On the mass spectrum of heavy Higgs bosons in 2HDM in the light of the CDF *W*-mass anomaly

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PPC 2023: XVI International Conference

2023/06/15 @IBS

Main Ref.: 2204.05728 [hep-ph] Y.H., Dong-Won Jung, Jae Sik Lee
2212.07620 [hep-ph] Y.H., Dong-Won Jung, Jae Sik Lee

1. Introduction

- We analyze the implications of the large positive deviation of the S and T parameters from their standard model (SM) values, as indicated by the high-precision CDF measurement of the W boson mass. We consider these implications in the context of the mass spectrum of heavy Higgs bosons, taking into account the most general CP-violating 2HDM.
- Outline
  1. Introduction
  2. Higgs basis 2HDM
  3. Constraints
  4. Analysis
  5. Conclusions

2. Φ basis 2HDM

- The general Φ basis 2HDM

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + i a_1) \end{pmatrix}, \quad \Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + i a_2) \end{pmatrix}$$

  - $v = \sqrt{v_1^2 + v_2^2}$, $\tan\beta = \frac{v_2}{v_1}$, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$

- Potential $V_\Phi = \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) + m_{12}^{*2}(\Phi_2^\dagger\Phi_1)$

$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*(\Phi_2^\dagger\Phi_1)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

2. Higgs basis 2HDM

- We have considered the most general 2HDM potential with explicit CP violation.
- Higgs basis

$$\mathcal{H}_1 = c_\beta \Phi_1 + e^{-i\xi} s_\beta \Phi_2 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_1 + iG^0) \end{pmatrix}, \quad \mathcal{H}_2 = -s_\beta \Phi_1 + e^{-i\xi} c_\beta \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_2 + ia) \end{pmatrix}$$

  - $\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$, $\begin{pmatrix} G^0 \\ a \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$,
  - $v = \sqrt{v_1^2 + v_2^2}$, $t_\beta = \tan(\beta) = v_2/v_1$

- Potential $V_{\mathcal{H}} = Y_1(\mathcal{H}_1^\dagger \mathcal{H}_1) + Y_2(\mathcal{H}_2^\dagger \mathcal{H}_2) + Y_3(\mathcal{H}_1^\dagger \mathcal{H}_2) + Y_3^*(\mathcal{H}_2^\dagger \mathcal{H}_1)$

$$+Z_1(\mathcal{H}_1^\dagger \mathcal{H}_1)^2 + Z_2(\mathcal{H}_2^\dagger \mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_2^\dagger \mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$
$$+Z_5(\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + Z_5^*(\mathcal{H}_2^\dagger \mathcal{H}_1)^2 + Z_6(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_6^*(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$
$$+Z_7(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_7^*(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$

2. Mass terms in the Higgs Basis 2HDM

- Mass terms

$$V_{\mathcal{H}\,\mathrm{mass}} = M_{H^\pm}^2 H^+ H^- + \frac{1}{2}(\varphi_1 \;\; \varphi_2 \;\; a)\, \mathcal{M}_0^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ a \end{pmatrix}$$

  - The charged Higgs boson mass: $M_{H^\pm}^2 = Y_2 + \frac{1}{2} Z_3 v^2$
  - The neutral Higgs bosons mass: $\mathcal{M}_0^2 = M_A^2 \,\mathrm{diag}(0,1,1) + \mathcal{M}_Z^2$
    - $M_A^2 = M_{H^\pm}^2 + \left[\frac{1}{2} Z_4 - \Re\mathrm{e}(Z_5)\right] v^2, \;\; \mathcal{M}_Z^2 = v^2 \begin{pmatrix} 2Z_1 & \Re\mathrm{e}(Z_6) & -\Im\mathrm{m}(Z_6) \\ \Re\mathrm{e}(Z_6) & 2\Re\mathrm{e}(Z_5) & -\Im\mathrm{m}(Z_5) \\ -\Im\mathrm{m}(Z_6) & -\Im\mathrm{m}(Z_5) & 0 \end{pmatrix}$
    - 3×3 real and symmetric mass-squared matrix $\mathcal{M}_0^2$:
      - $(\varphi_1 \;\; \varphi_2 \;\; a)_\alpha^T = O_{\alpha i} (H_1 \;\; H_2 \;\; H_3)_i^T, \;\; O^T \mathcal{M}_0^2 O = \mathrm{diag}\left(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2\right)$
      - $O = O_\gamma O_\eta O_\omega \equiv \begin{pmatrix} c_\gamma c_\eta & s_\gamma c_\omega - c_\gamma s_\eta s_\omega & s_\gamma s_\omega + c_\gamma s_\eta c_\omega \\ -s_\gamma c_\eta & c_\gamma c_\omega + s_\gamma s_\eta s_\omega & c_\gamma s_\omega - s_\gamma s_\eta c_\omega \\ -s_\eta & -c_\eta s_\omega & c_\eta c_\omega \end{pmatrix}$

2. Parameter set in CPV 2HDM

- Cubic interactions: $\mathcal{L}_{HVV} = gM_W\left(W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} Z_\mu Z^\mu\right)\sum_i g_{H_iVV} H_i\,,\quad g_{H_iVV} = O_{\varphi_1 i}$
- The quartic couplings
  - $Z_1 = \frac{1}{2v^2}\left(M_{H_1}^2 O_{\varphi_1 1}^2 + M_{H_2}^2 O_{\varphi_1 2}^2 + M_{H_3}^2 O_{\varphi_1 3}^2\right)$
  - $Z_4 = \frac{1}{v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 + O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 + O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 + O_{a3}^2\right) - 2M_{H^\pm}^2\right]$
  - $Z_5 = \frac{1}{2v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 - O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 - O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 - O_{a3}^2\right)\right] - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_2 1}O_{a1} + M_{H_2}^2 O_{\varphi_2 2}O_{a2} + M_{H_3}^2 O_{\varphi_2 3}O_{a3}\right)$
  - $Z_6 = \frac{1}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1}O_{\varphi_2 1} + M_{H_2}^2 O_{\varphi_1 2}O_{\varphi_2 1} + M_{H_3}^2 O_{\varphi_1 3}O_{\varphi_2 3}\right) - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1}O_{a1} + M_{H_1}^2 O_{\varphi_1 2}O_{a2} + M_{H_3}^2 O_{\varphi_1 3}O_{a3}\right)$
- Tadpole conditions: $Y_1 + Z_1 v^2 = 0;\quad Y_3 + \frac{1}{2}Z_6 v^2 = 0.$
- We use the more physical set of input parameters

$$\{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\} \to \left\{v; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \{O_{3\times 3}\}; Z_3; Z_2, Z_7\right\}$$

  - Phase rotation: $\mathcal{H}_2 \to e^{+i\zeta}\mathcal{H}_2;\quad Z_5 \to Z_5 e^{-2i\zeta},\ Z_6 \to Z_6 e^{-i\zeta},\ Z_7 \to Z_7 e^{-i\zeta}$
  - physical CPV parameters: $\theta_1 \equiv \mathrm{Arg}\left[Z_6\ (Z_5^*)^{1/2}\right], \theta_2 \equiv \mathrm{Arg}\left[Z_7\ (Z_5^*)^{1/2}\right]$

3. Constraints – UNIT⊕BFB

- Perturbative unitarity (UNIT) conditions

$Z_{\{6,7\}} = 0$	$Z_{\{1,2,3,4,5\}} = 0$
$\lvert Z_3 \pm Z_4 \rvert < 4\pi,$ $\lvert Z_3 \pm 2\lvert Z_5 \rvert \rvert < 4\pi,$ $\lvert Z_3 + 2Z_4 \pm 6\lvert Z_5 \rvert \rvert < 4\pi,$ $\left\lvert Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + 4\lvert Z_5 \rvert^2} \right\rvert < 4\pi,$ $\left\lvert Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + Z_4^2} 2\lvert Z_5 \rvert \right\rvert < 4\pi,$ $\left\lvert 3Z_1 + 3Z_2 \pm \sqrt{9(Z_1 - Z_2)^2 + (2Z_3 + Z_4)^2} \right\rvert < 4\pi.$	$\sqrt{\lvert Z_6 \rvert^2 + \lvert Z_7 \rvert^2} < 2\sqrt{2}\pi,$ $\sqrt{\lvert Z_6 \rvert^2 + \lvert Z_7 \rvert^2 + \lvert Z_6^2 + Z_7 \rvert^2} < \frac{4\pi}{3}.$

- Bounded from below (BFB)

$$Z_1 \geq 0, \quad Z_2 \geq 0;$$

$$2\sqrt{Z_1 Z_2} + Z_3 \geq 0, \quad 2\sqrt{Z_1 Z_2} + Z_3 + Z_4 - 2\lvert Z_5 \rvert \geq 0;$$

$$Z_1 + Z_2 + Z_3 + Z_4 + 2\lvert Z_5 \rvert - 2\lvert Z_6 + Z_7 \rvert \geq 0.$$

3. Constraints – EWP

- The electroweak (ELW) oblique corrections, S, T and U.
  - Fixing U = 0 which is suppressed by an additional factor M_Z^2/M_{BSM}^2
  - the S and T parameters are constrained as follow

M. E. P. and T. T., Phys. Rev. Lett. 65 (1990), 964-967
M. E. P. and T. T. , Phys. Rev. D 46 (1992), 381-409

$$\frac{(S-\widehat{S_0})^2}{\sigma_S^2}+\frac{(T-\widehat{T_0})^2}{\sigma_T^2}-2\rho_{ST}\frac{(S-\widehat{S_0})(T-\widehat{T_0})}{\sigma_S\sigma_T}\leq R^2(1-\rho_{ST}^2)$$

$(R^2 = 9.21 \text{ at } 95\% \text{ CLs})$

- Performing a global fit of electroweak data together with the high-precision CDF measurement while fixing U = 0, one may find the large central values of the oblique parameters S and T together with the standard deviations such as

arXiv:2204.03796

- CDF $(\widehat{S_0},\sigma_S) = (0.15, 0.08),\ (\widehat{T_0},\sigma_T) = (0.27, 0.06),\ \rho_{ST} = 0.93$
- PDG $(\widehat{S_0},\sigma_S) = (0.00, 0.07),\ (\widehat{T_0},\sigma_T) = (0.05, 0.06),\ \rho_{ST} = 0.92$

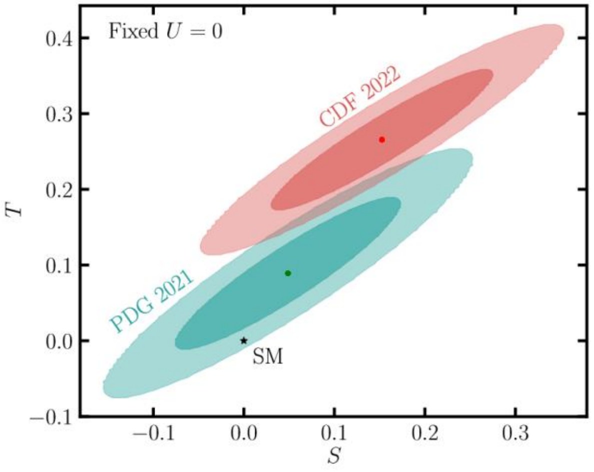


Figure 1. The 1- and 2-σ allowed regions in S-T plane from the electroweak fits using the PDG 2021 data set with the old value of m_W (green region) and the new CDF value of m_W (red region).

3. Expressions of S and T in CPV 2HDM

D. T., Phys. Rev. D 18 (1978), 1626
J. S. Lee and A. Pilaftsis, Phys. Rev. D 86 (2012), 03500

- The S and T parameters might be estimated as follows at the one-loop order

$$4\pi S = -F'_\Delta(M_{H^\pm}, M_{H^\pm}) + g^2_{H_3VV} F'_\Delta(M_{H_1}, M_{H_2}) + g^2_{H_2VV} F'_\Delta(M_{H_1}, M_{H_3}) + g^2_{H_1VV} F'_\Delta(M_{H_2}, M_{H_3})$$

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\mathrm{EM}}}} = (1 - g^2_{H_1VV}) F_\Delta(M_{H_1}, M_{H^\pm}) + (1 - g^2_{H_2VV}) F_\Delta(M_{H_2}, M_{H^\pm}) + (1 - g^2_{H_3VV}) F_\Delta(M_{H_3}, M_{H^\pm})$$

$$+ g^2_{H_3VV} F_\Delta(M_{H_1}, M_{H_2}) + g^2_{H_2VV} F_\Delta(M_{H_1}, M_{H_3}) + g^2_{H_1VV} F_\Delta(M_{H_2}, M_{H_3})$$

  - One-loop functions

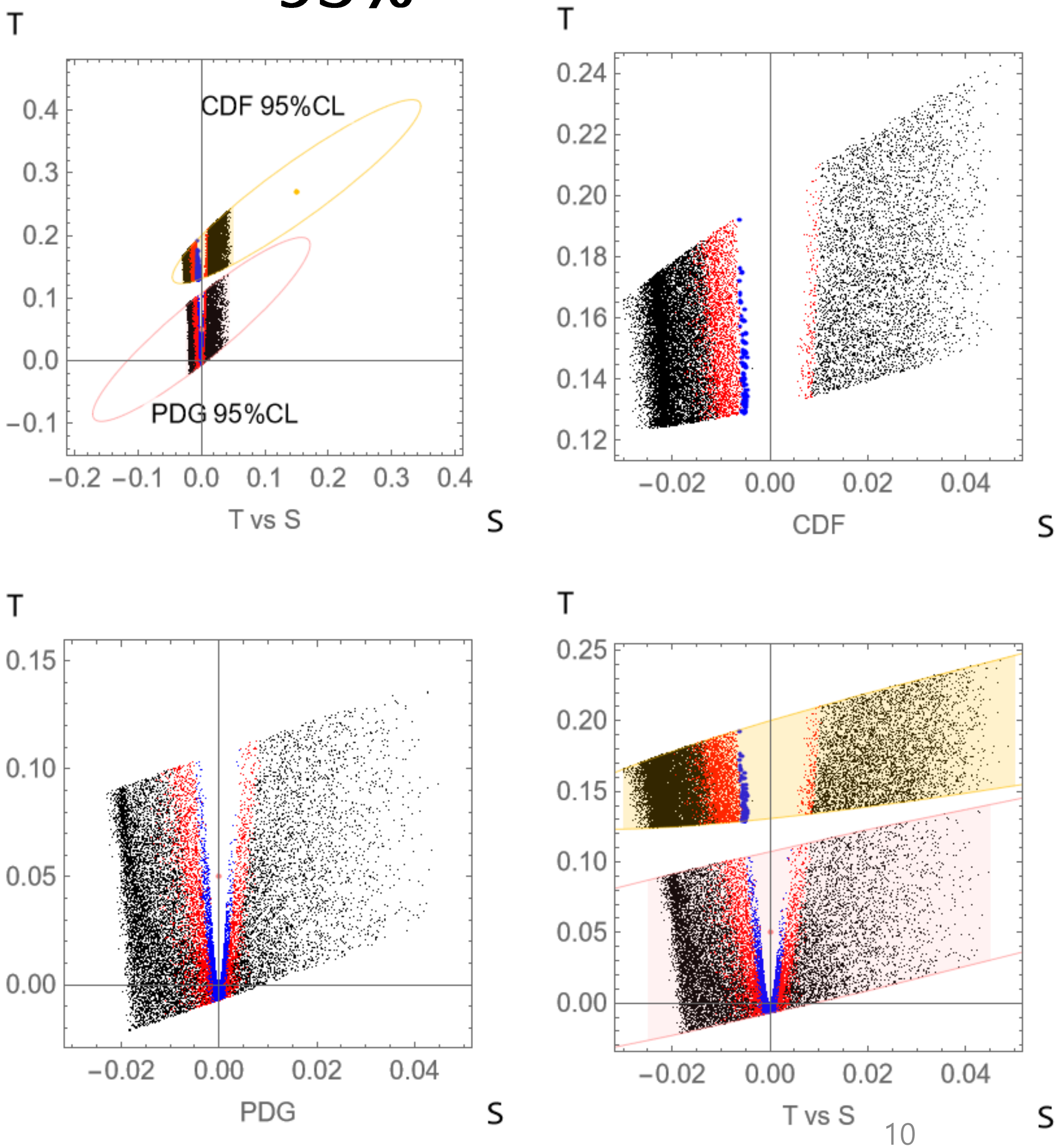
$$F_\Delta(m_0, m_1) = F_\Delta(m_1, m_0) = \frac{m_0^2 + m_1^2}{2} - \frac{m_0^2 m_1^2}{m_0^2 - m_1^2} \ln \frac{m_0^2}{m_1^2},$$

$$F'_\Delta(m_0, m_1) = F'_\Delta(m_1, m_0) = -\frac{1}{3}\left[\frac{4}{3} - \frac{m_0^2 \ln m_0^2 - m_1^2 \ln m_1^2}{m_0^2 - m_1^2} - \frac{m_0^2 + m_1^2}{(m_0^2 - m_1^2)^2} F_\Delta(m_0, m_1)\right]$$

$$F_\Delta(m, m) = 0, \quad F'_\Delta(m, m) = \frac{1}{3} \ln m^2$$

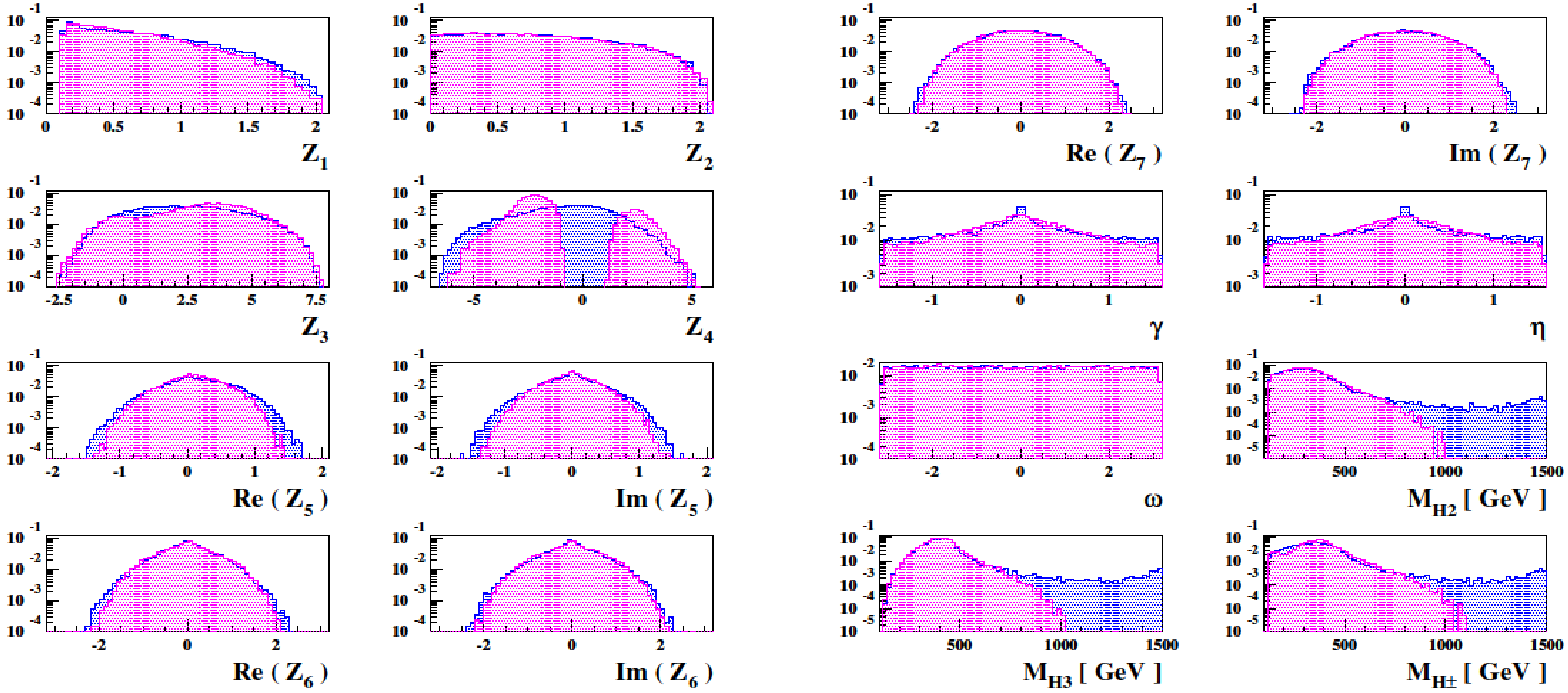
3. T versus S. UNIT⊕BFB⊕ELW$_{95\%}$

- The regions are obtained by imposing the combined UNIT⊕BFB⊕ELW$_{95\%}$
  - red dots $M_{H^\pm} > 500$ GeV,
    blue dots $M_{H^\pm} > 900$ GeV.
- Parameter range
  - S = −0.03~0.05, the narrow region around 0 with radius about 0.005 is not allowed,
  - T = 0.12~0.24, T is positive definite and sizable.



3. Parameter Scan

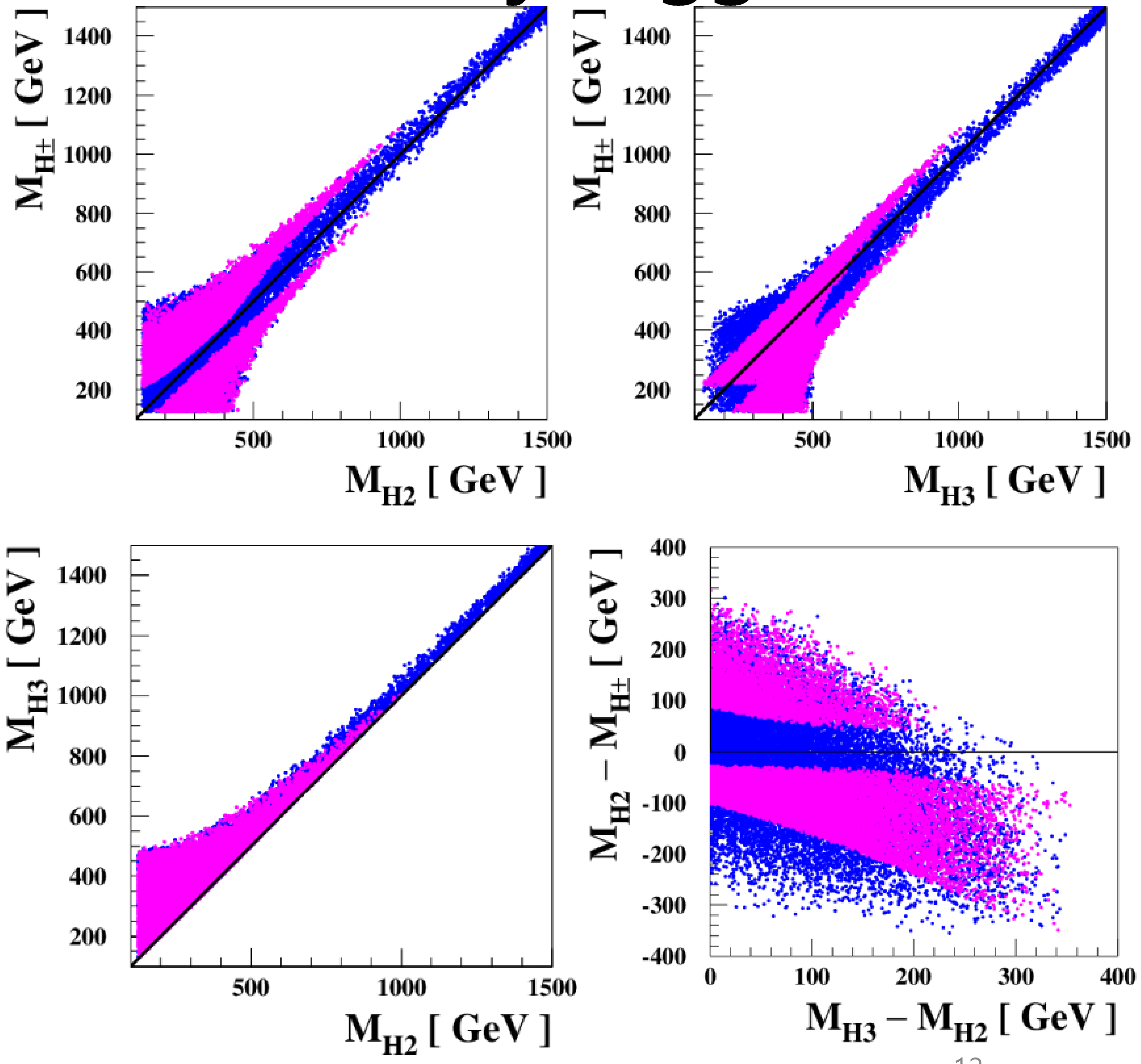
- The normalized distributions (magenta) of the seven quartic couplings, the three mixing angles, and the masses of heavy Higgs bosons using the CDF values, UNIT⊕BFB⊕ELW$_{95\%}$.



3. The correlations between the heavy Higgs masses

- In the upper-left and lower-right panels, it is clearly shown that there should be sizable mass difference between the charged Higgs boson and the second heaviest neutral one to accommodate the CDF W-mass anomaly.

- $M_{H^\pm} > M_{H_2}$
  - $M_{H_2} \simeq M_{H_3} \simeq 1000$ GeV
  - $M_{H^\pm} \simeq 1100$ GeV
- $M_{H^\pm} < M_{H_2}$
  - $M_{H_2} \simeq M_{H_3} \simeq 900$ GeV
  - $M_{H^\pm} \simeq 800$ GeV



4. Analysis

- Introduce the following two measures for the mass splitting of the heavy Higgs bosons

$$\Delta \equiv M_{H_2} - M_{H^\pm}, \qquad \delta \equiv M_{H_3} - M_{H_2}, \qquad M \equiv \frac{M_{H_2} + M_{H^\pm}}{2}$$

- the masses of heavy Higgs bosons are given by

$$M_{H^\pm} = M - \frac{\Delta}{2}, \qquad M_{H_2} = M + \frac{\Delta}{2}, \qquad M_{H_3} = M + \frac{\Delta}{2} + \delta$$

- In the heavy mass region where $M \gtrsim 500\ \mathrm{GeV},\ |\Delta| > \delta,\ M \gg M_{H_1} \sim |\Delta| \gtrsim \delta,$

  there are two types of mass spectra

  — 23C： $M_{H_2} < M_{H_3} < M_{H^\pm},$ when $\Delta < 0, \delta < |\Delta|$

  — C23： $M_{H^\pm} < M_{H_2} < M_{H_3},$ when $\Delta > 0$

4. Analysis

- Approximated expressions for the S and T

$(M \gg M_{H_1} \sim |\Delta| \gtrsim \delta,\ \ g_{H_1VV} \equiv 1-\epsilon,\quad g_{23}^2 \equiv \frac{g_{H_2VV}^2 + g_{H_3VV}^2}{2} = \frac{1-g_{H_1VV}^2}{2} = \epsilon - \frac{\epsilon^2}{2})$

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}} \simeq \frac{4}{3}(\Delta^2+\delta\Delta) + \left[-\left(2g_{23}^2\Delta + g_{H_2VV}^2\delta\right)M - \frac{4}{3}g_{23}^2\Delta^2 - \left(\frac{1}{2}g_{H_2VV}^2 + \frac{4}{3}g_{H_3VV}^2\right)\delta\Delta + \frac{1}{6}g_{H_2VV}^2\delta^2\right]$$

$$4\pi S \simeq \frac{1}{3}\left(\frac{2\Delta}{M} + \frac{\delta}{M}\right) + \left[-\frac{5}{9}g_{23}^2 + \frac{1}{3}\left(g_{H_2VV}^2 - g_{H_3VV}^2\right)\frac{\delta}{M}\right]$$

- The quantities in the square brackets are vanishing in the alignment limit where the lighter CP-even neutral Higgs boson behaves exactly as the SM Higgs boson. $\left(g_{H_2VV}^2 = g_{H_3VV}^2 = g_{23}^2 = 0\right)$.

4. Analysis

$$\frac{T}{\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\mathrm{EM}}}} = T' \simeq \frac{4}{3}(\Delta^2 + \delta\Delta) + \left[-\left(2g_{23}^2\Delta + g_{H_2VV}^2\delta\right)M - \frac{4}{3}g_{23}^2\Delta^2 - \left(\frac{1}{2}g_{H_2VV}^2 + \frac{4}{3}g_{H_3VV}^2\right)\delta\Delta + \frac{1}{6}g_{H_2VV}^2\delta^2\right]$$

- **Variation of the T parameter**
  - When $\delta = 0$, leaving only the terms with the average coupling. $T'|_{\delta=0} \simeq \frac{4}{3}\Delta^2 + \left[-(2g_{23}^2\Delta)M - \frac{4}{3}g_{23}^2\Delta^2\right]$
  - When $\delta = 0$ and $g_{23}^2 = 0$, T still have positive contributions proportional to Δ^2. $T'|_{\delta=g_{23}^2=0} \simeq \frac{4}{3}\Delta^2$
  - When $\Delta = 0$ and $g_{23}^2 = 0$, it is given by $g_{H_2VV}^2\,\delta\left(\frac{1}{6}\delta - M\right)$, which is negative unless $\delta > 6M$.
  - When $\Delta = \delta$, T is vanishing.
  - Non-vanishing Δ is necessary to produce the sizable and positive definite value of the T.

4. Analysis

- To strengthen our claim, we extend our discussion to the case where $\Delta \neq 0$.

  In this case, the expression for T is further simplified by dropping the terms suppressed

  by the factor Δ/M, δ/M : $\quad T' \simeq \frac{4}{3}(\Delta^2 + \delta\Delta) - \left(2g_{23}^2\Delta + g_{H_2VV}^2\delta\right)M,$

  It can be rewritten as : $\quad \left[\Delta - \left(\frac{3}{4}g_{23}^2 - \frac{\delta}{2}\right)\right]^2 \simeq T' + \frac{3}{4}g_{H_2VV}^2\delta M + \left(\frac{3}{4}g_{23}^2 - \frac{\delta}{2}\right)^2$

  - $T > 0,\ T' \simeq \frac{T}{0.18}(100\ \mathrm{GeV})^2$ using $\sqrt{2}\,G_F/16\pi^2\alpha_{EM} \simeq 1.337\times10^{-5}/\mathrm{GeV}^2$
  - Right−hand side of the above equation is positive definite unless $T < 0$,
    $\Delta = 0$ can not satisfy the above relation and the region of Δ with a radius smaller than $\sqrt{T'}$ around the point $\Delta = \frac{3g_{23}^2 M}{4} - \frac{\delta}{2}$ should not be allowed.
  - if $T \gtrsim 0.18$, the region of $|\Delta| \lesssim 100$ GeV should be excluded in the limit of $\delta = g_{23}^2 = 0$

4. Analysis

- the quartic coupling Z_4 is critical to understand the upper limit on the mass scale of heavy Higgs bosons.

$$\rightarrow Z_4 v^2 = 2(2\Delta + \delta)M + (\Delta + \delta)\delta - \left[2g_{23}^2 M + 2\left(g_{23}^2 \Delta + g_{H_3VV}^2 \delta\right)M + g_{23}^2\left(\frac{\Delta^2}{2} - 2M_{H_1}^2\right) + g_{H_3VV}^2 (\Delta + \delta)\delta\right]$$

- The expression of Z_4 leading terms is approximated as follows,

$$Z_4 v^2 \cong (4\Delta + 2\delta)M - 2g_{23}^2 M^2,$$

each term of the above equation involves its own physics origin.

  - The term containing Z_4 is constrained by the UNIT⊕BFB condition.
  - The first term in the right–hand side involves the mass splittings among the heavy Higgs bosons.
  - The second term in the right–hand side is proportional to M^2 and it quickly increases when g_{23}^2 deviates from the alignment limit of $g_{23}^2 = 0$.

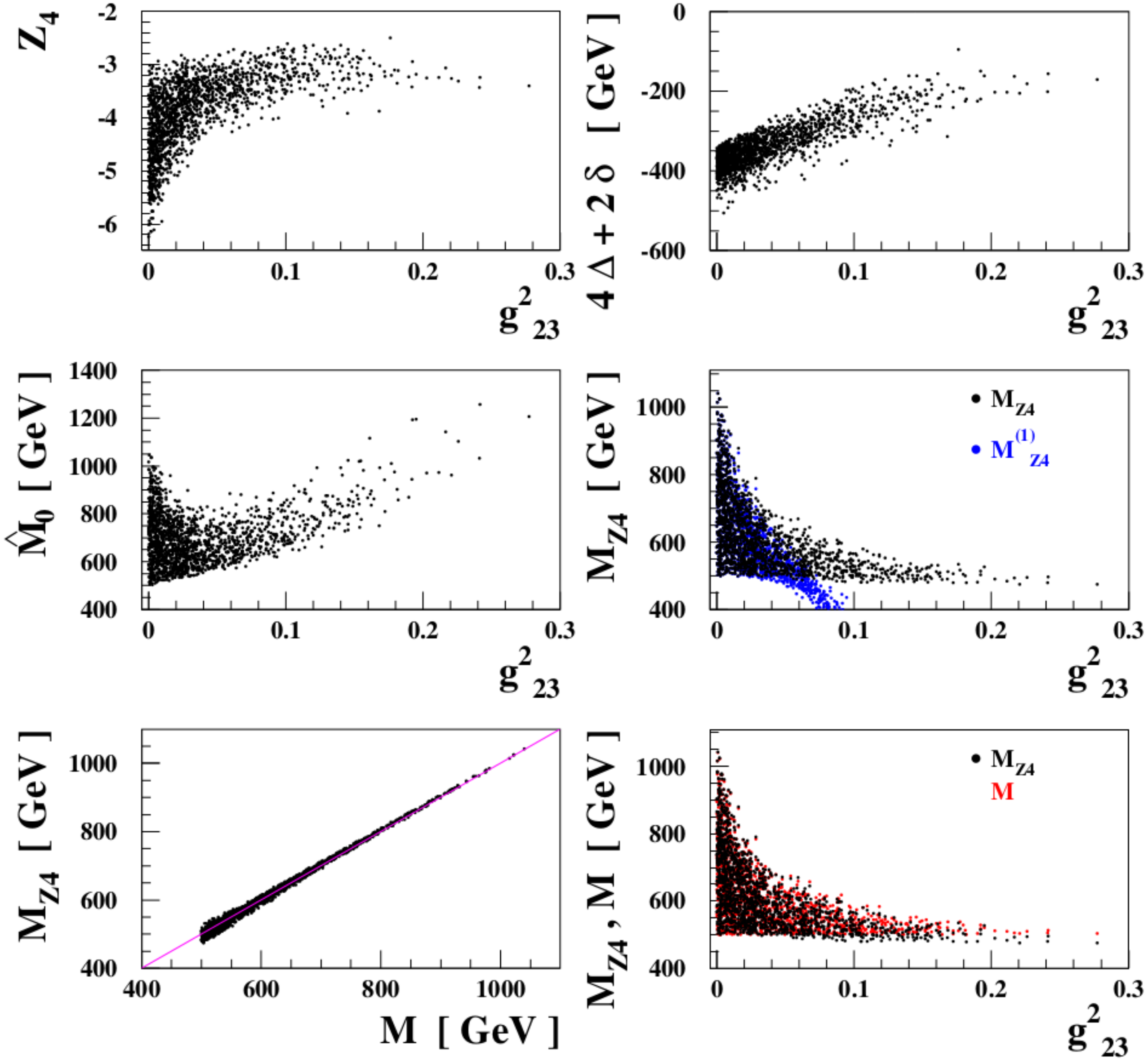
- Further, It can be rewritten as $M_{Z_4} = \widehat{M_0}\left[1 + \frac{g_{23}^2}{2}\frac{\widehat{M_0}}{\hat{\Delta}} + \frac{g_{23}^4}{2}\left(\frac{\widehat{M_0}}{\hat{\Delta}}\right)^2 + \mathcal{O}\left(g_{23}^2\frac{\widehat{M_0}}{\hat{\Delta}}\right)^2\right]$,

the leading term $M_{Z_4}^{(1)} \equiv \widehat{M_0}\left[1 + \frac{g_{23}^2}{2}\frac{\widehat{M_0}}{\hat{\Delta}}\right]$,

by introducing $g_{23}^2 M - 2\hat{\Delta}M + 2\hat{\Delta}\widehat{M_0} \cong 0,\ \hat{\Delta} = \Delta + \frac{\delta}{2},\ \widehat{M_0} = \frac{Z_4 v^2}{4\hat{\Delta}}$

4. Analysis

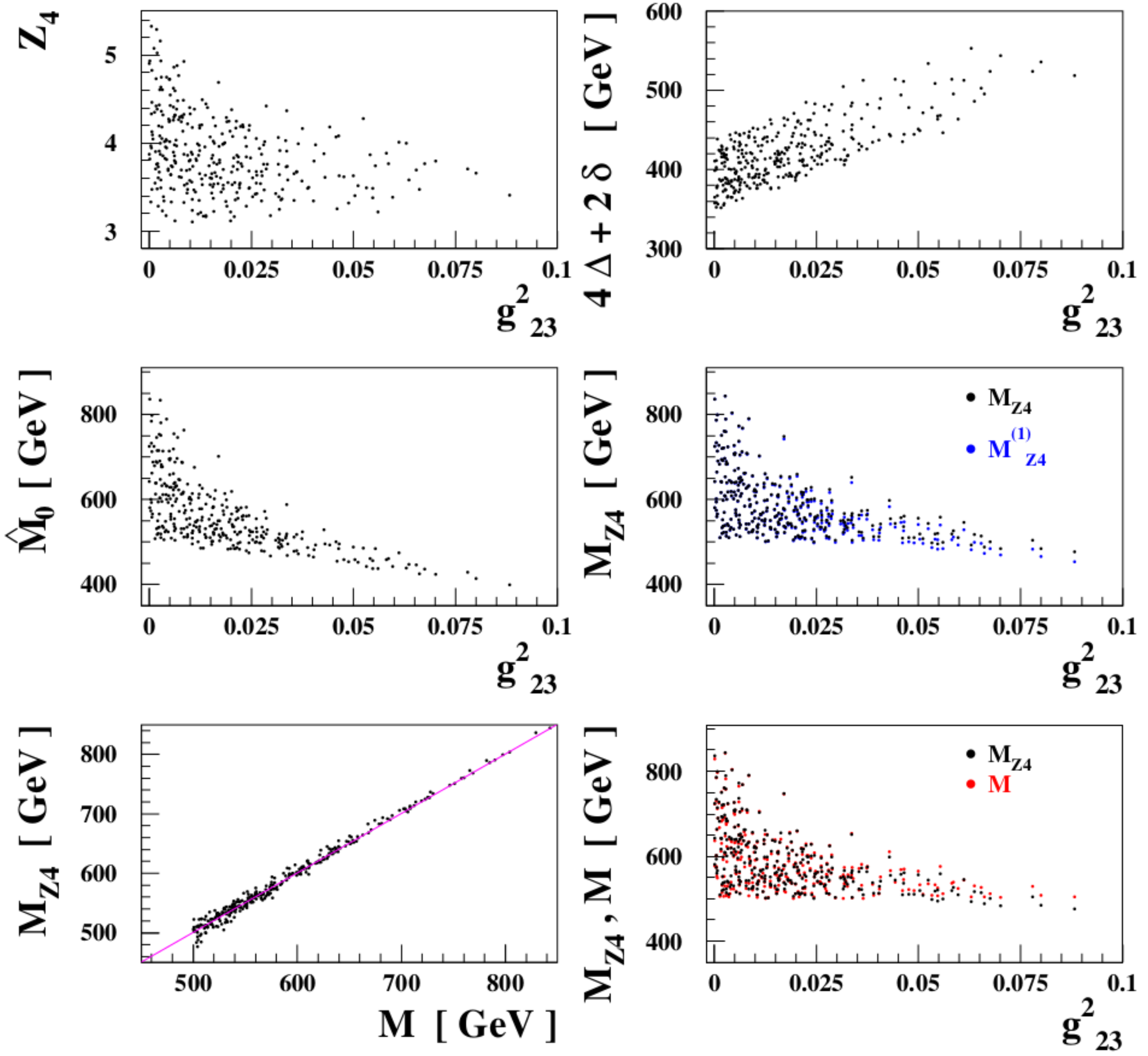
- 23C, $\Delta < 0,\ M > 500$ GeV
- For $|Z_4| = 6.3$ (max),
  $g_{23}^2 = 0$ (alignment limit),
  where $350 \lesssim |4\Delta + 2\delta| \lesssim 450$
  which leads to $\widehat{M_0} = \frac{Z_4 v^2}{4\hat{\Delta}} \lesssim 1050$ GeV
- the upper mass limit changes from about 1 TeV to about 700 GeV as g_{23}^2 deviates from 0 to 0.05



23C, $\Delta < 0$, $M > 500$ GeV

4. Analysis

- C23, $\Delta > 0,\ M > 500$ GeV
- $Z_4 \lesssim 5.3$, leading tov
  the smaller values of $\widehat{M_0} \lesssim 850$ GeV
- g_{23}^2 is constrained to be smaller
  than about 0.1
- We find that $|M_{Z_4} - M| \lesssim 30$ GeV and,
  M_{Z_4} excellently represents
  the heavy−Higgs mass scale M
  for the small values of g_{23}^2



23C, $\Delta < 0$, $M > 500$ GeV

4. Analysis

- To conclude, for a given value of g_{23}^2, the upper limit on the average mass scale $M \equiv (M_{H_2} + M_{H^\pm})/2$ is given by the maximum value of M_{Z_4} ,
- Within the parameter region that reproduces the large T value of the CDF W-mass anomaly, the mass spectrum of the Heavy Higgs and its upper mass limit are determined for each mass hierarchy as follows

$$\max\left(\widehat{M_0} = \frac{Z_4 v^2}{4\hat{\Delta}}\right) = \max\left(M_{Z_4}^{(1)} = \widehat{M_0}\left[1 + \frac{g_{23}^2}{2}\frac{\widehat{M_0}}{\hat{\Delta}}\right]\right) \cong \begin{cases} 1050\ \mathrm{GeV}\ \ \mathrm{for}\ \ 23\mathrm{C}\ (\Delta < 0) \\ 850\ \ \ \mathrm{GeV}\ \ \mathrm{for}\ \ \mathrm{C}23(\Delta > 0) \end{cases}$$

5. Conclusions

1. By imposing the combined UNIT$\bigoplus$BFB$\bigoplus$ELW$_{95\%}$ constraints and taking the CDF values for the S and T parameters, we find

$$0.1 \lesssim Z_1 \lesssim 2.0, \qquad 0 \lesssim Z_2 \lesssim 2.1, \qquad -2.6 \lesssim Z_3 \lesssim 8.0,$$

$$-6.3 \lesssim Z_4 \lesssim -0.8 \;\cup\; 1.1 \lesssim Z_4 \lesssim 5.4$$

$$|Z_5| \lesssim 1.7, \qquad |Z_6| \lesssim 2.4, \qquad |Z_7| \lesssim 2.7$$

2. We figure out that there could be the following two types of mass spectra

— **23C** : $M_{H_2} < M_{H_3} < M_{H^{\pm}}$, when $\Delta < 0, \delta < |\Delta|$

— **C23** : $M_{H^{\pm}} < M_{H_2} < M_{H_3}$, when $\Delta > 0$

5. Conclusions

3. The upper limit on the masses of heavy Higgs bosons becomes stronger when the effects of deviation from the alignment limit are taken into account. In the CDF case, we show that the upper limit on M changes from about 1 TeV to about 700 GeV as g_{23}^2 deviates from 0 to 0.05 for the 23C hierarchy.

4. We have fully figured out the physics origins relevant for the mass scale of heavy Higgs bosons.
   (i) the UNIT⊕BFB condition, (ii) the mass splittings among the heavy Higgs bosons, and (iii) the deviation from the alignment limit.

$$Z_4 v^2 \cong (4\Delta + 2\delta)M - 2g_{23}^2 M^2$$

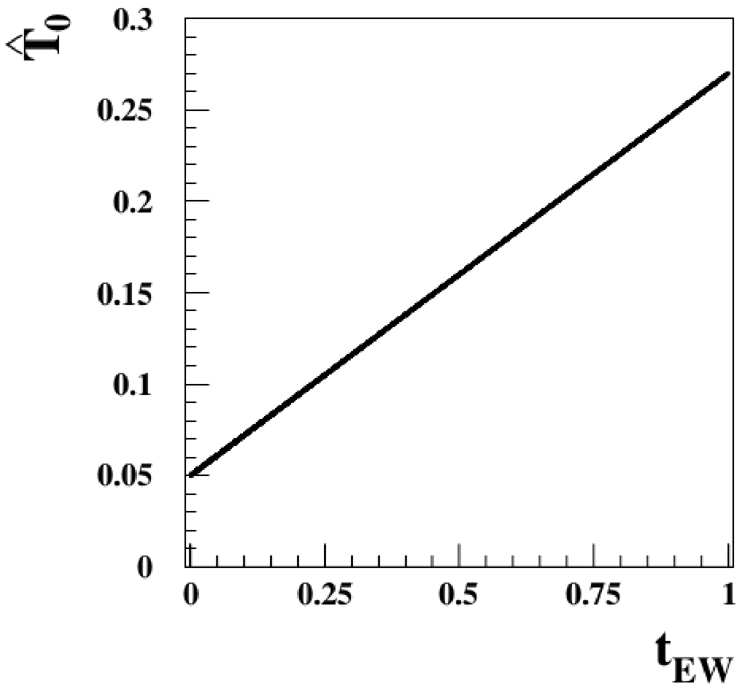
5. In the alignment limit, we confirmed the Heavy Higgs mass upper limit as

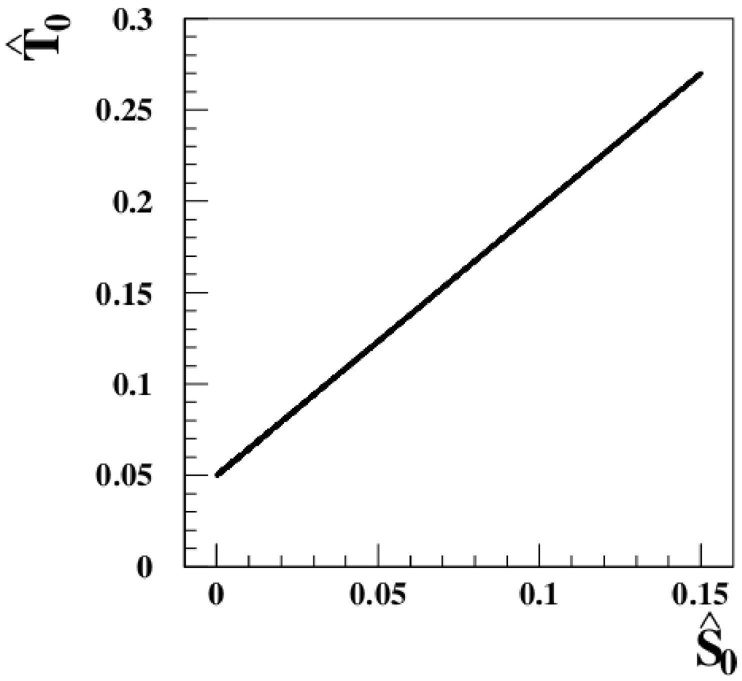
$$\begin{cases} 1050\ \text{GeV for } 23\text{C}\ (\Delta < 0) \\ 850\ \ \text{GeV for } \text{C}23(\Delta > 0) \end{cases}$$

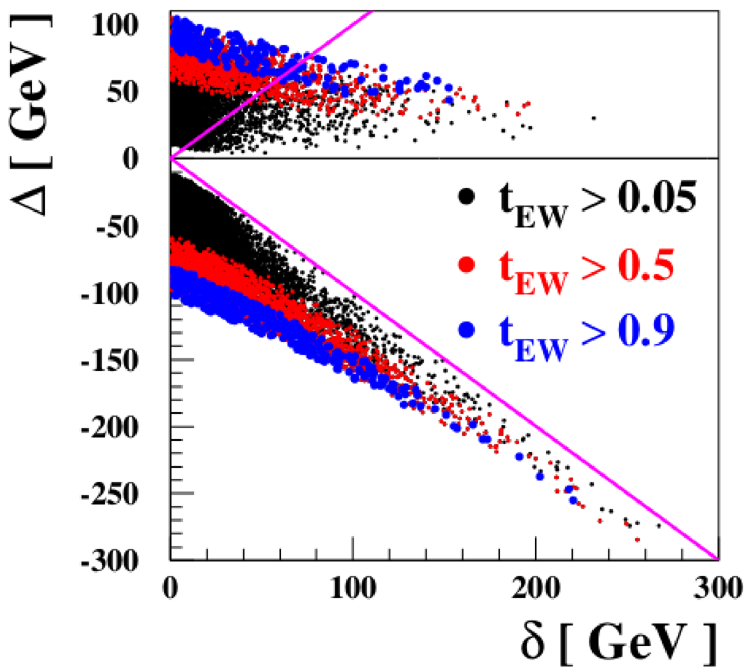
Analysis

- Taking PDG and CDF as the two extreme limits, we discuss how the upper limit on the heavy-Higgs mass scale M changes by varying the values for the S and T parameters between PDG and CDF.
  - CDF $\left(\widehat{S_0}, \sigma_S\right) = (0.15, 0.08),\ \left(\widehat{T_0}, \sigma_T\right) = (0.27, 0.06),\ \rho_{ST} = 0.93$
  - PDG $\left(\widehat{S_0}, \sigma_S\right) = (0.00, 0.07),\ \left(\widehat{T_0}, \sigma_T\right) = (0.05, 0.06),\ \rho_{ST} = 0.92$

$$S_0 = 0.15\, t_{EW},\ T_0 = 0.22\, t_{EW} + 0.05,\ \ (0 \leq t_{EW} \leq 1)$$







Analysis

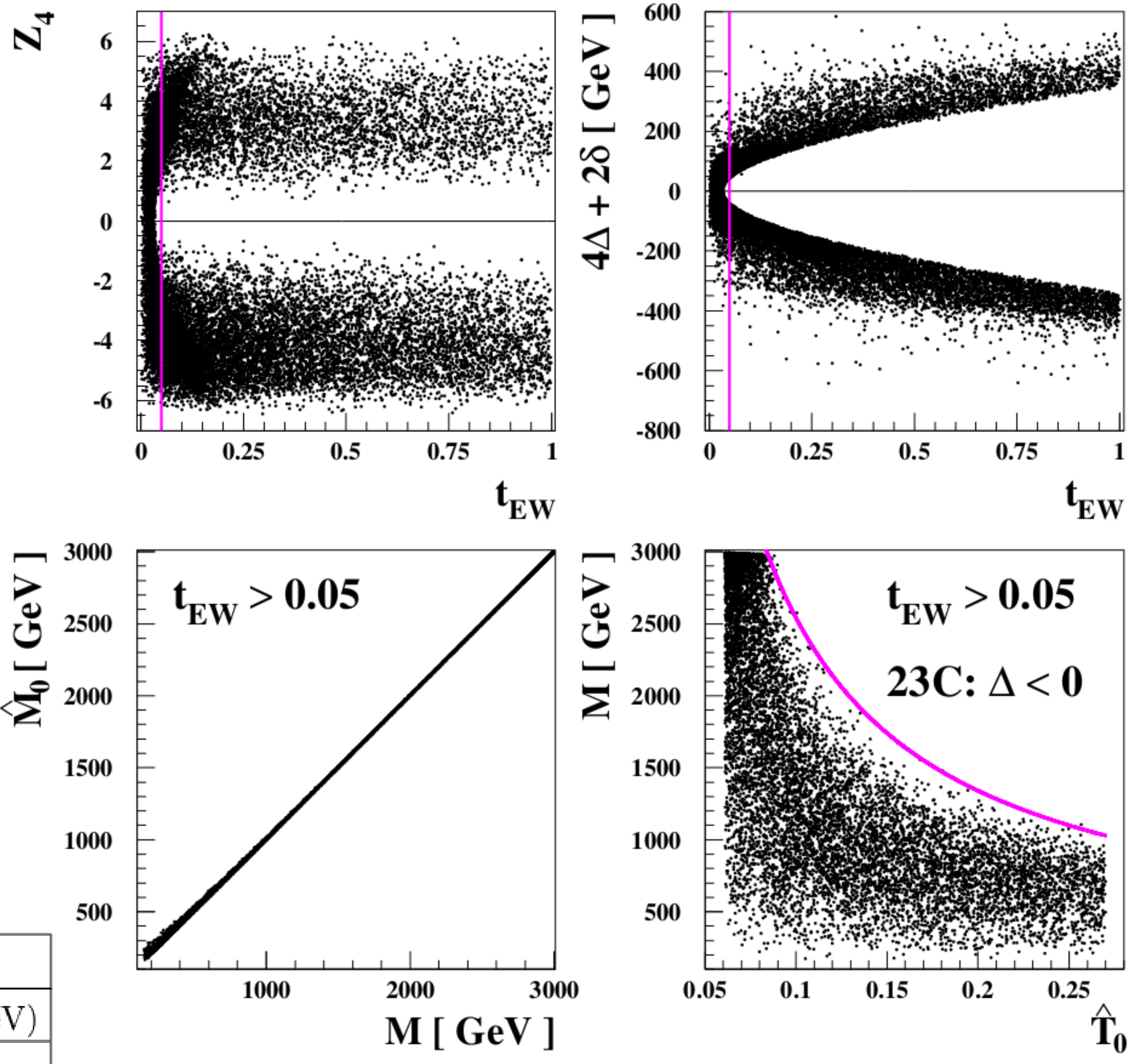
- Incidentally, we find that

$$M \lesssim \left(\frac{240}{T_0} + 140\right)\,\text{GeV}$$

as denoted by the magenta curve in the lower−right frame.
This empirical relation could be used to derive the conservative T_0−dependent upper limit on M more conveniently.

- Benchmarking scenarios in which the masses of the heavy Higgs bosons saturate their upper limits for a given value of T_0

$\widehat{T}_0$	23C		C23	
	$M_{H_2} \simeq M_{H_3}$ (GeV)	$M_{H^\pm}$ (GeV)	$M_{H_2} \simeq M_{H_3}$ (GeV)	$M_{H^\pm}$ (GeV)
0.27	1000	1100	900	800
0.21	1100	1180	1090	1010
0.15	1640	1700	1540	1480
0.09	2730	2760	2630	2600



Appendix - Higgs basis 2HDM potential

- The potential parameters $Y_{1,2,3}$ and Z_{1-7} in the Higgs basis
  - $Y_1 = \mu_1^2 c_\beta^2 + \mu_2^2 s_\beta^2 \; + \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) s_{2\beta}$,
  - $Y_2 = \mu_1^2 s_\beta^2 + \mu_2^2 c_\beta^2 - \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) s^{2\beta}$,
  - $Y_3 = -(\mu_1^2 - \mu_2^2) c_\beta s_\beta + \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) c_{2\beta} + i\Im\mathrm{m}\left(m_{12}^2 e^{i\xi}\right)$,

$\lambda_{345} = \frac{(\lambda_3+\lambda_4)}{2} + \Re\mathrm{e}\left(\lambda_5 e^{2i\xi}\right)$

  - $Z_1 = \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2\lambda_{345} c_\beta^2 s_\beta^2 + \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) c_\beta^2 + \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right) s_\beta^2\right] s_{2\beta}$,
  - $Z_3 = \lambda_3 + 2(\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 - \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) - \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\right] c_{2\beta} s_{2\beta}$,
  - $Z_6 = \left(-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2\right) s_{2\beta} + \; 2\lambda_{345} c_{2\beta} c_\beta s_\beta + \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right)\left(c_\beta^2 - 3s_\beta^2\right) c_\beta^2 + \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\left(3c_\beta^2 - s_\beta^2\right) s_\beta^2$
    $+i\,|\Im\mathrm{m}\left(\lambda_5 e^{2i\xi}\right) s_{2\beta} + \Im\mathrm{m}\left(\lambda_6 e^{i\xi}\right) c_\beta^2 + \Im\mathrm{m}\left(\lambda_7 e^{i\xi}\right) s_\beta^2]$,
    - $Z_1 \leftrightarrow Z_2,\;\; Z_3 \leftrightarrow Z_4,\;\; Z_6 \leftrightarrow Z_7;\; c_\beta \leftrightarrow s_\beta, \lambda_3 \leftrightarrow \lambda_4, \left(\lambda_5 e^{2i\xi}\right) \leftrightarrow \left(\lambda_5 e^{2i\xi}\right)^*, \left(\lambda_{6,7} e^{i\xi}\right) \leftrightarrow -\left(\lambda_{6,7} e^{i\xi}\right)^*$
  - $Z_5 = (\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 + \Re\mathrm{e}\left(\lambda_5 e^{2i\xi}\right) - \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) \; - \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\right] c_{2\beta} c_\beta s_\beta$
    $+i\,\left[\Im\mathrm{m}\left(\lambda_5 e^{2i\xi}\right) c_{2\beta} - \Im\mathrm{m}\left(\lambda_6 e^{i\xi}\right) c_\beta s_\beta + \Im\mathrm{m}\left(\lambda_7 e^{i\xi}\right) c_{\beta_{s_\beta}}\right]$

Appendix – Φ basis 2HDM potential

- Potential $V_\Phi = \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) + m_{12}^{*2}(\Phi_2^\dagger\Phi_1)$

$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*(\Phi_2^\dagger\Phi_1)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

- Tadpole conditions: the square of the charged Higgs-boson mass
  - $\mu_1^2 = -v^2\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}(\lambda_6 e^{i\xi})\right] + s_\beta^2 M_{H^\pm}^2$,
  - $\mu_1^2 = -v^2\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}(\lambda_6 e^{i\xi})\right] + s_\beta^2 M_{H^\pm}^2$,
  - $\Im\mathrm{m}(m_{12}^2 e^{i\xi}) = -\frac{v^2}{2}\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}(\lambda_6 e^{i\xi})\right] + s_\beta^2 M_{H^\pm}^2$.
- The square of the charged Higgs-boson mass
  - $M_{H^\pm}^2 = -\frac{\Re\mathrm{e}(m_{12}^2 e^{i\xi})}{c_\beta s_\beta} = -\frac{v^2}{2 c_\beta s_\beta}\left[\lambda_4 c_\beta s_\beta + 2 c_\beta s_\beta \Re\mathrm{e}(\lambda_5 e^{2i\xi}) + c_\beta^2\, \Re\mathrm{e}(\lambda_6 e^{i\xi}) + s_\beta^2 \Re\mathrm{e}(\lambda_7 e^{i\xi})\right]$

Appx. - Interactions with massive vector bosons

- The cubic interactions of the neutral and charged Higgs bosons with the massive gauge bosons Z and $W^{\pm}$ are described by the three interaction lagrangians:

$$\mathcal{L}_{HVV} = gM_W \left(W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} Z_\mu Z^\mu \right) \sum_i g_{H_i VV} H_i \,,$$

$$\mathcal{L}_{HHZ} = \frac{g}{2c_W} \sum_{i>j} g_{H_i H_j Z} Z^\mu \left(H_i \overleftrightarrow{\partial}_\mu H_j \right),$$

$$\mathcal{L}_{HH^\pm W^\mp} = -\frac{g}{2} \sum_i g_{H_i H^+ W^-} W^{-\mu} \left(H_i i \overleftrightarrow{\partial} H^+ \right) + \mathrm{h.c.}$$

  - The normalized couplings
    - $g_{H_i VV} = O_{\varphi_1 i}, \quad g_{H_i H_j Z} = \mathrm{sign}[\det(O)] \epsilon_{ijk} g_{H_k VV} = \mathrm{sign}[\det(O)] \epsilon_{ijk} O_{\varphi_1 k}, \quad g_{H_i H^+ W^-} = -O_{\varphi_2 i} + i O_{ai}$
    - Sum rules

$$\sum_{i=1}^{3} g_{H_i VV}^2 = 1 \quad \text{and} \quad g_{H_i VV}^2 + \left| g_{H_i H^+ W^-} \right|^2 = 1 \quad \text{for each} \quad i = 1,2,3$$

Appx. - Input parameters in CPV

- Using the tadpole conditions and Higgs masses

$$\{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\} \to \mathcal{I}' = \left\{v; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \{O_{3\times 3}\}; Z_3; Z_2, Z_7\right\}$$

  - $O = O_\gamma O_\eta O_\omega \equiv \begin{pmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_\eta & 0 & s_\eta \\ 0 & 1 & 0 \\ -s_\eta & 0 & c_\eta \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\omega & s_\omega \\ 0 & -s_\omega & c_\omega \end{pmatrix} = \begin{pmatrix} c_\gamma c_\eta & s_\gamma c_\omega - c_\gamma s_\eta s_\omega & s_\gamma s_\omega + c_\gamma s_\eta c_\omega \\ -s_\gamma c_\eta & c_\gamma c_\omega + s_\gamma s_\eta s_\omega & c_\gamma s_\omega - s_\gamma s_\eta c_\omega \\ -s_\eta & -c_\eta s_\omega & c_\eta c_\omega \end{pmatrix}$

- $Z_1 = \frac{1}{2v^2}\left(M_{H_1}^2 O_{\varphi_1 1}^2 + M_{H_2}^2 O_{\varphi_1 2}^2 + M_{H_3}^2 O_{\varphi_1 3}^2\right)$
- $Z_4 = \frac{1}{v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 + O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 + O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 + O_{a3}^2\right) - 2M_{H^\pm}^2\right]$
- $Z_5 = \frac{1}{2v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 - O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 - O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 - O_{a3}^2\right)\right]$

  $\qquad - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_2 1} O_{a1} + M_{H_2}^2 O_{\varphi_2 2} O_{a2} + M_{H_3}^2 O_{\varphi_2 3} O_{a3}\right)$
- $Z_6 = \frac{1}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1} O_{\varphi_2 1} + M_{H_2}^2 O_{\varphi_1 2} O_{\varphi_2 1} + M_{H_3}^2 O_{\varphi_1 3} O_{\varphi_2 3}\right)$

  $\qquad - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1} O_{a1} + M_{H_1}^2 O_{\varphi_1 2} O_{a2} + M_{H_3}^2 O_{\varphi_1 3} O_{a3}\right)$

Yukawa couplings in the Higgs basis

- The Yukawa couplings

$$-\mathcal{L}_Y = \sum_{k=1,2} \overline{Q_L^0} \boldsymbol{y}_k^u \widetilde{\mathcal{H}_k} u_R^0 + \overline{Q_L^0} \boldsymbol{y}_k^d \mathcal{H}_k d_R^0 + \overline{L_L^0} \boldsymbol{y}_k^e \mathcal{H} e_R^0 + \mathrm{h.c.}$$

$$\widetilde{\mathcal{H}_i} = i\tau_2 \mathcal{H}_i^*$$

- The mass terms in the Yukawa interactions

$$-\mathcal{L}_{Y_{\mathrm{mass}}} = \frac{v}{\sqrt{2}} \left(\overline{u_L^0} \boldsymbol{y}_1^u u_R^0 + \overline{d_L^0} \boldsymbol{y}_1^d d_R^0 + \overline{e_L^0} \boldsymbol{y}_1^e e_R^0 + \mathrm{h.c.} \right)$$

$$u_L^0 = \mathcal{U}_{u_L} u_L, \quad d_L^0 = \mathcal{U}_{d_L} d_L, \quad e_L^0 = \mathcal{U}_{e_L} e_L, \quad u_R^0 = \mathcal{U}_{u_R} u_R, \quad d_R^0 = \mathcal{U}_{d_R} d_R, \quad e_R^0 = \mathcal{U}_{e_R} e_R$$

$$\mathbf{M}_u = \frac{v}{\sqrt{2}} \mathcal{U}_{u_L}^\dagger \boldsymbol{y}_1^u \mathcal{U}_{u_R} = \mathrm{diag}\,(m_u, m_c, m_t),$$

$$\mathbf{M}_d = \frac{v}{\sqrt{2}} \mathcal{U}_{d_L}^\dagger \boldsymbol{y}_1^d \mathcal{U}_d = \mathrm{diag}\,(m_d, m_s, m_b),$$

$$\mathbf{M}_e = \frac{v}{\sqrt{2}} \mathcal{U}_{e_L}^\dagger \boldsymbol{y}_1^e \mathcal{U}_{e_R} = \mathrm{diag}\left(m_e, m_\mu, m_\tau\right)$$

- Finally $-\mathcal{L}_{Y_{\mathrm{mass}}} = \overline{u_L}\, \mathbf{M}_u u_R + \overline{d_L}\, \mathbf{M}_d d_R + \overline{e_L} \mathbf{M}_e e_R + \mathrm{h.c.}$

Couplings of Higgs bosons

- The couplings of the neutral Higgs bosons to two fermions

$$-\mathcal{L}_{H\bar{f}f} = \frac{1}{v}[\bar{u}\mathbf{M}_u u]\varphi_1 + [\bar{u}(\mathbf{h}_u^H + \mathbf{h}_u^A\gamma_5)u]\varphi_2 + [\bar{u}(-i\mathbf{h}_u^A - i\mathbf{h}_u^H\gamma_5)u]a$$

$$+\frac{1}{v}\left[\bar{d}\mathbf{M}_d d\right]\varphi_1 + \left[\bar{d}\left(\mathbf{h}_d^H + \mathbf{h}_d^A\gamma_5\right)d\right]\varphi_2 + \left[\bar{d}\left(i\mathbf{h}_d^A + i\mathbf{h}_d^H\gamma_5\right)d\right]a$$

$$+\frac{1}{v}[\bar{e}\mathbf{M}_e e]\varphi_1 + [\bar{e}(\mathbf{h}_e^H + \mathbf{h}_e^A\gamma_5)e]\varphi_2 + [\bar{e}(i\mathbf{h}_e^A + i\mathbf{h}_e^H\gamma_5)e]a$$

  - Hermitian and three anti-Hermitian Yukawa coupling matrices

$$\mathbf{h}_f^H \equiv \frac{\mathbf{h}_f + \mathbf{h}_f^\dagger}{2}, \qquad \mathbf{h}_f^A \equiv \frac{\mathbf{h}_f - \mathbf{h}_f^\dagger}{2}, \qquad \mathbf{h}_f \equiv \frac{1}{\sqrt{2}}\mathcal{U}_{f_L}^\dagger \boldsymbol{y}_2^f\, \mathcal{U}_{f_R}, \quad (f = u, d, e)$$

- The couplings of the charged Higgs bosons to two fermions

$$-\mathcal{L}_{H^\pm\overline{f_\uparrow}f_\downarrow} = -\sqrt{2}\left[\overline{u_R}\left(\boldsymbol{h}_u^\dagger V\right)d_L\right]H^+ + \sqrt{2}[\overline{u_L}(V\boldsymbol{h}_d)d_R]H^+ + \sqrt{2}\,[\overline{\nu_L}\boldsymbol{h}_e e_R]H^+ + \mathrm{h.c.}.$$

  - $\mathcal{U}_{u_L}^\dagger\mathcal{U}_{d_L} = V_{\mathrm{CKM}} \equiv V$ and $\boldsymbol{h}_u = t_\beta^{-1}\frac{\boldsymbol{M}_u}{v},\ \boldsymbol{h}_d = -t_\beta\frac{\boldsymbol{M}_d}{v},\ \boldsymbol{h}_e = -t_\beta\frac{\boldsymbol{M}_e}{v}$