acceleration:** $\dot{P}_b \propto a_{LOS} = \ddot{x}_{LOS} \sim h_0 f^2$
- Jerk:** $\ddot{P} \propto j_{LOS} = \dddot{x}_{LOS} \sim h_0 f^3$



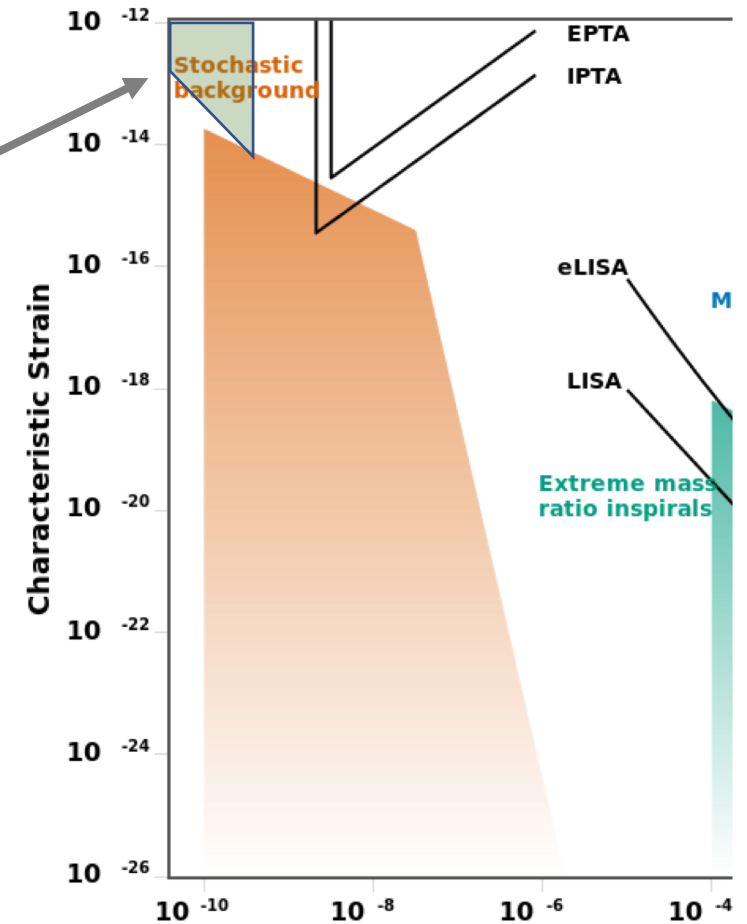
Back-of-the-envelope estimate

- Jerk $\sim 10^{-31} \text{ s}^{-2}$

$$j_{LOS} \sim h_0(\pi f)^2 \Rightarrow h_0 \sim \mathbf{10^{-14}} \left(\frac{f}{\text{nHz}} \right)^2$$

- Acceleration $\sim 10^{-19} \text{ s}^{-1}$

$$a_{LOS} \sim h_0(\pi f) \Rightarrow h_0 \sim \mathbf{10^{-11}} \left(\frac{f}{\text{nHz}} \right)$$



Statistical methodology

- Log-likelihood ratio test for averaged all-sky search

$$\hat{q}(h_0) = 2 \log \left(\frac{\mathcal{L}_M(h_0, f_{\text{GW}})}{\mathcal{L}_M(0, f_{\text{GW}})} \right)$$

likelihood in presence
of signal h_0

likelihood in presence
of no GW signal

- Null distribution obeys Wilks' theorem

