Light Cold Dark Matter from Non-thermal Decay

arXiv:2304.07462

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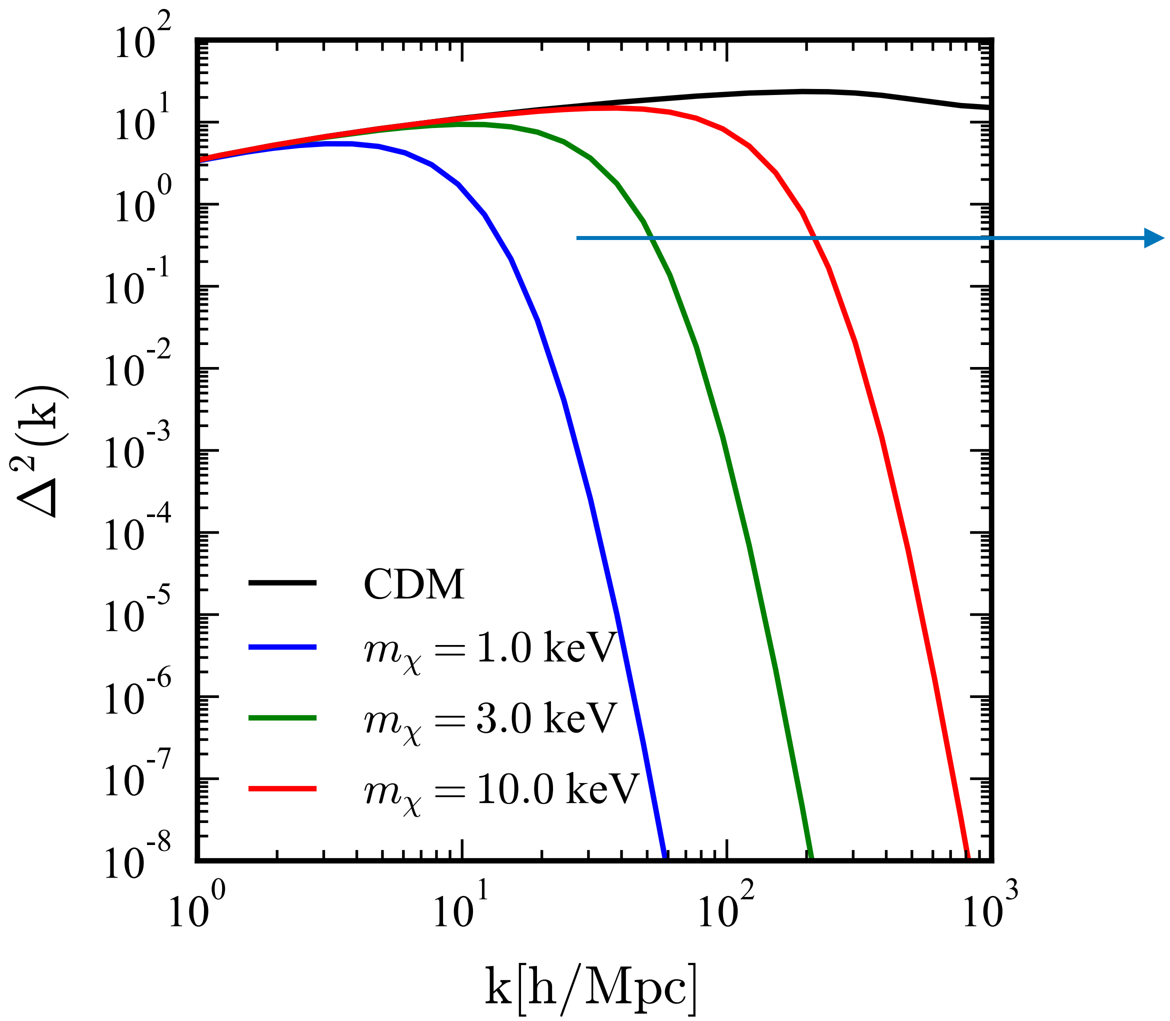
2023-06-15

Matter Power Spectrum of Warm Dark Matter

Mohammadtaher Safarzadeh *et al* 2018 *ApJL* **859** L18

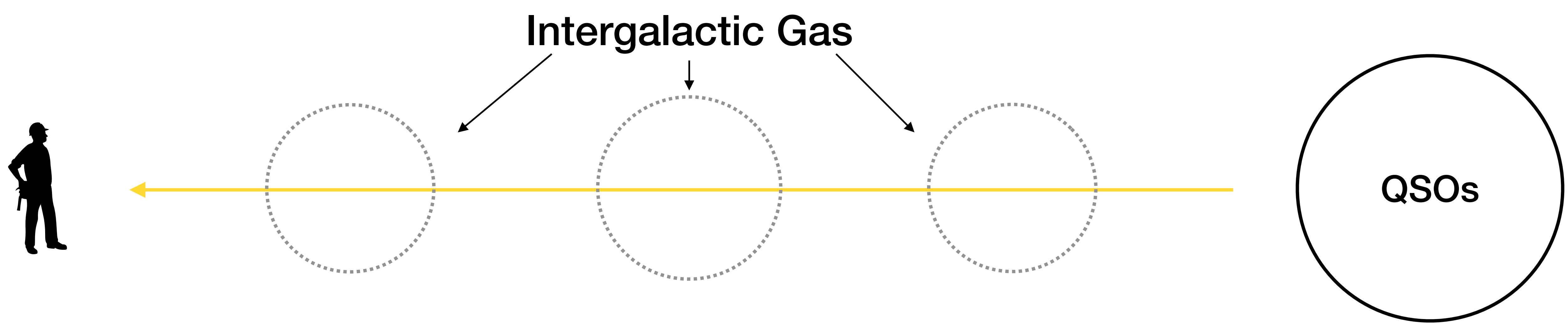


Warm dark matter with a small mass erases the perturbation of the small-scale structure since the warm dark matter has a large velocity and free-streaming length.

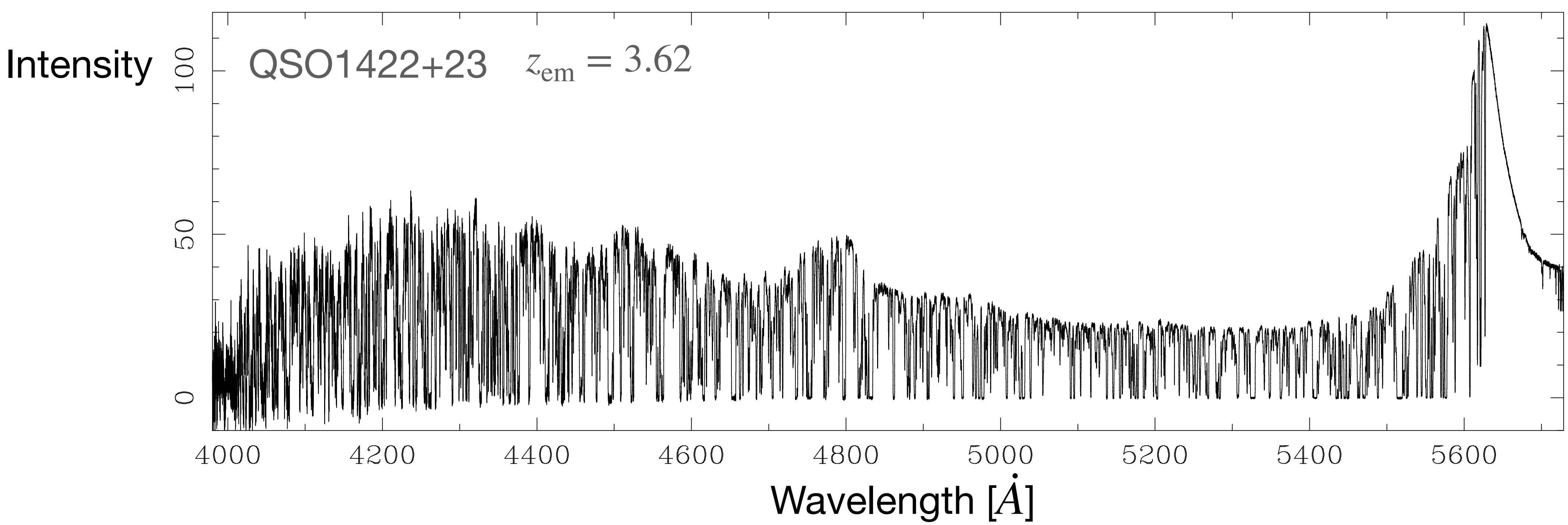
$m_{\mathrm{WDM}} > 5.3$ keV at 2σ

Viel et al, Phys. Rev. D 96, 023522

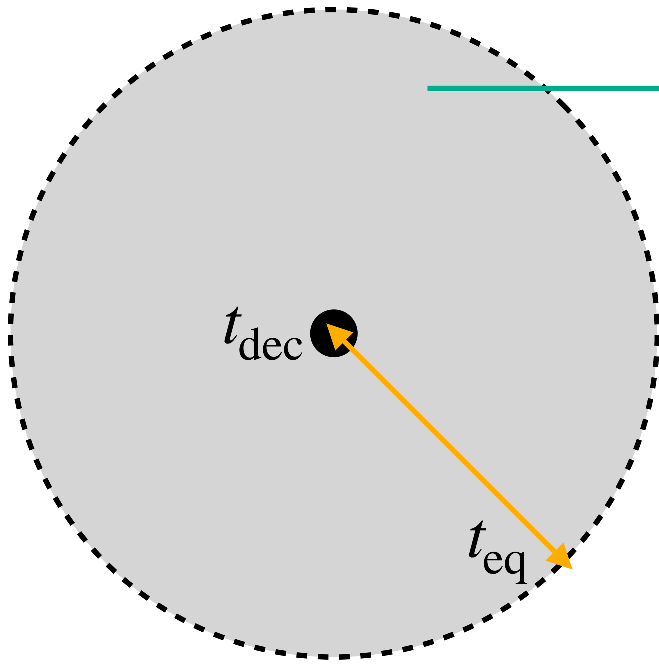
Lyman-α observation



Michael Rauch, Annual Review of Astronomy and Astrophysics Vol. 36:267-316



Free-streaming Length



Perturbation in the region is erased by DM

$\Rightarrow$ Free-streaming length of DM should be smaller than the size of the perturbation of galaxy $L_\alpha \sim 0.1\,\mathrm{Mpc}$

Free-streaming length:

$$\lambda_{fs} \equiv \int_{t_{dec}}^{t_{eq}} \frac{v(t')}{a(t')} dt' \simeq \left(\frac{t_{NR}}{a_{NR}}\right) \left[2 + \ln(t_{eq}/t_{NR})\right]$$

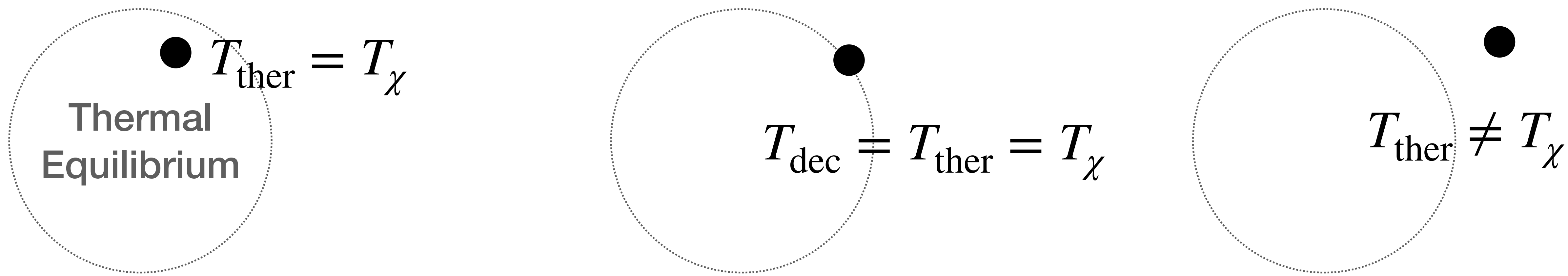
Edward Kolb et al, The Early Universe

$$\lambda_{fs} \simeq 0.1\,\mathrm{Mpc} \left(\frac{8.15\,\mathrm{keV}}{\mathrm{T_{NR}}}\right)$$

Assumption for log-term: $t_{\rm NR} = 10^6\,\mathrm{sec}$ & $t_{\rm eq} = 51.1\,\mathrm{kyr}$

Thermally Produced DM

Time



When DM is thermalized | When DM is decoupled | After DM is decoupled

Entropy conservation:

$$g_{*s}(T_{\text{dec}})T_{\text{dec}}^3 a(T_{\text{dec}})^3 = g_{*s}(T)T^3 a(T)^3$$

Thermalization:

$$\langle p_\chi \rangle / T_\chi \simeq 3.151$$

$$\frac{T_\chi}{T} = \left(\frac{g_{*s}(T)}{g_{*s}(T_{\text{dec}})} \right)^{1/3}$$

Red shift:

$$p_\chi(T_{\text{dec}})a(T_{\text{dec}}) = p_\chi(T_\chi)a(T_\chi)$$

Ratio of temperature between DM and background

Thermally Produced DM

Time

Non-relativistic

$$T = T_{\rm NR}$$

$$p_\chi \simeq m_\chi \simeq 3.151 \cdot T_{\chi,\rm NR}$$

$$\frac{T_\chi}{T} = \left(\frac{g_{*s}(T)}{\underbrace{g_{*s}(T_{\rm dec})}_{\simeq 106.75}} \right)^{1/3}$$

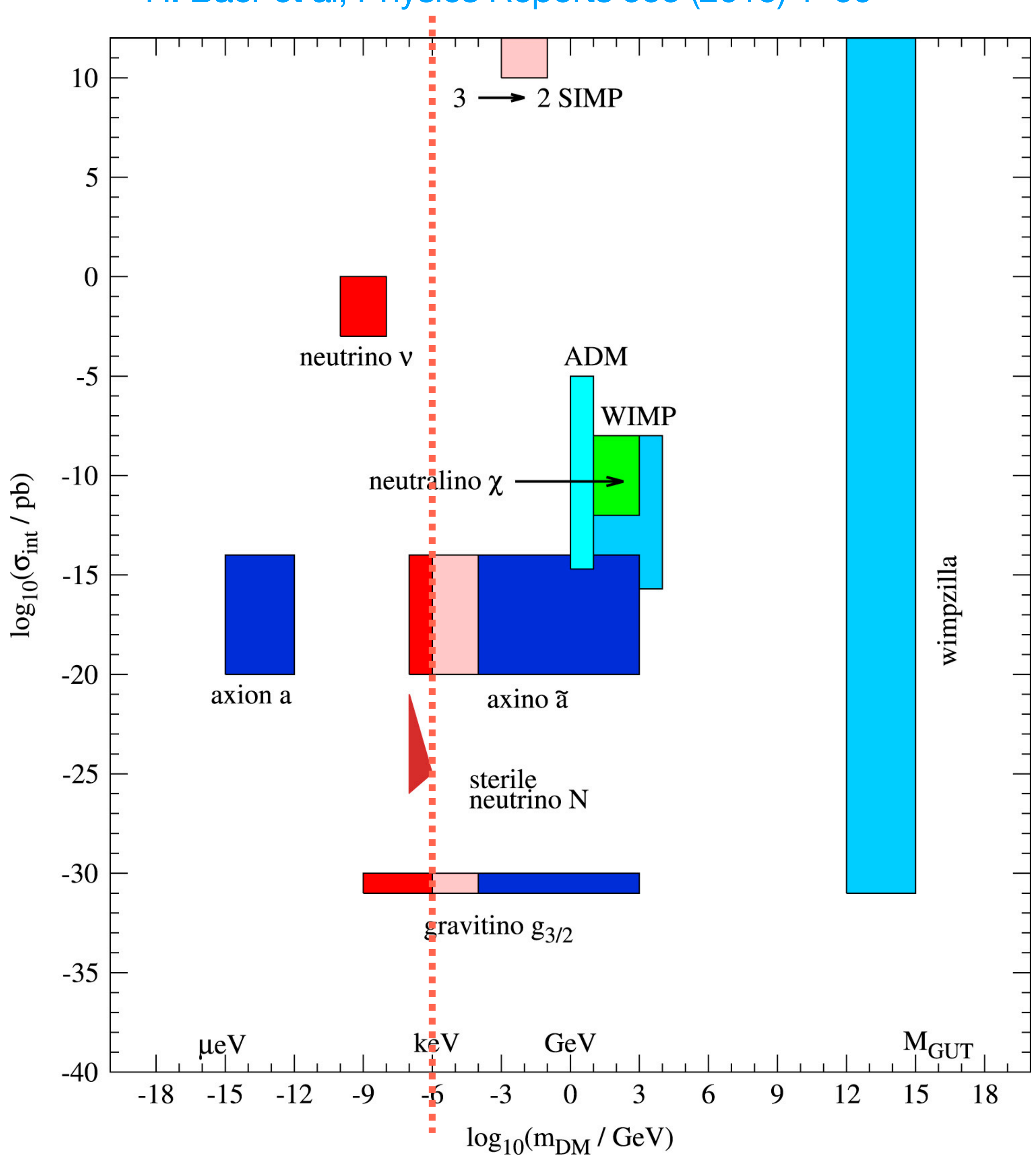
$\Rightarrow$ Free-streaming length: $\lambda_{fs} \simeq 0.32\,\mathrm{Mpc} \left(\frac{8.15\,\mathrm{keV}}{\mathrm{m}_\chi} \right) \left(\frac{\mathrm{T}_\chi}{\mathrm{T}} \right)_{\rm NR} < \mathrm{L}_\alpha$

$$\Rightarrow m_\chi \gtrsim 8.66\,\mathrm{keV} \left(\frac{0.1\,\mathrm{Mpc}}{\mathrm{L}_\alpha} \right)$$

Candidates of Dark Matter

H. Baer et al, Physics Reports 555 (2015) 1–60

Interaction with ordinary matter



$m_\chi \sim \mathrm{keV}$

(red)	Hot DM	$\lambda_{fs} > L_\alpha$
(pink)	Warm DM	$\lambda_{fs} \sim L_\alpha$
(blue)	Cold DM	$\lambda_{fs} < L_\alpha$

λ_{fs}: Free-streaming length of DM

L_α: Length scale of Lyman-α

How can we make the lower bound of DM mass lighter than keV?

DM from Decay of Heavy Particle

Non-thermally produced DM

Redshift: $$p_X = \frac{M}{2}\left(\frac{a_{\rm prod}}{a}\right) = \frac{M}{2}\left(\frac{T}{T_{\rm prod}}\right)\left(\frac{g_{*s}(T)}{g_{*s}(T_{\rm prod})}\right)^{1/3}$$

Entropy conservation: $$T_{\rm NR} = T_{\rm prod}\left(\frac{2m_\chi}{M}\right)\left(\frac{g_{*s}(T_{\rm NR})}{g_{*s}(T_{\rm prod})}\right)^{-1/3}$$

We don't need to consider the relation with plasma

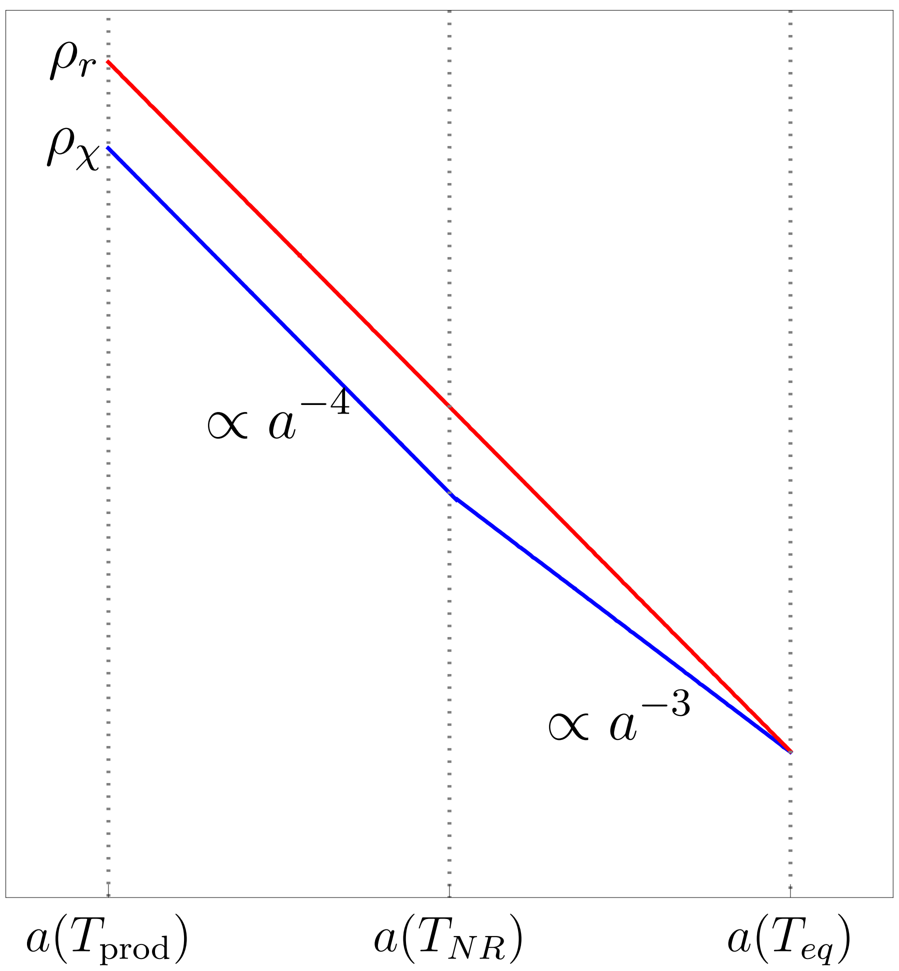
$\Rightarrow$ Free-streaming length: $$\lambda_{fs} \simeq 0.27\,{\rm Mpc}\left(\frac{\rm keV}{{\rm m}_\chi}\right)\frac{\rm M}{2{\rm T}_{\rm prod}} < {\rm L}_\alpha$$

$$\Rightarrow m_\chi \gtrsim 1.35\,{\rm keV}\left(\frac{\rm M}{{\rm T}_{\rm prod}}\right)\left(\frac{0.1\,{\rm Mpc}}{{\rm L}_\alpha}\right)$$

If $M \ll T_{\rm prod}$, CDM mass can be smaller than $1\,{\rm keV}$

Relic Density of DM

Assumption: DM produced explains 100% of DM at present



$$\alpha_\chi B_\chi \equiv \left(\frac{\rho_\chi}{\rho_r}\right)_{T_{\rm prod}} = \left(\frac{\rho_\chi}{\rho_r}\right)_{T_{\rm NR}} = \frac{T_{\rm eq}}{T_{\rm NR}}$$

α_χ : possible enhancement factor due to Bose-Einstein enhancement

Moroi, T., Yin, W. *J. High Energ. Phys.* **2021**, 301 (2021).

$$B_\chi \equiv \Gamma_{\phi\to\chi\chi}/\Gamma$$

$$\Rightarrow \lambda_{fs} \simeq 1.3\times 10^2\,\mathrm{Mpc}\;\alpha_\chi \mathrm{B}_\chi \left[1 + \ln\left(\alpha_\chi \mathrm{B}_\chi\right)\right]$$

Ex1: DM from Visible Decay of Inflaton

Basic Conditions

$\left\{\begin{array}{l}\text{Perturbative decay of the inflaton field}\\ \text{Sudden decay at the end of the reheating epoch}\end{array}\right. \longrightarrow \left\{\begin{array}{l}\Gamma \sim H\\ T_{\text{prod}} \simeq T_R\end{array}\right.$

Constraint 1

To avoid thermalization of DM:

$$\Gamma_{\phi\to\chi\chi} < H\Big|_{M<T<T_R}$$

$$\left\{\begin{array}{l} B_\chi = \dfrac{\Gamma_{\phi\to\chi\chi}}{\Gamma} < \dfrac{H\big|_{T=M}}{H(T_R)} \simeq \dfrac{M^2}{T_R^2} \\ B_\chi = \dfrac{1}{\alpha_\chi}\dfrac{T_{\text{eq}}}{T_{\text{NR}}} \end{array}\right.$$

Constraint 2

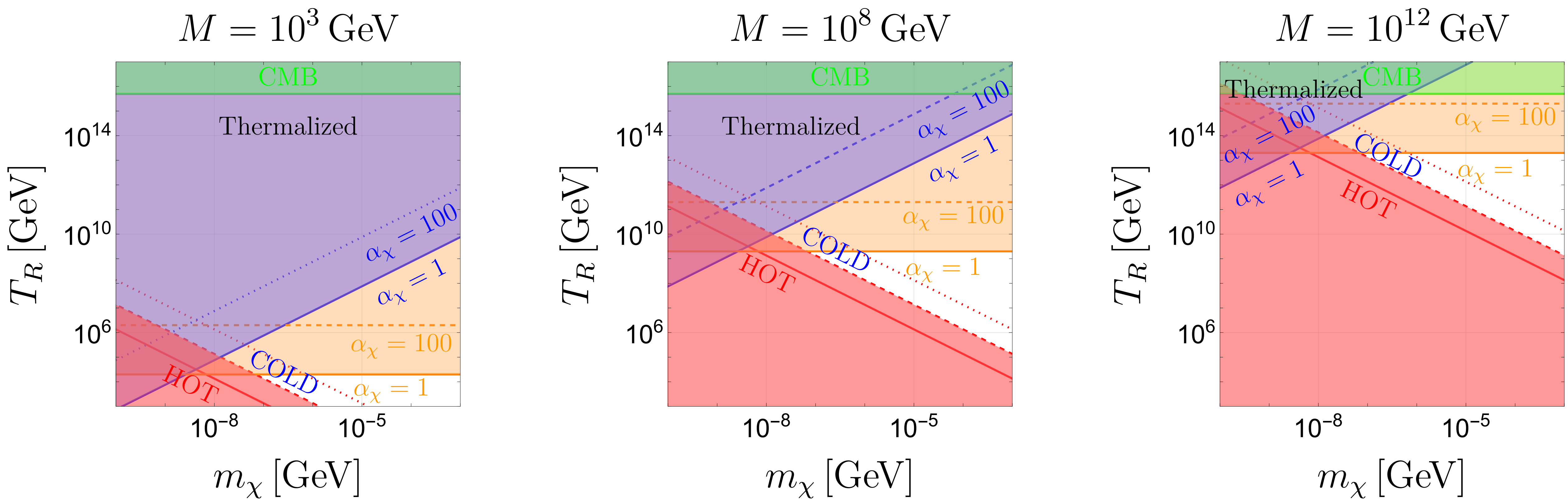
To avoid dominance
of thermally produced DM:

$$Y_\chi^{\text{non-th}} > Y_\chi^{\text{th}}\Big|_{T=M/3}$$

$$\left\{\begin{array}{l} Y_\chi^{\text{th}}\Big|_{T=M/3} \simeq \dfrac{2.2}{g_*^{3/2}(M/3)}\dfrac{\Gamma_{\phi\to\chi\chi}m_{Pl}}{M^2} \\ Y_\chi^{\text{non-th}} \simeq \dfrac{3\alpha_\chi B_\chi T_R}{2M} \end{array}\right.$$

Ex1: DM from Visible Decay of Inflaton

Results



arXiv:2304.07462

$\alpha_\chi \lesssim 27.3$

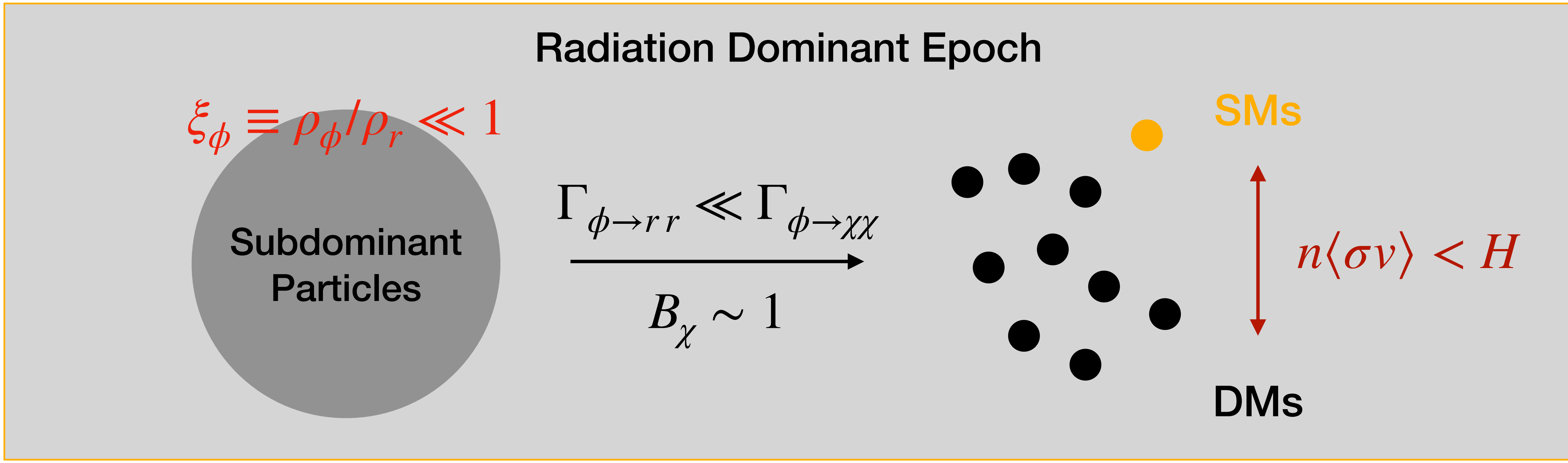
$$\Rightarrow m_\chi \gtrsim 68\,\mathrm{eV}\left(\frac{1}{\alpha_\chi}\right)\left(\frac{0.1\,\mathrm{Mpc}}{\lambda_{\mathrm{fs}}}\right)$$

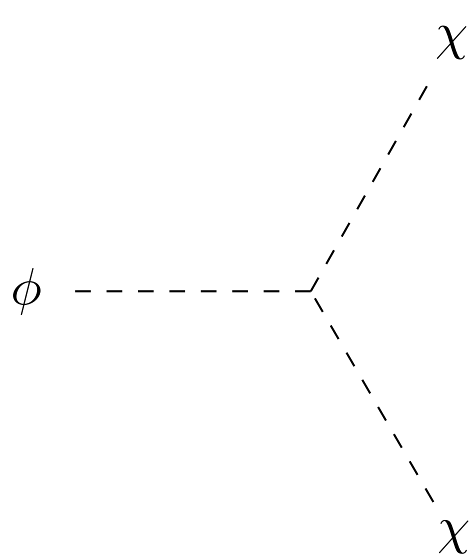
$\alpha_\chi \gtrsim 27.3$

$$\Rightarrow m_\chi \gtrsim 13\,\mathrm{eV}\left(\frac{1}{\alpha_\chi}\right)^{1/2}\left(\frac{0.1\,\mathrm{Mpc}}{\lambda_{\mathrm{fs}}}\right)^{1/2}$$

- Red line: $\lambda_{fs} = 1, 0.1, 0.01\,\mathrm{Mpc}$ (Solid, Dashed, Dotted line respectively)
- Orange region: dominance of thermally produced DM

DM from Hidden Decay of Heavy Particles



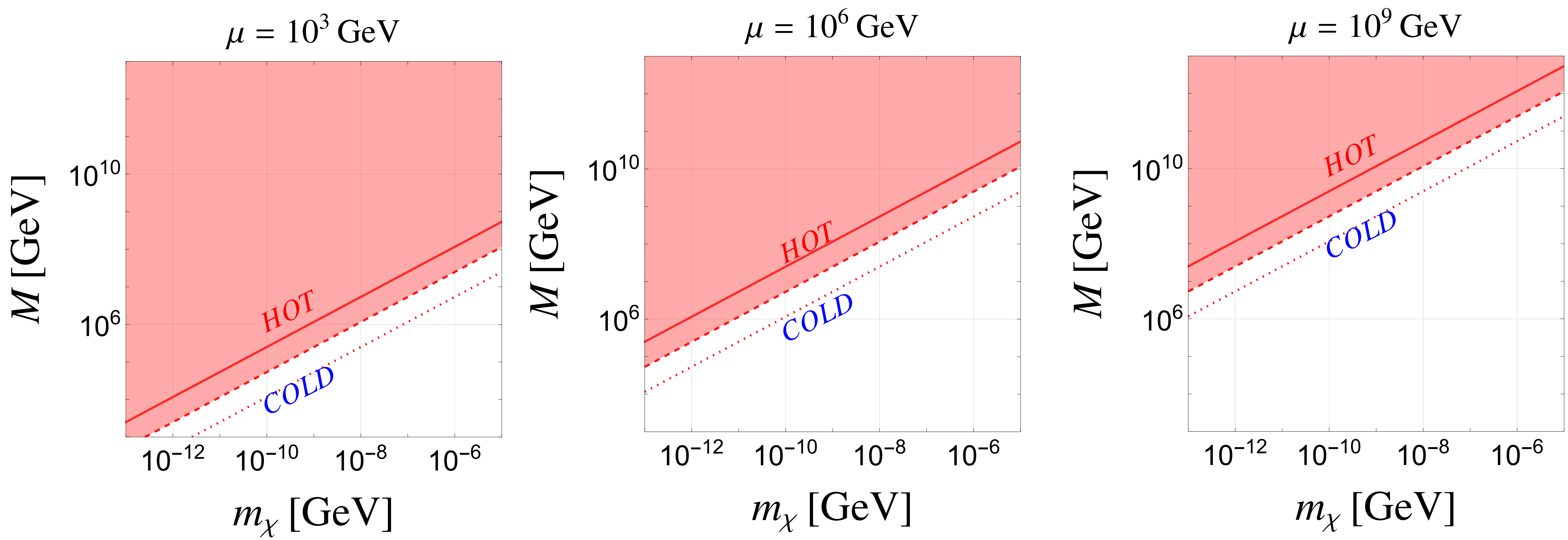


$$\mathscr{L}_{int} = \mu \phi \chi^2$$

$$\Gamma \simeq \Gamma_\phi = \mu^2/(8\pi M)$$

DM from Hidden Decay of Heavy Particles

Results with hidden decay



arXiv:2304.07462

$$\lambda_{fs} \simeq 8 \times 10^{-2}\,\mathrm{Mpc} \left(\frac{\mathrm{keV}}{\mathrm{m}_\chi}\right) \left(\frac{\mathrm{M}}{10^{12}\,\mathrm{GeV}}\right)^{3/2} \left(\frac{10^{10}\,\mathrm{GeV}}{\mu}\right) \lesssim \mathrm{L}_\alpha$$

- Red line: $\lambda_{fs} = 1, 0.1, 0.01\ \mathrm{Mpc}$ (Solid, Dashed, Dotted line respectively)

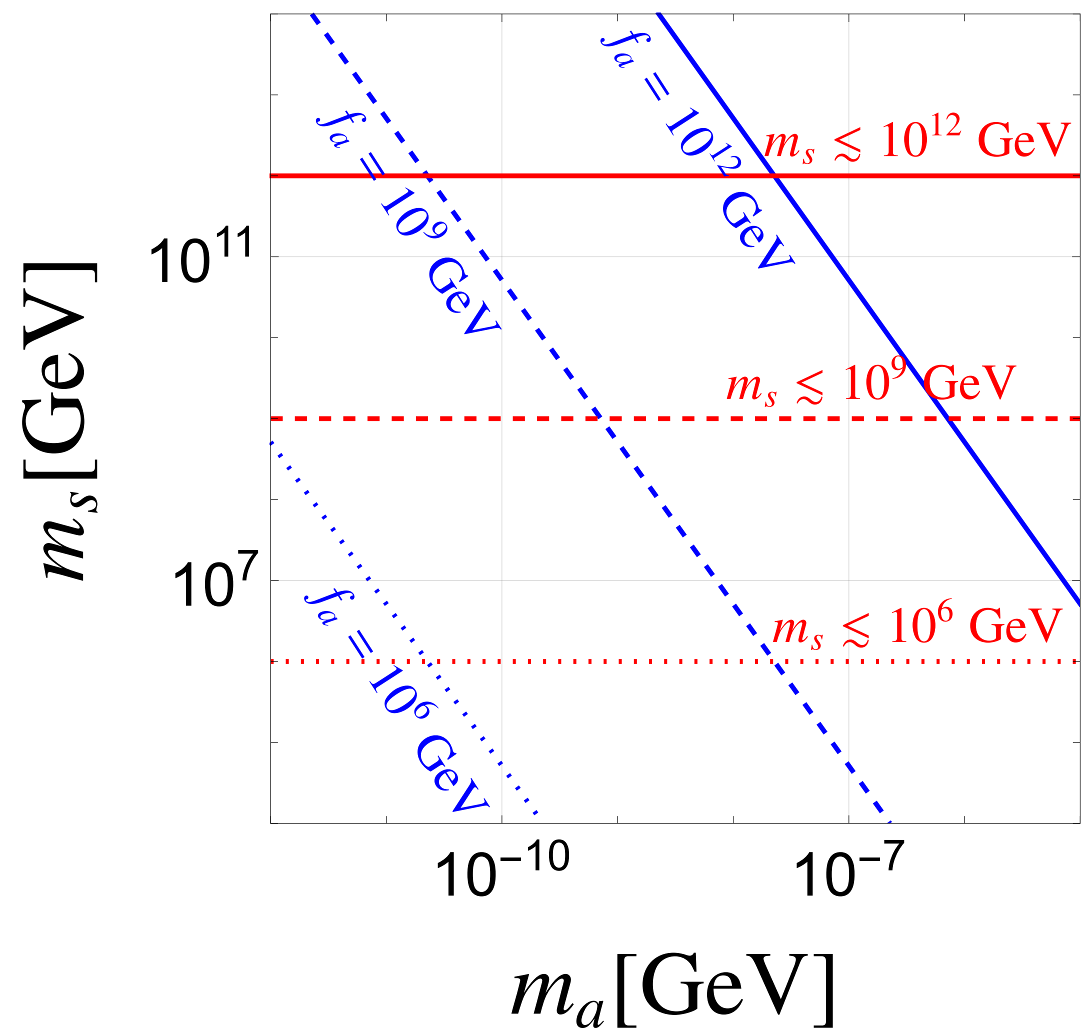
Ex2: Hidden Axion DM from Saxion Decay

$$\mathscr{L}_{int} \propto \left(1 + \frac{\sqrt{2}x}{f_a}s\right)\left(\frac{1}{2}\partial_\mu a \partial^\mu a\right) \qquad x \sim 1$$

$$\Gamma_{s\to aa} \simeq \frac{x^2}{64\pi}\frac{m_s^3}{f_a^2}$$

$$m_a \gtrsim 10\,\mathrm{eV}\left(\frac{0.1\,\mathrm{Mpc}}{\mathrm{L}_\alpha}\right)\left(\frac{\mathrm{f_a}}{10^{12}\,\mathrm{GeV}}\right)^{1/2}$$

arXiv:2304.07462

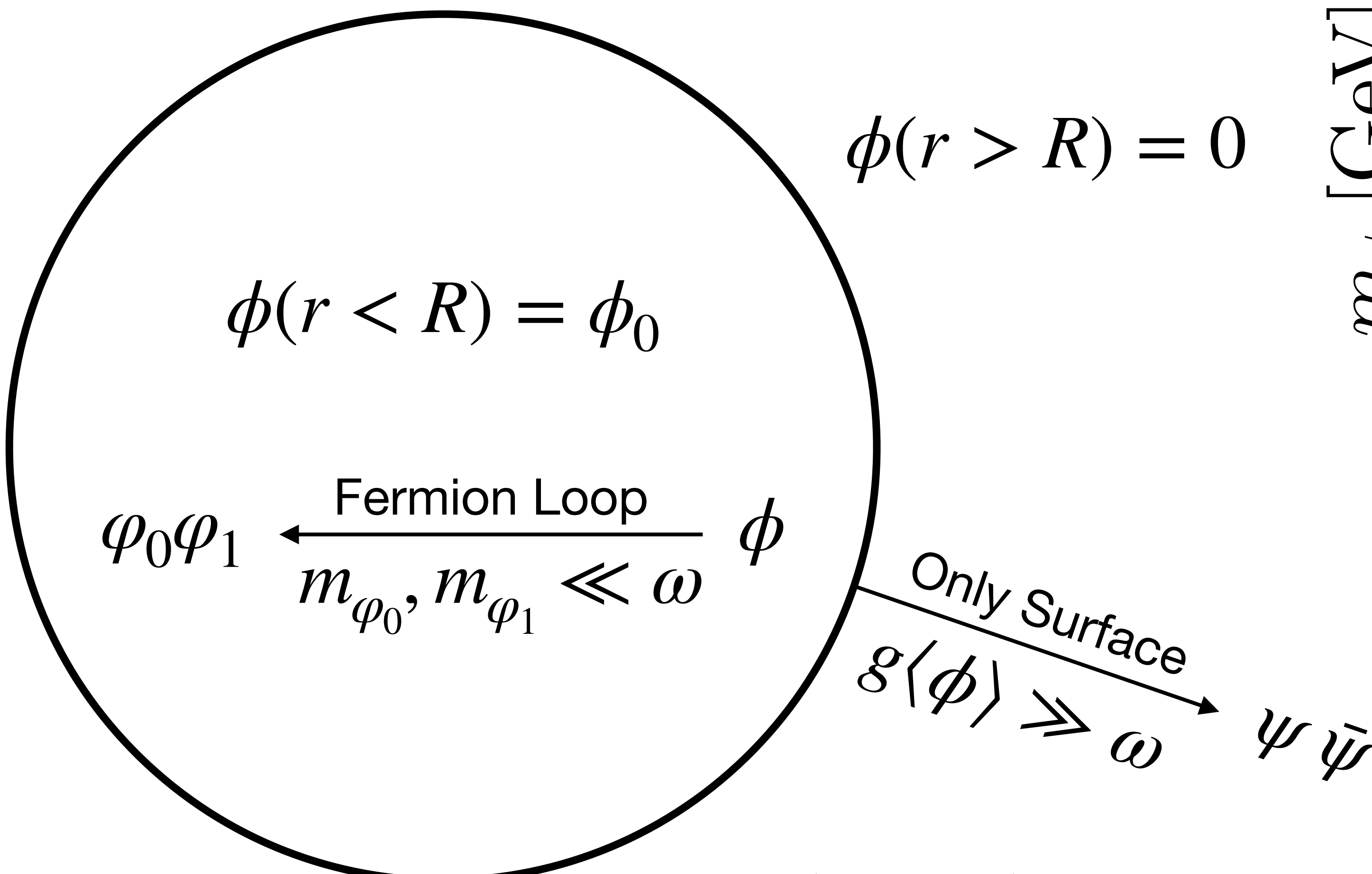


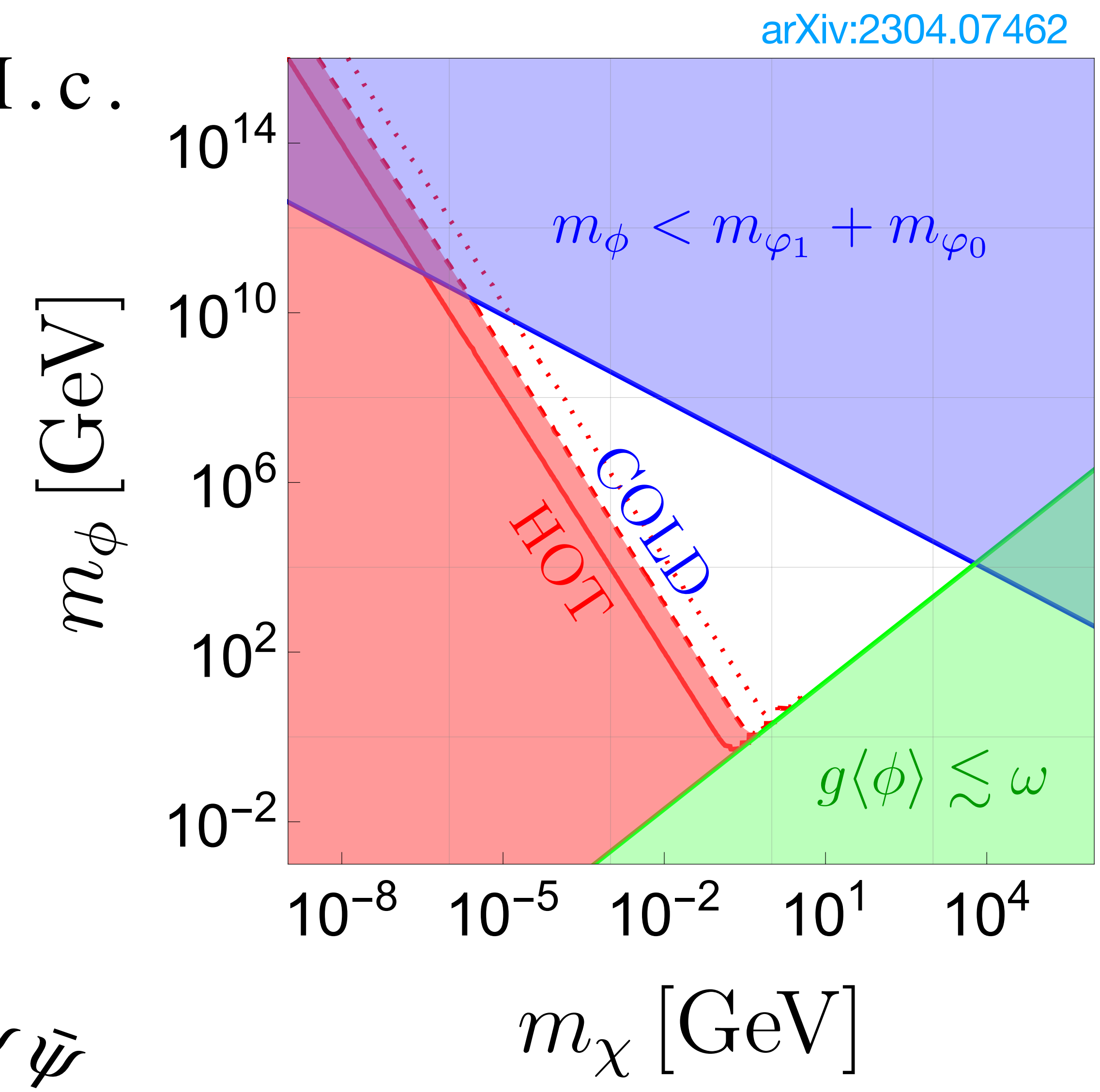
- Red line: Saxion mass is the same as Peccei-Quinn scale $m_s \simeq f_a$
- Blue line: Free-streaming length is the same as the scale of perturbation $\lambda_{fs} \simeq L_\alpha$

Ex3: DM from Q-ball decay

$$\mathscr{L}_{\text{int}} = -\psi_1^\dagger(g\phi\psi_0 + y\varphi_1\psi_0') - y\varphi_0\psi_0^T\psi_0' + \text{H.c.}$$

$$\phi(r,t) = \phi(r)e^{-i\omega t}$$





For dominance of scalar: $\dfrac{(dQ/dt)_{Q\to\varphi_1\varphi_0}}{(dQ/dt)_{Q\to\psi\bar{\psi}}} \gg 1 \Rightarrow R \gg 1/\omega \qquad m_\chi \gtrsim 10\,\text{keV}$

Conclusion

- Non-thermally produced particles can be cold even with lighter mass than thermally produced particles.
- The mass of cold DM from the visible decay of inflaton can become around 10eV.
- However, when the heavy particle decay dominantly to dark matter, the mass of cold DM can be much lighter.
- For hidden axion DM, the mass for coldness can be small down to $10^{-3}\mathrm{eV}$ for Peccei-Quinn scale around TeV.
- For DM produced from Q-ball, the mass can be light around keV to be cold.

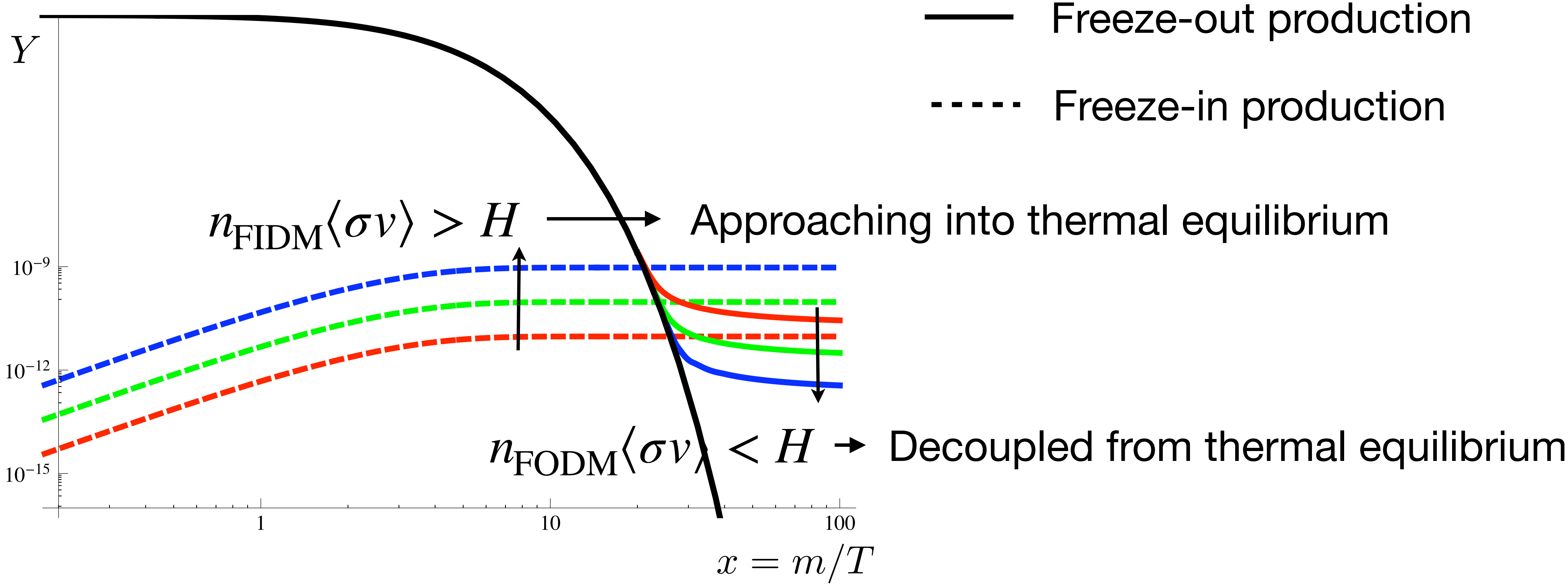
Thanks

Appendix

Production of Dark Matter

- Thermal production: produced from thermal particles

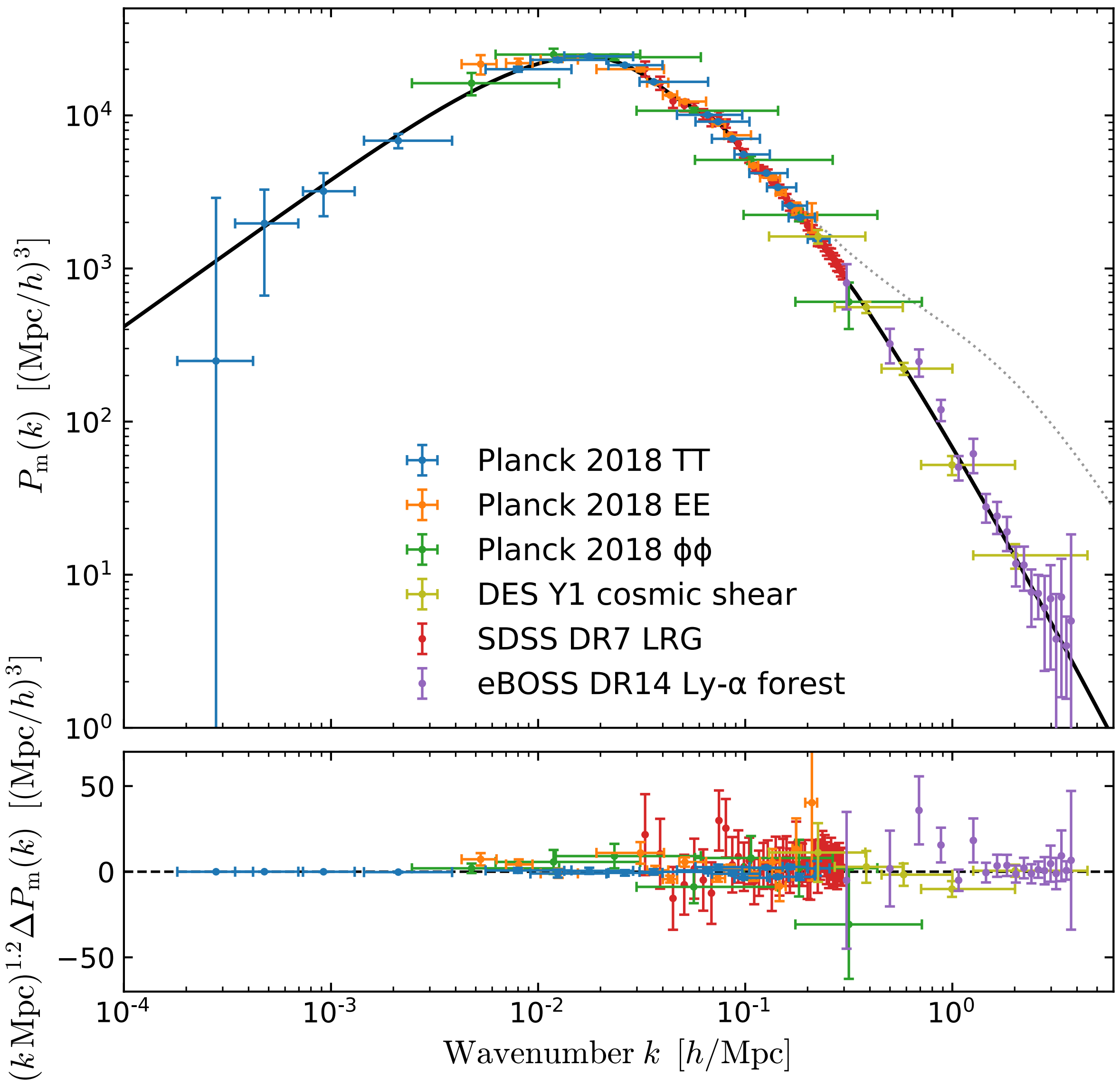
Lawrence J. Hall et al, Journal of High Energy Physics 2010



- Non-thermal production: no interaction with thermal particles

Appendix

Observation of the matter power spectrum



Solène Chabanier and others, Matter power spectrum: from Ly *α* forest to CMB scales, *Monthly Notices of the Royal Astronomical Society*, Volume 489, Issue 2, October 2019, Pages 2247–2253

Appendix

Possible enhancement factor

Moroi, T., Yin, W. *J. High Energ. Phys.* **2021**, 301 (2021).

$$\dot{f}_{\vec{k}} = H\vec{k}\frac{\partial f_{\vec{k}}}{\partial \vec{k}} + \dot{f}^{(\mathrm{coll})}_{\vec{k}}$$

$$\dot{f}^{(\mathrm{coll})}_{\vec{k}} = 2n_\phi \Gamma^{(0)}_{\phi\to\chi\chi}\left[(1\pm f_{\vec{k}})(1\pm f_{-\vec{k}}) - f_{\vec{k}}f_{-\vec{k}}\right]\delta(|\vec{k}| - p_\chi)\left(\frac{p_\chi^2}{2\pi^2}\right)^{-1}$$

$$\dot{\hat{f}}_{\hat{k}} = \frac{2n_\phi \Gamma^{(0)}_{\phi\to\chi\chi}}{Hp_\chi}(1\pm 2\hat{f}_{\hat{k}})\delta(t - t_{\hat{k}})\left(\frac{p_\chi^2}{2\pi^2}\right)^{-1}$$

$$\hat{f}_{\hat{k}}(t) \equiv f_k(t)$$

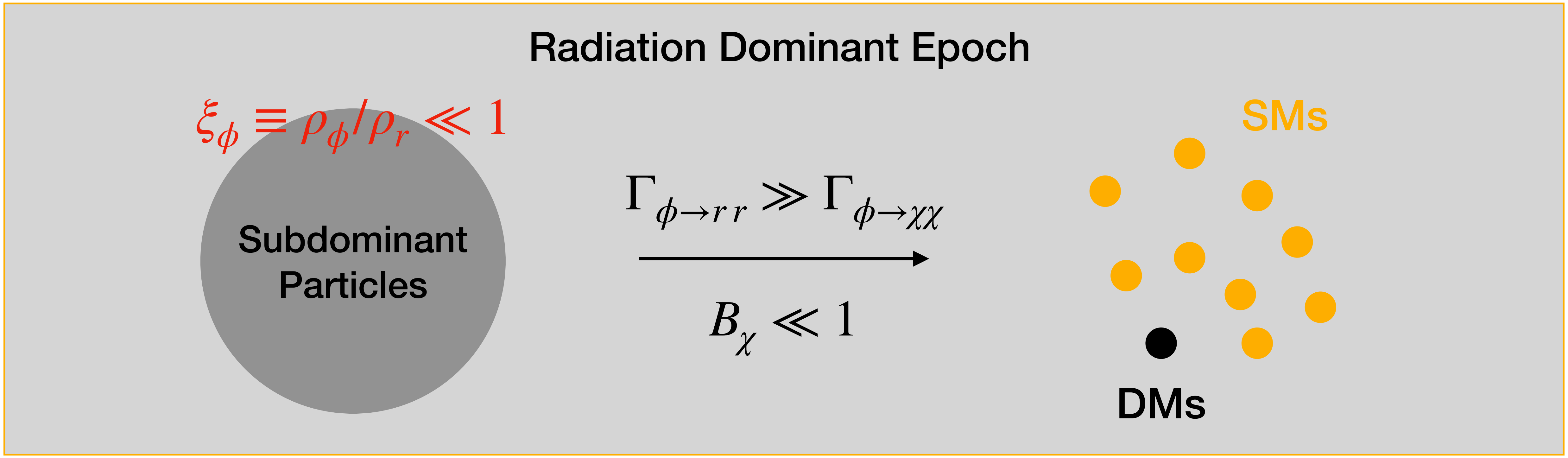
$$f_k(t\to\infty) = \pm\frac{1}{2}\left(e^{\pm 2\bar{f}(t_k)} - 1\right)\theta(p_\chi - k)$$

$$\bar{f}(t_k) \equiv \left.\frac{4\pi^2\Gamma^{(0)}_{\phi\to\chi\chi}n_\phi}{Hp_\chi^3}\right|_{t=t_{\hat{k}}}$$

If $\bar{f} \gtrsim 1$, the production of DM is enhanced exponentially due to stimulated emission

We consider this case by using α_χ

Appendix
Visible decay

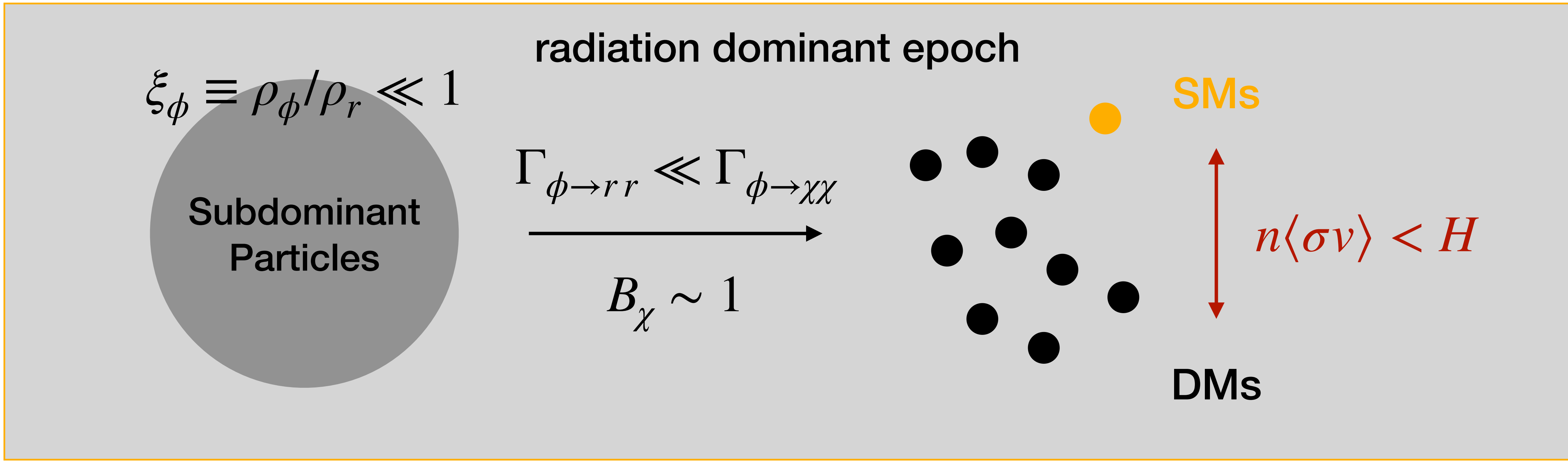


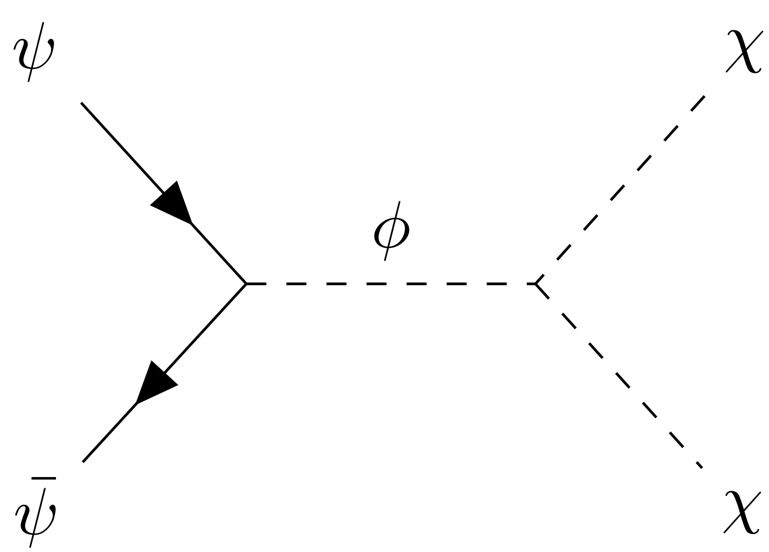
$$\Rightarrow m_\chi \gtrsim 68\,\mathrm{eV}\left(\frac{1}{\alpha_\chi \xi_\phi}\right)\left(\frac{0.1\,\mathrm{Mpc}}{\lambda_{\mathrm{fs}}}\right)$$

$\Rightarrow$ DM mass should be much bigger than $1\,\mathrm{keV}$

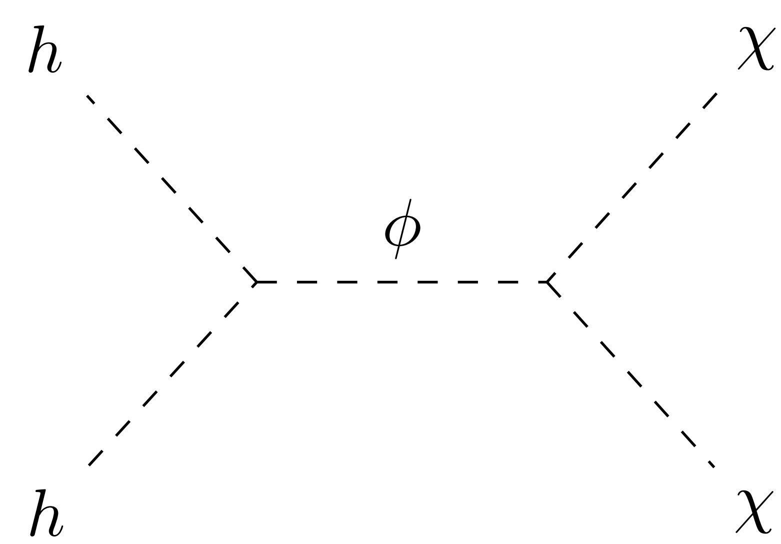
Appendix

Example of general case of subdominant particle





$$\mathscr{L}_{int} = \mu \phi \chi^2 + g \bar{\psi} \psi \phi$$

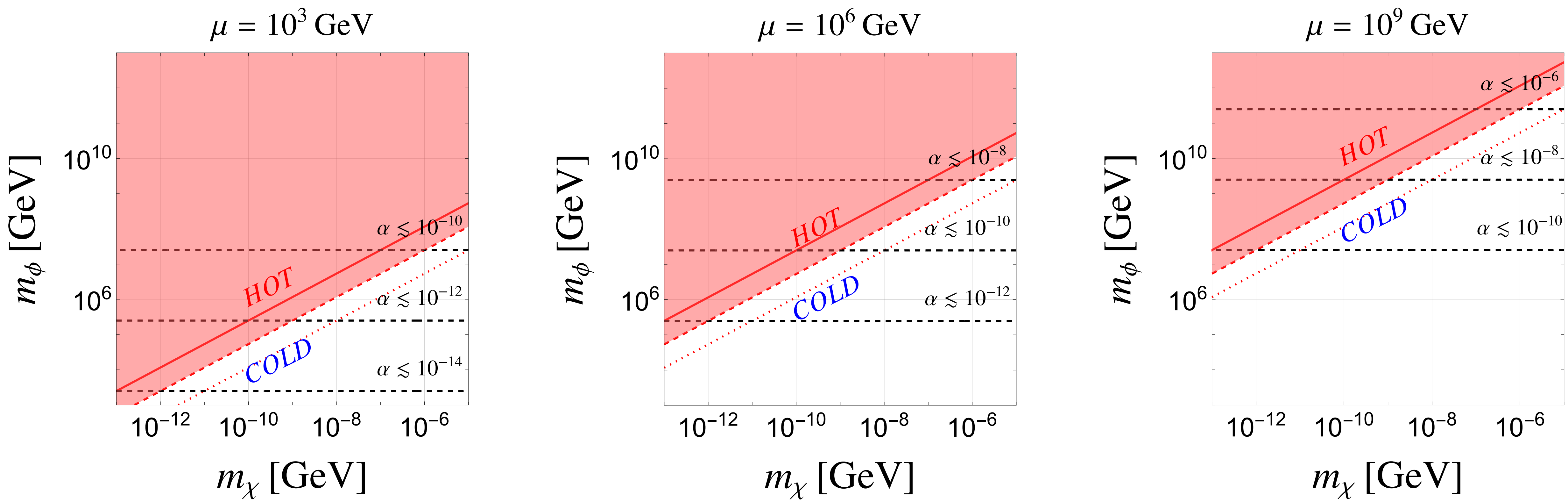


$$\mathscr{L}_{int} = \mu \phi \chi^2 + g \phi h^2$$

Appendix

Results with fermion SM particles

$$\mathcal{L}_{int} = \mu\phi\chi^2 + g\bar{\psi}\psi\phi \qquad \alpha \equiv g^2/4\pi$$



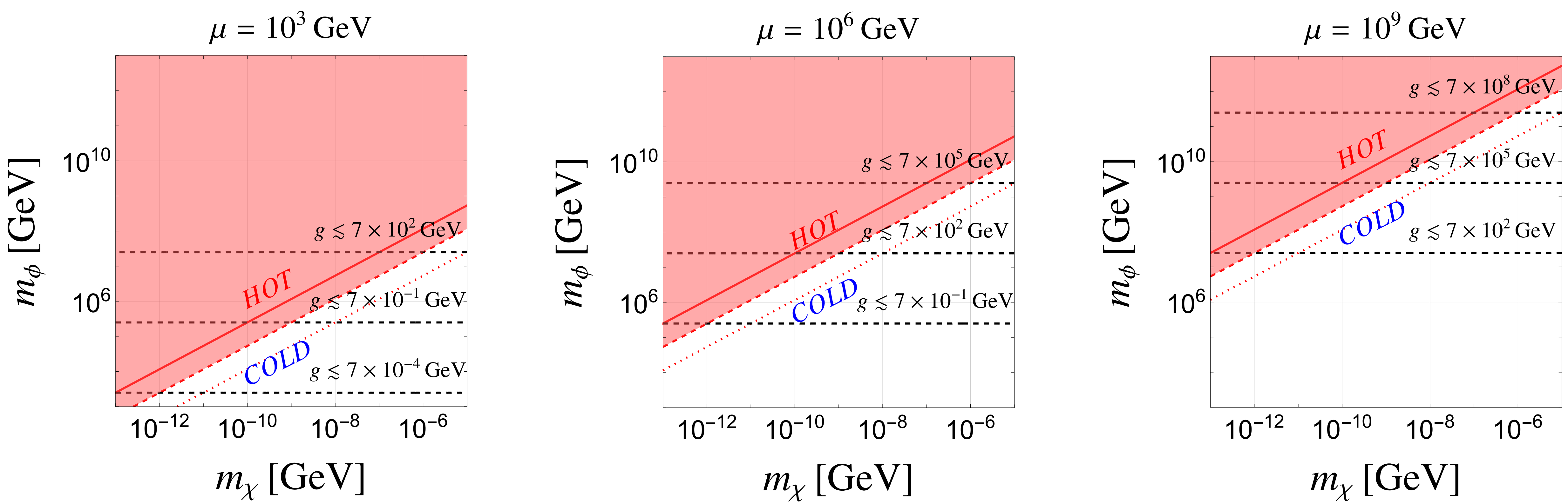
DM mass can be small

when coupling constant between SM particles and subdominant particle is small

Appendix

Results with scalar SM particles

$$\mathscr{L}_{int} = \mu\phi\chi^2 + g\phi h^2$$



DM mass can be small

when coupling constant between SM particles and subdominant particle is small

Appendix
Q-ball conditions

- Relic density:

$$\frac{\rho_\chi}{s} \simeq \left(\frac{m_\chi}{m_\phi}\right) \frac{45}{2\pi^2 g_{*S}} \left(\frac{90}{\pi^2 g_*}\right)^{-3/4} \frac{m_{CP}\phi_i^2}{m_\phi^{1/2} M_P^{3/2}}$$

$$R \simeq \frac{1}{m_\phi \sqrt{|K|}}$$

$$\phi_0 \simeq \frac{|K|^{3/4}}{\sqrt{2\pi^{3/2}}} m_\phi Q^{1/2}$$

$$\omega \simeq m_\phi$$

$$Q \simeq \left(\frac{\pi}{2}\right)^{3/2} \omega \phi_0^2 R^3$$

Condition of figure

$|K| = 10^{-4}\$, \$g = 1\$, \$y = 1\$$ and $\$m_{CP} = m_\phi$

Appendix

Q-ball decay rate

- Decay rate into fermion:

$$-\left(\frac{dQ}{dt}\right)_{Q\to\psi\bar{\psi}} = -\left(\frac{dQ}{dt}\right)_{\mathrm{sat}} \equiv \frac{\omega^3 R^2}{24\pi}$$

- Decay rate into scalar:

$$\Gamma(\phi \to \varphi_1\varphi_0) = \frac{\sqrt{(\omega^2-(m_{\varphi_1}+m_{\varphi_0})^2)(\omega^2-(m_{\varphi_1}-m_{\varphi_0})^2)}}{16\pi\omega^3}\left|\frac{gy^2}{16\pi^2}mF\right|^2$$

where $F \simeq \dfrac{5}{18}\dfrac{\omega^2}{m^2}$ is the auxiliary function.

- Auxiliary function:

- $$F \equiv \int_0^1\int_0^1 2ydydx\frac{2l^2+(p+2q)\cdot l+p\cdot q}{l^2+q^2(-1+y)+\left[m^2+p^2(-1+x)\right]y}$$