

Multi-dimensional optimization methods to analyze the null results of WIMP direct detection experiments with conservative bounds

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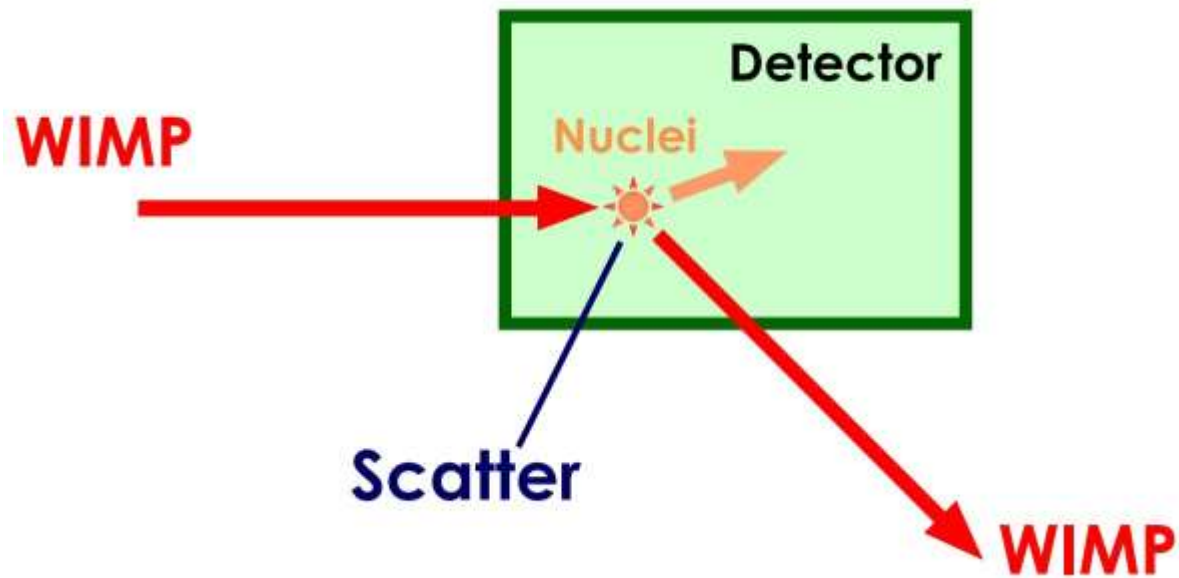
Introduction

- Weakly Interacting Massive Particles (WIMPs)
- 27% of the total mass density of the universe
- 90% of the halo of our galaxy
- GeV-TeV range

Introduction

- Scattering (Direct Detection)
- Measurement of the nuclear recoils

A WIMP scattering off normal nuclei in a dark matter direct detection experiment

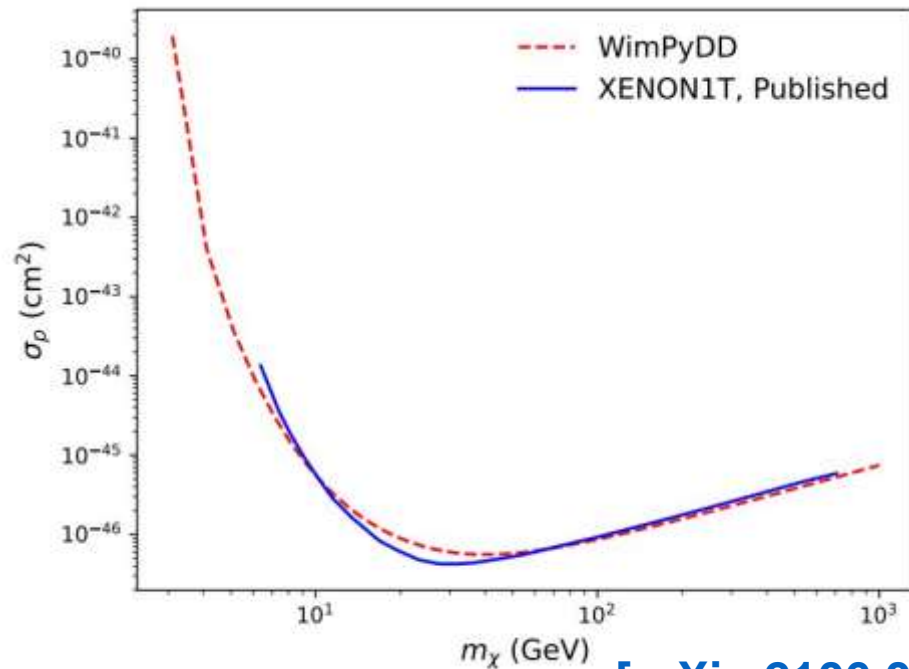


[\[arXiv:1111.0710\]](https://arxiv.org/abs/1111.0710)

Introduction

- Absence of detection, null results
- Exclusion plot (Upper bound of cross section)

XENON1T exclusion plot as a function of the WIMP mass m_χ



[\[arXiv:2106.06207\]](https://arxiv.org/abs/2106.06207)

Introduction

- $1/2$ spin WIMP
- Galilean-invariant
- WIMP-nucleon effective Hamiltonian

\vec{q} : transferred momentum

\vec{S}_N : nucleon spin

\vec{S}_χ : wimp spin

\vec{v}^\perp : transverse velocity

$\mathcal{O}_1 = 1_\chi 1_N$	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$	$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$
$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

$$\mathcal{H} = \sum_{\tau=0,1} \sum_{j=1}^{15} c_i^\tau \mathcal{O}_i t^\tau \quad \mathcal{O}_2 = (v^\perp)^2 : \text{vanished}$$

$$t^0 = \mathbb{1} , t^1 = \tau^3$$

$$c_j^0 = c_j^p + c_j^n$$

$$c_j^1 = c_j^p - c_j^n$$

Introduction

- 28 independent Wilson coefficients
- Expected rate R

$$R_{[E'_1, E'_2]} = MT \int_{E'_1}^{E'_2} \frac{dR}{dE'} dE',$$

$$\frac{dR}{dE'} = \sum_T \left(\frac{dR}{dE'} \right)_T = \sum_T \int_0^\infty \frac{dR_{\chi T}}{dE_{ee}} \mathcal{G}_T(E', E_{ee}) \epsilon(E') dE_{ee}$$

$$E_{ee} = q(E_R) E_R,$$

$$\frac{dR_{\chi T}}{dE_R}(t) = \sum_T N_T \frac{\rho_\chi}{m_\chi} \int_{v_{min}} d^3 v_T f(\vec{v}_T, t) v_T \frac{d\sigma_T}{dE_R}$$

Introduction

- Semi-analytic methods
- 28 dimensional vector \mathbf{c}
- 28×28 dimensional matrix \mathcal{R}
- Multi-dimensional ellipsoid \mathbf{R}

$$R = \mathbf{c}^t \cdot \mathcal{R} \cdot \mathbf{c}$$

$$R < N^{max}$$

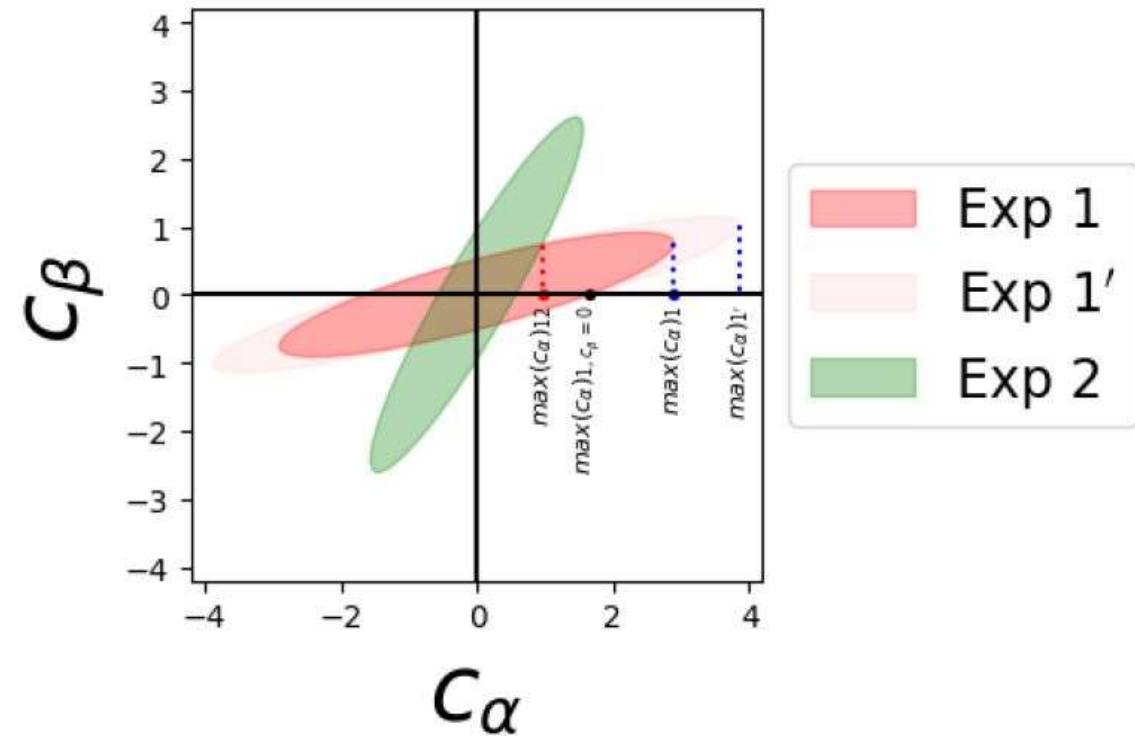
$$\mathcal{H} = \sum_{\tau=0,1} \sum_{j=1}^{15} \left(c_i^\tau + \frac{\alpha_i^\tau}{q^2} \right) \mathcal{O}_i t^\tau = \sum_{\tau=0,1} \sum_{j=1}^{15} c_i^\tau \mathcal{O}_i t^\tau + \sum_{\tau=0,1} \sum_{j=1}^{15} \alpha_i^\tau \frac{\mathcal{O}_i}{q^2} t^\tau \quad \rightarrow \quad 56 \text{ Wilson coefficients}$$

Conservative Bounds

- Ellipsoid in two-dimensional parameter space
- Combining experiments

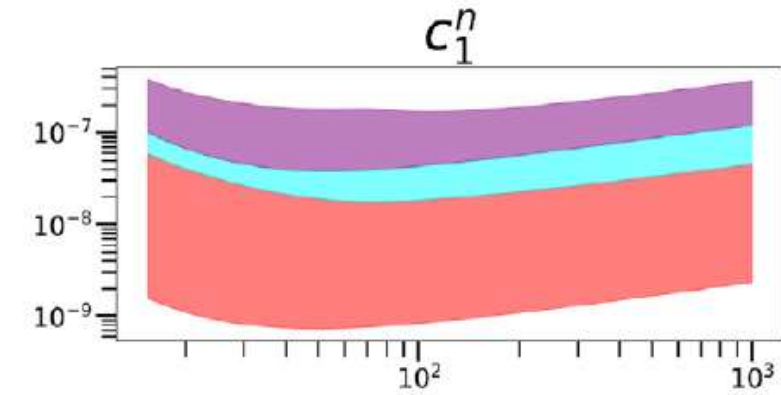
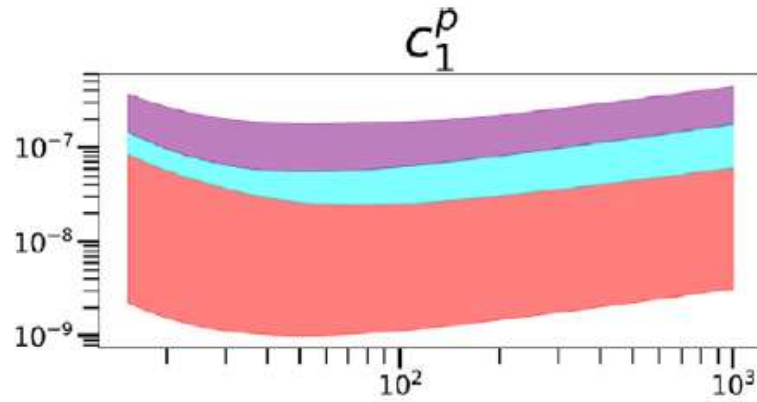
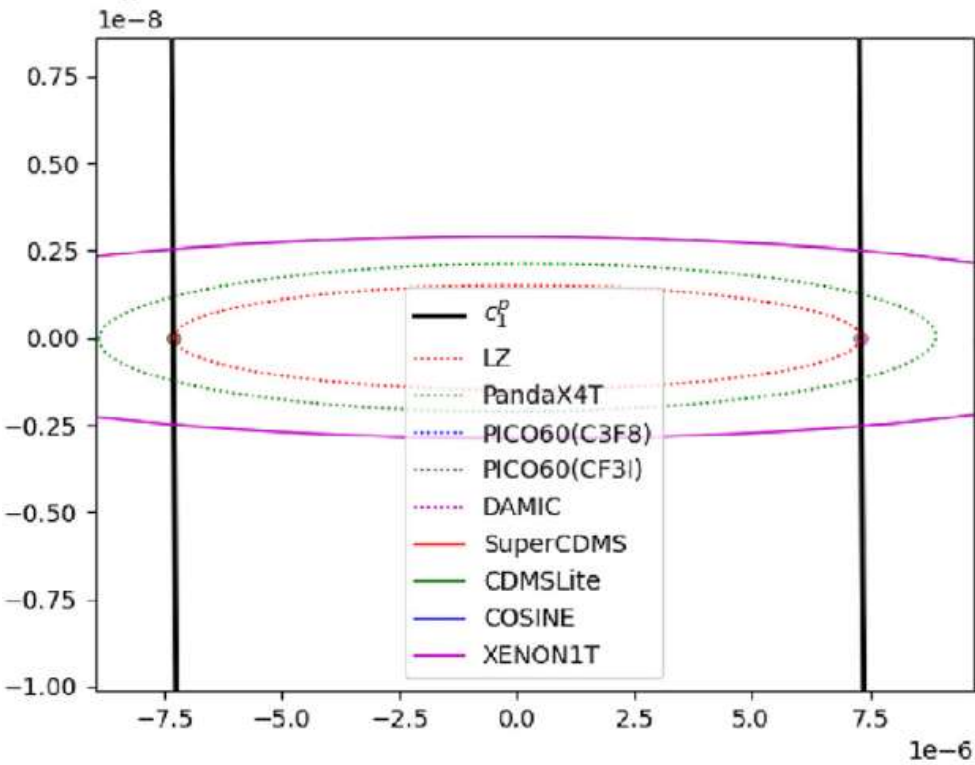
$$R_{Exp1} = \mathbf{c}^t \cdot \mathcal{R}_{Exp1} \cdot \mathbf{c} = \sum_{i,j=\alpha,\beta} c_i (\mathcal{R}_{Exp1})_{ij} c_j < 1$$

$$R_{Exp2} = \mathbf{c}^t \cdot \mathcal{R}_{Exp2} \cdot \mathbf{c} = \sum_{i,j=\alpha,\beta} c_i (\mathcal{R}_{Exp2})_{ij} c_j < 1$$



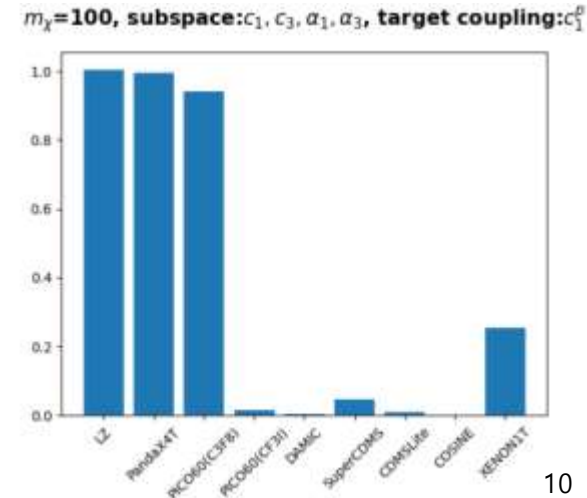
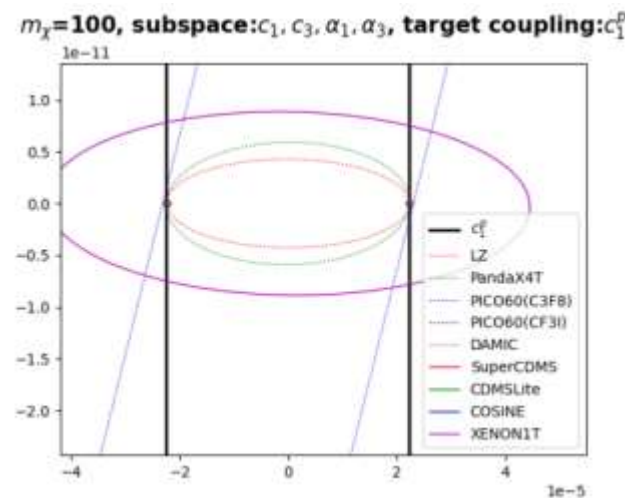
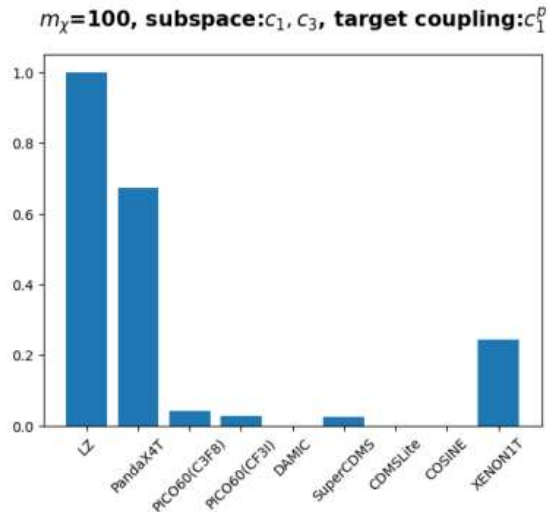
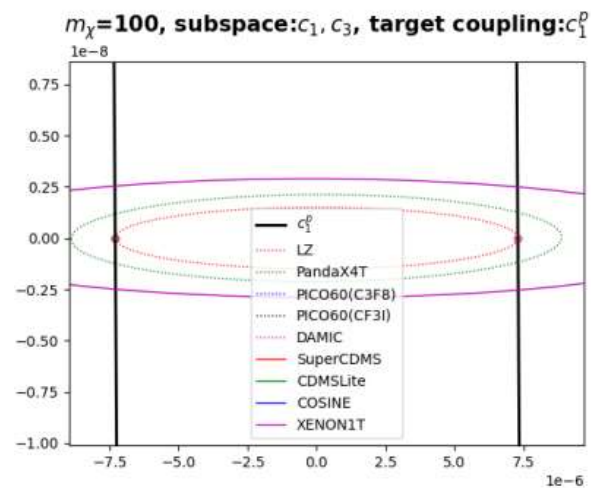
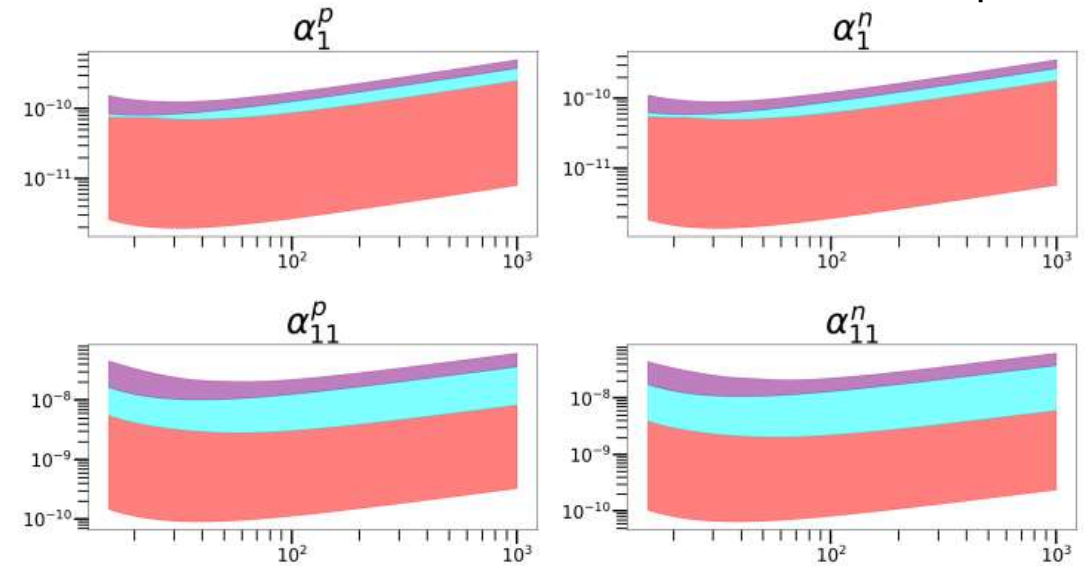
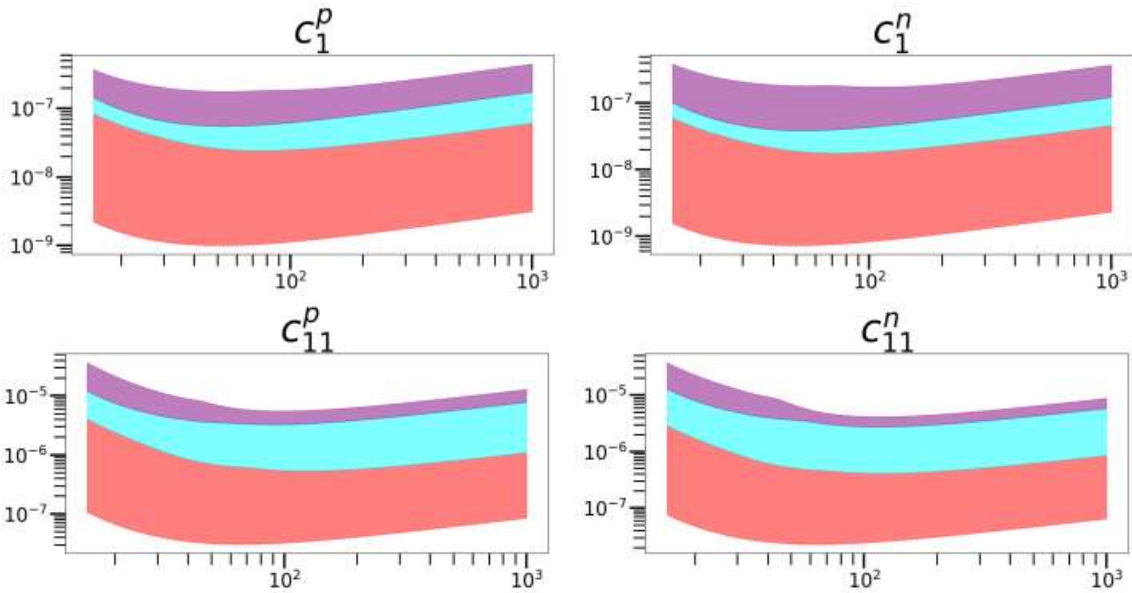
Exclusion bands

$m_\chi = 100$, subspace: c_1, c_3 , target coupling: c_1^p

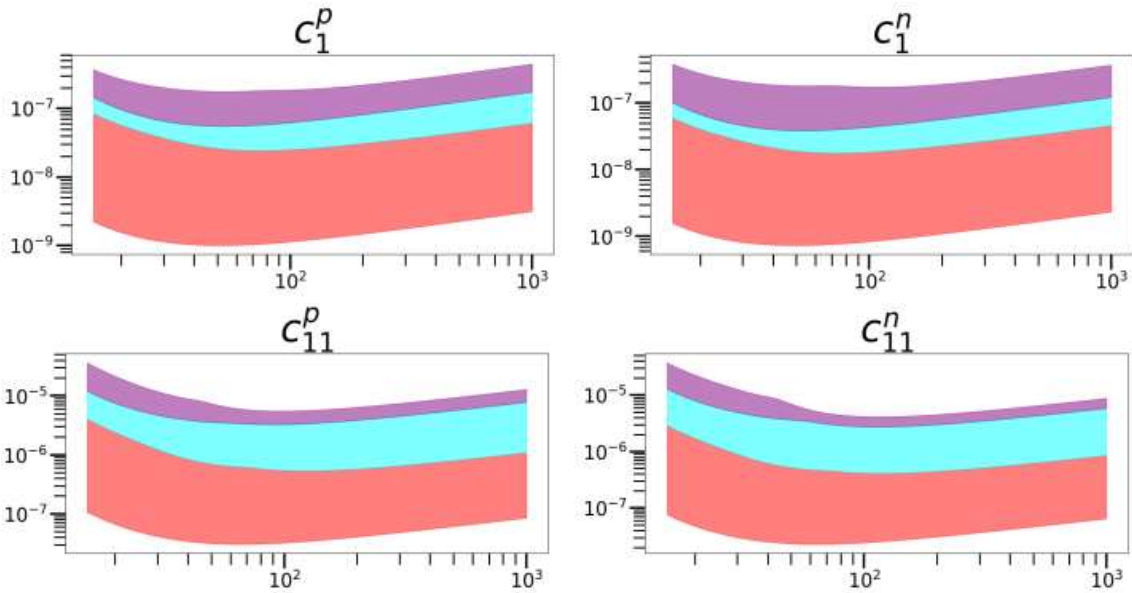


Spin independent

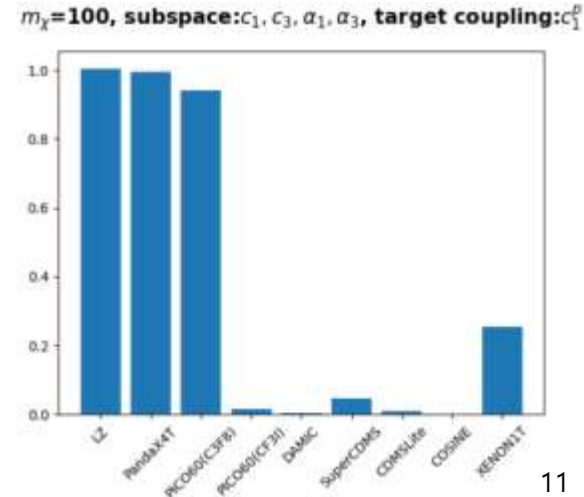
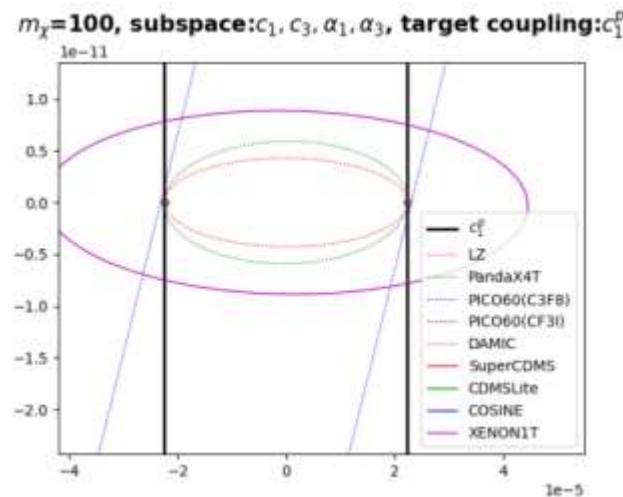
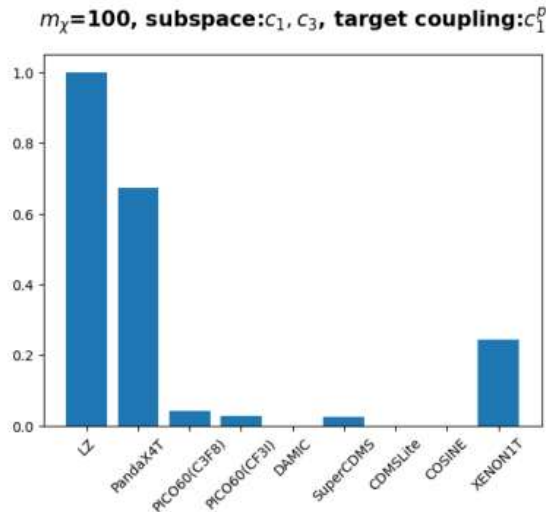
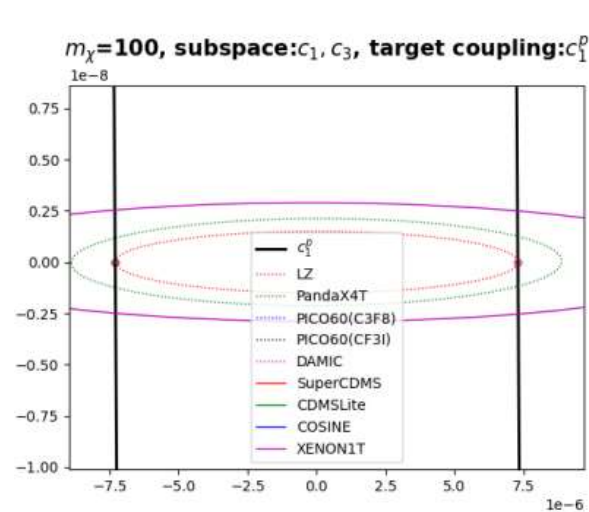
Variation $\sim 10^2$, up to 10^3



Spin independent

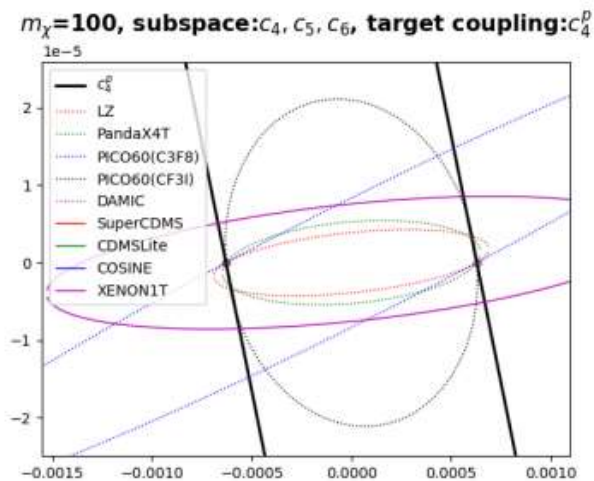
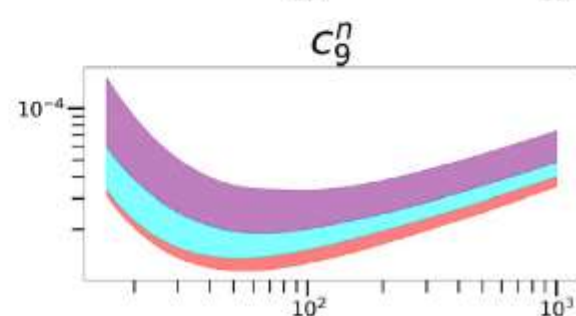
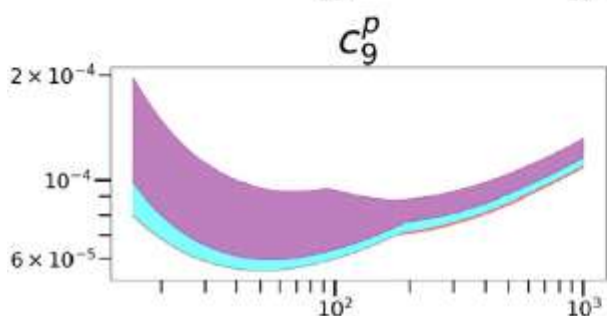
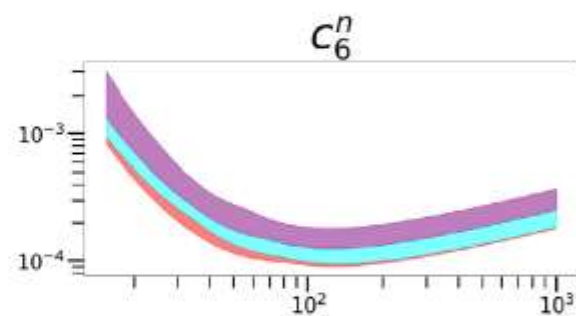
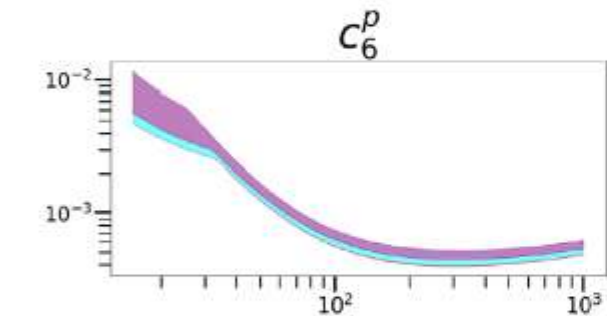
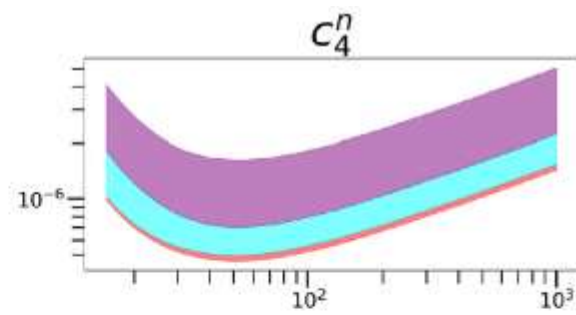
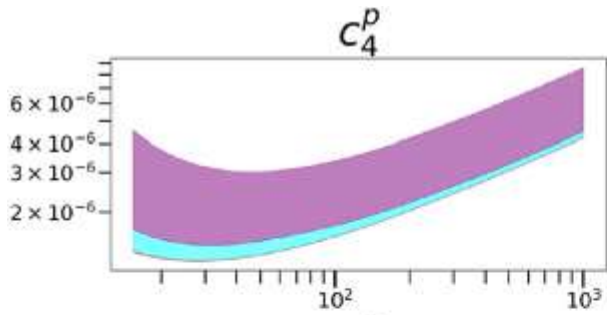


$$\sigma_{\chi N} \propto [c^p Z + c^n (A - Z)]^2$$



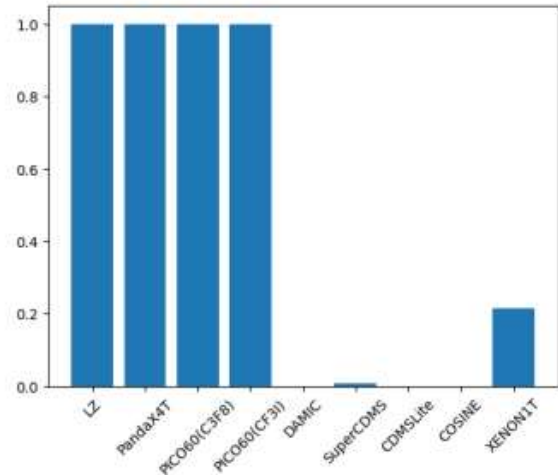
Spin dependent

- Inside the nucleons, spins tend to cancel



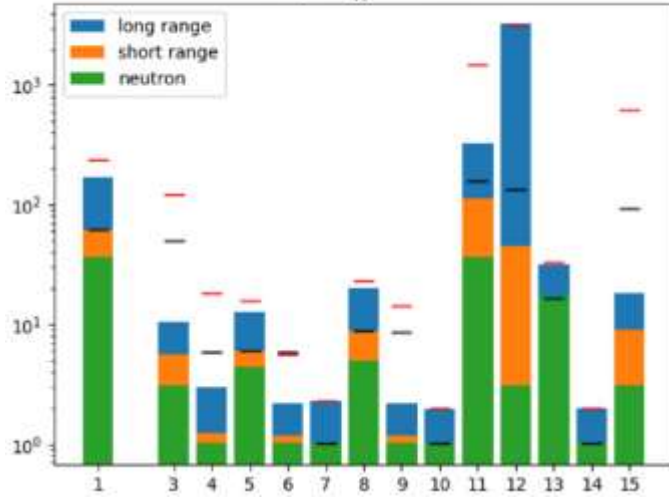
- only n → proton-neutron interf
- proton-neutron interf → only contact interf
- only contact interf → all couplings (contact+long range) interf

$m_\chi=100$, subspace: C_4, C_5, C_6 , target coupling: C_4^P

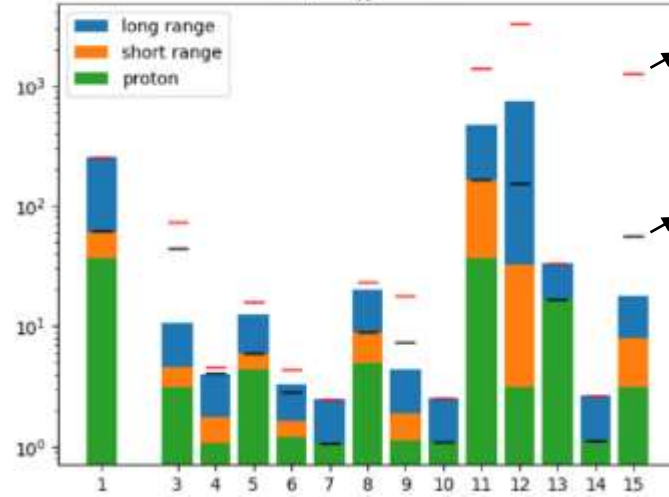


Sensitivity of the results

$c^p, m_\chi = 20$



$c^n, m_\chi = 20$



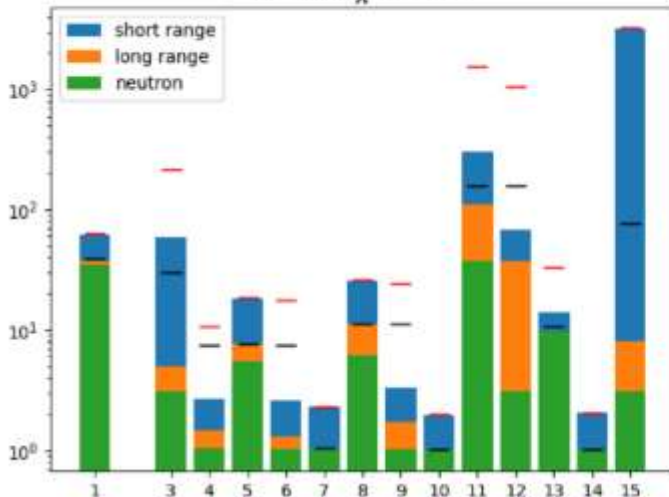
Both interaction

Only contact interaction

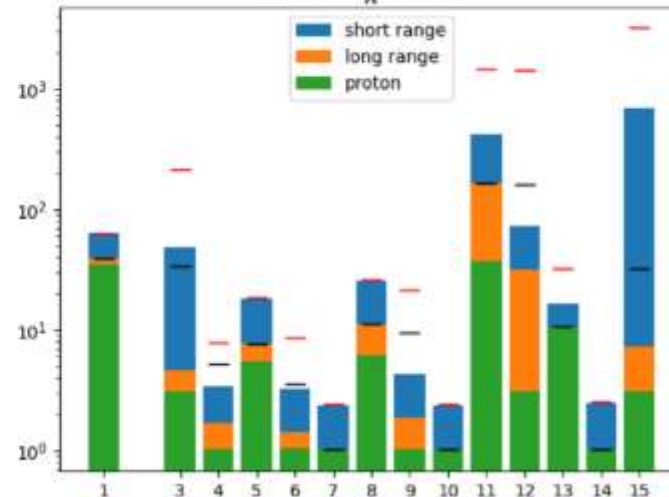
$$r = c_k^{less} / c_k^{most} \quad (\alpha_k^{less} / \alpha_k^{most})$$

$$\xi \equiv \max \left(\left| [c^{less}]_i \mathcal{M}_{ij} [c^{less}]_j \right| \right) = \left| [c^{less}]_\gamma \mathcal{M}_{\gamma\delta} [c^{less}]_\delta \right| \geq r^2$$

$\alpha^p, m_\chi = 20$



$\alpha^n, m_\chi = 20$

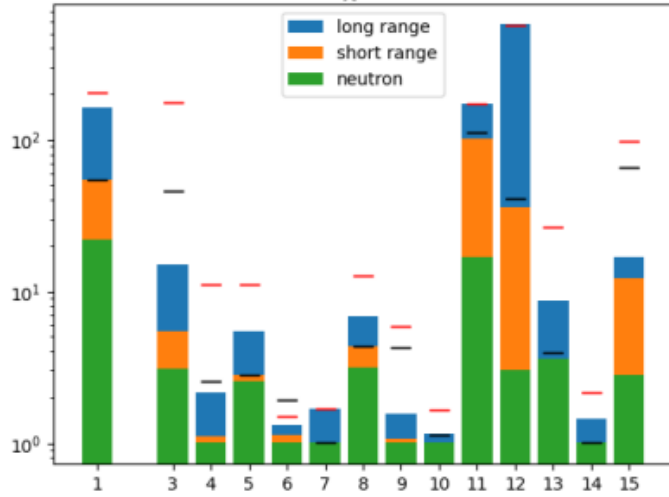


$$\mathcal{M}_{\gamma\delta} \rightarrow \mathcal{M}_{\gamma\delta}(1+\epsilon)$$

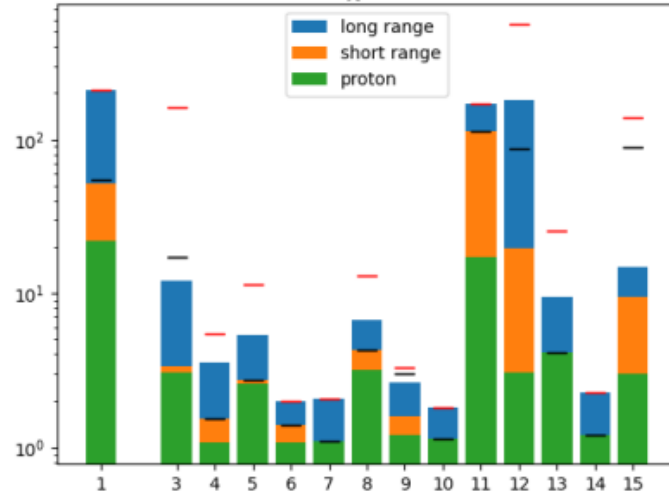
$$|\epsilon| = 1/\xi$$

Sensitivity of the results

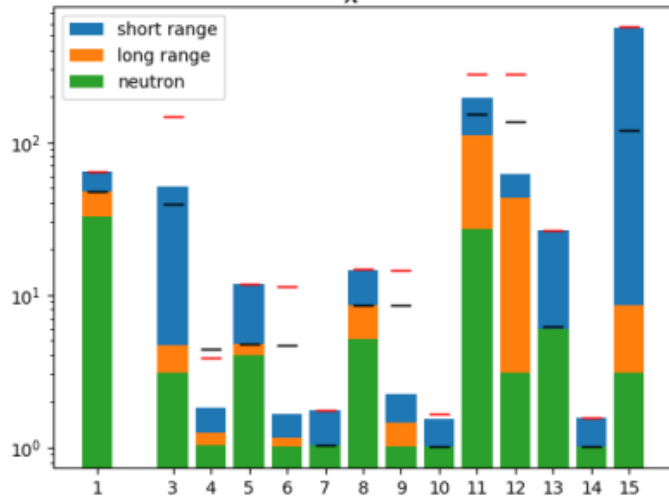
$c^p, m_\chi = 100$



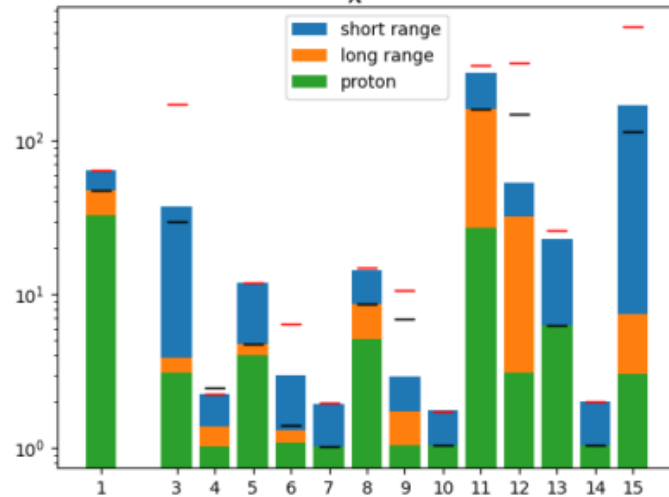
$c^n, m_\chi = 100$



$\alpha^p, m_\chi = 100$



$\alpha^n, m_\chi = 100$



$\epsilon \gtrsim \mathcal{O}(10^{-3})$ for all the “spin-dependent” couplings

$\epsilon \gtrsim \mathcal{O}(10^{-1})$ for $\mathcal{O}_7, \mathcal{O}_{10}$ and \mathcal{O}_{14}

$\mathcal{O}(10^{-6}) < \epsilon < \mathcal{O}(10^{-4})$ for $\mathcal{O}_{11}, \mathcal{O}_{12}$ and \mathcal{O}_{15}

$\mathcal{O}(10^{-5}) < \epsilon < \mathcal{O}(10^{-3})$ for \mathcal{O}_1 and \mathcal{O}_3

$\mathcal{O}(10^{-4}) < \epsilon < \mathcal{O}(10^{-2})$ for \mathcal{O}_{13}

$\epsilon \simeq \mathcal{O}(10^{-4})$

Conclusion

- Xe, F or I plays a role.
- To improve existing bounds will require to increase sensitivity of experiments that use other targets.
- To add new nuclear targets for the use in direct detection.



Thank you